Date: 02/05/2022

Aakash + BYJU'S

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Time: 90 Min.

MATHEMATICS

Max. Marks: 40

ICSE Board Class X (Semester-2) Exams

Answers & Solutions

GENERAL INSTRUCTIONS

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 10 minutes.
- This time is to be spent in reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers.
- Attempt **all** questions from **Section A** and **any three** questions from **Section B**.
- The marks intended for questions are given in brackets [ ].
- Mathematical tables are provided.

SECTION-A

*(Attempt **all** questions)*

Question 1

Choose the correct answers to the questions from the given options. (Do not copy the question. Write the correct answer only.) **[10]**

(i) The probability of getting a number divisible by 3 in throwing a dice is:

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Answer (b) **[1]**

Sol. : P(getting a number divisible by 3) $= \dfrac{\text{No. of favourable outcomes}}{\text{No. of total outcomes}}$

$$= \frac{2}{6} = \frac{1}{3}$$

(ii) The volume of a conical tent is 462 m^3 and the area of the base is 154 m^2. The height of the cone is:

(a) 15 m (b) 12 m

(c) 9 m (d) 24 m

Answer (c) **[1]**

Sol. : Given

$V = 462\ m^3$

$S = 154\ m^2$

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$$\frac{1}{3}\pi r^2 h = 462 \quad \ldots\text{(i)}$$

and area of base = $\pi r^2 = 154$ …(ii)

Put (ii) in (i)

$$\frac{1}{3}(154)h = 462$$

$$\Rightarrow \quad h = \frac{462 \times 3}{154} = 9\text{ m}$$

(iii) The median class for the given distribution is:

Class Interval	0 – 10	10 – 20	20 – 30	30 – 40
Frequency	2	4	3	5

(a) 0 – 10 (b) 10 – 20

(c) 20 – 30 (d) 30 – 40

Answer (c) **[1]**

Sol. :

Class interval	Frequency	Cumulative frequency
0 – 10	2	2
10 – 20	4	6
20 – 30	3	9
30 – 40	5	14 = N

Here, $\frac{N}{2} = \frac{14}{2} = 7,$ which lies in the class 20 – 30 (median class)

(iv) If two lines are perpendicular to one another then the relation between their slopes m_1 and m_2 is:

(a) $m_1 = m_2$ (b) $m_1 = \frac{1}{m_2}$

(c) $m_1 = -m_2$ (d) $m_1 \times m_2 = -1$

Answer (d) **[1]**

Sol. : Relation between slopes of perpendicular lines

$m_1 \times m_2 = -1$

(v) A lighthouse is 80 m high. The angle of elevation of its top from a point 80 m away from its foot along the same horizontal line is:

(a) 60° (b) 45°

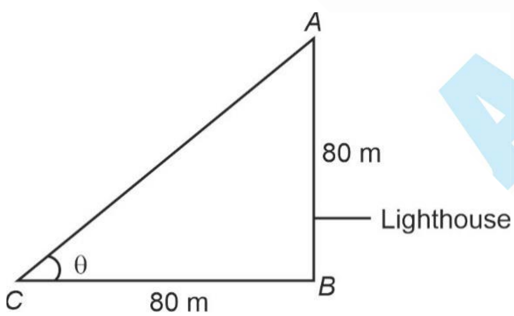
(c) 30° (d) 90°

Answer (b) **[1]**

Sol. : Height of lighthouse = 80 m

i.e., AB = 80 m

BC = 80 m



Let θ be the angle of elevation

Applying,

$$\tan\theta = \frac{AB}{BC} = \frac{80}{80} = 1$$

θ = 45°

(vi) The modal class of a given distribution always corresponds to the:

(a) interval with highest frequency

(b) interval with lowest frequency

(c) the first interval

(d) the last interval

Answer (a) **[1]**

Sol. : As we know, the modal class of a given distribution always corresponds to the interval with highest frequency.

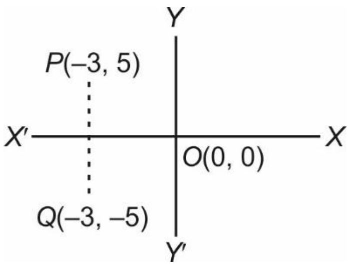
(vii) The coordinates of the point $P(-3, 5)$ on reflecting on the x axis are:

(a) (3, 5) (b) (–3, –5)

(c) (3, –5) (d) (–3, 5)

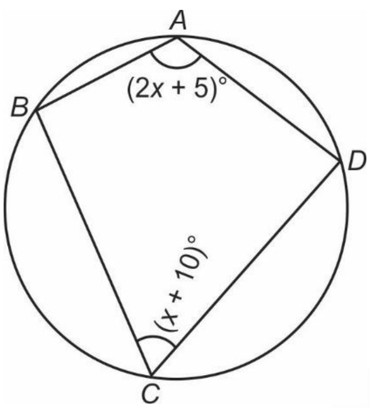
Answer (b) **[1]**

Sol. :



$Q(-3, -5)$ is reflecting point of $P(-3, 5)$ with respect to X-axis

(viii) $ABCD$ is a cyclic quadrilateral. If $\angle BAD = (2x + 5)°$ and $\angle BCD = (x + 10)°$ then x is equal to:

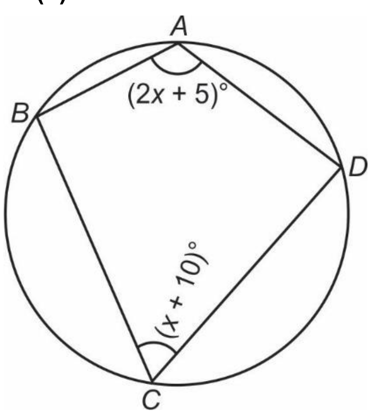


(a) 65° (b) 45°

(c) 55° (d) 5°

Answer (c) **[1]**

Sol. :



From figure, $ABCD$ is cyclic quadrilateral

$\angle A + \angle C = 180°$

$2x + 5° + x + 10° = 180°$

or $3x + 15° = 180°$

or $3x = 180° - 15°$

$= 165°$

$$x = \frac{165°}{3} = 55°$$

(ix) $A(1, 4)$, $B(4, 1)$ and $C(x, 4)$ are the vertices of $\Delta \boldsymbol{ABC}$. If the centroid of the triangle is $G(4, 3)$ then x is equal to

(a) 2 (b) 1

(c) 7 (d) 4

Answer (c) **[1]**

Sol. : Given $A(1, 4)$, $B(4, 1)$ and $C(x, 4)$ are the vertices of ΔABC

Co-ordinate of centroid of triangle (ΔABC) is $G(4, 3)$

Now,

X- Coordinate of centroid

$$\frac{1+4+x}{3} = 4 \qquad \left[\because \text{Co-ordinate of centroid} \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)\right]$$

or $5 + x = 12$

or $x = 7$

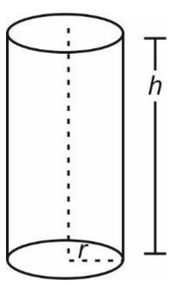
(x) The radius of a roller 100 cm long is 14 cm. The curved surface area of the roller is $\left(\text{Take } \pi = \frac{22}{7}\right)$

(a) 13200 cm^2 (b) 15400 cm^2

(c) 4400 cm^2 (d) 8800 cm^2

Answer (d) **[1]**

Sol. :



Given

$h = 100$ cm

$r = 14$ cm

Curved surface area of the roller = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 100$$

$$= 8800 \text{ cm}^2$$

SECTION-B

*(Attempt **any three** questions from this Section)*

Question 2

(i) Prove that: **[2]**

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

Sol. : L.H.S.

$$\Rightarrow \quad \frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$$ **[1]**

$$\Rightarrow \quad \frac{2}{\cos^2\theta} \qquad \Rightarrow \quad 2\sec^2\theta = \text{R.H.S.}$$ **[1]**

(ii) Find 'a', if $A(2a + 2, 3)$, $B(7, 4)$ and $C(2a + 5, 2)$ are collinear. **[2]**

Sol. : A, B and C are collinear

$A(2a + 2, 3)$ $B(7, 4)$ $C(2a + 5, 2)$ **[1]**

$m_{AB} = m_{BC}$ $\{m \Rightarrow \text{slope}\}$

$$\Rightarrow \quad \frac{4-3}{7-(2a+2)}=\frac{2-4}{2a+5-7}$$

$$\Rightarrow \quad \frac{1}{7-2a-2}=\frac{-2}{2a-2}$$

$\Rightarrow$ $2a - 2 = -2[5 - 2a]$

$\Rightarrow$ $2a - 2 = -10 + 4a$

$2a = 8$

$\boxed{a=4}$ **[1]**

(iii) Calculate the mean of the following frequency distribution. **[3]**

Class Interval	5-15	15-25	25-35	35-45	45-55
Frequency	2	6	4	8	4

Sol. :

Class Interval	**Frequency (f_i)**	**Class Mark (x_i)**	**($f_i x_i$)**
5 – 15	2	10	20
15 – 25	6	20	120
25 – 35	4	30	120
35 – 45	8	40	320
45 – 55	4	50	200
Total	N = 24		780

[1]

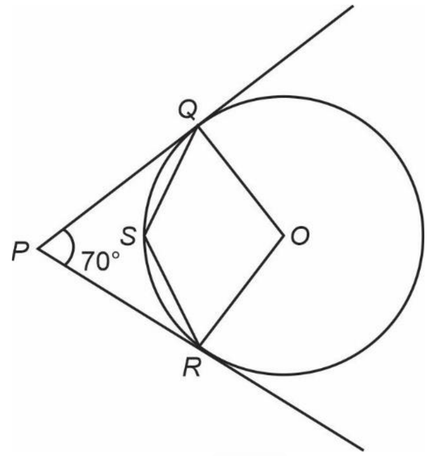
$$\text{Mean } \left(\bar{x}\right)=\frac{1}{N}\sum_{i=1}^{k}f_i x_i=\frac{1}{24}\sum_{i=1}^{5}f_i x_i$$ **[1]**

$$\frac{780}{24}=32.5$$ **[1]**

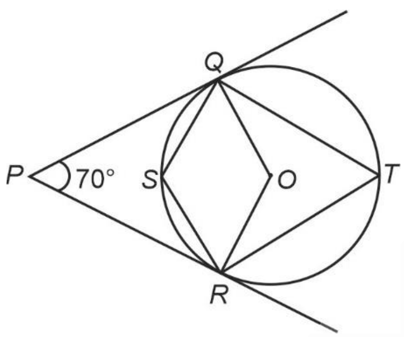
(iv) In the given figure O is the centre of the circle. PQ and PR are tangents and $\angle QPR = 70°$. Calculate: **[3]**

(a) $\angle QOR$

(b) $\angle QSR$



Sol. :



[Take any point T on circle and join QT and RT]

(a) $\angle QPR + \angle PRO + \angle QOR + \angle OQP = 360°$ **[½]**

$70° + 90° + \angle QOR + 90° = 360°$

$\Rightarrow \angle QOR = 360° - 250°$

$= 110°$ **[1]**

(b) $\angle QSR + \angle QTR = 180°$ **[½]**

$\angle QTR = \dfrac{1}{2}\angle QOR = 55°$ [Angle subtended by an arc of a circle at any point on the circle is half of angle subtended by it at centre of circle]

$\angle QSR + 55° = 180°$

$\angle QSR = 125°$ **[1]**

Question 3

(i) A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball? **[2]**

Sol. : Total number of balls in the bag = 10

$P(\text{ball drawn is a red ball}) = \dfrac{\text{Number of Red balls}}{\text{Total number of balls}}$ **[1]**

$= \dfrac{2}{10} = \dfrac{1}{5}$ **[1]**

(ii) A solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0.5 cm and height 1.5 cm. Find the number of cylinders so formed. **[2]**

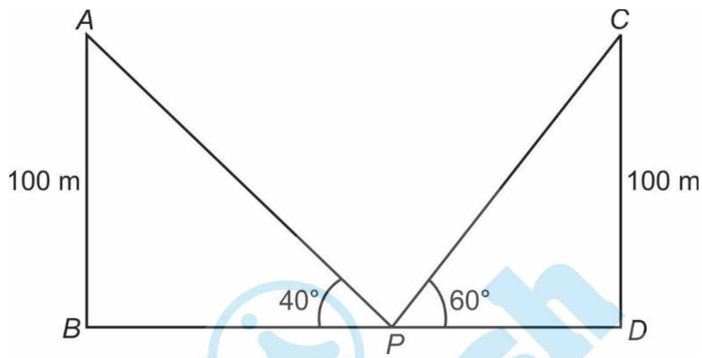
Sol. : Let the number of cylinders formed be 'n'.

According to question,

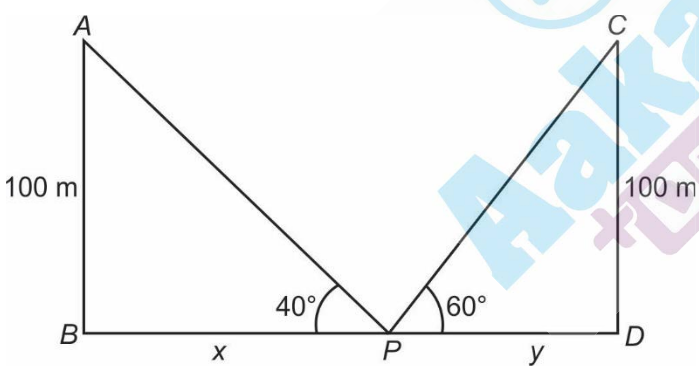
$$\frac{1}{3}\pi(5)^2 9 = n\left(\pi(0.5)^2(1.5)\right)$$ **[1]**

$n = 200$ **[1]**

(iii) Two lamp posts AB and CD each of height 100 m are on either side of the road. P is a point on the road between the two lamp posts. The angles of elevation of the top of the lamp posts from the point P are 60° and 40°. Find the distances PB and PD. **[3]**



Sol. : Let the distance PB be x and PD be y.



In ΔABP,

$$\tan 40° = \frac{100}{x}$$

$$\Rightarrow \quad x = \frac{100}{\tan 40°} = \frac{100}{0.839}$$

$\Rightarrow$ $x = PB = 119.19$ m **[1½]**

In ΔPDC,

$$\tan 60° = \frac{100}{y}$$

$$\Rightarrow \quad y = \frac{100}{\tan 60°} = \frac{100}{1.73} = 57.80$$

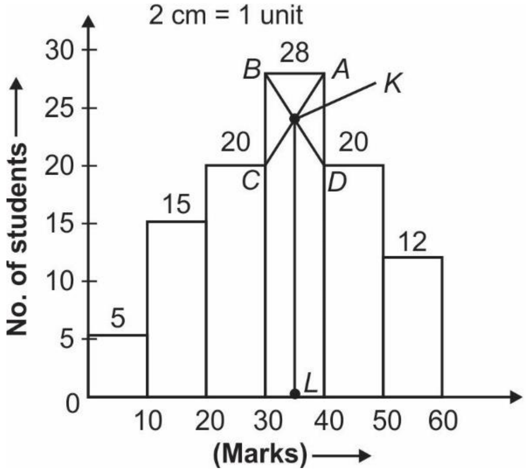
$\Rightarrow$ $PD = 57.8$ m **[1½]**

(iv) Marks obtained by 100 students in an examination are given below. **[3]**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	5	15	20	28	20	12

Draw a histogram for the given data using a graph paper and find the mode. Take 2 cm = 10 marks along one axis and 2 cm = 10 students along the other axis.

Sol. :



(Marks) Class interval	**(No. of students) frequency**
0–10	5
10–20	15
20–30	20
30–40	28
40–50	20
50–60	12

[2]

From the histogram, we can see that class interval 30–40 is the modal class

Steps:

(1) Draw a histogram of given distribution.

(2) Inside the modal class, draw the lines *AC* and *BD* diagonally from upper corners C and D of adjacent rectangles, intersecting at point *K*.

(3) Through the point *K* (the point of intersection of diagonals *AC* and *BD*) draw $KL \perp$ to horizontal axis.

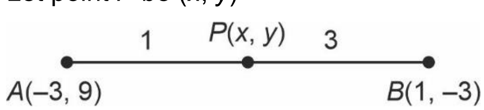
(4) The value of point *L* on horizontal axis represents value of mode

Mode = 35 **[1]**

Question 4

(i) Find a point *P* which divides internally the line segment joining the points *A*(–3, 9) and *B*(1, –3) in the ratio 1 : 3. **[2]**

Sol. : Let point *P* be (*x*, *y*)



[1]

$$P(x, y) = P\left(\frac{1\times 1+3(-3)}{1+3}, \frac{1(-3)+3(9)}{1+3}\right)$$

$$= P(-2, 6)$$ **[1]**

(ii) A letter of the word 'SECONDARY' is selected at random. What is the probability that the letter selected is not a vowel? **[2]**

Sol. : The word SECONDARY, contains *E*, *O* and *A* as vowel and *S*, *C*, *N*, *D*, *R* and *Y* as consonant, then

$$P(\text{letter selected is not a vowel}) = \frac{\text{Number of letters which is not a vowel}}{\text{Total number of vowels}}$$ **[1]**

$$= \frac{6}{9} = \frac{2}{3}$$ **[1]**

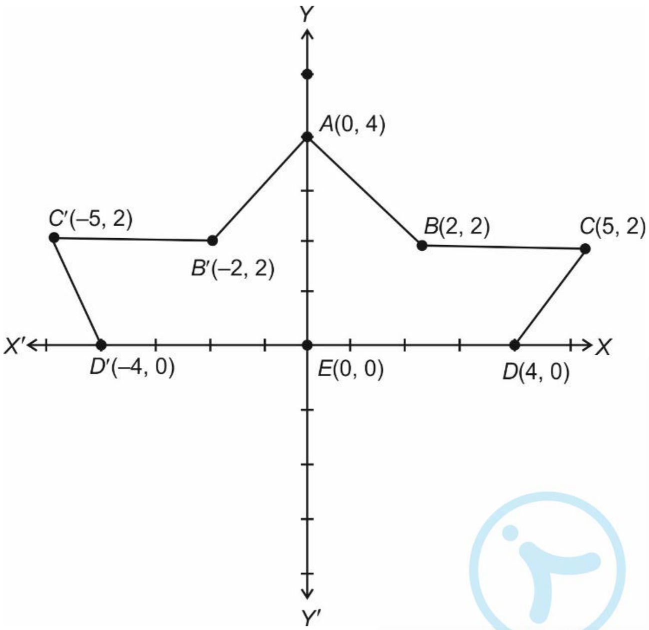
(iii) Use a graph paper for this question. Take 2 cm = 1 unit along both the axes. **[3]**

(a) Plot the points $A(0, 4)$, $B(2, 2)$, $C(5, 2)$ and $D(4, 0)$. $E(0, 0)$ is the origin.

(b) Reflect B, C, D on the y-axis and name them as B', C' and D' respectively.

(c) Join the points $ABCDD'C'B'$ and A in order and give a geometrical name to the closed figure.

Sol. : Take 2 cm = 1 unit

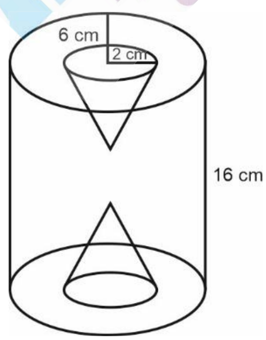


[2]

The figure thus obtained is a 'BOAT'. **[1]**

(iv) A solid wooden cylinder is of radius 6 cm and height 16 cm. Two cones each of radius 2 cm and height 6 cm are drilled out of the cylinder. Find the volume of the remaining solid. **[3]**

Take $\pi = \frac{22}{7}$



Sol. : Volume of remaining solid = Volume of cylinder – 2(Volume of each cone)

$= \pi(6)^2(16) - 2\left(\frac{1}{3}\pi(2)^2(6)\right)$ **[1]**

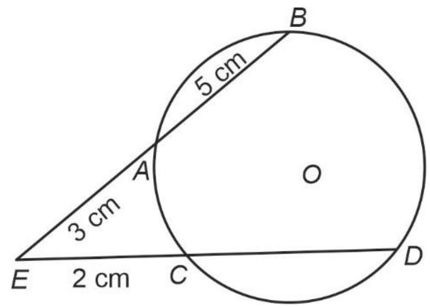
$= 560 \times \frac{22}{7}$ cm^3 $= 560 \times \frac{22}{7}$ **[1]**

$= 80 \times 22$

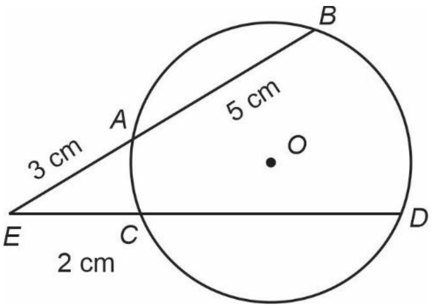
$= 1760$ cm^3 **[1]**

Question 5

(i) Two chords AB and CD of a circle intersect externally at E. If EC = 2 cm, EA = 3 cm and AB = 5 cm, find the length of CD. **[2]**



Sol. :



[1]

$EB \times EA = ED \times EC$

$\Rightarrow$ $8 \times 3 = (EC + CD) \times EC$

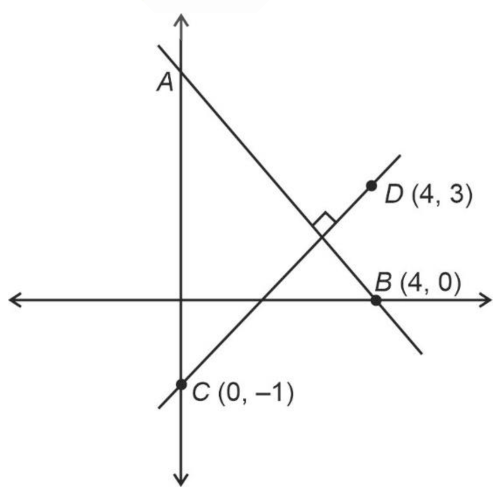
$\Rightarrow$ $8 \times 3 = (2 + CD) \times 2$ **[½]**

$24 = 4 + 2CD$

$2CD = 20$

$CD = \dfrac{20}{2} = 10\text{ cm}$ **[½]**

(ii) Line AB is perpendicular to CD. Coordinates of B, C and D are respectively (4, 0), (0, –1) and (4, 3).

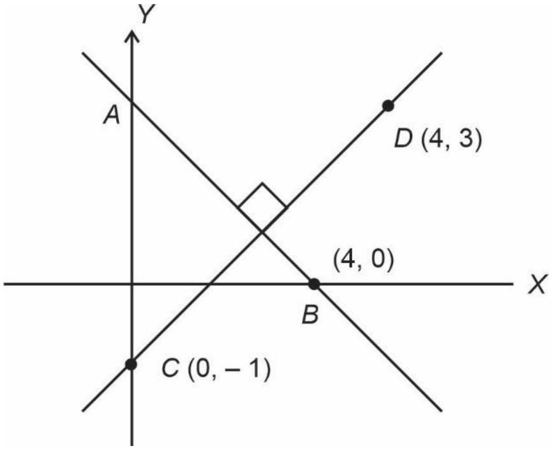


Find: **[2]**

(a) Slope of CD

(b) Equation of AB

Sol. :



(a) Slope of CD = $\frac{3-(-1)}{4-0} = \frac{4}{4} = 1$ **[½]**

$\boxed{m_{CD} = 1}$ **[½]**

(b) Equation of AB

$\Rightarrow$ $(y - y_1) = m_{AB}(x - x_1)$

$\Rightarrow$ $y - 0 = m_{AB}(x - 4)$

We know $AB \perp CD$

$\Rightarrow$ $m_{AB} \cdot m_{CD} = -1$ **[½]**

$\Rightarrow$ $m_{AB} = \frac{-1}{m_{CD}} = \frac{-1}{1} = -1$

Equation of AB

$\Rightarrow$ $y - 0 = -1(x - 4)$ **[½]**

$y = -x + 4$

$\boxed{y + x - 4 = 0}$

(iii) Prove that: **[3]**

$$\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta} = \sec^2\theta + \tan^2\theta$$

Sol. : L.H.S.

$$\frac{1+\sin^2\theta + 2\sin\theta + 1 + \sin^2\theta - 2\sin\theta}{2\cos^2\theta}$$ **[1]**

$$\Rightarrow \frac{2\left[1+\sin^2\theta\right]}{2\cos^2\theta} = \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$ **[1]**

$\Rightarrow$ $\sec^2\theta + \tan^2\theta$ (R.H.S) **[1]**

L.H.S. = R.H.S

Hence proved

(iv) The mean of the following distribution is 50. Find the unknown frequency. **[3]**

Class Interval	Frequency
0–20	6
20–40	f
40–60	8
60–80	12
80–100	8

Sol. :

Class Interval	Frequency (f_i)	Class mark (x_i)	Product (f_ix_i)
0 – 20	6	10	60
20 – 40	f	30	30f
40 – 60	8	50	400
60 – 80	12	70	840
80 – 100	8	90	720
	N = (34 + f)		**(2020 + 30f)**

[1]

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{k} f_i x_i$$

$$\Rightarrow \quad \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i$$ **[1]**

$$\Rightarrow \quad \frac{2020+30f}{34+f} = 50$$ **[½]**

$\Rightarrow$ 2020 + 30f = 1700 + 50f

20f = 2020 – 1700

$$f = \frac{320}{20} = 16$$ **[½]**

Question 6

(i) Prove that: **[2]**

$$1+\frac{\tan^2\theta}{1+\sec\theta} = \sec\theta$$

Sol. : From L.H.S

$$1+\frac{\tan^2\theta}{1+\sec\theta}$$

$$= 1+\frac{\sec^2\theta-1}{1+\sec\theta},$$ [$\because$ $\tan^2\theta = \sec^2\theta - 1$] **[1]**

$$= 1+\frac{(\sec\theta-1)(\sec\theta+1)}{(\sec\theta+1)},$$ [$\because a^2 - b^2 = (a+b)(a-b)$] **[½]**

= 1 + secθ – 1

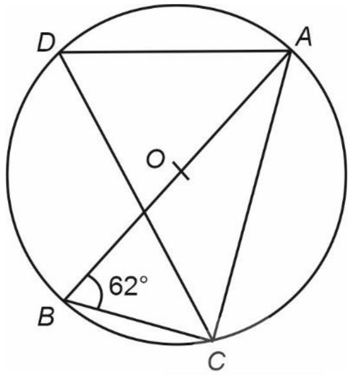
= secθ **[½]**

(ii) In the given figure *A*, *B*, *C* and *D* are points on the circle with centre *O*. Given $\angle ABC = 62°$. **[2]**

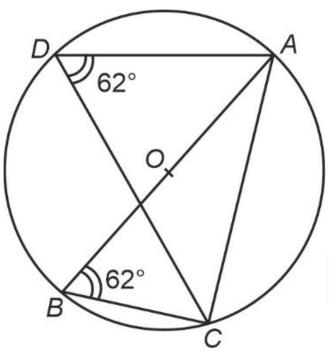
Find:

(a) $\angle ADC$

(b) $\angle CAB$



Sol. :



(a) $\angle ADC = \angle ABC$ [Angles on same segment] **[½]**

$\therefore\ \angle ADC = 62°$ **[½]**

(b) $\angle ADC = 62°$

In ΔACB

From figure

$\angle ABC = 62°$

and $\angle BCA = 90°$ [Angle in a semicircle] **[½]**

$\angle ABC + \angle BCA + \angle CAB = 180°$

$62° + 90° + \angle CAB = 180°$

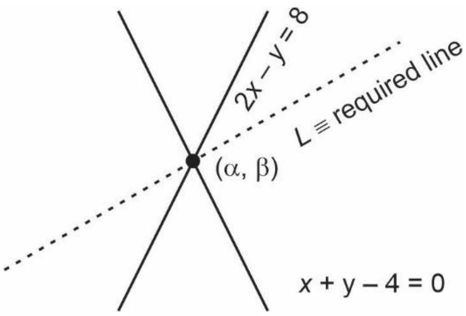
$\angle CAB = 180° - 152°$

$= 28°$ **[½]**

(iii) Find the equation of a line parallel to the line $2x + y - 7 = 0$ and passing through the intersection of the lines $x + y - 4 = 0$ and $2x - y = 8$. **[3]**

Sol. : Let (α,β) be the point of intersection of lines $2x - y = 8$ and $x + y - 4 = 0$

and (α, β) satisfies the lines $2x - y = 8$ and $x + y = 4$



i.e. $2\alpha - \beta = 8$ ...(i)

$\alpha + \beta = 4$ ...(ii)

Adding (i) + (ii), we get

$3\alpha = 12$

or $\alpha = 4$ **[1]**

Now,

$\alpha + \beta = 4$

$\beta = 4 - \alpha$

$\beta = 0$

$\therefore$ $(\alpha, \beta) \equiv (4, 0)$ **[1]**

Equation of required line

L is parallel to $2x + y - 7 = 0$

$L : 2x + y + k = 0$

Point (4, 0) satisfies the line $2x + y + k = 0$

i.e. $2 \times 4 + 0 + k = 0$

or $k = -8$

$\therefore$ Required line

$2x + y - 8 = 0$ **[1]**

(iv) Marks obtained by 40 students in an examination are given below.

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
No. of Students	3	8	14	9	4	2

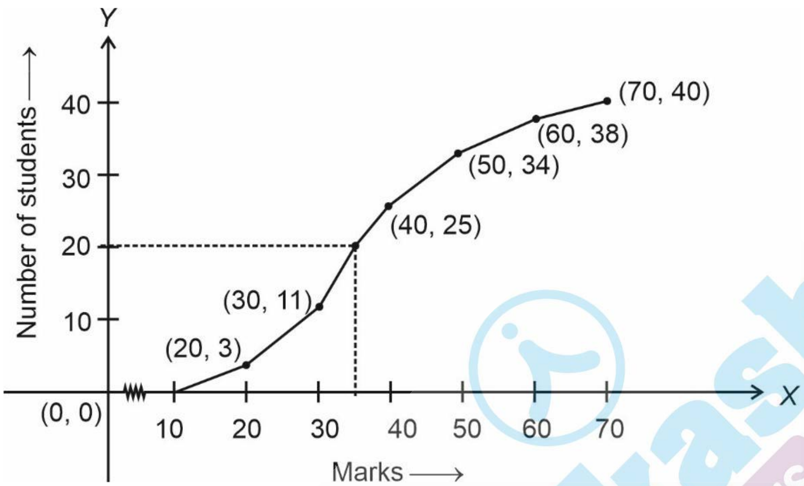
Using graph paper draw an ogive and estimate the median marks. Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis. **[3]**

Sol. :

Marks	No. of students (f_i)	Cumulative frequency (C.F.)
10 – 20	3	3
20 – 30	8	11
30 – 40	14	25
40 – 50	9	34
50 – 60	4	38
60 – 70	2	40

[1]

Plot the points (20, 3), (30, 11), (40, 25), (50, 34), (60, 38) and (70, 40) on a graph paper by taking 2 cm = 10 marks on '*X*'-axis and 2 cm = 5 students on *Y*-axis



[1]

Draw a free hand curve passing through points marked, starting from the lower limit of first class and terminating at upper limit of last class.

Median $= \left(\frac{n}{2}\right)^{th}$ term $= \left(\frac{40}{2}\right)^{th}$ term = 20^{th} term

= 36.4 (approximately) **[1]**

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