CHAPTER 18

Definite Integrals and Applications of Integrals

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. $f(x)=\begin{vmatrix} \sec x & \cos x & \sec^2 x+\cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$.

Then $\int_0^{\pi/2} f(x)\,dx=$ .... *(1987 - 2 Marks)*

2. The integral $\int_0^{1.5}[x^2]dx$, *(1988 - 2 Marks)*

Where [ ] denotes the greatest integer function, equals ....

3. The value of $\int_{-2}^{2}|1-x^2|\,dx$ is....... *(1989 - 2 Marks)*

4. The value of $\int_{\pi/4}^{3\pi/4}\frac{\phi}{1+\sin\phi}d\phi$ is .... *(1993 - 2 Marks)*

5. The value of $\int_2^3\frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ is .... *(1994 - 2 Marks)*

6. If for nonzero x, $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-5$ where $a\neq b$, then

$\int_1^2 f(x)\,dx=$....... *(1996 - 2 Marks)*

7. For $n>0$, $\int_0^{2\pi}\frac{x\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}dx=$ ...... *(1996 - 1 Mark)*

8. The value of $\int_1^{e^{37}}\frac{\pi\sin(\pi \ln x)}{x}dx$ is .... *(1997 - 2 Marks)*

9. Let $\frac{d}{dx}F(x)=\frac{e^{\sin x}}{x}$, $x>0$. If $\int_1^4\frac{2e^{\sin x^2}}{x}dx=F(k)-F(1)$

then one of the possible values of k is ....

(1997 - 2 Marks)

B True / False

1. The value of the integral $\int_0^{2a}\left[\frac{f(x)}{\{f(x)+f(2a-x)\}}\right]dx$ is equal to a. *(1988 - 1 Mark)*

C MCQs with One Correct Answer

1. The value of the definite integral $\int_0^1(1+e^{-x^2})\,dx$ is

(a) -1
(b) 2 *(1981 - 2 Marks)*
(c) $1+e^{-1}$
(d) none of these

2. Let a, b, c be non-zero real numbers such that

$\int_0^1(1+\cos^8 x)(ax^2+bx+c)\,dx=\int_0^2(1+\cos^8 x)(ax^2+bx+c)\,dx.$

Then the quadratic equation $ax^2+bx+c=0$ has

(1981 - 2 Marks)

(a) no root in (0, 2)
(b) at least one root in (0, 2)
(c) a double root in (0, 2)
(d) two imaginary roots

3. The area bounded by the curves $y=f(x)$, the x-axis and the ordinates $x=1$ and $x=b$ is $(b-1)\sin(3b+4)$. Then $f(x)$ is

(1982 - 2 Marks)

(a) $(x-1)\cos(3x+4)$
(b) $\sin(3x+4)$
(c) $\sin(3x+4)+3(x-1)\cos(3x+4)$
(d) none of these

4. The value of the integral $\int_0^{\pi/2}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx$ is

(a) $\pi/4$
(b) $\pi/2$ *(1983 - 1 Mark)*
(c) π
(d) none of these

5. For any integer n the integral —

$\int_0^{\pi}e^{\cos^2 x}\cos^3(2n+1)x\,dx$ has the value *(1985 - 2 Marks)*

(a) π
(b) 1
(c) 0
(d) none of these

6. Let $f: R \to R$ and $g: R \to R$ be continous functions. Then the value of the integral

$$\int_{-\pi/2}^{\pi/2} [f(x)+f(-x)][g(x)-g(-x)]dx$$ is *(1990 - 2 Marks)*

(a) π (b) 1 (c) -1 (d) 0

7. The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is *(1993 - 1 Marks)*

(a) 0 (b) 1 (c) $\pi/2$ (d) $\pi/4$

8. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then constants A and B are *(1995S)*

(a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$

(c) 0 and $\frac{-4}{\pi}$ (d) $\frac{4}{\pi}$ and 0

9. The value of $\int_\pi^{2\pi} [2\,Sinx]\,dx$ where [.] represents the greatest integer function is *(1995S)*

(a) $\frac{-5\pi}{3}$ (b) $-\pi$ (c) $\frac{5\pi}{3}$ (d) -2π

10. If $g(x) = \int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals *(1997 - 2 Marks)*

(a) $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$

(c) $g(x)\,g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

11. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to *(1999 - 2 Marks)*

(a) 2 (b) -2 (c) 1/2 (d) $-1/2$

12. If for a real number y, $[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2} [2\sin x]\,dx$ is *(1999 - 2 Marks)*

(a) $-\pi$ (b) 0 (c) $-\pi/2$ (d) $\pi/2$

13. Let $g(x) = \int_0^x f(t)dt$, where f is such that $\frac{1}{2} \le f(t) \le 1$, for $t \in [0,1]$ and $0 \le f(t) \le \frac{1}{2}$, for $t \in [1,2]$. Then $g(2)$ satisfies the inequality *(2000S)*

(a) $-\frac{3}{2} \le g(2) < \frac{1}{2}$ (b) $0 \le g(2) < 2$

(c) $\frac{3}{2} < g(2) \le \frac{5}{2}$ (d) $2 < g(2) < 4$

14. If $f(x) = \begin{cases} e^{\cos x}\sin x, & \text{for } |x| \le 2 \\ 2, & \text{otherwise,} \end{cases}$ then $\int_{-2}^{3} f(x)dx =$ *(2000S)*

(a) 0 (b) 1 (c) 2 (d) 3

15. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is: *(2000S)*

(a) 3/2 (b) 5/2 (c) 3 (d) 5

16. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, $a > 0$, is *(2001S)*

(a) π (b) $a\pi$ (c) $\pi/2$ (d) 2π

17. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is *(2002S)*

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) 4

18. Let $f(x) = \int_1^x \sqrt{2-t^2}\,dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are *(2002S)*

(a) ± 1 (b) $\pm\frac{1}{\sqrt{2}}$ (c) $\pm\frac{1}{2}$ (d) 0 and 1

19. Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in R$, $f(x+T) = f(x)$.

If $I = \int_0^T f(x)dx$ then the value of $\int_3^{3+3T} f(2x)dx$ is *(2002S)*

(a) $3/2I$ (b) $2I$ (c) $3I$ (d) $6I$

20. The integral $\int_{-1/2}^{1/2} \left([x] + \ell n\left(\frac{1+x}{1-x}\right)\right) dx$ equal to *(2002S)*

(a) $-\frac{1}{2}$ (b) 0 (c) 1 (d) $2\ell n\left(\frac{1}{2}\right)$

21. If $l(m,n) = \int_0^1 t^m(1+t)^n\,dt$, then the expression for $l(m,n)$ in terms of $l(m+1, n-1)$ is *(2003S)*

(a) $\frac{2^n}{m+1} - \frac{n}{m+1} l(m+1, n-1)$

(b) $\frac{n}{m+1} l(m+1, n-1)$

(c) $\frac{2^n}{m+1} + \frac{n}{m+1} l(m+1, n-1)$

(d) $\frac{m}{n+1} l(m+1, n-1)$

22. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$, then $f(x)$ increases in *(2003S)*

(a) $(-2, 2)$ (b) no value of x
(c) $(0, \infty)$ (d) $(-\infty, 0)$

23. The area bounded by the curves $y=\sqrt{x}$, $2y+3=x$ and x-axis in the 1st quadrant is (2003S)
(a) 9 (b) 27/4 (c) 36 (d) 18

24. If $f(x)$ is differentiable and $\int_0^{t^2} xf(x)dx=\frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals (2004S)
(a) 2/5 (b) –5/2 (c) 1 (d) 5/2

25. The value of the integral $\int_0^1\sqrt{\frac{1-x}{1+x}}dx$ is (2004S)
(a) $\frac{\pi}{2}+1$ (b) $\frac{\pi}{2}-1$ (c) -1 (d) 1

26. The area enclosed between the curves $y=ax^2$ and $x=ay^2$ $(a>0)$ is 1 sq. unit, then the value of a is (2004S)
(a) $1/\sqrt{3}$ (b) 1/2 (c) 1 (d) 1/3

27. $\int_{-2}^{0}\{x^3+3x^2+3x+3+(x+1)\cos(x+1)\}\,dx$ is equal to (2005S)
(a) –4 (b) 0 (c) 4 (d) 6

28. The area bounded by the parabolas $y=(x+1)^2$ and $y=(x-1)^2$ and the line $y=1/4$ is (2005S)
(a) 4 sq. units (b) 1/6 sq. units
(c) 4/3 sq. units (d) 1/3 sq. units

29. The area of the region between the curves $y=\sqrt{\frac{1+\sin x}{\cos x}}$ and $y=\sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x=0$ and $x=\frac{\pi}{4}$ is (2008)
(a) $\int_0^{\sqrt{2}-1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$ (b) $\int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$
(c) $\int_0^{\sqrt{2}+1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$ (d) $\int_0^{\sqrt{2}+1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

30. Let f be a non-negative function defined on the interval $[0,1]$. If $\int_0^x\sqrt{1-(f'(t))^2}\,dt=\int_0^x f(t)\,dt$, $0\le x\le 1$, and $f(0)=0$, then (2009)
(a) $f\left(\frac{1}{2}\right)<\frac{1}{2}$ and $f\left(\frac{1}{3}\right)>\frac{1}{3}$
(b) $f\left(\frac{1}{2}\right)>\frac{1}{2}$ and $f\left(\frac{1}{3}\right)>\frac{1}{3}$
(c) $f\left(\frac{1}{2}\right)<\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$
(d) $f\left(\frac{1}{2}\right)>\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$

31. The value of $\lim_{x\to 0}\frac{1}{x^3}\int_0^x\frac{t\,ln(1+t)}{t^4+4}dt$ is (2010)
(a) 0 (b) $\frac{1}{12}$ (c) $\frac{1}{24}$ (d) $\frac{1}{64}$

32. Let f be a real-valued function defined on the interval $(-1,1)$ such that $e^{-x}f(x)=2+\int_0^x\sqrt{t^4+1}\,dt$, for all $x\in(-1,1)$, and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to (2010)
(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

33. The value of $\int_{\sqrt{\ell n2}}^{\sqrt{\ell n3}}\frac{x\sin x^2}{\sin x^2+\sin(\ell n6-x^2)}dx$ is (2011)
(a) $\frac{1}{4}\ell n\frac{3}{2}$ (b) $\frac{1}{2}\ell n\frac{3}{2}$ (c) $\ell n\frac{3}{2}$ (d) $\frac{1}{6}\ell n\frac{3}{2}$

34. Let the straight line $x=b$ divide the area enclosed by $y=(1-x)^2$, $y=0$, and $x=0$ into two parts R_1 $(0\le x\le b)$ and R_2 $(b\le x\le 1)$ such that $R_1-R_2=\frac{1}{4}$. Then b equals (2011)
(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

35. Let $f:[-1,2]\to[0,\infty)$ be a continuous function such that $f(x)=f(1-x)$ for all $x\in[-1,2]$
Let $R_1=\int_{-1}^{2}xf(x)dx$, and R_2 be the area of the region bounded by $y=f(x)$, $x=-1$, $x=2$, and the x-axis. Then (2011)
(a) $R_1=2R_2$ (b) $R_1=3R_2$
(c) $2R_1=R_2$ (d) $3R_1=R_2$

36. The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is (2012)
(a) 0 (b) $\frac{\pi^2}{2}-4$ (c) $\frac{\pi^2}{2}+4$ (d) $\frac{\pi^2}{2}$

37. The area enclosed by the curves $y=\sin x+\cos x$ and $y=|\cos x-\sin x|$ over the interval $\left[0,\frac{\pi}{2}\right]$ is (JEE Adv. 2013)
(a) $4(\sqrt{2}-1)$ (b) $2\sqrt{2}(\sqrt{2}-1)$
(c) $2(\sqrt{2}+1)$ (d) $2\sqrt{2}(\sqrt{2}+1)$

38. Let $f:\left[\frac{1}{2},1\right]\to R$ (the set of all real number) be a positive, non-constant and differentiable function such that $f'(x)<2f(x)$ and $f\left(\frac{1}{2}\right)=1$. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval *(JEE Adv. 2013)*

(a) $(2e-1, 2e)$ (b) $(e-1, 2e-1)$

(c) $\left(\frac{e-1}{2}, e-1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

39. The following integral $\int_{\pi/4}^{\pi/2}(2\operatorname{cosec} x)^{17}\,dx$ is equal to *(JEE Adv. 2014)*

(a) $\int_0^{\log(1+\sqrt{2})} 2\left(e^u+e^{-u}\right)^{16} du$

(b) $\int_0^{\log(1+\sqrt{2})} \left(e^u+e^{-u}\right)^{17} du$

(c) $\int_0^{\log(1+\sqrt{2})} \left(e^u-e^{-u}\right)^{17} du$

(d) $\int_0^{\log(1+\sqrt{2})} 2\left(e^u-e^{-u}\right)^{16} du$

40. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2\cos x}{1+e^x}dx$ is equal to *(JEE Adv. 2016)*

(a) $\frac{\pi^2}{4}-2$ (b) $\frac{\pi^2}{4}+2$

(c) $\pi^2-e^{\frac{\pi}{2}}$ (d) $\pi^2+e^{\frac{\pi}{2}}$

41. Area of the region $\left\{(x,y)\in\mathbb{R}^2 : y\ge\sqrt{|x+3|}, 5y\le x+9\le 15\right\}$ is equal to *(JEE Adv. 2016)*

(a) $\frac{1}{6}$ (b) $\frac{4}{3}$ (c) $\frac{3}{2}$ (d) $\frac{5}{3}$

42. The area of the region $\left\{(x,y : xy\le 8, 1\le y\le x^2\right\}$ is *(JEE Adv. 2018)*

(a) $8\log_e 2-\frac{14}{3}$ (b) $16\log_e 2-\frac{14}{3}$

(c) $8\log_e 2-\frac{7}{3}$ (d) $16\log_e 2-6$

D MCQs with One or More than One Correct

1. If $\int_0^x f(t)\,dt = x+\int_x^1 t\,f(t)\,dt$, then the value of $f(1)$ is *(1998 - 2 Marks)*

(a) 1/2 (b) 0 (c) 1 (d) $-1/2$

2. Let $f(x)=x-[x]$, for every real number x, where $[x]$ is the integral part of x. Then $\int_{-1}^{1} f(x)\,dx$ is *(1998 - 2 Marks)*

(a) 1 (b) 2 (c) 0 (d) 1/2

3. For which of the following values of m, is the area of the region bounded by the curve $y=x-x^2$ and the line $y=mx$ equals 9/2? *(1999 - 3 Marks)*

(a) -4 (b) -2 (c) 2 (d) 4

4. Let $f(x)$ be a non-constant twice differentiable function definied on $(-\infty,\infty)$ such that $f(x)=f(1-x)$ and $f'\left(\frac{1}{4}\right)=0$. Then, *(2008)*

(a) $f''(x)$ vanishes at least twice on [0, 1]

(b) $f'\left(\frac{1}{2}\right)=0$

(c) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x\,dx=0$

(d) $\int_0^{1/2} f(t)\,e^{\sin\pi t}\,dt=\int_{1/2}^{1} f(1-t)\,e^{\sin\pi t}\,dt$

5. Area of the region bounded by the curve $y=e^x$ and lines $x=0$ and $y=e$ is *(2009)*

(a) $e-1$ (b) $\int_1^e \ln(e+1-y)\,dy$

(c) $e-\int_0^1 e^x dx$ (d) $\int_1^e \ln y\,dy$

6. If $I_n=\int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$ $n=0,1,2,\ldots$, then *(2009)*

(a) $I_n=I_{n+2}$ (b) $\sum_{m=1}^{10} I_{2m+1}=10\pi$

(c) $\sum_{m=1}^{10} I_{2m}=0$ (d) $I_n=I_{n+1}$

7. The value(s) of $\int_0^1\frac{x^4(1-x)^4}{1+x^2}dx$ is (are) *(2010)*

(a) $\frac{22}{7}-\pi$ (b) $\frac{2}{105}$

(c) 0 (d) $\frac{71}{15}-\frac{3\pi}{2}$

8. Let f be a real-valued function defined on the interval $(0, \infty)$ by $f(x) = \ln x + \int_0^x \sqrt{1+\sin t}\, dt$. Then which of the following statement(s) is (are) true? *(2010)*

(a) $f''(x)$ exists for all $x \in (0,\infty)$

(b) $f'(x)$ exists for all $x \in (0,\infty)$ and f' is continuous on $(0,\infty)$, but not differentiable on $(0,\infty)$

(c) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha,\infty)$

(d) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \le \beta$ for all $x \in (0,\infty)$

9. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0, x = 0$ and $x = 1$; then *(2012)*

(a) $S \ge \frac{1}{e}$ (b) $S \ge 1 - \frac{1}{e}$

(c) $S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ (d) $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

10. The option(s) with the values of a and L that satisfy the following equation is(are) *(JEE Adv. 2015)*

$$\frac{\int_0^{4\pi} e^t\left(\sin^6 at + \cos^4 at\right)dt}{\int_0^{\pi} e^t\left(\sin^6 at + \cos^4 at\right)dt} = L?$$

(a) $a = 2, L = \frac{e^{4\pi}-1}{e^{\pi}-1}$ (b) $a = 2, L = \frac{e^{4\pi}+1}{e^{\pi}+1}$

(c) $a = 4, L = \frac{e^{4\pi}-1}{e^{\pi}-1}$ (d) $a = 4, L = \frac{e^{4\pi}+1}{e^{\pi}+1}$

11. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression(s) is(are) *(JEE Adv. 2015)*

(a) $\int_0^{\pi/4} xf(x)dx = \frac{1}{12}$ (b) $\int_0^{\pi/4} f(x)dx = 0$

(c) $\int_0^{\pi/4} xf(x)dx = \frac{1}{6}$ (d) $\int_0^{\pi/4} f(x)dx = 1$

12. Let $f'(x) = \frac{192x^3}{2+\sin^4 \pi x}$ for all $x \in \mathbb{R}$ with $f\left(\frac{1}{2}\right) = 0$.

If $m \le \int_{1/2}^{1} f(x)dx \le M$, then the possible values of m and M are *(JEE Adv. 2015)*

(a) $m = 13, M = 24$ (b) $m = \frac{1}{4}, M = \frac{1}{2}$

(c) $m = -11, M = 0$ (d) $m = 1, M = 12$

13. Let $f(x) = \lim\limits_{n\to\infty}\left(\frac{n^n (x+n)\left(x+\frac{n}{2}\right)\ldots\left(x+\frac{n}{n}\right)}{n!\left(x^2+n^2\right)\left(x^2+\frac{n^2}{4}\right)\ldots\left(x^2+\frac{n^2}{n^2}\right)}\right)^{\frac{x}{n}}$, for all $x > 0$. Then *(JEE Adv. 2016)*

(a) $f\left(\frac{1}{2}\right) \ge f(1)$ (b) $f\left(\frac{1}{3}\right) \le f\left(\frac{2}{3}\right)$

(c) $f'(2) \le 0$ (d) $\frac{f'(3)}{f(3)} \ge \frac{f'(2)}{f(2)}$

14. Let $f: R \to (0,1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)? *(JEE Adv. 2017)*

(a) $x^9 - f(x)$ (b) $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\, dt$

(c) $e^x - \int_0^x f(t)\sin t\, dt$ (d) $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t\, dt$

15. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt$, then *(JEE Adv. 2017)*

(a) $g'\left(\frac{\pi}{2}\right) = -2\pi$ (b) $g'\left(-\frac{\pi}{2}\right) = 2\pi$

(c) $g'\left(\frac{\pi}{2}\right) = 2\pi$ (d) $g'\left(-\frac{\pi}{2}\right) = -2\pi$

16. If the line $x = \alpha$ divides the area of region $R = \left\{(x,y) \in R^2 : x^3 \le y \le x, 0 \le x \le 1\right\}$ into two equal parts, then *(JEE Adv. 2017)*

(a) $0 < \alpha \le \frac{1}{2}$ (b) $\frac{1}{2} < \alpha < 1$

(c) $2\alpha^4 - 4\alpha^2 + 1 = 0$ (d) $\alpha^4 + 4\alpha^2 - 1 = 0$

17. If $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)}dx$, then *(JEE Adv. 2017)*

(a) $I > \log_e 99$ (b) $I < \log_e 99$

(c) $I < \frac{49}{50}$ (d) $I > \frac{49}{50}$

18. For, $a \in R, |a| > 1$, let

$$\lim_{x\to\infty}\left(\frac{1+\sqrt[3]{2}+\ldots\ldots+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+\ldots+\frac{1}{(an+n)^2}\right)}\right) = 54$$

Then the possible value(s) of a is/are *(JEE Adv. 2019)*

(a) -9 (b) 7

(c) -6 (d) 8

E Subjective Problems

1. Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$. *(1981 - 4 Marks)*

2. Show that : $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{6n}\right) = \log 6$ *(1981 - 2 Marks)*

3. Show that $\int_0^{\pi} xf(\sin x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$. *(1982 - 2 Marks)*

4. Find the value of $\int_{-1}^{3/2} |x\sin\pi x|\,dx$ *(1982 - 3 Marks)*

5. For any real t, $x = \frac{e^t + e^{-t}}{2}$, $y = \frac{e^t - e^{-t}}{2}$ is a point on the hyperbola $x^2 - y^2 = 1$. Show that the area bounded by this hyperbola and the lines joining its centre to the points corresponding to t_1 and $-t_1$ is t_1. *(1982 - 3 Marks)*

6. Evaluate : $\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx$ *(1983 - 3 Marks)*

7. Find the area bounded by the x-axis, part of the curve $y = \left(1+\frac{8}{x^2}\right)$ and the ordinates at $x = 2$ and $x = 4$. If the ordinate at $x = a$ divides the area into two equal parts, find a. *(1983 - 3 Marks)*

8. Evaluate the following $\int_0^{\frac{1}{2}}\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ *(1984 - 2 Marks)*

9. Find the area of the region bounded by the x-axis and the curves defined by *(1984 - 4 Marks)*

$y = \tan x,\ -\frac{\pi}{3}\le x\le\frac{\pi}{3};\ y = \cot x,\ \frac{\pi}{6}\le x\le\frac{3\pi}{2}$

10. Given a function $f(x)$ such that *(1984 - 4 Marks)*
(i) it is integrable over every interval on the real line and
(ii) $f(t+x) = f(x)$, for every x and a real t, then show that the integral $\int_a^{a+t} f(x)\,dx$ is independent of a .

11. Evaluate the following : $\int_0^{\pi/2}\frac{x\sin x\cos x}{\cos^4 x+\sin^4 x}dx$ *(1985 - 2½ Marks)*

12. Sketch the region bounded by the curves $y = \sqrt{5-x^2}$ and $y = |x-1|$ and find its area. *(1985 - 5 Marks)*

13. Evaluate: $\int_0^{\pi}\frac{x\,dx}{1+\cos\alpha\sin x}$, $0<\alpha<\pi$ *(1986 - 2½ Marks)*

14. Find the area bounded by the curves, $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$ above the x-axis. *(1987 - 6 Marks)*

15. Find the area of the region bounded by the curve $C : y = \tan x$, tangent drawn to C at $x = \frac{\pi}{4}$ and the x-axis. *(1988 - 5 Marks)*

16. Evaluate $\int_0^1 \log[\sqrt{1-x}+\sqrt{1+x}]dx$ *(1988 - 5 Marks)*

17. If f and g are continuous function on $[0, a]$ satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx$ *(1989 - 4 Marks)*

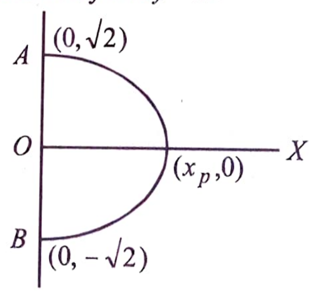
18. Show that $\int_0^{\pi/2} f(\sin 2x)\sin x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$ *(1990 - 4 Marks)*

19. Prove that for any positive integer k,
$\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + \ldots\ldots\ldots\ldots + \cos(2k-1)x]$
Hence prove that $\int_0^{\pi/2}\sin 2kx\cot x\,dx = \frac{\pi}{2}$ *(1990 - 4 Marks)*

20. Compute the area of the region bounded by the curves $y = ex\ \text{In}\ x$ and $y = \frac{\text{In}\ x}{ex}$ where $\text{In}\ e = 1$. *(1990 - 4 Marks)*

21. Sketch the curves and identify the region bounded by $x = \frac{1}{2}$, $x = 2$, $y = \text{In}\ x$ and $y = 2^x$. Find the area of this region. *(1991 - 4 Marks)*

22. If 'f' is a continous function with $\int_0^x f(t)\,dt \to \infty$ as $|x| \to \infty$, then show that every line $y = mx$



intersects the curve $y^2 + \int_0^x f(t)\,dt = 2!$ *(1991 - 4 Marks)*

23. Evaluate $\int_0^{\pi}\frac{x\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}dx$ *(1991 - 4 Marks)*

24. Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$. Find the area. *(1992 - 4 Marks)*

25. Determine a positive integer $n \le 5$, such that $\int_0^1 e^x(x-1)^n dx = 16 - 6e$ *(1992 - 4 Marks)*

26. Evalute $\int_2^3 \frac{2x^5+x^4-2x^3+2x^2+1}{(x^2+1)(x^4-1)}dx$. *(1993 - 5 Marks)*

27. Show that $\int\limits_0^{n\pi+v} |\sin x|\, dx = 2n+1-\cos v$ where n is a positive integer and $0 \le v < \pi$. *(1994 - 4 Marks)*

28. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$? *(1994 - 5 Marks)*

29. Let $I_m = \int\limits_0^{\pi} \frac{1-\cos mx}{1-\cos x}\, dx$. Use mathematical induction to prove that $I_m = m\,\pi, m = 0, 1, 2, ...$ *(1995 - 5 Marks)*

30. Evaluate the definite integral :

$$\int\limits_{-1/\sqrt{3}}^{1/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right)\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx$$ *(1995 - 5 Marks)*

31. Consider a square with vertices at (1, 1), (– 1, 1), (– 1, – 1) and (1, – 1). Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area. *(1995 - 5 Marks)*

32. Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0, y = 0$ and $x = \frac{\pi}{4}$. Prove that for $n > 2$,

$A_n + A_{n-2} = \frac{1}{n-1}$ and deduce $\frac{1}{2n+2} < A_n < \frac{1}{2n-2}$. *(1996 - 3 Marks)*

33. Determine the value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$. *(1997 - 5 Marks)*

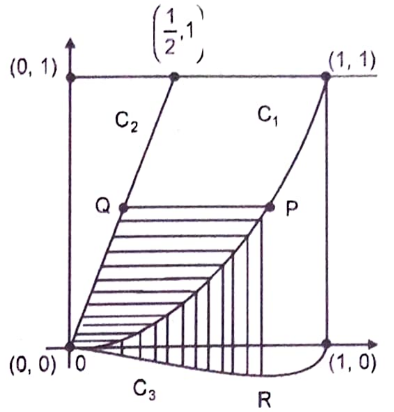
34. Let $f(x) =$ Maximum $\{x^2, (1-x)^2, 2x(1-x)\}$, where $0 \le x \le 1$. Determine the area of the region bounded by the curves $y = f(x)$, x-axis, $x = 0$ and $x = 1$. *(1997 - 5 Marks)*

35. Prove that $\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = 2\int_0^1 \tan^{-1} x\, dx$.

Hence or otherwise, evaluate the integral

$\int_0^1 \tan^{-1}(1-x+x^2)\, dx$. *(1998 - 8 Marks)*

36. Let C_1 and C_2 be the graphs of the functions $y = x^2$ and $y = 2x$, $0 \le x \le 1$ respectively. Let C_3 be the graph of a function $y = f(x)$, $0 \le x \le 1$, $f(0) = 0$. For a point P on C_1, let the lines through P, parallel to the axes, meet C_2 and C_3 at Q and R respectively (see figure.) If for every position of P (on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function $f(x)$. *(1998 - 8 Marks)*



37. Integrate $\int\limits_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$. *(1999 - Marks)*

38. Let $f(x)$ be a continuous function given by

$f(x) = \begin{cases} 2x, & |x| \le 1 \\ x^2+ax+b, & |x|>1 \end{cases}$ *(1999 - 10 Marks)*

Find the area of the region in the third quadrant bounded by the curves $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$.

39. For $x>0$, let $f(x) = \int\limits_e^x \frac{\ln t}{1+t}dt$. Find the function

$f(x) + f\left(\frac{1}{x}\right)$ and show that $f(e) + f\left(\frac{1}{e}\right) = \frac{1}{2}$.

Here, $\ln t = \log_e t$. *(2000 - 5 Marks)*

40. Let $b \ne 0$ and for $j = 0, 1, 2, \ldots, n$, let S_j be the area of the region bounded by the y–axis and the curve $xe^{ay} = \sin by$, $\frac{jr}{b} \le y \le \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \ldots, S_n$ are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$. *(2001 - 5 Marks)*

41. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$, which lies to the right of the line $x = 1$. *(2002 - 5 Marks)*

42. If f is an even function then prove that *(2003 - 2 Marks)*

$$\int\limits_0^{\pi/2} f(\cos 2x)\cos x\, dx = \sqrt{2}\int\limits_0^{\pi/4} f(\sin 2x)\cos x\, dx.$$

43. If $y(x) = \int\limits_{\pi^2/16}^{x^2} \frac{\cos x \cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$, then find $\frac{dy}{dx}$ at $x = \pi$ *(2004 - 2 Marks)*

44. Find the value of $\int\limits_{-\pi/3}^{\pi/3} \frac{\pi+4x^3}{2-\cos\left(|x|+\frac{\pi}{3}\right)}dx$ *(2004 - 4 Marks)*

45. Evaluate $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\,dx$

(2005 - 2 Marks)

46. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$. *(2005 - 4 Marks)*

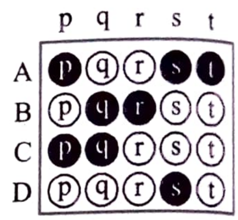
47. $f(x)$ is a differentiable function and $g(x)$ is a double differentiable function such that $|f(x)| \le 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 9$. Prove that there exists some $c \in (-3, 3)$ such that $g(c).g''(c) < 0$. *(2005 - 6 Marks)*

48. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB. *(2005 - 6 Marks)*

49. The value of $5050\dfrac{\int_0^1 (1-x^{50})^{100}dx}{\int_0^1 (1-x^{50})^{101}dx}$ is. *(2006 - 6M)*

F Match the Following

DIRECTIONS (Q. 1 and 2) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*



If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

1. Match the following : *(2006 - 6M)*

Column I	*Column II*
(A) $\int_0^{\pi/2} (\sin x)^{\cos x}\left(\cos x \cot x - \log(\sin x)^{\sin x}\right)dx$	(p) 1
(B) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$	(q) 0
(C) Cosine of the angle of intersection of curves $y = 3^{x-1}\log x$ and $y = x^x - 1$ is	(r) $6 \ln 2$
(D) Let $\dfrac{dy}{dx} = \dfrac{6}{x+y}$ where $y(0) = 0$ then value of y when $x + y = 6$ is	(s) $\dfrac{4}{3}$

2. Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the *ORS*. *(2007 - 6 marks)*

Column I	*Column II*
(A) $\int_{-1}^{1} \dfrac{dx}{1+x^2}$	(p) $\dfrac{1}{2}\log\left(\dfrac{2}{3}\right)$
(B) $\int_0^1 \dfrac{dx}{\sqrt{1-x^2}}$	(q) $2\log\left(\dfrac{2}{3}\right)$
(C) $\int_2^3 \dfrac{dx}{1-x^2}$	(r) $\dfrac{\pi}{3}$
(D) $\int_1^2 \dfrac{dx}{x\sqrt{x^2-1}}$	(s) $\dfrac{\pi}{2}$

DIRECTIONS (Q. 3) : *Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

3.

List - I	List - II
P. The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0)=0$ and $\int_0^1 f(x)\,dx = 1$, is	**1.** 8
Q. The number of points in the interval $\left[-\sqrt{13}, \sqrt{13}\right]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	**2.** 2
R. $\displaystyle\int_{-2}^{2} \frac{3x^2}{\left(1+e^x\right)} dx$ equals	**3.** 4
S. $\dfrac{\left(\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\displaystyle\int_{0}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}$	**4.** 0

(JEE Adv. 2014)

	P	Q	R	S
(a)	3	2	4	1
(b)	2	3	4	1
(c)	3	2	1	4
(d)	2	3	1	4

G Comprehension Based Questions

PASSAGE - 1

Let the definite integral be defined by the formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$. For more accurate result for $c \in (a, b)$, we can use $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx = F(c)$ so that for $c = \frac{a+b}{2}$, we get $\int_a^b f(x)dx = \frac{b-a}{4}\left(f(a)+f(b)+2f(c)\right)$.

1. $\displaystyle\int_0^{\pi/2} \sin x \, dx =$

(2006 - 5M, –2)

(a) $\frac{\pi}{8}\left(1+\sqrt{2}\right)$ (b) $\frac{\pi}{4}\left(1+\sqrt{2}\right)$

(c) $\frac{\pi}{8\sqrt{2}}$ (d) $\frac{\pi}{4\sqrt{2}}$

2. If $\displaystyle\lim_{x\to a} \frac{\int_a^x f(x)dx - \left(\frac{x-a}{2}\right)\left(f(x)+f(a)\right)}{(x-a)^3} = 0$, then $f(x)$ is of maximum degree *(2006 - 5M, –2)*

(a) 4 (b) 3 (c) 2 (d) 1

3. If $f''(x) < 0 \; \forall \, x \in (a, b)$ and c is a point such that $a < c < b$, and $(c, f(c))$ is the point lying on the curve for which $F(c)$ is maximum, then $f'(c)$ is equal to *(2006 - 5M, –2)*

(a) $\frac{f(b)-f(a)}{b-a}$ (b) $\frac{2\left(f(b)-f(a)\right)}{b-a}$

(c) $\frac{2f(b)-f(a)}{2b-a}$ (d) 0

PASSAGE - 2

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

4. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$ *(2008)*

(a) $\frac{4\sqrt{2}}{7^3 3^2}$ (b) $-\frac{4\sqrt{2}}{7^3 3^2}$ (c) $\frac{4\sqrt{2}}{7^3 3}$ (d) $-\frac{4\sqrt{2}}{7^3 3}$

5. The area of the region bounded by the curve $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is *(2008)*

(a) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)} dx + bf(b) - af(a)$

(b) $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)} dx + bf(b) - af(a)$

(c) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)} dx - bf(b) + af(a)$

(d) $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)} dx - bf(b) + af(a)$

6. $\int_{-1}^{1} g'(x)\, dx =$ *(2008)*

(a) $2g(-1)$ (b) 0
(c) $-2g(1)$ (d) $2g(1)$

PASSAGE - 3

Consider the function $f: (-\infty, \infty) \to (-\infty, \infty)$ defined by

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, 0 < a < 2.$$

7. Which of the following is true? *(2008)*

(a) $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$
(b) $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$
(c) $f'(1) f'(-1) = (2-a)^2$
(d) $f'(1) f'(-1) = -(2+a)^2$

8. Which of the following is true? *(2008)*

(a) $f(x)$ is decreasing on $(-1, 1)$ and has a local minimum at $x = 1$
(b) $f(x)$ is increasing on $(-1, 1)$ and has a local minimum at $x = 1$
(c) $f(x)$ is increasing on $(-1, 1)$ but has neither a local maximum nor a local minimum at $x = 1$
(d) $f(x)$ is decreasing on $(-1, 1)$ but has neither a local maximum nor a local minimum at $x = 1$

9. Let $g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2} dt$. Which of the following is true? *(2008)*

(a) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$
(b) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$
(c) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$
(d) $g'(x)$ does not change sign on $(-\infty, \infty)$

PASSAGE - 4

Consider the polynomial *(2010)*
$f(x) = 1 + 2x + 3x^2 + 4x^3$.
Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$.

10. The real numbers lies in the interval

(a) $\left(-\frac{1}{4}, 0\right)$ (b) $\left(-11, -\frac{3}{4}\right)$
(c) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (d) $\left(0, \frac{1}{4}\right)$

11. The area bounded by the curve $y = f(x)$ and the lines $x = 0$, $y = 0$ and $x = t$, lies in the interval

(a) $\left(\frac{3}{4}, 3\right)$ (b) $\left(\frac{21}{64}, \frac{11}{16}\right)$
(c) $(9, 10)$ (d) $\left(0, \frac{21}{64}\right)$

12. The function $f'(x)$ is

(a) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$
(b) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$
(c) increasing in $(-t, t)$
(d) decreasing in $(-t, t)$

PASSAGE - 5

Given that for each $a \in (0, 1)$, $\lim_{h \to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1} dt$ exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on $(0, 1)$. *(JEE Adv. 2014)*

13. The value of $g\left(\frac{1}{2}\right)$ is

(a) π (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

14. The value of $g'\left(\frac{1}{2}\right)$ is

(a) $\frac{\pi}{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) 0

PASSAGE - 6

Let $F : \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. Suppose that $F(1) = 0$, $F(3) = -4$ and $F(x) < 0$ for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all $x \in \mathbb{R}$. *(JEE Adv. 2015)*

15. The correct statement(s) is(are)

(a) $f'(1) < 0$
(b) $f(2) < 0$
(c) $f'(x) \neq 0$ for any $x \in (1, 3)$
(d) $f'(x) = 0$ for some $x \in (1, 3)$

16. If $\int_1^3 x^2F'(x)dx = -12$ and $\int_1^3 x^3F''(x)dx = 40$, then the correct expression(s) is (are)

(a) $9f'(3)+f'(1)-32=0$ (b) $\int_1^3 f(x)dx = 12$

(c) $9f'(3)-f'(1)+32=0$ (d) $\int_1^3 f(x)dx = -12$

I Integer Value Correct Type

1. Let f: **R** → **R** be a continuous function which satisfies

$$f(x)=\int_0^x f(t)dt.$$

Then the value of $f(\ln 5)$ is *(2009)*

2. For any real number x, let $[x]$ denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by

$$f(x)=\begin{cases} x-[x] & \text{if } [x] \text{ is odd,} \\ 1+[x]-x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is *(2010)*

3. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\}dx$ is

(JEE Adv. 2014)

4. Let $f: R \to R$ be a function defined by $f(x)=\begin{cases}[x], & x \le 2\\ 0, & x>2\end{cases}$

where $[x]$ is the greatest integer less than or equal to x, if $I=\int_{-1}^{2}\frac{xf(x^2)}{2+f(x+1)}dx$, then the value of $(4I-1)$ is

(JEE Adv. 2015)

5. Let $F(x)=\int_x^{x^2+\frac{\pi}{6}} 2\cos^2 t(dt)$ for all $x \in R$ and $f:\left[0,\frac{1}{2}\right]\to[0,\infty)$ be a continuous function. For $a\in\left[0,\frac{1}{2}\right]$, if $F'(a)+2$ is the area of the region bounded by $x=0, y=0, y=f(x)$ and $x=a$, then $f(0)$ is *(JEE Adv. 2015)*

6. If $\alpha=\int_0^1 \left(e^{9x+3\tan^{-1}x}\right)\left(\frac{12+9x^2}{1+x^2}\right)dx$ where $\tan^{-1}x$ takes only principal values, then the value of $\left(\log_e|1+\alpha|-\frac{3\pi}{4}\right)$ is *(JEE Adv. 2015)*

7. Let $f: \mathbb{R}\to\mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^{x} f(t)dt$ for all $x\in[-1, 2]$ and $G(x)=\int_{-1}^{x} t|f(f(t))|dt$ for all $x\in[-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is *(JEE Adv. 2015)*

8. The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt = 2x-1$ is *(JEE Adv. 2016)*

9. Let $f: R \to R$ be a differentiable function such that $f(0)=0$, $f\left(\frac{\pi}{2}\right)=3$ and $f'(0)=1$.

If $g(x)=\int_x^{\frac{\pi}{2}}[f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t)]dt$ for $x\in\left(0,\frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x)=$ *(JEE Adv. 2018)*

10. For each positive integer n, let

$$y_n=\frac{1}{n}(n+1)(n+2)\ldots(n+n)^{\frac{1}{n}}$$

For $x\in\mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is ______.

(JEE Adv. 2018)

11. A farmer F_1 has a land in the shape of a triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y=x^n\ (n>1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is ______. *(JEE Adv. 2018)*

12. The value of the integral

$$\int_0^{\frac{1}{2}}\frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}}dx$$

is ______. *(JEE Adv. 2018)*

13. If $I=\frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$, then $27I^2$ equals ______, *(JEE Adv. 2019)*

14. The value of the integral

$\int_0^{\pi/2}\frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$ equals ______ *(JEE Adv. 2019)*

Section-B JEE Main / AIEEE

1. $\int_0^{10\pi} |\sin x|\, dx$ is [2002]
(a) 20 (b) 8 (c) 10 (d) 18

2. $I_n = \int_0^{\pi/4} \tan^n x\, dx$ then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ equals [2002]
(a) $\frac{1}{2}$ (b) 1 (c) ∞ (d) zero

3. $\int_0^2 [x^2]dx$ is [2002]
(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$
(c) $\sqrt{2}-1$ (d) $-\sqrt{2}-\sqrt{3}+5$

4. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is [2002]
(a) $\frac{\pi^2}{4}$ (b) π^2 (c) zero (d) $\frac{\pi}{2}$

5. If $y=f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of 3/4 square unit with the axes then $\int_0^2 xf'(x)dx$ is [2002]
(a) 3/2 (b) 1 (c) 5/4 (d) –3/4

6. The area bounded by the curves $y = \ln x$, $y = \ln|x|$, $y = |\ln x|$ and $y = |\ln|x||$ is [2002]
(a) 4 sq. units (b) 6 sq. units
(c) 10 sq. units (d) none of these

7. The area of the region bounded by the curves $y = |x-1|$ and $y = 3-|x|$ is [2003]
(a) 6 sq. units (b) 2 sq. units
(c) 3 sq. units (d) 4 sq. units.

8. If $f(a+b-x) = f(x)$ then $\int_a^b xf(x)dx$ is equal to [2003]
(a) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$ (b) $\frac{a+b}{2}\int_a^b f(b-x)dx$
(c) $\frac{a+b}{2}\int_a^b f(x)dx$ (d) $\frac{b-a}{2}\int_a^b f(x)dx$.

9. Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x)+g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$, is [2003]
(a) $e+\frac{e^2}{2}+\frac{5}{2}$ (b) $e-\frac{e^2}{2}-\frac{5}{2}$
(c) $e+\frac{e^2}{2}-\frac{3}{2}$ (d) $e-\frac{e^2}{2}-\frac{3}{2}$.

10. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is [2003]
(a) $\frac{1}{n+1}+\frac{1}{n+2}$ (b) $\frac{1}{n+1}$
(c) $\frac{1}{n+2}$ (d) $\frac{1}{n+1}-\frac{1}{n+2}$.

11. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is [2004]
(a) $e+1$ (b) $e-1$ (c) $1-e$ (d) e

12. The value of $\int_{-2}^{3} |1-x^2|dx$ is [2004]
(a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$

13. The value of $I = \int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is [2004]
(a) 3 (b) 1 (c) 2 (d) 0

14. If $\int_0^{\pi} xf(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$, then A is [2004]
(a) 2π (b) π (c) $\frac{\pi}{4}$ (d) 0

15. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is [2004]
(a) 1 (b) –3 (c) –1 (d) 2

16. The area of the region bounded by the curves $y = |x-2|, x = 1, x = 3$ and the x-axis is [2004]
(a) 4 (b) 2 (c) 3 (d) 1

17. If $I_1 = \int_0^1 2^{x^2}dx$, $I_2 = \int_0^1 2^{x^3}dx$, $I_3 = \int_1^2 2^{x^2}dx$ and $I_4 = \int_1^2 2^{x^3}dx$ then [2005]
(a) $I_2 > I_1$ (b) $I_1 > I_2$ (c) $I_3 = I_4$ (d) $I_3 > I_4$

18. The area enclosed between the curve $y = \log_e(x+e)$ and the coordinate axes is [2005]
(a) 1 (b) 2 (c) 3 (d) 4

19. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is [2005]
(a) 1:2:1 (b) 1:2:3 (c) 2:1:2 (d) 1:1:1

20. Let $f(x)$ be a non – negative continuous function such that the area bounded by the curve $y = f(x)$, x - axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is [2005]
$\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is
(a) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ (b) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$
(c) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ (d) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

21. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0, is [2005]
(a) a π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{a}$ (d) 2π

22. The value of integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$ is
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 1

23. $\int_0^{\pi} xf(\sin x)dx$ is equal to [2006]
(a) $\pi\int_0^{\pi} f(\cos x)dx$ (b) $\pi\int_0^{\pi} f(\sin x)dx$
(c) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (d) $\pi\int_0^{\pi/2} f(\cos x)dx$

24. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} [(x+\pi)^3 + \cos^2(x+3\pi)]dx$ is equal to [2006]
(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32} + \frac{\pi}{2}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4} - 1$

25. The value of $\int_1^a [x]f'(x)dx$, $a > 1$ where $[x]$ denotes the greatest integer not exceeding x is [2006]
(a) $af(a) - \{f(1) + f(2) + ...f([a])\}$
(b) $[a]f(a) - \{f(1) + f(2) + ...f([a])\}$
(c) $[a]f([a]) - \{f(1) + f(2) + ...f(a)\}$
(d) $af([a]) - \{f(1) + f(2) + ...f(a)\}$

26. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_l^x \frac{\log t}{1+t} dt$, Then $F(e)$ equals [2007]
(a) 1 (b) 2 (c) 1/2 (d) 0

27. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is [2007]
(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$ (c) 2 (d) None

28. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is [2007]
(a) 1/6 (b) 1/3 (c) 2/3 (d) 1

29. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true?
(a) $I > \frac{2}{3}$ and $J > 2$ (b) $I < \frac{2}{3}$ and $J < 2$
(c) $I < \frac{2}{3}$ and $J > 2$ (d) $I > \frac{2}{3}$ and $J < 2$

30. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to [2008]
(a) $\frac{5}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

31. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent of the parabola at the point (2, 3) and the x-axis is: [2009]
(a) 6 (b) 9 (c) 12 (d) 3

32. $\int_0^{\pi} [\cot x]dx$, where [.] denotes the greatest integer function, is equal to : [2009]
(a) 1 (b) –1 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

33. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is [2010]
(a) $4\sqrt{2}+2$ (b) $4\sqrt{2}-1$ (c) $4\sqrt{2}+1$ (d) $4\sqrt{2}-2$

34. Let $p(x)$ be a function defined on **R** such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)dx$ equals [2010]
(a) 21 (b) 41 (c) 42 (d) $\sqrt{41}$

35. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2} dx$ is [2011]
(a) $\frac{\pi}{8}\log 2$ (b) $\frac{\pi}{2}\log 2$
(c) log 2 (d) π log 2

36. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive x-axis is [2011]
(a) 1 square unit (b) $\frac{3}{2}$ square units
(c) $\frac{5}{2}$ square units (d) $\frac{1}{2}$ square unit

37. The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is : [2012]

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$ (c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

38. If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x+\pi)$ equals [2012]

(a) $\frac{g(x)}{g(\pi)}$ (b) $g(x)+g(\pi)$

(c) $g(x)-g(\pi)$ (d) $g(x).g(\pi)$

39. **Statement-1 :** The value of the integral

$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\pi/6$ [JEE M 2013]

Statement-2 : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

40. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is : [JEE M 2013]

(a) 9 (b) 36 (c) 18 (d) $\frac{27}{4}$

41. The integral $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\, dx$ equals: [JEE M 2014]

(a) $4\sqrt{3}-4$ (b) $4\sqrt{3}-4-\frac{\pi}{3}$

(c) $\pi-4$ (d) $\frac{2\pi}{3}-4-4\sqrt{3}$

42. The area of the region described by $A = \{(x,y): x^2+y^2 \le 1 \text{ and } y^2 \le 1-x\}$ is: [JEE M 2014]

(a) $\frac{\pi}{2}-\frac{2}{3}$ (b) $\frac{\pi}{2}+\frac{2}{3}$ (c) $\frac{\pi}{2}+\frac{4}{3}$ (d) $\frac{\pi}{2}-\frac{4}{3}$

43. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is [JEE M 2015]

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$ (c) $\frac{7}{32}$ (d) $\frac{5}{64}$

44. The integral

$\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)} dx$ is equal to : [JEE M 2015]

(a) 1 (b) 6 (c) 2 (d) 4

45. The area (in sq. units) of the region $\{(x, y) : y^2 \ge 2x$ and $x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is : [JEE M 2016]

(a) $\pi-\frac{4\sqrt{2}}{3}$ (b) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$

(c) $\pi-\frac{4}{3}$ (d) $\pi-\frac{8}{3}$

46. The area (in sq. units) of the region $\{(x, y) : x \ge 0, x + y \le 3, x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\}$ is : [JEE M 2017]

(a) $\frac{5}{2}$ (b) $\frac{59}{12}$ (c) $\frac{3}{2}$ (d) $\frac{7}{3}$

47. The integral $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to : [JEE M 2017]

(a) -1 (b) -2

(c) 2 (d) 4

48. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and $\alpha, \beta\ (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is : [JEE M 2018]

(a) $\frac{1}{2}(\sqrt{3}+1)$ (b) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(c) $\frac{1}{2}(\sqrt{2}-1)$ (d) $\frac{1}{2}(\sqrt{3}-1)$

49. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is : [JEE M 2018]

(a) $\frac{\pi}{2}$ (b) 4π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

50. The value of $\int_0^{\pi} |\cos x|^3\, dx$ is: [JEE M 2019 – 9 Jan (M)]

(a) 0 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

51. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is: [JEE M 2019 – 9 Jan (M)]

(a) $\frac{8}{3}$ (b) $\frac{32}{3}$ (c) $\frac{56}{3}$ (d) $\frac{14}{3}$

52. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is: [JEE M 2019 – 9 April (M)]

(a) $\frac{\pi-2}{8}$ (b) $\frac{\pi-1}{4}$

(c) $\frac{\pi-2}{4}$ (d) $\frac{\pi-1}{2}$

53. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is: [JEE M 2019 – 9 April (M)]

(a) $\frac{10}{3}$ (b) $\frac{9}{2}$ (c) $\frac{31}{6}$ (d) $\frac{13}{6}$