CHAPTER 11

Limits, Continuity and Differentiability

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Let $f(x)=\begin{cases}(x-1)^2\sin\dfrac{1}{(x-1)}-|x| & if\ x\neq 1\\ -1, & if\ x=1\end{cases}$

be a real-valued function. Then the set of points where $f(x)$ is not differentiable is .... *(1981 - 2 Marks)*

2. Let $f(x)=\begin{cases}\dfrac{(x^3+x^2-16x+20)}{(x-2)^2}, & if\ x\neq 2\\ k, & if\ x=2\end{cases}$

If $f(x)$ is continuous for all x, then $k=$ .... *(1981 - 2 Marks)*

3. A discontinuous function $y=f(x)$ satisfying $x^2+y^2=4$ is given by $f(x)=$ .... *(1982 - 2 Marks)*

4. $\lim\limits_{x\to 1}(1-x)\tan\dfrac{\pi x}{2}=$ .... *(1984 - 2 Marks)*

5. If $f(x)=\sin x,\ x\neq n\pi,\ n=0,\pm1,\pm2,\pm3,$ ....
$=2$, otherwise

and $g(x)=x^2+1,\ x\neq 0,\ 2$
$=4, x=0$
$=5, x=2,$

then $\lim\limits_{x\to 0} g[f(x)]$ is .... *(1986 - 2 Marks)*

6. $\lim\limits_{x\to-\infty}\left[\dfrac{x^4\sin\left(\frac{1}{x}\right)+x^2}{(1+|x|^3)}\right]=$ .... *(1987 - 2 Marks)*

7. If $f(9)=9,\ f'(9)=4$, then $\underset{x\to 9}{Lim}\dfrac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ equals........... *(1988 - 2 Marks)*

8. ABC is an isosceles triangle inscribed in a circle of radius r. If $AB=AC$ and h is the altitude from A to BC then the triangle ABC has perimeter $P=2(\sqrt{2hr-h^2})+\sqrt{2hr})$ and area $A=$ .... also $\lim\limits_{h\to 0}\dfrac{A}{P^3}=$ .... *(1989 - 2 Marks)*

9. $\underset{x\to\infty}{Lt}\left(\dfrac{x+6}{x+1}\right)^{x+4}=$ .... *(1990 - 2 Marks)*

10. Let $f(x)=x\,|\,x\,|$. The set of points where $f(x)$ is twice differentiable is .... *(1992 - 2 Marks)*

11. Let $f(x)=[x]\sin\left(\dfrac{\pi}{[x+1]}\right)$, where $[\bullet]$ denotes the greatest integer function. The domain of f is... and the points of discontinuity of f in the domain are..... *(1996 - 2 Marks)*

12. $\lim\limits_{x\to 0}\left(\dfrac{1+5x^2}{1+3x^2}\right)^{1/x^2}=$ ... *(1996 - 1 Mark)*

13. Let $f(x)$ be a continuous function defined for $1\le x\le 3$. If $f(x)$ takes rational values for all x and $f(2)=10$, then $f(1.5)=$............ *(1997 - 2 Marks)*

B True / False

1. If $\underset{x\to a}{Lt}[f(x)g(x)]$ exists then both $\underset{x\to a}{Lt}f(x)$ and $\underset{x\to a}{Lt}g(x)$ exist. *(1981 - 2 Marks)*

C MCQs with One Correct Answer

1. If $f(x)=\sqrt{\dfrac{x-\sin x}{x+\cos^2 x}}$, then $\lim\limits_{x\to\infty}f(x)$ is *(1979)*
(a) 0 (b) ∞
(c) 1 (d) none of these

2. For a real number y, let $[y]$ denotes the greatest integer less than or equal to y: Then the function $f(x)=\dfrac{\tan(\pi[x-\pi])}{1+[x]^2}$ is *(1981 - 2 Marks)*
(a) discontinuous at some x
(b) continuous at all x, but the derivative $f'(x)$ does not exist for some x
(c) $f'(x)$ exists for all x, but the second derivative $f''(x)$ does not exist for some x
(d) $f'(x)$ exists for all x

3. There exist a function $f(x)$, satisfying $f(0)=1, f'(0)=-1$, $f(x)>0$ for all x, and *(1982 - 2 Marks)*
(a) $f''(x)>0$ for all x (b) $-1<f''(x)<0$ for all x
(c) $-2\le f''(x)\le -1$ for all (d) $f''(x)<-2$ for all x

4. If $G(x)=-\sqrt{25-x^2}$ then $\lim\limits_{x\to 1}\dfrac{G(x)-G(1)}{x-1}$ has the value *(1983 - 1 Mark)*
(a) $\dfrac{1}{24}$ (b) $\dfrac{1}{5}$
(c) $-\sqrt{24}$ (d) none of these

5. If $f(a)=2,\ f'(a)=1,\ g(a)=-1,\ g'(a)=2$, then the value of $\lim\limits_{x\to a}\dfrac{g(x)f(a)-g(a)f(x)}{x-a}$ is *(1983 - 1 Mark)*
(a) -5 (b) $\dfrac{1}{5}$
(c) 5 (d) none of these

6. The function $f(x)=\dfrac{\ln(1+ax)-\ln(1-bx)}{x}$ is not defined at $x=0$. The value which should be assigned to f at $x=0$ so that it is continuous at $x=0$, is *(1983 - 1 Mark)*
(a) $a-b$ (b) $a+b$
(c) $\ln a-\ln b$ (d) none of these

7. $\lim\limits_{n\to\infty}\left\{\dfrac{1}{1-n^2}+\dfrac{2}{1-n^2}+\dots+\dfrac{n}{1-n^2}\right\}$ is equal to *(1984 - 2 Marks)*
(a) 0 (b) $-\dfrac{1}{2}$
(c) $\dfrac{1}{2}$ (d) none of these

8. If $f(x)=\dfrac{\sin[x]}{[x]},\quad [x]\neq 0$ *(1985 - 2 Marks)*
$\quad = 0, \quad [x]=0$
Where $[x]$ denotes the greatest integer less than or equal to x. then $\lim\limits_{x\to 0}f(x)$ equals –
(a) 1 (b) 0
(c) -1 (d) none of these

9. Let $f: R\to R$ be a differentiable function and $f(1)=4$. Then the value of $\lim\limits_{x\to 1}\displaystyle\int_4^{f(x)}\frac{2t}{x-1}\,dt$ is *(1990 - 2 Marks)*
(a) $8f'(1)$ (b) $4f'(1)$ (c) $2f'(1)$ (d) $f'(1)$

10. Let [.] denote the greatest integer function and $f(x)=[\tan^2 x]$, then: *(1993 - 1 Mark)*
(a) $\lim\limits_{x\to 0}f(x)$ does not exist
(b) $f(x)$ is continuous at $x=0$
(c) $f(x)$ is not differentiable at $x=0$
(d) $f'(0)=1$

11. The function $f(x)=[x]\cos\left(\dfrac{2x-1}{2}\right)\pi$, [.] denotes the greatest integer function, is discontinuous at *(1995S)*
(a) All x (b) All integer points
(c) No x (d) x which is not an integer

12. $\lim\limits_{n\to\infty}\dfrac{1}{n}\sum\limits_{r=1}^{2n}\dfrac{r}{\sqrt{n^2+r^2}}$ equals *(1997 - 2 Marks)*
(a) $1+\sqrt{5}$ (b) $-1+\sqrt{5}$ (c) $-1+\sqrt{2}$ (d) $1+\sqrt{2}$

13. The function $f(x)=[x]^2-[x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at *(1999 - 2 Marks)*
(a) all integers
(b) all integers except 0 and 1
(c) all integers except 0
(d) all integers except 1

14. The function $f(x)=(x^2-1)\,|x^2-3x+2|+\cos(|x|)$ is NOT differentiable at *(1999 - 2 Marks)*
(a) -1 (b) 0 (c) 1 (d) 2

15. $\lim\limits_{x\to 0}\dfrac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ is *(1999 - 2 Marks)*
(a) 2 (b) -2 (c) $1/2$ (d) $-1/2$

16. For $x\in R$, $\lim\limits_{x\to\infty}\left(\dfrac{x-3}{x+2}\right)^x=$ *(2000S)*
(a) e (b) e^{-1} (c) e^{-5} (d) e^5

17. $\lim\limits_{x\to 0}\dfrac{\sin(\pi\cos^2 x)}{x^2}$ equals *(2001S)*
(a) $-\pi$ (b) π (c) $\pi/2$ (d) 1

18. The left-hand derivative of $f(x)=[x]\sin(\pi x)$ at $x=k$, k an integer, is *(2001S)*
(a) $(-1)^k(k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$
(c) $(-1)^k k\pi$ (d) $(-1)^{k-1}k\pi$

19. Let $f: R\to R$ be a function defined by $f(x)=\max\{x, x^3\}$. The set of all points where $f(x)$ is NOT differentiable is *(2001S)*
(a) $\{-1,1\}$ (b) $\{-1,0\}$ (c) $\{0,1\}$ (d) $\{-1,0,1\}$

20. Which of the following functions is differentiable at $x=0$? *(2001S)*
(a) $\cos(|x|)+|x|$ (b) $\cos(|x|)-|x|$
(c) $\sin(|x|)+|x|$ (d) $\sin(|x|)-|x|$

21. The domain of the derivative of the function
$$f(x)=\begin{cases}\tan^{-1}x & \text{if } |x|\le 1\\ \dfrac{1}{2}(|x|-1) & \text{if } |x|>1\end{cases}$$ is *(2002S)*
(a) $R-\{0\}$ (b) $R-\{1\}$
(c) $R-\{-1\}$ (d) $R-\{-1,1\}$

22. The integer n for which $\lim\limits_{x\to 0}\dfrac{(\cos x-1)(\cos x-e^x)}{x^n}$ is a finite non-zero number is *(2002S)*
(a) 1 (b) 2 (c) 3 (d) 4

23. Let $f: R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals *(2002S)*

(a) 1 (b) $e^{1/2}$ (c) e^2 (d) e^3

24. If $\lim_{x\to 0}\frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to *(2003S)*

(a) 0 (b) $\frac{n+1}{n}$ (c) n (d) $n+\frac{1}{n}$

25. $\lim_{h\to 0}\frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$, given that $f'(2) = 6$ and $f'(1) = 4$

(a) does not exist (b) is equal to – 3/2
(c) is equal to 3/2 (d) is equal to 3 *(2003S)*

26. If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is *(2004S)*

(a) 1 (b) 0 (c) –1 (d) 2

27. The function given by $y = ||x|-1|$ is differentiable for all real numbers except the points *(2005S)*

(a) $\{0, 1, -1\}$ (b) ± 1 (c) 1 (d) –1

28. If $f(x)$ is continuous and differentiable function and $f(1/n) = 0 \;\forall\; n \ge 1$ and $n \in I$, then *(2005S)*

(a) $f(x) = 0, x \in (0, 1]$
(b) $f(0) = 0, f'(0) = 0$
(c) $f(0) = 0 = f'(0),\; x \in (0, 1]$
(d) $f(0) = 0$ and $f'(0)$ need not to be zero

29. The value of $\lim_{x\to 0}\left((\sin x)^{1/x} + (1+x)^{\sin x}\right)$, where $x > 0$ is *(2006 - 3M, –1)*

(a) 0 (b) –1 (c) 1 (d) 2

30. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t\to x}\frac{t^2 f(x) - x^2 f(t)}{t-x} = 1$ for each $x > 0$. Then $f(x)$ is *(2007 - 3 marks)*

(a) $\frac{1}{3x}+\frac{2x^2}{3}$ (b) $\frac{-1}{3x}+\frac{4x^2}{3}$ (c) $\frac{-1}{x}+\frac{2}{x^2}$ (d) $\frac{1}{x}$

31. $\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2-\frac{\pi^2}{16}}$ equals *(2007 - 3 marks)*

(a) $\frac{8}{\pi}f(2)$ (b) $\frac{2}{\pi}f(2)$ (c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (d) $4f(2)$

32. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \ne 0,\; n > 0$, and let p be the left hand derivative of $|x-1|$ at $x = 1$. If $\lim_{x\to 1^+} g(x) = p$, then *(2008)*

(a) $n = 1, m = 1$ (b) $n = 1, m = -1$
(c) $n = 2, m = 2$ (d) $n > 2, m = n$

33. If $\lim_{x\to 0}\left[1 + x\,\ell n(1+b^2)\right]^{1/x} = 2b\sin^2\theta, b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is *(2011)*

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

34. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$, then *(2012)*

(a) $a = 1, b = 4$ (b) $a = 1, b = -4$
(c) $a = 2, b = -3$ (d) $a = 2, b = 3$

35. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \ne 0 \\ 0, & x = 0 \end{cases}, x \in R$ then f is *(2012)*

(a) differentiable both at $x = 0$ and at $x = 2$
(b) differentiable at $x = 0$ but not differentiable at $x = 2$
(c) not differentiable at $x = 0$ but differentiable at $x = 2$
(d) differentiable neither at $x = 0$ nor at $x = 2$

36. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2 + \left(\sqrt{1+a}-1\right)x + \left(\sqrt[6]{1+a}-1\right) = 0$ where $a > -1$. Then $\lim_{a\to 0^+}\alpha(a)$ and $\lim_{a\to 0^+}\beta(a)$ are *(2012)*

(a) $-\frac{5}{2}$ and 1 (b) $-\frac{1}{2}$ and – 1
(c) $-\frac{7}{2}$ and 2 (d) $-\frac{9}{2}$ and 3

D MCQs with One or More than One Correct

1. If $x + |y| = 2y$, then y as a function of x is *(1984 - 3 Marks)*

(a) defined for all real x
(b) continuous at $x = 0$
(c) differentiable for all x
(d) such that $\frac{dy}{dx} = \frac{1}{3}$ for $x < 0$

2. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then– *(1985 - 2 Marks)*

(a) $f(x)$ is continuous but not differentiable at $x = 0$
(b) $f(x)$ is differentiable at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$
(d) none of these

3. The function $f(x) = 1 + |\sin x|$ is *(1986 - 2 Marks)*

(a) continuous nowhere
(b) continuous everywhere
(c) differentiable nowhere
(d) not differentiable at $x = 0$
(e) not differentiable at infinite number of points.

4. Let $[x]$ denote the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then $f(x)$ is *(1986 - 2 Marks)*
(a) continuous at $x = 0$ (b) continuous in $(-1, 0)$
(c) differentiable at $x = 1$ (d) differentiable in $(-1, 1)$
(e) none of these

5. The set of all points where the function $f(x) = \dfrac{x}{(1+|x|)}$ is differentiable, is *(1987 - 2 Marks)*
(a) $(-\infty, \infty)$ (b) $[0, \infty)$
(c) $(-\infty, 0) \cup (0, \infty)$ (d) $(0, \infty)$
(e) None

6. The function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4}, & x < 1 \end{cases}$ is *(1988 - 2 Marks)*
(a) continous at $x = 1$ (b) differentiable at $x = 1$
(c) continous at $x = 3$ (d) differentiable at $x = 3$.

7. If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$ *(1989 - 2 Marks)*
(a) $\tan[f(x)]$ and $1/f(x)$ are both continuous
(b) $\tan[f(x)]$ and $1/f(x)$ are both discontinuous
(c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous
(d) $\tan[f(x)]$ is continuous but $1/f(x)$ is not.

8. The value of $\lim\limits_{x \to 0} \dfrac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$ *(1991 - 2 Marks)*
(a) 1 (b) -1
(c) 0 (d) none of these

9. The following functions are continuous on $(0, \pi)$. *(1991 - 2 Marks)*
(a) $\tan x$
(b) $\displaystyle\int_0^x t \sin \frac{1}{t}\, dt$
(c) $\begin{cases} 1, & 0 < x \leq \dfrac{3\pi}{4} \\ 2\sin\dfrac{2}{9}x, & \dfrac{3\pi}{4} < x < \pi \end{cases}$
(d) $\begin{cases} x \sin x, & 0 < x \leq \dfrac{\pi}{2} \\ \dfrac{\pi}{2}\sin(\pi + x), & \dfrac{\pi}{2} < x < \pi \end{cases}$

10. Let $f(x) = \begin{cases} 0, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ then for all x *(1994)*
(a) f' is differentiable (b) f is differentiable
(c) f' is continuous (d) f is continuous

11. Let $g(x) = x f(x)$, where $f(x) = \begin{cases} x \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. At $x = 0$ *(1994)*
(a) g is differentiable but g' is not continuous
(b) g is differentiable while f is not
(c) both f and g are differentiable
(d) g is differentiable and g' is continuous

12. The function $f(x) = \max\{(1-x), (1+x), 2\}$, $x \in (-\infty, \infty)$ is *(1995)*
(a) continuous at all points
(b) differentiable at all points
(c) differentiable at all points except at $x = 1$ and $x = -1$
(d) continuous at all points except at $x = 1$ and $x = -1$, where it is discontinuous

13. Let $h(x) = \min\{x, x^2\}$, for every real number of x, Then *(1998 - 2 Marks)*
(a) h is continuous for all x
(b) h is differentiable for all x
(c) $h'(x) = 1$, for all $x > 1$
(d) h is not differentiable at two values of x.

14. $\lim\limits_{x \to 1} \dfrac{\sqrt{1 - \cos 2(x-1)}}{x-1}$ *(1998 - 2 Marks)*
(a) exists and it equals $\sqrt{2}$
(b) exists and it equals $-\sqrt{2}$
(c) does not exist because $x - 1 \to 0$
(d) does not exist because the left hand limit is not equal to the right hand limit.

15. If $f(x) = \min\{1, x^2, x^3\}$, then *(2006 - 5M, -1)*
(a) $f(x)$ is continuous $\forall x \in R$
(b) $f(x)$ is continuous and differentiable everywhere.
(c) $f(x)$ is not differentiable at two points
(d) $f(x)$ is not differentiable at one point

16. Let $L = \lim\limits_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4}$, $a > 0$.
If L is finite, then *(2009)*
(a) $a = 2$ (b) $a = 1$ (c) $L = \dfrac{1}{64}$ (d) $L = \dfrac{1}{32}$

17. Let $f: R \to R$ be a function such that $f(x+y) = f(x) + f(y)$, $\forall\, x, y \in R$. If $f(x)$ is differentiable at $x = 0$, then *(2011)*
(a) $f(x)$ is differentiable only in a finite interval containing zero
(b) $f(x)$ is continuous $\forall\, x \in R$
(c) $f'(x)$ is constant $\forall\, x \in R$
(d) $f(x)$ is differentiable except at finitely many points.

18. If $f(x) = \begin{cases} -x - \dfrac{\pi}{2}, & x \leq -\dfrac{\pi}{2} \\ -\cos x, & -\dfrac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ell n x, & x > 1 \end{cases}$, then *(2011)*

(a) $f(x)$ is continuous at $x=-\frac{\pi}{2}$
(b) $f(x)$ is not differentiable at $x=0$
(c) $f(x)$ is differentiable at $x=1$
(d) $f(x)$ is differentiable at $x=-\frac{3}{2}$

19. For every integer n, let a_n and b_n be real numbers. Let function $f: IR \to IR$ be given by *(2012)*

$$f(x)=\begin{cases} a_n+\sin \pi x, & for \quad x\in[2n,2n+1] \\ b_n+\cos \pi x, & for \quad x\in(2n-1,2n) \end{cases}$$

for all integers n. If f is continuous, then which of the following hold(s) for all n?
(a) $a_{n-1}-b_{n-1}=0$ (b) $a_n-b_n=1$
(c) $a_n-b_{n+1}=1$ (d) $a_{n-1}-b_n=-1$

20. For $a\in\mathbb{R}$ (the set of all real numbers), $a\neq -1$, *(JEE Adv. 2013)*

$$\lim_{n\to\infty}\frac{(1^a+2^a+...+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+...+(na+n)]}=\frac{1}{60}.$$

Then $a=$
(a) 5 (b) 7 (c) $\frac{-15}{2}$ (d) $\frac{-17}{2}$

21. Let $f: [a, b] \to [1,\infty)$ be a continuous function and let $g: R\to R$ be defined as *(JEE Adv. 2014)*

$$g(x)=\begin{cases} 0, & \text{if } x<a, \\ \int_a^x f(t)dt, & \text{if } a\le x\le b; \text{ then} \\ \int_a^b f(t)dt, & \text{if } x>b. \end{cases}$$

(a) $g(x)$ is continuous but not differentiable at a
(b) $g(x)$ is differentiable on R
(c) $g(x)$ is continuous but not differentiable at b
(d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

22. For every pair of continuous functions $f, g: [0, 1]\to R$ such that $\max\{f(x): x\in[0,1]\} = \max\{g(x): x\in[0,1]\}$, the correct statement(s) is (are): *(JEE Adv. 2014)*
(a) $(f(c))^2+3f(c)=(g(c))^2+3g(c)$ for some $c\in[0, 1]$
(b) $(f(c))^2+f(c)=(g(c))^2+3g(c)$ for some $c\in[0, 1]$
(c) $(f(c))^2+3f(c)=(g(c))^2+g(c)$ for some $c\in[0, 1]$
(d) $(f(c))^2=(g(c))^2$ for some $c\in[0, 1]$

23. Let $g: R\to R$ be a differentiable function with $g(0)=0$, $g'(0)=0$ and $g'(1)\neq 0$. Let $f(x)=\begin{cases}\frac{x}{|x|}g(x), & x\neq 0\\ 0, & x=0\end{cases}$ and $h(x)=e^{|x|}$ for all $x\in R$. Let $(foh)(x)$ denote $f(h(x))$ and $(hof)(x)$ denote $h(f(x))$. Then which of the following is (are) true? *(JEE Adv. 2015)*
(a) f is differentiable at $x=0$
(b) h is differentiable at $x=0$
(c) foh is differentiable at $x=0$
(d) hof is differentiable at $x=0$

24. Let a, b $\in\mathbb{R}$ and $f:\mathbb{R}\to\mathbb{R}$ be defined by
$$f(x)=a\cos(|x^3-x|)+b|x|\sin(|x^3+x|).$$
Then f is *(JEE Adv. 2016)*
(a) differentiable at x=0 if a=0 and b=1
(b) differentiable at x=1 if a=1 and b=0
(c) NOT differentiable at x=0 if a=1 b=0
(d) NOT differentiable at x=1 if a=1 and b=1

25. Let $f:\left[-\frac{1}{2},2\right]\to\mathbb{R}$ and $g:\left[-\frac{1}{2},2\right]\to\mathbb{R}$ be functions defined by $f(x)=[x^2-3]$ and $g(x)=|x|f(x)+|4x-7|f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y\in R$. Then *(JEE Adv. 2016)*
(a) f is discontinuous exactly at three points in $\left[-\frac{1}{2},2\right]$
(b) f is discontinuous exactly at four points in $\left[-\frac{1}{2},2\right]$
(c) g is NOT differentiable exactly at four points in $\left(-\frac{1}{2},2\right)$
(d) g is NOT differentiable exactly at five points in $\left(-\frac{1}{2},2\right)$

26. Let [x] be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x)=x\cos(\pi(x+[x]))$ is discontinuous? *(JEE Adv. 2017)*
(a) x=–1 (b) x=0
(c) x=1 (d) x=2

27. Let $f(x)=\frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x\neq 1$. Then *(JEE Adv. 2017)*
(a) $\lim_{x\to 1^-} f(x)=0$
(b) $\lim_{x\to 1^-} f(x)$ does not exist
(c) $\lim_{x\to 1^+} f(x)=0$
(d) $\lim_{x\to 1^+} f(x)$ does not exist

28. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be two non-constant differentiable functions. If
$$f'(x)=(e^{(f(x)-g(x))})g'(x) \text{ for all } x\in\mathbb{R},$$
and $f(1)=g(2)=1$, then which of the following statement (s) is (are) TRUE? *(JEE Adv. 2018)*
(a) $f(2)<1-\log_e 2$ (b) $f(2)>1-\log_e 2$
(c) $g(1)>1-\log_e 2$ (d) $g(1)<1-\log_e 2$

29. Let $f: R \to R$ given by

$$f(x)\begin{cases} x^5+5x^4+10x^3+10x^2+3x+1, & x<0; \\ x^2-x+1, & 0\le x<1; \\ \frac{2}{3}x^3-4x^2+7x-\frac{8}{3}, & 1\le x<3; \\ (x-2)\log_e(x-2)-x+\frac{10}{3}, & x\ge 3 \end{cases}$$

Then which of the following options is/are correct ? *(JEE Adv. 2019)*

(a) f' has a local maximum at $x=1$
(b) f is increasing on $(-\infty, 0)$
(c) f' is NOT differentiable at $x=1$
(d) f is onto

30. Let $f: R \to R$ be a function. We say that f has

PROPERTY 1 if $\lim_{h\to 0}\frac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h\to 0}\frac{f(h)-f(0)}{h^2}$ exists and is finite

Then which of the following options is/are correct? *(JEE Adv. 2019)*

(a) $f(x)=x^{2/3}$ has **PROPERTY 1**
(b) $f(x)=\sin x$ has **PROPERTY 2**
(c) $f(x)=|x|$ has **PROPERTY 1**
(d) $f(x)=x|x|$ has **PROPERTY 2**

E Subjective Problems

1. Evaluate $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}, (a\ne 0)$ *(1978)*

2. $f(x)$ is the integral of $\frac{2\sin x-\sin 2x}{x^3}, x\ne 0$, find $\lim_{x\to 0} f'(x)$ *(1979)*

3. Evaluate : $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ *(1980)*

4. Let $f(x+y)=f(x)+f(y)$ for all x and y. If the function $f(x)$ is continuous at $x=0$, then show that $f(x)$ is continuous at all x. *(1981 - 2 Marks)*

5. Use the formula $\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$ to find $\lim_{x\to 0}\frac{2^x-1}{(1+x)^{1/2}-1}$ *(1982 - 2 Marks)*

6. Let $f(x)=\begin{cases}1+x, 0\le x\le 2\\ 3-x, 2\le x\le 3\end{cases}$ *(1983 - 2 Marks)*

Determine the form of $g(x)=f[f(x)]$ and hence find the points of discontinuity of g, if any

7. Let $f(x)=\begin{cases}\frac{x^2}{2}, & 0\le x<1\\ 2x^2-3x+\frac{3}{2}, & 1\le x\le 2\end{cases}$ *(1983 - 2 Marks)*

Discuss the continuity of f, f' and f'' on $[0, 2]$.

8. Let $f(x)=x^3-x^2+x+1$ and

$$g(x)=\max\{f(t); 0\le t\le x\}, \quad 0\le x\le 1$$
$$=3-x \qquad 0\le x\le 2$$ *(1985 - 5 Marks)*

Discuss the continuity and differentiability of the function $g(x)$ in the interval $(0, 2)$.

9. Let $f(x)$ be defined in the interval $[-2, 2]$ such that

$$f(x)=\begin{cases}-1, -2\le x\le 0\\ x-1, 0<x\le 2\end{cases}$$

and $g(x)=f(|x|)+|f(x)|$

Test the differentiability of $g(x)$ in $(-2, 2)$. *(1986 - 5 Marks)*

10. Let $f(x)$ be a continuous and $g(x)$ be a discontinuous function. prove that $f(x)+g(x)$ is a discontinuous function. *(1987 - 2 Marks)*

11. Let $f(x)$ be a function satisfying the condition $f(-x)=f(x)$ for all real x. If $f'(0)$ exists, find its value. *(1987 - 2 Marks)*

12. Find the values of a and b so that the function

$$f(x)=\begin{cases}x+a\sqrt{2}\sin x, & 0\le x<\pi/4\\ 2x\cot x+b & \pi/4\le x\le \pi/2\\ a\cos 2x-b\sin x & \pi/2<x\le\pi\end{cases}$$

is continuous for $0\le x\le\pi$. *(1989 - 2 Marks)*

13. Draw a graph of the function $y=[x]+|1-x|, -1\le x\le 3$. Determine the points, if any, where this function is not differentiable. *(1989 - 4 Marks)*

14. Let $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2}, & x<0\\ a, & x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x>0\end{cases}$ *(1990 - 4 Marks)*

Determine the value of a, if possible, so that the function is continuous at $x=0$

15. A function $f: R\to R$ satisfies the equation $f(x+y)=f(x)f(y)$ for all x, y in R and $f(x)\ne 0$ for any x in R. Let the function be differentiable at $x=0$ and $f'(0)=2$. Show that $f'(x)=2f(x)$ for all x in R. Hence, determine $f(x)$. *(1990 - 4 Marks)*

16. Find $\lim_{x\to 0}\{\tan(\pi/4+x)\}^{1/x}$ *(1993 - 2 Marks)*

17. Let $f(x)=\begin{cases}\{1+|\sin x|\}^{a/|\sin x|} & ; \quad -\dfrac{\pi}{6}<x<0\\ b & ; \quad x=0\\ e^{\tan 2x/\tan 3x} & ; \quad 0<x<\dfrac{\pi}{6}\end{cases}$

(1994 - 4 Marks)

Determine a and b such that $f(x)$ is continuous at $x=0$

18. Let $f\left(\dfrac{x+y}{2}\right)=\dfrac{f(x)+f(y)}{2}$ for all real x and y. If $f'(0)$ exists and equals -1 and $f(0)=1$, find $f(2)$. *(1995 - 5 Marks)*

19. Determine the values of x for which the following function fails to be continuous or differentiable: *(1997 - 5 Marks)*

$f(x)=\begin{cases}1-x, & x<1\\ (1-x)(2-x), & 1\le x\le 2\\ 3-x, & x>2\end{cases}$ Justify your answer.

20. Let $f(x), x\ge 0$, be a non–negative continuous function, and let $F(x)=\int_0^x f(t)\,dt, x\ge 0$. If for some $c>0, f(x)\le cF(x)$ for all $x\ge 0$, then show that $f(x)=0$ for all $x\ge 0$. *(2001 - 5 Marks)*

21. Let $\alpha\in R$. Prove that a function $f: R\to R$ is differentiable at α if and only if there is a function $g: R\to R$ which is continuous at α and satisfies $f(x)-f(\alpha)=g(x)(x-\alpha)$ for all $x\in R$. *(2001 - 5 Marks)*

22. Let $f(x)=\begin{cases}x+a & \text{if } x<0\\ |x-1| & \text{if } x\ge 0,\end{cases}$ and *(2002 - 5 Marks)*

$g(x)=\begin{cases}x+1 & \text{if } x<0\\ (x-1)^2+b & \text{if } x\ge 0,\end{cases}$ where a and b are non-negative real numbers. Determine the composite function $g\,o\,f$. If $(g\,o\,f)(x)$ is continuous for all real x, determine the values of a and b. Further, for these values of a and b, is $g\,o\,f$ differentiable at $x=0$? Justify your answer.

23. If a function $f:[-2a, 2a]\to R$ is an odd function such that $f(x)=f(2a-x)$ for $x\in[a, 2a]$ and the left hand derivative at $x=a$ is 0 then find the left hand derivative at $x=-a$. *(2003 - 2 Marks)*

24. $f'(0)=\lim_{n\to\infty} nf\left(\dfrac{1}{n}\right)$ and $f(0)=0$. Using this find

$\lim_{n\to\infty}\left((n+1)\dfrac{2}{\pi}\cos^{-1}\left(\dfrac{1}{n}\right)-n\right), \left|\cos^{-1}\dfrac{1}{n}\right|<\dfrac{\pi}{2}$

(2004 - 2 Marks)

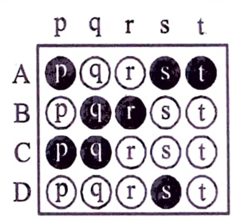
25. If $|c|\le\dfrac{1}{2}$ and $f(x)$ is a differentiable function at $x=0$ given by $f(x)=\begin{cases}b\sin^{-1}\left(\dfrac{c+x}{2}\right) & , \quad -\dfrac{1}{2}<x<0\\ \dfrac{1}{2} & , \quad x=0\\ \dfrac{e^{ax/2}-1}{x} & , \quad 0<x<\dfrac{1}{2}\end{cases}$.

Find the value of 'a' and prove that $64\,b^2=4-c^2$ *(2004 - 4 Marks)*

26. If $f(x-y)=f(x)\cdot g(y)-f(y)\cdot g(x)$ and $g(x-y)=g(x)\cdot g(y)-f(x)\cdot f(y)$ for all $x, y\in R$. If right hand derivative at $x=0$ exists for $f(x)$. Find derivative of $g(x)$ at $x=0$ *(2005 - 4 Marks)*

F Match the Following

DIRECTIONS (Q. 1 and 2) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. In this questions there are entries in columns I and II. Each entry in column I is related to exactly one entry in column II. Write the correct letter from column II against the entry number in column I in your answer book. *(1992 - 2 Marks)*

Column I	*Column II*
(A) $\sin(\pi[x])$	(p) differentiable everywhere
(B) $\sin(\pi(x-[x]))$	(q) nowhere differentiable
	(r) not differentiable at 1 and -1

2. In the following $[x]$ denotes the greatest integer less than or equal to x.

Match the functions in **Column I** with the properties in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. *(2007 - 6 marks)*

Column I	*Column II*
(A) $x\|x\|$	(p) continuous in $(-1, 1)$
(B) $\sqrt{\|x\|}$	(q) differentiable in $(-1, 1)$
(C) $x+[x]$	(r) strictly increasing in $(-1, 1)$
(D) $\|x-1\|+\|x+1\|$	(s) not differentiable at least at one point in $(-1, 1)$

DIRECTIONS (Q. 3) : Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

3. Let $f_1 : R \to R$, $f_2 : [0,\infty) \to R$, $f_3 : R \to R$ and $f_4 : R \to [0,\infty)$ be defined by $f_1(x) = \begin{cases} |x| & \text{if } x < 0, \\ e^x & \text{if } x \ge 0; \end{cases}$

$f_2(x) = x^2$; $f_3(x) = \begin{cases} \sin x & \text{if } x < 0, \\ x & \text{if } x \ge 0; \end{cases}$ and $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0, \\ f_2(f_1(x)) - 1 & \text{if } x \ge 0. \end{cases}$ *(JEE Adv. 2014)*

List-I	**List-II**
P. f_4 is	**1.** Onto but not one-one
Q. f_3 is	**2.** Neither continuous nor one-one
R. $f_2 o f_1$ is	**3.** Differentiable but not one-one
S. f_2 is	**4.** Continuous and one-one

	P	Q	R	S
(a)	3	1	4	2
(b)	1	3	4	2
(c)	3	1	2	4
(d)	1	3	2	4

4. Let $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$, $f_3 : \left(-1, e^{\frac{\pi}{2}} - 2\right) \to \mathbb{R}$ and $f_4 : \mathbb{R} \to \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$,

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x} & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where, for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \ne 0 \\ 0 & \text{if } x = 0 \end{cases}$.

LIST - I	**LIST - II**
P. The function f_1 is	**1.** **NOT** continuous at $x = 0$
Q. The function f_2 is	**2.** continuous at $x = 0$ and **NOT** differentiable at $x = 0$
R. The function f_3 is	**3.** differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
S. The function f_4 is	**4.** differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is: *(JEE Adv. 2018)*

(a) $P \to 2$; $Q \to 3$; $R \to 1$; $S \to 4$

(b) $P \to 4$; $Q \to 1$; $R \to 2$; $S \to 3$

(c) $P \to 4$; $Q \to 2$; $R \to 1$; $S \to 3$

(d) $P \to 2$; $Q \to 1$; $R \to 4$; $S \to 3$

I Integer Value Correct Type

1. Let $f:[1,\infty)\to[2,\infty)$ be a differentiable function such that $f(1)=2$. If $6\int_1^x f(t)dt=3xf(x)-x^3$ for all $x\ge1$, then the value of $f(2)$ is (2011)

2. The largest value of non-negative integer a for which
$$\lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$$ is (JEE Adv. 2014)

3. Let $f:R\to R$ and $g:R\to R$ be respectively given by $f(x)=|x|+1$ and $g(x)=x^2+1$. Define $h:R\to R$ by
$$h(x)=\begin{cases}\max\{f(x),g(x)\} & \text{if } x\le0,\\ \min\{f(x),g(x)\} & \text{if } x>0.\end{cases}$$
The number of points at which $h(x)$ is not differentiable is (JEE Adv. 2014)

4. Let m and n be two positive integers greater than 1. If $\lim_{\alpha\to0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is (JEE Adv. 2015)

5. Let $\alpha,\beta\in\mathbb{R}$ be such that $\lim_{x\to0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$. Then $6(\alpha+\beta)$ equals. (JEE Adv. 2016)

Section-B JEE Main / AIEEE

1. $\lim_{x\to0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is [2002]
(a) 1 (b) –1
(c) zero (d) does not exist

2. $\underset{x\to\infty}{\text{Lim}}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^x$ [2002]
(a) e^4 (b) e^2 (c) e^3 (d) 1

3. Let $f(x)=4$ and $f'(x)=4$. Then $\lim_{x\to2}\frac{xf(2)-2f(x)}{x-2}$ is given by [2002]
(a) 2 (b) –2 (c) –4 (d) 3

4. $\lim_{n\to\infty}\frac{1^p+2^p+3^p+.....+n^p}{n^{p+1}}$ is [2002]
(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$ (c) $\frac{1}{p}-\frac{1}{p-1}$ (d) $\frac{1}{p+2}$

5. $\lim_{x\to0}\frac{\log x^n-[x]}{[x]}$, $n\in N$, ($[x]$ denotes greatest integer less than or equal to x) [2002]
(a) has value –1 (b) has value 0
(c) has value 1 (d) does not exist

6. If $f(1)=1, f^1(1)=2$, then $\lim_{x\to1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is [2002]
(a) 2 (b) 4 (c) 1 (d) 1/2

7. f is defined in [-5, 5] as [2002]
$f(x)=x$ if x is rational
$=-x$ if x is irrational. Then
(a) $f(x)$ is continuous at every x, except $x=0$
(b) $f(x)$ is discontinuous at every x, except $x=0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere

8. $f(x)$ and $g(x)$ are two differentiable functions on [0, 2] such that $f''(x)-g''(x)=0$, $f'(1)=2g'(1)=4f(2)=3g(2)=9$ then $f(x)-g(x)$ at $x=3/2$ is [2002]
(a) 0 (b) 2 (c) 10 (d) 5

9. If $f(x+y)=f(x).f(y)\forall x.y$ and $f(5)=2, f'(0)=3$, then $f'(5)$ is [2002]
(a) 0 (b) 1 (c) 6 (d) 2

10. $\lim_{n\to\infty}\frac{1+2^4+3^4+...n^4}{n^5}-\lim_{n\to\infty}\frac{1+2^3+3^3+...n^3}{n^5}$ [2003]
(a) $\frac{1}{5}$ (b) $\frac{1}{30}$ (c) Zero (d) $\frac{1}{4}$

11. If $\lim_{x\to0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is [2003]
(a) $-\frac{2}{3}$ (b) 0 (c) $-\frac{1}{3}$ (d) $\frac{2}{3}$

12. The value of $\lim_{x\to0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is [2003]
(a) 0 (b) 3 (c) 2 (d) 1

13. Let $f(a)=g(a)=k$ and their nth derivatives $f^n(a), g^n(a)$ exist and are not equal for some n. Further if

$$\lim_{x \to a} \frac{f(a)g(x)-f(a)-g(a)f(x)+f(a)}{g(x)-f(x)}=4$$

then the value of k is [2003]

(a) 0 (b) 4 (c) 2 (d) 1

14. $\lim_{x \to \frac{\pi}{2}} \frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is [2003]

(a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

15. If $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & , x \neq 0 \\ 0 & , x=0 \end{cases}$ then f(x) is [2003]

(a) discontinuous every where
(b) continuous as well as differentialble for all x
(c) continuous for all x but not differentiable at $x=0$
(d) neither differentiable nor continuous at $x=0$

16. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b, are [2004]

(a) $a=1$ and $b=2$ (b) $a=1, b \in \mathbf{R}$
(c) $a \in \mathbf{R}, b=2$ (d) $a \in \mathbf{R}, b \in \mathbf{R}$

17. Let $f(x)=\frac{1-\tan x}{4x-\pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is [2004]

(a) -1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

18. $\operatorname{Lim}_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}\text{..........}+\frac{1}{n}\sec^2 1\right]$ equals [2005]

(a) $\frac{1}{2}\sec 1$ (b) $\frac{1}{2}$ cosec 1
(c) $\tan 1$ (d) $\frac{1}{2}\tan 1$

19. Let α and β be the distinct roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to [2005]

(a) $\frac{a^2}{2}(\alpha-\beta)^2$ (b) 0
(c) $\frac{-a^2}{2}(\alpha-\beta)^2$ (d) $\frac{1}{2}(\alpha-\beta)^2$

20. Suppose $f(x)$ is differentiable at $x=1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h)=5$, then $f'(1)$ equals [2005]

(a) 3 (b) 4 (c) 5 (d) 6

21. Let f be differentiable for all x. If $f(1)=-2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then [2005]

(a) $f(6) \geq 8$ (b) $f(6)<8$ (c) $f(6)<5$ (d) $f(6)=5$

22. If f is a real valued differentiable function satisfying $|f(x)-f(y)| \leq (x-y)^2, x, y \in R$ and $f(0)=0$, then $f(1)$ equals [2005]

(a) -1 (b) 0 (c) 2 (d) 1

23. Let $f: R \to R$ be a function defined by $f(x)=\min\{x+1, |x|+1\}$, Then which of the following is true? [2007]

(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x=0$
(c) $f(x) \geq 1$ for all $x \in R$
(d) $f(x)$ is not differentiable at $x=1$

24. The function $f: R/\{0\} \to R$ given by [2007]

$$f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$$

can be made continuous at $x=0$ by defining $f(0)$ as

(a) 0 (b) 1 (c) 2 (d) -1

25. Let $f(x)=\begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x=1 \end{cases}$ [2008]

Then which one of the following is true?

(a) f is neither differentiable at $x=0$ nor at $x=1$
(b) f is differentiable at $x=0$ and at $x=1$
(c) f is differentiable at $x=0$ but not at $x=1$
(d) f is differentiable at $x=1$ but not at $x=0$

26. Let f : **R** $\to$ **R** be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$ [2010]

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 1

27. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ [2011]

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$
(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

28. The values of p and q for which the function [2011]

$$f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$$ is continuous for all x in R, are

(a) $p=\frac{5}{2}, q=\frac{1}{2}$ (b) $p=-\frac{3}{2}, q=\frac{1}{2}$

(c) $p=\frac{1}{2}, q=\frac{3}{2}$ (d) $p=\frac{1}{2}, q=-\frac{3}{2}$

29. Let $f: R \to [0,\infty)$ be such that $\lim_{x\to5} f(x)$ exists and $\lim_{x\to5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$. Then $\lim_{x\to5} f(x)$ equals :

(a) 0 (b) 1 (c) 2 (d) 3

30. If $f: R \to R$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is . [2012]

(a) continuous for every real x.
(b) discontinuous only at $x=0$
(c) discontinuous only at non-zero integral values of x.
(d) continuous only at $x=0$.

31. Consider the function, $f(x)=|x-2|+|x-5|, x\in R$.
Statement-1 : $f'(4)=0$
Statement-2 : f is continuous in [2,5], differentiable in (2,5) and $f(2)=f(5)$. [2012]
(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

32. $\lim_{x\to0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to [JEE M 2013]

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

33. $\lim_{x\to0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$ is equal to: [JEE M 2014]

(a) $-\pi$ (b) π (c) $\frac{\pi}{2}$ (d) 1

34. $\lim_{x\to0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to : [JEE M 2015]

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) 3

35. If the function.

$$g(x)=\begin{cases}k\sqrt{x+1}, & 0\le x\le 3\\ mx+2, & 3<x\le 5\end{cases}$$ is differentiable, then the value of $k+m$ is : [JEE M 2015]

(a) $\frac{10}{3}$ (b) 4 (c) 2 (d) $\frac{16}{5}$

36. For x I R, $f(x)=|\log 2-\sin x|$ and $g(x)=f(f(x))$, then : [JEE M 2016]

(a) $g'(0)=-\cos(\log 2)$
(b) g is differentiable at $x=0$ and $g'(0)=-\sin(\log 2)$
(c) g is not differentiable at $x=0$
(d) $g'(0)=\cos(\log 2)$

37. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)\dots3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to: [JEE M 2016]

(a) $\frac{9}{e^2}$ (b) $3\log 3-2$

(c) $\frac{18}{e^4}$ (d) $\frac{27}{e^2}$

38. Let $p=\lim_{x\to0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then $\log p$ is equal to : [JEE M 2016]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 2 (d) 1

39. $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$ equals : [JEE M 2017]

(a) $\frac{1}{4}$ (b) $\frac{1}{24}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

40. For each $t\in R$, let $[t]$ be the greatest integer less than or equal to t. Then [JEE M 2018]

$$\lim_{x\to0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\dots+\left[\frac{15}{x}\right]\right)$$

(a) is equal to 15. (b) is equal to 120.
(c) does not exist (in R). (d) is equal to 0.

41. Let S = $\{t \in R : f(x) = |x-\pi|(e^{|x|}-1)\sin|x|$ is not differentiable at t$\}$. Then the set S is equal to : [JEE M 2018]
(a) $\{0\}$ (b) $\{\pi\}$
(c) $\{0, \pi\}$ (d) ϕ (an empty set)

42. $\lim\limits_{y \to 0} \dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ [JEE M 2019 – 9 Jan (M)]
(a) exists and equals $\dfrac{1}{4\sqrt{2}}$
(b) exists and equals $\dfrac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$
(c) exists and equals $\dfrac{1}{2\sqrt{2}}$
(d) does not exist

43. Let f : R → R be a function defined as [JEE M 2019 – 9 Jan (M)]

$$f(x) = \begin{cases} 5, & \text{if} \quad x \le 1 \\ a + bx, & \text{if} \quad 1 < x < 3 \\ b + 5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$$

Then, f is :
(a) continuous if a = 5 and b = 5
(b) continuous if a = – 5 and b = 10
(c) continous if a = 0 and b = 5
(d) not continuous for any values of a and b

44. If the function f defined on $\left(\dfrac{\pi}{6}, \dfrac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \dfrac{\pi}{4} \\ k, & x = \dfrac{\pi}{4} \end{cases}$$

is continuous, then k is equal to: [JEE M 2019 – 9 April (M)]
(a) 2 (b) $\dfrac{1}{2}$ (c) 1 (d) $\dfrac{1}{\sqrt{2}}$

45. Let f(x) = 15 – |x – 10|; x R. Then the set of all values of x, at which the function, g(x) = f (f(x)) is not differentiable, is: [JEE M 2019 – 9 April (M)]
(a) {5, 10, 15} (b) {10, 15}
(c) {5, 10, 15, 20} (d) {10}