CHAPTER 8 Circle

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If A and B are points in the plane such that $PA/PB = k$ (constant) for all P on a given circle, then the value of k cannot be equal to .... *(1982 - 2 Marks)*

2. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are .... and .... *(1983 - 2 Marks)*

3. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle. The radius of this circle is .... *(1984 - 2 Marks)*

4. Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a circle. A pair of tangents from the point (4, 5) with a pair of radii form a quadrilateral of area .... *(1985 - 2 Marks)*

5. From the origin chords are drawn to the circle $(x - 1)^2 + y^2 = 1$. The equation of the locus of the mid-points of these chords is .... *(1985 - 2 Marks)*

6. The equation of the line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$ is .... *(1986 - 2 Marks)*

7. From the point $A(0, 3)$ on the circle $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M such that $AM = 2AB$. The equation of the locus of M is .... *(1986 - 2 Marks)*

8. The area of the triangle formed by the tangents from the point (4, 3) to the the circle $x^2 + y^2 = 9$ and the line joining their points of contact is .... *(1987 - 2 Marks)*

9. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that common chord is of maximum length and has a slope equal to 3/4, then the coordinates of the centre of C_2 are .... *(1988 - 2 Marks)*

10. The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is, ... *(1989 - 2 Marks)*

11. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of λ = ... *(1991 - 2 Marks)*

12. The equation of the locus of the mid-points of the circle $4x^2 + 4y^2 - 12x + 4y + 1 = 0$ that subtend an angle of $2\pi/3$ at its centre is .... *(1993 - 2 Marks)*

13. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as a diameter is .... *(1996 - 1 Mark)*

14. For each natural number k, let C_k denote the circle with radius k centimetres and centre at the origin. On the circle C_k, α-particle moves k centimetres in the counter-clockwise direction. After completing its motion on C_k, the particle moves to C_{k+1} in the radial direction. The motion of the particle continues in this manner. The particle starts at (1, 0). If the particle crosses the positive direction of the x-axis for the first time on the circle C_n then n = ... *(1997 - 2 Marks)*

15. The chords of contact of the pair of tangents drawn from each point on the line $2x+y=4$ to circle $x^2+y^2 = 1$ pass through the point .... *(1997 - 2 Marks)*

B True / False

1. No tangent can be drawn from the point (5/2, 1) to the circumcircle of the triangle with vertices $(1, \sqrt{3})$ $(1, -\sqrt{3})$, $(3, -\sqrt{3})$. *(1985 - 1 Mark)*

2. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 - 6x + 2y = 0$. *(1989 - 1 Mark)*

C MCQs with One Correct Answer

1. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the coordinate axes. The one vertex of the square is *(1980)*
   (a) $(1 + \sqrt{2}, -2)$ (b) $(1 - \sqrt{2}, -2)$
   (c) $(1, -2 + \sqrt{2})$ (d) none of these

2. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is *(1980)*
   (a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$
   (c) $x^2 + y^2 - 4y + 2 = 0$ (d) none of these

3. The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is *(1983 - 1 Mark)*

(a) $\left(\frac{-16}{5}, \frac{27}{10}\right)$ (b) $\left(\frac{-16}{7}, \frac{53}{10}\right)$

(c) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ (d) none of these

4. The equation of the circle passing through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is *(1983 - 1 Mark)*

(a) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$

(b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

(c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$

(d) none of these

5. The locus of the mid-point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is *(1984 - 2 Marks)*

(a) $x + y = 2$ (b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = 2$ (d) $x + y = 1$

6. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is *(1988 - 2 Marks)*

(a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$

(b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$

(c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$

(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$.

7. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then *(1989 - 2 Marks)*

(a) $2 < r < 8$ (b) $r < 2$ (c) $r = 2$ (d) $r > 2$

8. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq. units. Then the equation of this circle is *(1989 - 2 Marks)*

(a) $x^2 + y^2 + 2x - 2y = 62$

(b) $x^2 + y^2 + 2x - 2y = 47$

(c) $x^2 + y^2 - 2x + 2y = 47$

(d) $x^2 + y^2 - 2x + 2y = 62$

9. The centre of a circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is *(1992 - 2 Marks)*

(a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, -2^{\frac{1}{2}}\right)$

10. The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation: *(1993 - 1 Marks)*

(a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$

(c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$

11. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if *(1994)*

(a) $r < 2$ (b) $r > 8$ (c) $2 < r < 8$ (d) $2 \le r \le 8$

12. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is *(1996 - 1 Mark)*

(a) $x^2 + y^2 + 4x - 6y + 4 = 0$ (b) $x^2 + y^2 + 4x - 6y - 9 = 0$

(c) $x^2 + y^2 + 4x - 6y - 4 = 0$ (d) $x^2 + y^2 + 4x - 6y + 9 = 0$

13. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \ne 0$) are bisected by the x-axis, then *(1999 - 2 Marks)*

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$ (c) $P^2 < 8q^2$ (d) $p^2 > 8q^2$.

14. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) and (–4, 3) respectively, then $\angle QPR$ is equal to *(2000S)*

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

15. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$, $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is *(2000S)*

(a) 2 or $-\frac{3}{2}$ (b) -2 or $-\frac{3}{2}$ (c) 2 or $\frac{3}{2}$ (d) -2 or $\frac{3}{2}$

16. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle PAB as P moves on the circle is *(2001S)*

(a) a parabola (b) a circle

(c) an ellipse (d) a pair of straight lines

17. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals *(2001S)*

(a) $\sqrt{PQ.RS}$ (b) $(PQ + RS)/2$

(c) $2PQ.RS/(PQ + RS)$ (d) $\sqrt{(PQ^2 + RS^2)}/2$

18. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets a straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is *(2002S)*

(a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

19. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is *(2003S)*

(a) (4, 7) (b) (7, 4) (c) (9, 4) (d) (4, 9)

20. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is *(2004S)*

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 3 (d) 2

21. A circle is given by $x^2 + (y-1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is *(2005S)*

(a) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$

(b) $\{(x, y) : x^2 + (y-1)^2 = 4\} \cup \{(x, y) : y \le 0\}$

(c) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$

(d) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

22. Tangents drawn from the point $P(1, 8)$ to the circle
$x^2+y^2-6x-4y-11=0$
touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is *(2009)*
(a) $x^2+y^2+4x-6y+19=0$
(b) $x^2+y^2-4x-10y+19=0$
(c) $x^2+y^2-2x+6y-29=0$
(d) $x^2+y^2-6x-4y+19=0$

23. The circle passing through the point $(-1, 0)$ and touching the y-axis at $(0, 2)$ also passes through the point. *(2011)*
(a) $\left(-\frac{3}{2}, 0\right)$ (b) $\left(-\frac{5}{2}, 2\right)$ (c) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (d) $(-4, 0)$

24. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x-5y=20$ to the circle $x^2+y^2=9$ is *(2012)*
(a) $20(x^2+y^2)-36x+45y=0$
(b) $20(x^2+y^2)+36x-45y=0$
(c) $36(x^2+y^2)-20x+45y=0$
(d) $36(x^2+y^2)+20x-45y=0$

25. A line $y=mx+1$ intersectrs the circle $(x-3)^2+(y+2)^2=25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct ? *(JEE Adv. 2019)*
(a) $2 \le m < 4$ (b) $-3 \le m < -1$
(c) $4 \le m < 6$ (d) $6 \le m < 8$

D MCQs with One or More than One Correct

1. The equations of the tangents drawn from the origin to the circle $x^2+y^2-2rx-2hy+h^2=0$, are *(1988 - 2 Marks)*
(a) $x=0$ (b) $y=0$
(c) $(h^2-r^2)x-2rhy=0$ (d) $(h^2-r^2)x+2rhy=0$

2. The number of common tangents to the circles $x^2+y^2=4$ and $x^2+y^2-6x-8y=24$ is *(1998 - 2 Marks)*
(a) 0 (b) 1 (c) 3 (d) 4

3. If the circle $x^2+y^2=a^2$ intersects the hyperbola $xy=c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then *(1998 - 2 Marks)*
(a) $x_1+x_2+x_3+x_4=0$ (b) $y_1+y_2+y_3+y_4=0$
(c) $x_1x_2x_3x_4=c^4$ (d) $y_1y_2y_3y_4=c^4$

4. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) *(JEE Adv. 2013)*
(a) $x^2+y^2-6x+8y+9=0$ (b) $x^2+y^2-6x+7y+9=0$
(c) $x^2+y^2-6x-8y+9=0$ (d) $x^2+y^2-6x-7y+9=0$

5. A circle S passes through the point $(0, 1)$ and is orthogonal to the circles $(x-1)^2+y^2=16$ and $x^2+y^2=1$. Then *(JEE Adv. 2014)*
(a) radius of S is 8 (b) radius of S is 7
(c) centre of S is $(-7, 1)$ (d) centre of S is $(-8, 1)$

6. Let RS be the diameter of the circle $x^2+y^2=1$, where S is the point $(1, 0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) *(JEE Adv. 2016)*
(a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$
(c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

7. Let T be the line passing through the points $P(-2, 7)$ and $Q(2, -5)$. Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1, 1)$ be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE? *(JEE Adv. 2018)*
(a) The point $(-2, 7)$ lies in E_1
(b) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie in E_2
(c) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2
(d) The point $\left(0, \frac{3}{2}\right)$ does **NOT** lie in E_1

E Subjective Problems

1. Find the equation of the circle whose radius is 5 and which touches the circle $x^2+y^2-2x-4y-20=0$ at the point $(5, 5)$. *(1978)*

2. Let A be the centre of the circle $x^2+y^2-2x-4y-20=0$. Suppose that the tangents at the points $B(1, 7)$ and $D(4, -2)$ on the circle meet at the point C. Find the area of the quadrilateral $ABCD$. *(1981 - 4 Marks)*

3. Find the equations of the circle passing through $(-4, 3)$ and touching the lines $x+y=2$ and $x-y=2$. *(1982 - 3 Marks)*

4. Through a fixed point (h, k) secants are drawn to the circle $x^2+y^2=r^2$. Show that the locus of the mid-points of the secants intercepted by the circle is $x^2+y^2=hx+ky$. *(1983 - 5 Marks)*

5. The abscissa of the two points A and B are the roots of the equation $x^2+2ax-b^2=0$ and their ordinates are the roots of the equation $x^2+2px-q^2=0$. Find the equation and the radius of the circle with AB as diameter. *(1984 - 4 Marks)*

6. Lines $5x+12y-10=0$ and $5x-12y-40=0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts intercepts of length 8 on these lines. *(1986 - 5 Marks)*

7. Let a given line L_1 intersects the x and y axes at P and Q, respectively. Let another line L_2, perpendicular to L_1, cut the x and y axes at R and S, respectively. Show that the locus of the point of intersection of the lines PS and QR is a circle passing through the origin. *(1987 - 3 Marks)*

8. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{1/2} = 0$. Find k. *(1987 - 4 Marks)*

9. If $\left(m_i, \frac{1}{m_i}\right)$, $m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1m_2m_3m_4 = 1$ *(1989 - 2 Marks)*

10. A circle touches the line $y = x$ at a point P such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of the circle. *(1990 - 5 Marks)*

11. Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of their common tangent is $4x + 3y = 10$, find the equation of the circles. *(1991 - 4 Marks)*

12. Let a circle be given by $2x(x - a) + y(2y - b) = 0, (a \neq 0, b \neq 0)$. Find the condition on a and b if two chords, each bisected by the x- axis, can be drawn to the circle from $\left(a, \frac{b}{2}\right)$. *(1992 - 6 Marks)*

13. Consider a family of circles passing through two fixed points $A(3,7)$ and $B(6, 5)$. Show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinate of this point. *(1993 - 5 Marks)*

14. Find the coordinates of the point at which the circles $x^2 + y^2 - 4x - 2y = -4$ and $x^2 + y^2 - 12x - 8y = -36$ touch each other. Also find equations common tangents touching the circles in the distinct points. *(1993 - 5 Marks)*

15. Find the intervals of values of a for which the line $y + x = 0$ bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2 + 2y^2 - (1+\sqrt{2}a)x - (1-\sqrt{2}a)y = 0$. *(1996 - 5 Marks)*

16. A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively and the distance between the point A and the mid point of the line segment DC is d, prove that the area of the circle is

$$\frac{\pi d^2 \cos^2 \alpha}{\cos^2 \alpha + \cos^2 \beta + 2\cos\alpha\cos\beta\cos(\beta - \alpha)}$$

(1996 - 5 Marks)

17. Let C be any circle with centre $(0, \sqrt{2})$. Prove that at the most two rational points can be there on C. (A rational point is a point both of whose coordinates are rational numbers.) *(1997 - 5 Marks)*

18. C_1 and C_2 are two concentric circles, the radius of C_2 being twice that of C_1. From a point P on C_2, tangents PA and PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1. *(1998 - 8 Marks)*

19. Let T_1, T_2 be two tangents drawn from $(-2, 0)$ onto the circle $C: x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time. *(1999 - 10 Marks)*

20. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. *(2001 - 5 Marks)*

21. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C. *(2001 - 5 Marks)*

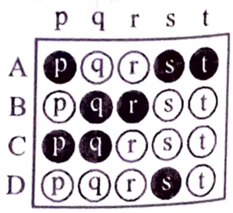
22. For the circle $x^2 + y^2 = r^2$, find the value of r for which the area enclosed by the tangents drawn from the point $P(6, 8)$ to the circle and the chord of contact is maximum. *(2003 - 2 Marks)*

23. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at $(1, -1)$ and cutting orthogonally the circle having line segment joining $(0, 3)$ and $(-2, -1)$ as diameter. *(2004 - 4 Marks)*

24. Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. *(2005 - 2 Marks)*

F Match the Following

DIRECTIONS (Q. 1): *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y. Suppose that another circle

$C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions:
(i) Centre of C_3 is collinear with the centres of C_1 and C_2
(ii) C_1 and C_2 both lie inside C_3, and
(iii) C_3 touches C_1 at M and C_2 at N
Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.
There are some expressions given in the List – I whose values are given in List – II below

Column I	Column II
(A) $2h + k$	(p) 6
(B) $\dfrac{\text{Length of } ZW}{\text{Length of } XY}$	(q) $\sqrt{6}$
(C) $\dfrac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$	(r) $\dfrac{5}{4}$
	(s) $\dfrac{21}{5}$
	(t) $2\sqrt{6}$
	(u) $\dfrac{10}{3}$

Which of the following is the only CORRECT combination?
(a) (I), (U)
(b) (I), (S)
(c) (II), (T)
(d) (II), (Q)

2. Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions:
(i) Centre of C_3 is collinear with the centres of C_1 and C_2
(ii) C_1 and C_2 both lie inside C_3, and
(iii) C_3 touches C_1 at M and C_2 at N
Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.
There are some expressions given in the List – I whose values are given in List – II below

Column I	Column II
(A) $2h + k$	(p) 6
(B) $\dfrac{\text{Length of } ZW}{\text{Length of } XY}$	(q) $\sqrt{6}$
(C) $\dfrac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$	(r) $\dfrac{5}{4}$
	(s) $\dfrac{21}{5}$
	(t) $2\sqrt{6}$
	(u) $\dfrac{10}{3}$

Which of the following is the only INCORRECT combination?
(a) (IV), (S)
(b) (I), (P)
(c) (III), (R)
(d) (IV), (U)

G Comprehension Based Questions

PASSAGE-1

$ABCD$ is a square of side length 2 units. C_1 is the circle touching all the sides of the square $ABCD$ and C_2 is the circumcircle of square $ABCD$. L is a fixed line in the same plane and R is a fixed point.

1. If P is any point of C_1 and Q is another point on C_2, then

$$\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2} \text{ is equal to}$$ *(2006 - 5M, –2)*

(a) 0.75 (b) 1.25 (c) 1 (d) 0.5

2. If a circle is such that it touches the line L and the circle C_1 externally, such that both the circles are on the same side of the line, then the locus of centre of the circle is *(2006 - 5M, –2)*

(a) ellipse
(b) hyperbola
(c) parabola
(d) pair of straight line

3. A line L' through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts L' at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is *(2006 - 5M, –2)*

(a) $\frac{1}{2}$ sq. units (b) $\frac{2}{3}$ sq. units
(c) 1 sq. units (d) 2 sq. units

PASSAGE-2

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x+y-6=0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

4. The equation of circle C is *(2008)*

(a) $(x-2\sqrt{3})^2+(y-1)^2=1$
(b) $(x-2\sqrt{3})^2+\left(y+\frac{1}{2}\right)^2=1$
(c) $(x-\sqrt{3})^2+(y+1)^2=1$
(d) $(x-\sqrt{3})^2+(y-1)^2=1$

5. Points E and F are given by *(2008)*

(a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$
(c) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

6. Equations of the sides QR, RP are *(2008)*

(a) $y=\frac{2}{\sqrt{3}}x+1,\ y=-\frac{2}{\sqrt{3}}x-1$
(b) $y=\frac{1}{\sqrt{3}}x,\ y=0$
(c) $y=\frac{\sqrt{3}}{2}x+1,\ y=-\frac{\sqrt{3}}{2}x-1$
(d) $y=\sqrt{3}x,\ y=0$

PASSAGE-3

A tangent PT is drawn to the circle $x^2+y^2=4$ at the point $P(\sqrt{3},1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2+y^2=1$. *(2012)*

7. A possible equation of L is

(a) $x-\sqrt{3}y=1$ (b) $x+\sqrt{3}y=1$
(c) $x-\sqrt{3}y=-1$ (d) $x+\sqrt{3}y=5$

8. A common tangent of the two circles is

(a) $x=4$ (b) $y=2$
(c) $x+\sqrt{3}y=4$ (d) $x+2\sqrt{2}y=6$

PASSAGE - 4

Let S be the circle in the xy-plane defined by the equation $x^2+y^2=4$.

9. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope –1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3, and G_3 lie on the curve *(JEE Adv. 2018)*

(a) $x+y=4$
(b) $(x-4)^2+(y-4)^2=16$
(c) $(x-4)(y-4)=4$
(d) $xy=4$

10. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve *(JEE Adv. 2018)*

(a) $(x+y)^2=3xy$ (b) $x^{2/3}+y^{2/3}=2^{4/3}$
(c) $x^2+y^2=2xy$ (d) $x^2+y^2=x^2y^2$

H Assertion & Reason Type Questions

1. Tangents are drawn from the point (17, 7) to the circle $x^2+y^2=169$.

STATEMENT-1 : The tangents are mutually perpendicular.

because

STATEMENT-2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2+y^2=338$. *(2007 -3 marks)*

(a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

2. Consider $L_1: 2x+3y+p-3=0$
$L_2: 2x+3y+p+3=0$
where p is a real number, and $C: x^2+y^2+6x-10y+30=0$

STATEMENT - 1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C

and

STATEMENT - 2 : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C. *(2008)*

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is** a correct explanation for Statement - 1
(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is NOT** a correct explanation for Statement - 1
(c) Statement - 1 is True, Statement - 2 is False
(d) Statement - 1 is False, Statement - 2 is True

I Integer Value Correct Type

1. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segement joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is *(2009)*
2. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts.

   If $S = \left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$ then the number of points (s) in S lying inside the smaller part is *(2011)*
3. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points? *(JEE Adv. 2017)*
4. Let the point B be the reflection of the point $A(2,3)$ with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centers A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is ____. *(JEE Adv. 2019)*

Section-B JEE Main / AIEEE

1. If the chord $y = mx + 1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45° at the major segment of the circle then value of m is [2002]
   (a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$
   (c) $-1 \pm \sqrt{2}$ (d) none of these
2. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is [2002]
   (a) $4 \le x^2+y^2 \le 64$ (b) $x^2+y^2 \le 25$
   (c) $x^2+y^2 \ge 25$ (d) $3 \le x^2+y^2 \le 9$
3. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2+y^2=9$ is [2002]
   (a) $\left(\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},-\sqrt{2}\right)$ (c) $\left(\frac{3}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\frac{3}{2}\right)$
4. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is [2002]
   (a) $x^2+y^2=9a^2$ (b) $x^2+y^2=16a^2$
   (c) $x^2+y^2=4a^2$ (d) $x^2+y^2=a^2$
5. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct point, then [2003]
   (a) $r > 2$ (b) $2 < r < 8$ (c) $r < 2$ (d) $r = 2$.
6. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq.units. Then the equation of the circle is [2003]
   (a) $x^2 + y^2 - 2x + 2y = 62$ (b) $x^2 + y^2 + 2x - 2y = 62$
   (c) $x^2 + y^2 + 2x - 2y = 47$ (d) $x^2 + y^2 - 2x + 2y = 47$
7. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is [2004]
   (a) $2ax - 2by - (a^2 + b^2 + 4) = 0$
   (b) $2ax + 2by - (a^2 + b^2 + 4) = 0$
   (c) $2ax - 2by + (a^2 + b^2 + 4) = 0$
   (d) $2ax + 2by + (a^2 + b^2 + 4) = 0$
8. A variable circle passes through the fixed point $A(p,q)$ and touches x-axis. The locus of the other end of the diameter through A is [2004]
   (a) $(y-q)^2 = 4px$ (b) $(x-q)^2 = 4py$
   (c) $(y-p)^2 = 4qx$ (d) $(x-p)^2 = 4qy$
9. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameter of a circle of circumference 10π, then the equation of the circle is [2004]
   (a) $x^2 + y^2 + 2x - 2y - 23 = 0$
   (b) $x^2 + y^2 - 2x - 2y - 23 = 0$
   (c) $x^2 + y^2 + 2x + 2y - 23 = 0$
   (d) $x^2 + y^2 - 2x + 2y - 23 = 0$
10. Intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is [2004]
    (a) $x^2 + y^2 + x - y = 0$
    (b) $x^2 + y^2 - x + y = 0$
    (c) $x^2 + y^2 + x + y = 0$
    (d) $x^2 + y^2 - x - y = 0$
11. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for [2005]
    (a) exactly one value of a
    (b) no value of a
    (c) infinitely many values of a
    (d) exactly two values of a
12. A circle touches the x- axis and also touches the circle with centre at (0,3) and radius 2. The locus of the centre of the circle is [2005]
    (a) an ellipse (b) a circle
    (c) a hyperbola (d) a parabola

13. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is [2005]

(a) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$

(b) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

(c) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$

(d) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

14. If the pair of lines $ax^2 + 2(a+b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then [2005]

(a) $3a^2 - 10ab + 3b^2 = 0$ (b) $3a^2 - 2ab + 3b^2 = 0$

(c) $3a^2 + 10ab + 3b^2 = 0$ (d) $3a^2 + 2ab + 3b^2 = 0$

15. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is [2006]

(a) $x^2 + y^2 + 2x - 2y - 47 = 0$

(b) $x^2 + y^2 + 2x - 2y - 62 = 0$

(c) $x^2 + y^2 - 2x + 2y - 62 = 0$

(d) $x^2 + y^2 - 2x + 2y - 47 = 0$

16. Let C be the circle with centre $(0, 0)$ and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is

(a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$ [2006]

(c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$

17. Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval [2007]

(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$

(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$

18. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is [2008]

(a) $(3, -4)$ (b) $(-3, 4)$ (c) $(-3, -4)$ (d) $(3, 4)$

19. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is

(a) $(x-2)y'^2 = 25 - (y-2)^2$
(b) $(y-2)y'^2 = 25 - (y-2)^2$
(c) $(y-2)^2 y'^2 = 25 - (y-2)^2$
(d) $(x-2)^2 y'^2 = 25 - (y-2)^2$

20. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P, Q and $(1, 1)$ for: [2009]

(a) all except one value of p
(b) all except two values of p
(c) exactly one value of p
(d) all values of p

21. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if [2010]

(a) $-35 < m < 15$ (b) $15 < m < 65$
(c) $35 < m < 85$ (d) $-85 < m < -35$

22. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c > 0)$ touch each other if [2011]

(a) $|a| = c$ (b) $a = 2c$
(c) $|a| = 2c$ (d) $2|a| = c$

23. The length of the diameter of the circle which touches the x-axis at the point $(1,0)$ and passes through the point $(2,3)$ is: [2012]

(a) $\frac{10}{3}$ (b) $\frac{3}{5}$ (c) $\frac{6}{5}$ (d) $\frac{5}{3}$

24. The circle passing through $(1, -2)$ and touching the axis of x at $(3, 0)$ also passes through the point [JEE M 2013]

(a) $(-5, 2)$ (b) $(2, -5)$ (c) $(5, -2)$ (d) $(-2, 5)$

25. Let C be the circle with centre at $(1, 1)$ and radius $= 1$. If T is the circle centred at $(0, y)$, passing through origin and touching the circle C externally, then the radius of T is equal to [JEE M 2014]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

26. Locus of the image of the point $(2, 3)$ in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0, k \in \mathbf{R}$, is a : [JEE M 2015]

(a) circle of radius $\sqrt{2}$.
(b) circle of radius $\sqrt{3}$.
(c) straight line parallel to x-axis
(d) straight line parallel to y-axis

27. The number of common tangents to the circles $x^2 + y^2 - 4x - 6x - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is : [JEE M 2015]

(a) 3 (b) 4 (c) 1 (d) 2

28. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on: [JEE M 2016]

(a) a hyperbola
(b) a parabola
(c) a circle
(d) an ellipse which is not a circle

29. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at $(-3, 2)$, then the radius of S is: [JEE M 2016]

(a) 5 (b) 10
(c) $5\sqrt{2}$ (d) $5\sqrt{3}$

30. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is: [JEE M 2019 - 9 April (M)]

(a) $x^2 + y^2 - 4x^2y^2 = 0$ (b) $x^2 + y^2 - 2xy = 0$
(c) $x^2 + y^2 - 16x^2y^2 = 0$ (d) $x^2 + y^2 - 2x^2y^2 = 0$