

# SM5083 - BASICS OF PROGRAMMING

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**Abstract**—This paper contains solution to problem no 9(i) of Examples II Section of Analytical Geometry by Hukum Chand. Links to Python codes are available below.

Download python codes at

<https://github.com/rsgirishkumar/SM5083/ASSIGNMENT1>

## 1 PROBLEM

Find the area of the quadrilateral formed by the points

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}. \quad (1.0.1)$$

## 2 SOLUTION

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}. \quad (2.0.1)$$

**Step1:** Let us check whether the given points form a quadrilateral or not. This can be ascertained by collinearity check of any two sets of points i.e. either (A, B, C) or (A, C, D) or (B, C, D) or (A, B, D).

## 2.1 Collinearity Check

Collinearity Check of points A, B, D i.e.

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \quad (2.1.1)$$

In Vector approach, If the rank of a matrix formed by the vectors of points  $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$  is **1 or 2** for any 3 X 3 matrix then the points are said to be **collinear**. If  $\rho(\text{matrix})=3$  then the points are non collinear.

$$\Rightarrow \rho \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} = 3 \quad (2.1.2)$$

Here the vectors are as below.

$$\mathbf{AB} = (\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (2.1.3)$$

$$\mathbf{BD} = (\mathbf{D} - \mathbf{B}) = \begin{pmatrix} -1 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ -10 \end{pmatrix} \quad (2.1.4)$$

$$\mathbf{DA} = (\mathbf{A} - \mathbf{D}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \quad (2.1.5)$$

The matrix formed by vectors is

$$\begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix}$$

Rank of the matrix: By reduction.

$$\Rightarrow \begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ -4 & -10 & 1 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_1 + R_2 + R_3} \begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ 0 & 0 & 3 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_2 - R_1} \begin{pmatrix} 2 & 4 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (2.1.6)$$

The number of non-zero rows in the matrix = 3.  
 $\Rightarrow \rho(\text{matrix}) = 3$ .

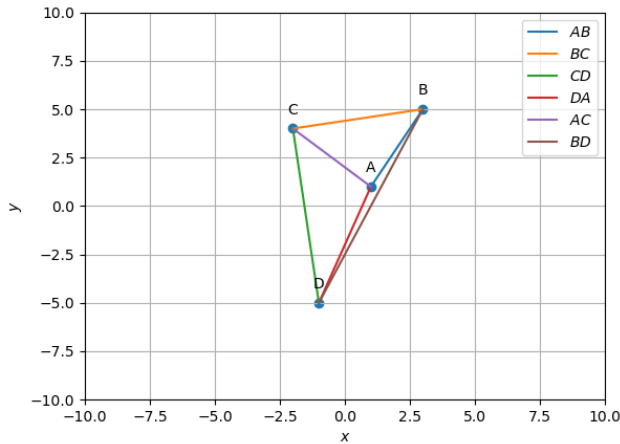
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Here the AB, BD, DA are not collinear.

is given by

Let us examine the lines generated by the given points in the Figure below:

$$\Delta ACD = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 4 & -5 \end{vmatrix} = 11.5 \quad (2.2.3)$$



The area of quadrilateral ABCD = 9 + 11.5 = 20.5

Fig. 0: Quadrilateral ABCD

The above figure clearly depicts the other set of points are not collinear. Hence the points given form a Quadrilateral.

## 2.2 Area of a Quadrilateral

$$\text{Area of } \square ABCD = \text{Area of } \triangle ACD + \text{Area of } \triangle ABC$$

$\therefore$  this does not fall into category of Square, Parallelogram or Rhombus, Trapezium.

Area of a  $\triangle ABC$  formed by points  $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ ,  $\mathbf{C} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$  is given by

$$\frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{vmatrix} \quad (2.2.1)$$

Area of  $\triangle ABC$  formed by  $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ ,  $\mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$  is given by

$$\Delta ABC = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & 5 & 4 \end{vmatrix} = 9 \quad (2.2.2)$$

Area of  $\triangle ACD$  formed by  $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $\mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ ,  $\mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$