Solutions to Analytical Plane Geometry by Hem Raj and Hukum Chand

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Contents

Abstract—**This book provides a vector approach to analytical geometry. The content and exercises are based on Hem Raj and hukum Chand's book on Analytical Plane Geometry.**

1 Coordinates

1.0.1. Find the distance between the following pair of points (11,16) and (23,21).
Let

$$\mathbf{A} = \begin{pmatrix} 11 \\ 16 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 23 \\ 21 \end{pmatrix} \tag{1.0.1.1}$$

Then,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 12 \\ 5 \end{pmatrix} \tag{1.0.1.2}$$

and

$$\|\mathbf{B} - \mathbf{A}\|^2 = (\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 12 & 5 \end{pmatrix} \begin{pmatrix} 12 \\ 5 \end{pmatrix} \tag{1.0.1.3}$$

$$\implies \|\mathbf{B} - \mathbf{A}\| = \sqrt{(12^2 + 5^2)} \tag{1.0.1.4}$$

$$= 13$$

1.0.2. Find the distance between the following pair of points (66,25) and (99,69).
Let

$$\mathbf{A} = \begin{pmatrix} 66 \\ 25 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 99 \\ 69 \end{pmatrix} \tag{1.0.2.1}$$

Then

$$\|\mathbf{B} - \mathbf{A}\| = \|\mathbf{C}\| \tag{1.0.2.2}$$

$$\|\mathbf{C}\|^2 = \mathbf{C}^\top \mathbf{C} \tag{1.0.2.3}$$

$$= \begin{pmatrix} 33 & 44 \end{pmatrix} \begin{pmatrix} 33 \\ 44 \end{pmatrix} \tag{1.0.2.4}$$

$$\implies \|\mathbf{C}\| = \sqrt{(33^2 + 44^2)} \tag{1.0.2.5}$$

$$= 55 \tag{1.0.2.6}$$

1.0.3. Find the area of the quadrilateral formed by the points

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}. \tag{1.0.3.1}$$

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}. \tag{1.0.3.2}$$

Then,

$$(\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{1.0.3.3}$$

$$(\mathbf{D} - \mathbf{B}) = \begin{pmatrix} -1 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ -10 \end{pmatrix} \tag{1.0.3.4}$$

$$(\mathbf{A} - \mathbf{D}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \tag{1.0.3.5}$$

Row reducing the matrix formed by the vec-

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tors,

$$\begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix} \tag{1.0.3.6}$$

$$\implies \begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ -4 & -10 & 1 \end{pmatrix} \xleftrightarrow{R_3 \leftrightarrow R_1 + R_2 + R_3} \begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ 0 & 0 & 3 \end{pmatrix} \xleftrightarrow{R_2 \leftrightarrow R_2 - R_1} \begin{pmatrix} 2 & 4 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{1.0.3.7}$$

The number of non-zero rows in the matrix = 3. Hence the matrix is full rank and AB, BD, DA are not collinear. See Fig. 1.0.3.1, which shows that the points given form a quadrilateral. Area

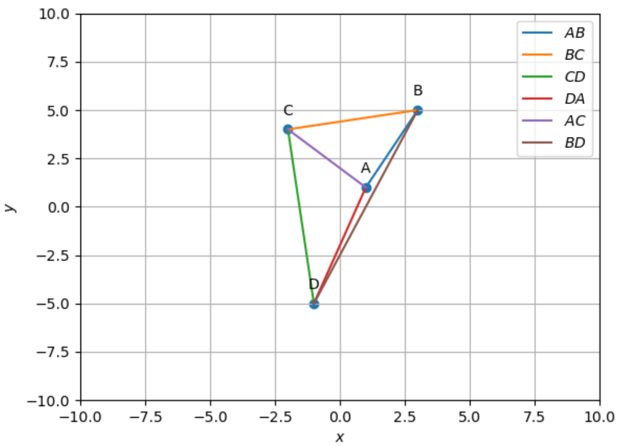

Fig. 1.0.3.1: Quadrilateral ABCD

of a $\triangle ABC$ is given by

$$\triangle ABC = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & 5 & 4 \end{vmatrix} = 9 \tag{1.0.3.8}$$

Area of $\triangle ACD$ is given by

$$\triangle ACD = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 4 & -5 \end{vmatrix} = 11.5 \tag{1.0.3.9}$$

Thus, the area of quadrilateral ABCD = 9+11.5 = 20.5.