MATRIX ANALYSIS

Through JEE

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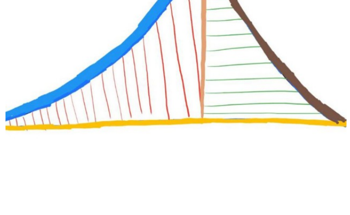

Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Straight Line and Pair of Straight Lines

Section-A [JEE Advanced/IIT-JEE]

A : Fill in the Blanks

1. The area enclosed with in the curves $|x| + |y|$ is......... (1981)

2. $y = 10^x$ is the reflection of $y = \log_1 0^x$ in the line whose equation is.........(1982)

3. The set of lines $ax + by + c = 0$ where $3a + 2b + 4c = 0$ is concurrent at the point..........(1982)

4. Given the points $\mathbf{A}(0,4)$ and $\mathbf{B}(0,-4)$, the equation of the locus of the point P(x,y) such that$|\mathbf{AP} - \mathbf{BP}| = 6$ is............(1983)

5. If $a, b and c$ are in A.P.. then the straight line $ax+by+c=0$ will always pass through a fixed point whose coordinates are........(1984)

6. The orthocenter of the triangle formed by the lines $x+y=1, 2x+3y=6 and 4x-y+4=0$ lies in quadrant number............(1985)

7. Let the algebraic sum of the perpendicular distances from the points (2,0), (0,2) and (1,1)to a variable straight line be zero; then the line passes through a fixed point whose cordinates are.........(1991)

8. The vertices of a triangle are $\mathbf{A}(-1,-7)$, $\mathbf{B}(5,1)$and $\mathbf{C}(1,4)$. The equation of the bisector of the angle $\angle ABC$ is........(1993)

B : True/False

1. The sraight line $5x+4y=0$ passes through the point of intersection of the straight lines $x+2y-10=0$ and $2x+5+6=0$.(1983)

2. The lines $2x+3y=19=0$ $9x+6y-17=0$ cut the coordinate axes in concyclic points.(1988)

C : MCQ'S with One Correct Answer

1. The points$(a, b), (0,0)(a, b)$ and (a^2, ab) are: (1979)

    (a) Collinear

    (b) Vertices of a parallelogram

    (c) Vertices of a rectangle

    (d) None of the above

2. The point $(4,1)$ undergose the following three transformations successively.

    (a) Reflection about the line y=x

    (b) Translation through a distance 2 unit along the positivedirection of x-axis.

    (c) Rotation through an angle $p/4$ about the origin the counter clockwise direction.

    then the final position of the point is given by the coordinates. (1980)

    (a) $\left[\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right]$

    (b) $\left(-\sqrt{2}, \sqrt[7]{2}\right)$

    (c) $\left[\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right]$

    (d) $\left(\sqrt{2}, \sqrt[7]{2}\right)$

3. The straight lines $x + y = 0$,$3x + y - 4 = 0$,$x + 3y - 4 = 0$ form a triangle which is (1932)

   (a) isosceles

   (b) equilateral

   (c) rightangled

   (d) none of these

4. If $\mathbf{P} = (1, 0), \mathbf{Q} = (-1, 0) and \mathbf{R} = (2, 0)$ are three given points,then the locus of the point S satisfying the relation $SQ^2 + SR^2 = SP^2$, is (1988)

   (a) a straight line parllel to x-axis

   (b) a circie passing hrough the origin

   (c) a circle with the center at the origin

   (d) a straight line parllel to y-axis

5. Line L has intercepts a and b on the coordinate axes. When the axes are rotate through a given angle, keeping up the origin fixed, the same line L has intercepts p and q then (1990)

   (a) $a^2 + b^2 = p^2 + q^2$

   (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

   (c) $a^2 + p^2 = b^2 + q^2$

   (d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

6. If the sum of the distance of point from two perpendicular lines in a plane is 1,then its locus is (1992)

   (a) square
   (b) circle
   (c) straight line
   (d) two intersecting lines

7. The locus of the variable point whose distance from from $(-2, 0)$ is 2/3 times it's distance from the line $x = \frac{-9}{2}$ is (1994)

   (a) ellipse
   (b) parabola
   (c) hyperbola
   (d) none of the above

8. The equation to a pair of opposite sides of parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$ the equations to it's diagonals are (1994)

   (a) $x + 4y = 13, y = 4x - 7$
   (b) $4x + y = 13, 4y = x - 7$
   (c) $4x + y = 13, y = 4x - 7$
   (d) $y - 4x = 13, y + 4x = 7$

9. The orthocenter of the lines formed by $xy = 0$ and $x+y = 1$ is (1995'S)

   (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(b) $\left(\frac{1}{3}, \frac{1}{3}\right)$

(c) (0,0)

(d) $\left(\frac{1}{4}, \frac{1}{4}\right)$

10. Let PQR be a rightangled isoscales triangle, right angled at $\mathbf{P}(2,1)$ if the equation of the line QR is $2x+y=3$, then the equation representing the pair of line PQ and PR is (1999)

(a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$

(b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$

(d) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

11. If x_1, x_2, x_3 as well as $y_1, y_2, y_3$are in GP with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) (1999 - 2 marks)

(a) lie on a straight line

(b) lie on a ellipse

(c) lie on a circle

(d) are vertices of triangle

12. Let PS median of the tringle with vertices $\mathbf{P}(2,2), \mathbf{Q}(6,1) and \mathbf{R}(7,3)$. The equation of the line passing through (1,-1) and parllel to PS is. (2000'S)

(a) $2x - 9y - 7 = 0$

(b) $2x - 9y - 11 = 0$

(c) $2x + 9y - 11 = 0$

(d) $2x + 9y + 7 = 0$

13. The incenter of the triangle with vertices $(1, \sqrt{3})$,$(0, 0)$ and $(2, 0)$ is (2000'S)

    (a) $\left[1, \frac{\sqrt{3}}{2}\right]$

    (b) $\left[\frac{2}{3}, \frac{\sqrt{3}}{2}\right]$

    (c) $\left[\frac{2}{3}, \frac{\sqrt{3}}{2}\right]$

    (d) $\left[1, \frac{1}{\sqrt{3}}\right]$

14. The number of integer values of m, for which the x-coordinate of the of intersection of line $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is (2001'S)

    (a) 2

    (b) 0

    (c) 4

    (d) 1

15. Area of parllelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals (2001'S)

    (a) $\frac{|m+n|}{(m-n)^2}$

    (b) $\frac{2}{|m+n|}$

(c) $\frac{1}{(|m+n|)}$

(d) $\frac{1}{(|m-n|)}$

16. Let $0 < a < \frac{\pi}{2}$ be fixed angle. If $\mathbf{P} = (\cos\theta, \sin\theta)$, $\mathbf{Q} = (\cos\alpha - \theta), (\sin\alpha - \theta)$, then Q is obtained from P by (2002S)

(a) clockwise wise rotation around origin through an angle α

(b) anticlockwise wise rotation around origin through an angle α

(c) reflection in the line through origin with slope $\tan\alpha$

(d) reflection in the line through origin with slope $\tan\alpha/2$

17. Let $\mathbf{P} = (-1, 0)$,$\mathbf{Q} = (0, 0)$ and $\mathbf{R} = (3, \sqrt[3]{3})$ be three points.

Then the equation of the bisector of the angle PQR is (2002'S)

(a) $\frac{\sqrt{3}}{2x} + y = 0$

(b) $x + \sqrt{3}y = 0$

(c) $\sqrt{3}x + y = 0$

(d) $x + \frac{\sqrt{3}}{2y} = 0$

18. A straight line through the origin O meets the parallel lines $4x+2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then the point O divides the segment PQ in the ratio (2002)

(a) $1 : 2$

(b) $3 : 4$

(c) $2 : 1$

(d) $4 : 3$

19. The number of integral points(integral points means both the coordinates should be integer) exactly in the interior of the triangle with vertices is $(0,0)(0,21)$ and $(21,0)$ is (2003)

    (a) 133

    (b) 190

    (c) 233

    (d) 105

20. Orthocenter of triangle with vertices $(0,0)(3,4)$ and $(4,0)$ is (2003)

    (a) $\left[3, \frac{5}{4}\right]$

    (b) $[3, 12]$

    (c) $\left[3, \frac{3}{4}\right]$

    (d) $[3, 9]$

21. Aear of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ (2004)

    (a) 2 sq. units

    (b) 4 sq. units

    (c) 6 sq. units

    (d) 8 sq. units

22. Let $\mathbf{O}(0,0)$,$\mathbf{P}(3,4)$,$\mathbf{Q}(6,0)$ be the vertices of the tiangles OPQ. The point R inside the triangle OPQ is such that the triangles OPR,PQR,OQR are of equal area. The coordinates of R are (2007)

(a) $\left[\frac{4}{3}, 3\right]$

(b) $\left[3, \frac{2}{3}\right]$

(c) $\left[3, \frac{4}{3}\right]$

(d) $\left[\frac{4}{3}, \frac{2}{3}\right]$

23. A straight line through the point $(3, 2)$ inclined at an angle $60°$ to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is (2011)

(a) $y + \sqrt{3}x + 2 + \sqrt[3]{3} = 0$

(b) $y - \sqrt{3}x + 2 + \sqrt[3]{3} = 0$

(c) $\sqrt{3}y - x + 3 + \sqrt[2]{3} = 0$

(d) $\sqrt{3}y + x - 3 + \sqrt[2]{3} = 0$

D : MCQ'S with One or More Than One Correct Answer

1. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent if (1985)

(a) $p + q + r = 0$

(b) $p^2 + q^2 + r^2 = qr + rp + pq$

(c) $p^3 + q^3 + r^3 = 3pqr$

(d) none of these

2. The points $\left[0, \frac{8}{3}\right], [1,3]$ and $[82,30]$ are vertices of (1986)

   (a) an obtuse angle triangle

   (b) an acute angle triangle

   (c) a right angled triangle

   (d) an isoscales triangle

   (e) none of these

3. All points lying inside the triangle are formed by the points$(1,3)$,$(5,0)$ and $(-1,2)$ satisfy (1986)

   (a) $3x + 2y \geq= 0$

   (b) $2x + 3y - 13 \geq= 0$

   (c) $2x - 3y - 12 \leq= 0$

   (d) $-2x + y \geq= 0$

   (e) none of these

4. A vector $\bar{a}$ has components of 2p and 1 with respect to a rectangular cartesian system. This system is rotted through a certain angle about origin in the counter clockwise sense. If,with respect the new system,$\bar{a}$ has components $p+1$ and 1, then (1986)

(a) $p = 0$

(b) $p = 1$ or $p = -1/3$

(c) $p = -1$ or $p = 1/3$

(d) $p = 1$ or $p = -1$

(e) none of these.

5. If $\mathbf{P}(1,2), \mathbf{Q}(4,6), vecR(5,7)$ and $\mathbf{S}(a,b)$ are the vertices of a parallelogram PQRS, /then (1998)

(a) $a = 2, b = 4$

(b) $a = 3, b = 4$

(c) $a = 2, b = 3$

(d) $a = 3, b = 5$

(e) none of these

6. The diagonals of a parallelogram PQRS are along the lines $x + 3y = 4$ and $6x - 2y = 7$ then PQRS must be a. (1998)

(a) rectangle

(b) square

(c) cyclic quadrilateral

(d) rhombus

7. If the vertices P,Q,R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) always rational point(s)? (1998)

   (a) centroid

   (b) incenter

   (c) circumcenter

   (d) orthocenter (A rational point is a point both of whose coordinates are rational numbers.)

8. Let $\mathbf{L_1}$ be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on $\mathbf{L_1}$ and $\mathbf{L_2}$ are equal, then which of the equation can represents $\mathbf{L_1}$? (1999)

   (a) $x + y = 0$

   (b) $x - y = 0$

   (c) $x + 7y = 0$

   (d) $x - 7y = 0$

9. For $a > b > c > 0$,the distance between (1,1)and the point of intersection of the lines $ax + by + c = 0$ and $ay + c = 0$ is less than $\sqrt[2]{2}$. Then (JEE Adv. 2013)

(a) $a + b - c > 0$

(b) $a - b + c < 0$

(c) $a + b - c > 0$

(d) $a + b - c < 0$

E : Subjective Problems

1. A straight line segment of length l moves with it's ends on two mutually perpendicular lines. Find the locus of the point which divides the line segment in the ratio 1 : 2. (1978)

2. The area of triangle is 5. Two of it's vertices are $\mathbf{A}(2, 1)$ and $\mathbf{B}(3-2)$. The third vertex C lies on $y = x + 3$. Finf C.(1978)

3. One side of the rectangle lies along the line $4x + 7y + 5 = 0$. Two of it's vertices are (-3,1) and (1,1). Find the equation of the other two sides.(1978)

4. (a) Two vertices of a triangle are (5,-1) and (-2,3) If the orthocenter of the triangle is the origin,find the coordinates of the third point. (1978) (b) Find the equation of the line which bisects the obtuse angle between the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$
(1979)

5. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find

the equation of the line.

(1980)

6. The end A,B of a straight line segment of constant length c slide upon he fixed rectangular axes (X, Y) respectively. If a rectanle OAPB are completed, then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$.(1983)

7. The vertices of the triangle are $[at_1t_2, a(t_1+t_2)]$,$[at_1t_3, a(t_1+t_3)]$ and $[at_3t_4, a(t_3+t_4)]$. Find the orthocenter of the triangle.

   (1983 - 2 marks)

8. The coordinates of A,B,C are (6,3),(3,5),(4,2) respectively, and P is any point (x,y). Show that the ratio of the area of the triangle $\triangle PBC$ and $\triangle ABC$ is $\mid \frac{(x+y-2)}{7} \mid$ (1983)

9. Two equal sides of a isoscales triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and it's third side passes through the point $(1, 10)$. Determaine the equation of the third side.(1984)

10. One of the diametes of the circle circumscribing the rectangle ABCD is $4y = x + 7$. If A and B are the ponts $(-3, 4)$ and $(5, 4)$ respetively then find the area of the rectangle.(1985)

11. Two sides of a rhombus ABCD are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersects at the point $(1, 2)$ and the vertex A on the y axis, find the possible coordinates of A.(1985)

12. Lines $L_1 = ax + by + c = 0$ and $L_2 = lx + my + n = 0$ intersects at the point P and make an angle θ with each other. Find the equation of a line L different from $\mathbf{L_2}$ which passes through P and makes the same angle θ with $\mathbf{L_1}$.(1989)

13. Let ABC be a triangle with $\mathbf{AB} - \mathbf{AC}$. If D is the pont of BC, E is the foot of the perpendicular drawn from D to AC and F the mid point of DE, prove that AF is perpendicular to BE.(1989)

14. Sraight lines $3x + 4y - 5$ and $4x + 3y - 5$ intersects at the point A. Points B and C are choosen on these two lines such that $\mathbf{AB} = \mathbf{AC}$. Determine the possible equaton of the line BC passing through the point (1,2).(1990)

15. A line cuts the x-axis at $\mathbf{A}(7, 0)$ and the y-axis at $\mathbf{B}(0, 5)$. A variable line PQ drawn perpendicular to AB cutting the x-axis in P and y-axis in Q. If $\mathbf{AQ}$ and $\mathbf{BP}$ intersets at R, find the locus of R.(1990)

16. Find the equation of the line passing through the point (2,3)and intersects of length 2 units between the lines $y + 2x = 3$ and $y + 2x = 5$.(1991)

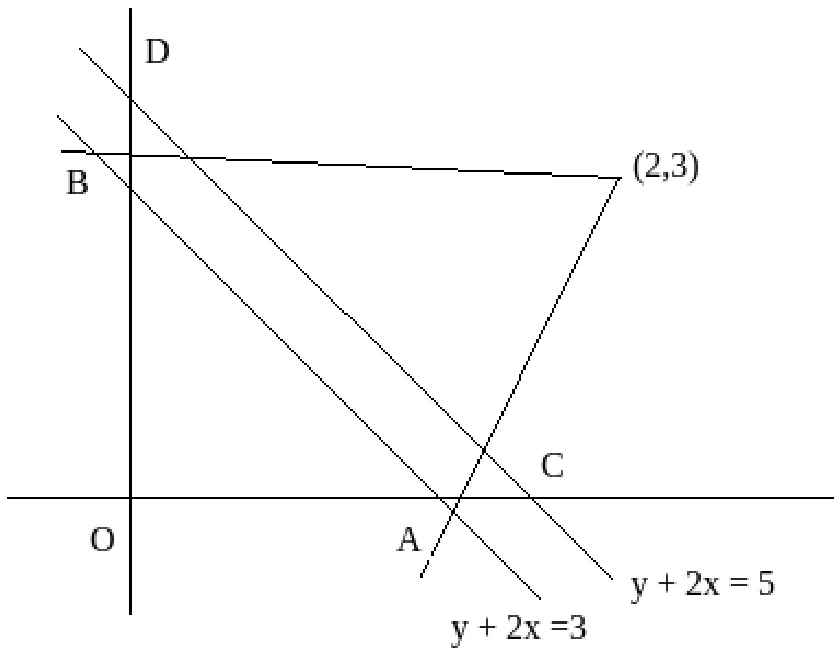

Figure 1.1:

17. Show that all chords of the curve $2x^2 - y^2 - 2x + 4y = 0$. Which subtend a right angle at the origin. Passes through a fixed point. Find the coordinates of the point.(1991)

18. Determine all values of a for which the point (a, a^2) lies inside the triangle formed by the lines

$$2x + 3y - 1 = 0 \tag{1.1}$$

$$x + 2y - 1 = 0 \;\; (1992) \tag{1.2}$$

$$5x - 6y - 1 = 0 \tag{1.3}$$

19. Tangent at a point $\mathbf{P_1}$ [other than (0,0)] on the curve $y - x^3$ meets the curve again at $\mathbf{P_2}$. The tangent at $\mathbf{P_1}$ meets the curve at $\mathbf{P_2}$ and so on. Show that the abscissae of $\mathbf{p_1} + \mathbf{p_2} + \mathbf{p_3} + ... + \mathbf{p_n}$ form a G.P. Also find the ratio.(1993)

20. A line through $\mathbf{A}(5, 4)$ meets the line $x + 3y + 2 = 0$ $2x + y + 4 = 0$ and $x - y - 5 = 0$ at points B,C and D respectively. If $\frac{15}{AB}^2 + \frac{10}{AC}^2 - \frac{6}{AD}^2$, find the equation of the line.(1993)

21. A triangle PQRS has it's side PQ parallel to the line $y - mx$ and vertices P,Q and S on the lines $y - a$,$x - b$ and $x - -b$, respectively find the locus of the vertex R. (1996)

22. Using co-ordinate geometry prove that the three altitudes of any triangle are concurrent (1998)

23. For points $P = (x_1, Y_1)$ and $Q = (x_2, y_2)$ of the coordinate palne, a new distance $d(P, Q)$ is defined by $d(P, Q) =\mid x_1 - x_2 \mid + \mid y_1 - y_2 \mid$. Let $\mathbf{O} = (0, 0)$ and $\mathbf{A} = (3, 2)$. Prove that the set of points in the first quadrant which are equidistance (with to line new distance) from O and A consists of the union of line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. (2000)

24. Let ABC and PQR be any two triangles in the same plane. Assume that the perpendicular from the points A,B,C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from P,Q,R to BC, CA, AB respectively are also concurrent. (2000)

25. Let a,b,c are real numbers with $a^2+b^2+c^2 = 1$. Show that the equation
$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & ax + by - c & cy + b \\ cx + a & cy + b & ax - by + c \end{vmatrix}$$
represents a straight line. (2001)

26. A straight line L through the origin meets the lines $x + y + 1$ and

$x + y = 3$ at P and Q respectively. Through P and Q two straight lines $\mathbf{L_1} and \mathbf{L_2}$ intersects at R . Show that the locus of R, as L varies, is a staight line. (2002)

27. A straight line negative slope passes through the points $(8, 2)$ cuts the positive coordnate axes at points P and Q. Find the absolute minimum value of $\mathbf{OP} + \mathbf{OQ}$, as L varies . Where O is the origin. (2002)

28. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through $\mathbf{p}(h, k)$ with the lines $y - x$ and $x + y - 2$ is $4h^2$. Find the locus of the point. (2002)

H : Assertion and Reason Type Questions

1. Lines $L_1 : Y - X = 0$ and $L_2 : 2x + y = 0$ intersects the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between $L_1 and L_2$ intersects L_3 at R.
   STATEMENT-1 : The ratio $PR : RQ$equals $\sqrt[2]{2} : \sqrt{5}$. because
   STATEMENT-2 :In any triangle, bisector of an angle divides the triangle into two triangles. (2007)

   (a) Statement-1 is True, Statement-2 is True;Satement-2 is not a correct explaination for Statement-1

(b) Statement-1 is True, Statement-2 is True;Satement-2 is NOT a correct explaination for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

I : Integer Value Correct Type

1. For a point P in the plane, let $\mathbf{d_1}(p)$ and $\mathbf{d_2}(p)$ be the distance of a point P from the lines $x - y = 0$ and $x = y = 0$ respectively. The area of the region R consistes of all points P lying in the first quadrant of the plane and satisfying $2 \leq \mathbf{d_1}(p) + \mathbf{d_2}(p) \leq$, is (JEE Adv. 2014)

Section-B [JEE Main/AIEE]

1. A triangle with vertices $(4, 0), (-1, -1), (3, 5)$ is (2002)

   (a) isoscales and right angled

   (b) isoscales but not right angled

   (c) right angled but not isoscales

   (d) neither right angled nor isoscales

2. Locus of mid point of the portion between the axes of $x \cos\alpha + y \sin\alpha = p$. Where p is constant is. (2002)

(a) $x^2 + y^2 = \frac{4}{p^2}$

(b) $x^2 + y^2 = 4p^2$

(c) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$

(d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

3. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersects on the y-axis then (2002)

   (a) $2fgh = bg^2 + ch^2$

   (b) $bg^2 \neq ch^2$

   (c) $abc = 2fgh$

   (d) none of these

4. The pair of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ are perpendicular to each other for (2002)

   (a) two values of a

   (b) $\forall a$

   (c) for one value of a

   (d) for no values of a

5. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha \left[0 < a < \frac{\Pi}{4}]\right]$ with the positive direction of x-axis. The equation of it's diagonal passing through the origin is (2003)

   (a) $y\left(\cos\alpha + \sin\alpha\right) + x\left(\cos\alpha - \sin\alpha\right)$=a

(b) $y\,(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha)$=a

(c) $y\,(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha)$=a

(d) $y\,(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha)$=a

6. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then (2003)

   (a) $pq = -1$

   (b) $p = q$

   (c) $p = -q$

   (d) $pq = 1$

7. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(a\sin t, -b\cos t)$ and (1,0) where t is a parameter, is (2003)

   (a) $(3x+1)^2 + (3y)^2 = a^2 - b^2$

   (b) $(3x-1)^2 + (3y)^2 = a^2 - b^2$

   (c) $(3x-1)^2 + (3y)^2 = a^2 + b^2$

   (d) $(3x+1)^2 + (3y)^2 = a^2 + b^2$

8. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P with the same common ratio then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) (2003)

   (a) are vertices of a triangle

   (b) lies on a straight line

   (c) lies on ellipse

(d) lies on circle

9. If the equation of the locus of a equidistance from the point (a_1, b_1) and (a_2, b_2) is $(a_1 - b_2)x + (a_1 - b_2)y + c = 0$, then the value of 'c' is (2003)

   (a) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

   (b) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

   (c) $a + 1^2 - a_2^2 + b_1^2 - b_2^2$

   (d) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$

10. Let $\mathbf{A}(2, -3)$ and $\mathbf{B}(-2, 3)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is in the line (2004)

    (a) $3x - 2y = 0$

    (b) $2x - 3y = 7$

    (c) $3x + 2y = 5$

    (d) $2x + = 3y = 9$

11. The equation of the straight line passing through the point $(4, 3)$ and making intercepts on the coordinate axes whose sum is -1 is (2004) t

    (a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

    (b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

    (c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$

    (d) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

12. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times the product c has the value (2004)

    (a) -2

    (b) -1

    (c) 2

    (d) 1

13. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals (2004)

    (a) -3

    (b) -1

    (c) 3

    (d) 1

14. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ (2005)

    (a) below the x-axis at a distance of $\frac{3}{2}$ from it

    (b) below the x-axis at a distance of $\frac{2}{3}$ from it

    (c) above the x-axis at a distance of $\frac{3}{2}$ from it

    (d) above the x-axis at a distance of $\frac{2}{3}$ from it

15. If a vertex of a triangle is (1,1) and the mid point of two sides of this vertex are (-1,2) and (3,2) then the centroid of the triangle is (2005)

(a) $\left[-1, \frac{7}{3}\right]$

(b) $\left[\frac{-1}{3}, \frac{7}{3}\right]$

(c) $\left[1, \frac{7}{3}\right]$

(d) $\left[\frac{1}{3}, \frac{7}{3}\right]$

16. A straigjt line through point $A(3,4)$ is such that it's intercept between the axes is bisected at A. It's equation is (2006)

    (a) $x + y = 7$

    (b) $3x - 4y + 7 = 0$

    (c) $4x + 3y = 24$

    (d) $3x + 4y = 25$

17. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}, x > 0$ and $y = 3x, x > 0$, then a belong to (2006)

    (a) $\left[0, \frac{1}{2}\right]$

    (b) $(3, \infty)$

    (c) $\left[\frac{1}{2}, 3\right]$

    (d) $\left[-3, \frac{1}{2}\right]$

18. Let $\mathbf{A}(h,k)$ and $\mathbf{B}(1,1)$ and $\mathbf{C}(2,1)$ be the vertices of a right angle triangle with AC as it's hypotenuse. If the area of the triangle is 1 square unit, then the set of values which 'k' can taken is given by (2007)

    (a) $(-1, 3)$

(b) $(-3,-2)$

(c) $(1,3)$

(d) $(0,2)$

19. Let $\mathbf{P} = (-1,0), \mathbf{Q} = (0,0) and \mathbf{R} = (3, \sqrt[3]{3})$ be three points. The equation of the bisector of the angle PQR is (2007)

(a) $\frac{\sqrt{3}}{2}x + y = 0$

(b) $x + \sqrt{3}y = 0$

(c) $\sqrt{3}x + y = 0$

(d) $x + \frac{\sqrt{3}}{2}y = 0.$

20. If one of the lines of $my^2 + (1-m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is (2007)

(a) 1

(b) 2

(c) $\frac{-1}{2}$

(d) -2

21. The perpendicular bisector of the line segment joining $P(1,4)$ and $Q(k,3)$ has y-intercept -4. Then a possible value of k is (2008)

(a) 1

(b) 2

(c) -2

(d) -4

22. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is (2009)

(a) $\frac{\sqrt{2}3}{8}$

(b) $\frac{\sqrt{3}2}{5}$

(c) $\frac{\sqrt{3}}{4}$

(d) $\frac{\sqrt{3}2}{8}$.

23. The lines $p(p^2+1)x - y + q = 0$ and $(p^2+1)^2 x + (p^2+1)y + 2q = 0$ are perpendicular to a common line for (2009)

(a) exactly one value of p

(b) exactly two values of p

(c) more than two values of p

(d) no value of p

24. Three distinct points A,B and C are given i the 2-dimentional coordinates plane such that the ratio of the distance of any one of them from the point (1,0) to the distance from the point (-1,0) is equal to $\frac{1}{3}$. Then the circumcenter of the triangle ABC is at the point; (2009)

(a) $\left[\frac{5}{4}, 0\right]$

(b) $\left[\frac{5}{2}, 0\right]$

(c) $\left[\frac{5}{3}, 0\right]$

(d) (0,0)

25. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13,32). The line K is parallel L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is. (2010)

(a) $\sqrt{17}$

(b) $\frac{17}{\sqrt{15}}$

(c) $\frac{23}{\sqrt{17}}$

(d) $\frac{23}{\sqrt{15}}$

26. The line $L_1 : y-x=0$ and $L_2 : 2x+=y=0$ intersects the line $L_3 : y+2=0$ at P and Q respectively. The bisector of the acute angle between $L_1 and L_2$ intersects L_3 at R STATEMENT-1: The ratio PR:RQ equals $\sqrt[2]{2}:\sqrt{5}$

STATEMENT-2: In any triangle,bisector of an angle divides the triangle into two similar triangles.(2011)

(a) Statement-1 is True, Statement-2 is True,Statement-2 is not a correct explaination for the Statement-1.

(b) Statement-1 is True, Statement-2 is False

(c) Statement-1 is False, Statement-2 is True

(d) Statement-1 is True, Statement-2 is True, tatement-2 is correct explaination for the Statement-1.

27. If the line $2x+y=k$ passes through the point which divides the line segment joining the points $(1,1)$ and (2,4) in the ratio $3:2$,then k equals: (2012)

(a) $\frac{29}{5}$

(b) 5

(c) 6

(d) $\frac{11}{5}$

28. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ get reflected upon reaching x-axis, the equation of the reflected ray is (JEE M 2013)

    (a) $y = x + \sqrt{3}$

    (b) $\sqrt{3}y = x - \sqrt{3}$

    (c) $y = \sqrt{3}x - \sqrt{3}$

    (d) $\sqrt{3}y = x - 1$

29. The coordinate of the incenter of the triangle that has the coordinates of mid points of it's sides as (0,1) (1,1) and (1,0) is; (JEE M 2013)

    (a) $2 + \sqrt{2}$

    (b) $2 - \sqrt{2}$

    (c) $1 + \sqrt{2}$

    (d) $1 - \sqrt{2}$

30. Let PS e the median of the triangle with vertices $\mathbf{P}(2,2)$,$\mathbf{Q}(6,-1)$ and $\mathbf{R}(7,3)$. The equation of the line passing through (1,-1)and parallel to PS is: (JEE M 2014)

    (a) $4x + 7y + 3 = 0$

    (b) $2x - 9y + 11 = 0$

(c) $4x - 7y + 11 = 0$

(d) $2x + 7y + 9 = 0$

31. Let a,b,c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and eqidistance from the two axes then (JEE M 2014)

(a) $3bc_2ad = 0$

(b) $3bc + 2ad = 0$

(c) $2b - 3ad = 0$

(d) $2bc + 3ad = 0$

32. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle ith vertices (0,0)(0,41) and(41,0)is. (JEE M 2015)

(a) 820

(b) 780

(c) 901

(d) 861

33. Two sides of a rhombus are alone the lines, $x-y+1 = 0$ and $7x+y-5 = 0$. If it's diagoals intersect at(-1,-2), then which one of the following is a vertex of this rhombus? (JEE M 2016)

(a) $\left(\frac{1}{3}, \frac{8}{3}\right)$

(b) $\left(\frac{10}{3}, \frac{7}{3}\right)$

(c) $(-3, -9)$

(d) $(-3, -8)$

34. A straight the thrugh a fixed point (2,3)intersects the coordinate axes at distinct point P and Q. If O is the origin and the rectangle OQPR is completed, then the locus of R is: (JEE M 2018)

(a) $2x + 3y = xy$

(b) $3x + 2y = xy$

(c) $3x + 2y = 6xy$

(d) $3x + 2y = 6$

35. consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true? [JEE M 2019-9 Jan (M)]

(a) The lines are concurent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

(b) Each the line passes through the origin.

(c) The lines are parallel.

(d) The lines are not concurrent.

36. Slope of line passing through $P(2, 3)$ and intersecting the line $x + y = 7$ at a distance of 4 units from P, is : [JEE M 2019-9 April (M)]

(a) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

(b) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

(c) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

(d) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

Chapter 2

Three Dimensional Geometry

Section-A [JEE Advanced/IIT-JEE]

A : Fill in the Blanks

1. Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ be vectors of length 3,4,5 respectively. Let $\mathbf{A}$ perpendicular to $\mathbf{B} + \mathbf{C}, \mathbf{B}$ to $\mathbf{C} + \mathbf{A}$ and $\mathbf{C}$ to $\mathbf{A} + \mathbf{B}$. Then the length of vector $\mathbf{A} + \mathbf{B} + \mathbf{C}$ is...... (1981)

2. The unit vector perpendicual to plane determined by $P(1, -1, 2), Q(2, 0, -1) and R(0, 2, 1)$is..... (1983)

3. The area of whose vertices are $A(1, -1, 2), B(2, 1, -1) C(3, -1, 2)$ is.....(1983)

4. A,B,C and D are four points in a plane with position vectors a,b,c and d respectively such that $(\mathbf{a} - \mathbf{d})(\mathbf{b} - \mathbf{c}) = (\mathbf{b} - \mathbf{d})(\mathbf{c} - \mathbf{a})$ (1984)
   The point D then,is the.......... of the triangle ABC.

5. If$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$ =0 and the vectors $\mathbf{A} = (1, a, a^2)$, $\mathbf{B} = (1, b, , b^2)$, $\mathbf{C} = (1, c, c^2)$, are non-coplanar, then the product abc =..........(1984)

6. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three non-coplanar vectors, then-$\frac{\mathbf{A \cdot B \times C}}{\mathbf{C \times A \cdot B}} + \frac{\mathbf{B \cdot A \times C}}{\mathbf{C \cdot A \times B}}$ =..........(1985)

7. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$, $(a \neq b \neq c \neq 1)$ are coplanar, hen the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ =..........(1987)

8. Let $b = 4\hat{i} + 3\hat{j}$ and $\mathbf{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors n the same plane having projections 1 and 2 along $\mathbf{b} and \mathbf{c}$,respectively are given by ....(1987)

9. The components of a vector $\mathbf{a}$ along and perpendicular t a non zero vector $\mathbf{b}$ are.....and.....respectively. (1988)

10. Given that $\mathbf{a} = (1, 1, 1)\mathbf{c} = (0, 1, -1), \mathbf{ab} = 3$ and $\mathbf{a} \times \mathbf{b} = \mathbf{c}$, ten $\mathbf{b}$=........(1991)

11. A unit vector coplanar with $\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $\mathbf{i}+2\mathbf{j} = \mathbf{k}$ and perpendicular to $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is.......(1992)

12. A unit vector perpendicular to the plane determined by the points $P(1, -1, 2), Q(2, 0, -1) and R(0, 2, 1)$ is.....(1994)

13. A nonzero vector $\mathbf{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$ and the plane determined by the

vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The angle between $\mathbf{a}$ and the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ is........(1996)

14. If $\mathbf{b}$ and $\mathbf{c}$ are any two nn collinear unit vectoers and $\mathbf{a}$ is any vector then $(\mathbf{a} \cdot \mathbf{b})\, \mathbf{b} + (\mathbf{a} \cdot \mathbf{c})\, \mathbf{c} + \frac{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})}{|\mathbf{b} \times \mathbf{c}|}\ (\mathbf{b} \times \mathbf{c})$=..........(1996)

15. Let $OA = a, OB = 10a + 2b$ and $OC = b$ where O,A and C are non collinear points. Let P denote the are of the qudrailateral OABC, and let q denote the area of the parallelogram with OA and OC as adjacent sides. IfP=kq, then K=...........(1997)

B : True/False

1. $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ be unit vectors suppose that $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C} = 0$, and that the angle between $\mathbf{B} and \mathbf{C}$ is $\frac{\pi}{6}$. Then $\mathbf{A} = + - 2\,(\mathbf{B} \times \mathbf{C})$. (1981)

2. If $X \cdot A = 0, X \cdot B = 0. X \cdot C = 0$ for some non-zero vectors X, then [A B C]=0(1983)

3. The points with position vectors $a + b, a - b and a + kb$ are collinear for all real values of k. (1984)

4. For any three vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$,$(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{b} - \mathbf{c}) \times (\mathbf{c} - \mathbf{a})$ =2$\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$

C : MCQ'S with One Correct Answer

1. The scalar $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) \times (\mathbf{A} + \mathbf{B} + \mathbf{C})$ equals: (1981)

(a) 0

(b) $[\mathbf{ABC}]+[\mathbf{BCA}]$

(c) $[\mathbf{ABC}]$

(d) None of these

2. For non-zero vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mid (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} \mid = \mid \mathbf{a} \mid\mid \mathbf{b} \mid\mid \mathbf{c} \mid$ holds if and only if (1982)

   (a) $\mathbf{a} \cdot \mathbf{b} = 0$,$\mathbf{b} \cdot \mathbf{c} = 0$

   (b) $\mathbf{b} \cdot \mathbf{c} = 0$,$\mathbf{c} \cdot \mathbf{a} = 0$

   (c) $\mathbf{c} \cdot \mathbf{a} = 0$,$\mathbf{a} \cdot \mathbf{b} = 0$

   (d) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$

3. The volume of the parallelopiped whose sides are given by $\overrightarrow{OA} = 2i - 2j$,$\overrightarrow{OB} = i + j - k$,$\overrightarrow{OC} = 3i - k$, is (1983)

   (a) $\frac{4}{13}$

   (b) 4

   (c) $\frac{2}{7}$

   (d) None of these

4. The two points with positiooning vectors $60 + 3j, 40i - 8j, ai - 52j$ are colinearr if (1983)

   (a) a=-40

   (b) a=40

   (c) a=20

(d) None of these

5. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three non-coplanar vectors and $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are vectors defined by the relations $\mathbf{p} = \frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\cdot\mathbf{b}\cdot\mathbf{c}]}, \mathbf{q} = \frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{a}\cdot\mathbf{b}\cdot\mathbf{c}]}, \mathbf{r} = \frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{a}\cdot\mathbf{b}\cdot\mathbf{c}]}$ then the value of the expression $\left[_{\mathbf{a}+\mathbf{b}\cdot\mathbf{p}}\right] + \left[_{\mathbf{b}+\mathbf{c}\cdot\mathbf{q}}\right] + \left[_{\mathbf{c}+\mathbf{a}\cdot\mathbf{r}}\right]$ is equal to (1988)

(a) 0

(b) 1

(c) 2

(d) 3

6. Let be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is (1993)

(a) The Arithmetic Mean of a and b

(b) The Geometric Mean of a and b

(c) The Harmonic Mean of a and b

(d) equal to zero

7. Let $\mathbf{P}$ and $\mathbf{Q}$ are position vectors of P and Q respectively, with respect to O and $\mathbf{p} = p, \mathbf{q} = q$. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If OR and OS are perpendicular then (1994)

(a) $9q^2 = 4q^2$

(b) $4p^2 = 9q^2$

(c) $9p = 4q$

(d) $4p = 9q$

8. Let α, β, γ be distinct real numbers. The point with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}, \gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$ (1994)

   (a) are collinear

   (b) from an equilateral triangle

   (c) from a scalene triangle

   (d) from a right angled triangle

9. Let $\mathbf{a} = \hat{i} - \hat{j}, \mathbf{b} = \hat{j} - \hat{k}, \mathbf{c} = \hat{k} - \hat{i}$. If $\mathbf{d}$ is a unit vector such that $\mathbf{a} \cdot \mathbf{d} = 0 = [\mathbf{bcd}]$,then $\mathbf{d}$ equals (1995)

   (a) $\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$

   (b) $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

   (c) $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

   (d) $\pm\hat{k}$

10. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non coplanar unit vectors such that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{(\mathbf{b}+\mathbf{c})}{\sqrt{2}}$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is (1995)

    (a) $\frac{3\pi}{4}$

    (b) $\frac{\pi}{4}$

    (c) $\frac{\pi}{2}$

    (d) π

11. Let $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ be vectors such that $\mathbf{u} + \mathbf{v} + \mathbf{w} = 0$. If $|\mathbf{u}| = 3, |\mathbf{v}| = 4, |\mathbf{w}| = 5$, then $\mathbf{u} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{w} + \mathbf{w} \cdot \mathbf{u} +$ is

(a) 47

(b) -25

(c) 0

(d) 25

12. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are three non coplanar vectors, then $(\mathbf{a}+\mathbf{b}+\mathbf{c}) \cdot [(\mathbf{a}+\mathbf{b}) \times (\mathbf{a}+\mathbf{c})]$ equals (1995)

    (a) 0

    (b) $[\mathbf{abc}]$

    (c) $2\,[\mathbf{abc}]$

    (d) $-\,[\mathbf{abc}]$

13. Let $a = 2i + j - 2k, b = i + j$. If c is a vector such that a. $c = \mid c \mid, \mid c - a \mid = \sqrt[2]{2}$ and the angle between $(a \times b)$ and c is $30\circ$ then $\mid (a \times b) \times c \mid =$ (1999)

    (a) $\frac{2}{3}$

    (b) $\frac{3}{2}$

    (c) 2

    (d) 3

14. Let $a = 2i + j + k, b = i + 2j - k$ and a unit vector c be coplanar. If c is perpendicular to a, then c = (1999)

    (a) $\frac{1}{\sqrt{2}}(-j + k)$

    (b) $\frac{1}{\sqrt{3}}(-i - j - k)$

(c) $\frac{1}{\sqrt{5}}(i-2j)$

(d) $\frac{1}{\sqrt{3}}(i-j-k)$

15. If the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ from the sides BC,CA and AB respectivelly of a triangle ABC, then (2000)

    (a) $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = 0$

    (b) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}$

    (c) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a}$

    (d) $\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a} = 0$

16. Let the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ be such that $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = 0$. Let $p_1 and P_2$ be planes determined by the pairs of vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}, \mathbf{d}$ respectvely. Then the angle between $p_1 and P_2$ is (2000)

    (a) 0

    (b) $\frac{\pi}{4}$

    (c) $\frac{\pi}{3}$

    (d) $\frac{\pi}{2}$

17. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit coplanar vectors, then the scalar triple product $[2\mathbf{a} - \mathbf{b}, 2\mathbf{b} - \mathbf{c}, 2\mathbf{c} - \mathbf{a}] =$ (2000)

    (a) 0

    (b) 1

    (c) -$\sqrt{3}$

    (d) $\sqrt{3}$

18. Let $\mathbf{a} = \mathbf{i} - \mathbf{k}, \mathbf{b} = x\mathbf{i} + \mathbf{j} + (1 - x)\,\mathbf{k}$ and $\mathbf{c} = y\mathbf{i} + x\mathbf{j} + (1 + x - y)\,\mathbf{k}$. Then $[\mathbf{abc}]$ depends on (2000)

    (a) only x

    (b) only y

    (c) neither x nor y

    (d) both x and y

19. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit vectors, then $\mid \mathbf{a} - \mathbf{b} \mid^2 + \mid \mathbf{b} - \mathbf{c} \mid^2 + \mid \mathbf{c} - \mathbf{a} \mid^2$ dose NOT exceed (2001)

    (a) 4

    (b) 9

    (c) 8

    (d) 6

20. If $\mathbf{a} and \mathbf{b}$ are two unit vectors such that $\mathbf{a} + 2\mathbf{b}$ and $5\mathbf{a} - 4\mathbf{b}$ are perpendicular to each other then the angle between $\mathbf{a}$ and $\mathbf{b}$ is (2002)

    (a) $45\circ$

    (b) $60\circ$

    (c) $\cos^{-1}\frac{1}{3}$

    (d) $\cos^{-1}\frac{2}{7}$

21. Let $\mathbf{V} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{W} = \mathbf{i} + 3\mathbf{k}$. If $\mathbf{U}$ is a unit vector, then the maxium value of the scalar triple product $\mid \mathbf{VVW} \mid$ is (2002)

    (a) -1

(b) $\sqrt{10}+\sqrt{6}$

(c) $\sqrt{59}$

(d) $\sqrt{60}$

22. The value of K such that $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$, is (2003)

(a) 7

(b) -7

(c) no real value

(d) 4

23. The value of 'a' such that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}, \hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is (2004)

(a) -3

(b) 3

(c) $\frac{1}{\sqrt{3}}$

(d) $\sqrt{3}$

24. If $\mathbf{a}=\left(\hat{i}+a\hat{j}+\hat{k}\cdot\mathbf{a}\cdot\mathbf{b}=1\right)$ and $\mathbf{a}\times\mathbf{b}=\hat{j}-\hat{k}$, then $\mathbf{b}$ is

(a) $\hat{i}-\hat{j}+\hat{k}$

(b) $2\hat{j}-\hat{k}$

(c) $\hat{i}$

(d) $2\hat{i}$

25. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is

(a) $\frac{3}{2}$

(b) $\frac{9}{2}$

(c) $\frac{2}{9}$

(d) $\frac{3}{2}$

26. The unit vector which is orthogonal to the vector $3\hat{\imath}+2\hat{\jmath}+6\hat{k}$ and is coplanar with the vectors $2\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\hat{\imath}-\hat{\jmath}+\hat{k}$ is

(a) $\frac{2\hat{\imath}-6\hat{\jmath}+\hat{k}}{\sqrt{41}}$

(b) $\frac{2\hat{\imath}-3\hat{\jmath}}{\sqrt{13}}$

(c) $\frac{3\hat{\imath}-\hat{k}}{\sqrt{10}}$

(d) $\frac{4\hat{\imath}+3\hat{\jmath}-3\hat{k}}{\sqrt{34}}$

27. A variable plane at the distance of the one unit from the origin cuts the coordinates axes at A,B and C. If the centroid $D\left(x,y,z\right)$ of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$ then the value of k is

(a) 3

(b) 1

(c) $\frac{1}{3}$

(d) 9

28. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are three non-zero and non coplanar vectors $\mathbf{b_1}=\mathbf{b}-\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$, $\mathbf{b_2}=\mathbf{b}+\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a},\ \mathbf{c_1}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}+\frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{c}|^2}\mathbf{b_1},\ \mathbf{c_2}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}\text{-}\frac{\mathbf{b_1}\cdot\mathbf{c}}{|\mathbf{b_1}|^2}\mathbf{b_1},\ \mathbf{c_3}=$

$\mathbf{c} - \frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{c}|^2}\mathbf{a} + \frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{c}|^2}\mathbf{b_1}$, $\mathbf{c_4} = \mathbf{c} - \frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} = \frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{b}|^2}\mathbf{b_1}$ then the seet of orthogonal vector is

(a) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_3})$

(b) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_2})$

(c) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_1})$

(d) $(\mathbf{a}, \mathbf{b_2}, \mathbf{c_2})$

29. A plane which is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$ passes through $(1, -2, 1)$. The distance of the plane from the point $(1, 2, 2)$ is

(a) 0

(b) 1

(c) $\sqrt{2}$

(d) $\sqrt[2]{2}$

30. Let $\mathbf{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}, \mathbf{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\mathbf{c} = \hat{\imath} + \hat{\jmath} - \hat{k}$. A vector in the plane of $\mathbf{a} and \mathbf{b}$ whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$ is

(a) $4\hat{\imath} - \hat{\jmath} + 4\hat{k}$

(b) $3\hat{\imath} + \hat{\jmath} - 3\hat{k}$

(c) $2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

(d) $4\hat{\imath} + \hat{\jmath} - 4\hat{k}$

31. The number of real distinct values of λ, for which the vectors $-\lambda^2\hat{\imath} + \hat{\jmath} + \hat{k}, \hat{\imath} - \lambda^2\hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \lambda^2\hat{k}$ are coplanar, is

(a) zero

(b) one

(c) two

(d) three

32. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit vectors such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$. Which one of the following is correct ?

(a) $\mathbf{a} \times \mathbf{b} = b \times \mathbf{c} = \mathbf{c} \times \mathbf{a} = \mathbf{0}$

(b) $\mathbf{a} \times \mathbf{b} = b \times \mathbf{c} = \mathbf{c} \times \mathbf{a} \neq \mathbf{0}$

(c) $\mathbf{a} \times \mathbf{b} = b \times \mathbf{c} = \mathbf{c} \times \mathbf{c} \neq \mathbf{0}$

(d) $\mathbf{a} \times \mathbf{b}, b \times \mathbf{c}, \mathbf{c} \times \mathbf{a} = \mathbf{0}$ are mutually perpendicular

33. The edges of the parallelopiped are of unit length and are parallel to non- coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{c} = \hat{c} \cdot \hat{a} = \frac{1}{2}$. Then the volume of parallelopiped is (2008)

(a) $\frac{1}{\sqrt{2}}$

(b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{1}{\sqrt{3}}$

34. Let two non-coplanar unit vectors $\hat{a} and \hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$(here O is the origin) is given by $\hat{a} \cos t + \hat{b} \sin t$. Then P is farthest from origin O,let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then, (2008)

(a) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = \left(1 + \hat{a} \cdot \hat{b}^{\frac{1}{2}}\right)$

(b) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = \left(1 + \hat{a} \cdot \hat{b}^{\frac{1}{2}}\right)$

(c) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = \left(1 + 2\hat{a} \cdot \hat{b}^{\frac{1}{2}}\right)$

(d) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = \left(1 + 2\hat{a} \cdot \hat{b}^{\frac{1}{2}}\right)$

35. Let P(3,2,6) be point in space and Q be a point on the line $\mathbf{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(-3\hat{i} + \hat{j} + 5\hat{k}\right)$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is (2009)

(a) $\frac{1}{4}$

(b) $-\frac{1}{4}$

(c) $\frac{1}{8}$

(d) $-\frac{1}{8}$

36. If $\mathbf{a}, \mathbf{b}, \mathbf{c} and \mathbf{d}$ are unit vectors such that $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = 1$ and $\mathbf{a} \cdot \mathbf{c} = \frac{1}{2}$, then (2009)

(a) $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar

(b) $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are non-coplanar

(c) $\mathbf{b}, \mathbf{d}$ are non-parallel

(d) $\mathbf{a}, \mathbf{d}$ are parallel and $\mathbf{b}, \mathbf{c}$ are parallel

37. A line wit positive direction cosines passes through the points P(2,-1,2) and makes equal angles with the coordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals(2009)

(a) 1

(b) $\sqrt{2}$

(c) $\sqrt{3}$

(d) 2

38. Let P,Q,R and S be points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a (2010)

(a) parallelogram, which is neither a rhombus nor a rectangle

(b) squrae

(c) rectangle, but not a square

(d) rhombus, not a square

39. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight line $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is (2010)

(a) $x+2y-2z=0$

(b) $x+2y-2z=0$

(c) $x-2y+z=0$

(d) $5x+2y-4z=0$

40. If the distance of the point P(1,-2,1) from the plane $x+2y-2z=\alpha$, where$\alpha>0$, is 5,then the foot of the perpendicular from P to the pane is (2010)

(a) $\left[\frac{8}{3}, \frac{4}{3}, \frac{7}{3}\right]$

(b) $\left[\frac{4}{3}, \frac{4}{3}, \frac{1}{3}\right]$

(c) $\left[\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right]$

(d) $\left[\frac{2}{3}, \frac{1}{3}, \frac{5}{2}\right]$

41. Two adjcent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{\imath} + 10\hat{0} + 11\hat{k}$ and $\overrightarrow{AD} = \hat{\imath} + 21\hat{0} + 2\hat{k}$ the side AD is rotated by an acute angle α, in the plane of the parallelogram so that AD becomes AD^1. If AD^1 makes a right angle with the side AB, then the cosine of the angle α is given by 2010)

(a) $\frac{8}{9}$

(b) $\frac{\sqrt{17}}{9}$

(c) $\frac{1}{9}$

(d) $\frac{4\sqrt{5}}{9}$

42. Let $\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$,$\mathbf{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\mathbf{a} = \hat{\imath} - \hat{\jmath} - \hat{k}$ be threevectors. A vector $\mathbf{v}$ in the plane of $\mathbf{a} and \mathbf{b}$, whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is given by (2011)

(a) $\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$

(b) $-3\hat{\imath} - 3\hat{\jmath} - \hat{k}$

(c) $3\hat{\imath} - \hat{\jmath} + 3\hat{k}$

(d) $\hat{\imath} - 3\hat{\jmath} - 3\hat{k}$

43. The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1,-1,4) with the plane $5x - 4y - z = 1$. If S is the foot

of the perpendicular drawn from the point T(2,1,4) to QR, then the length of the line segment PS is (2012)

(a) $\frac{1}{\sqrt{2}}$

(b) $\sqrt{2}$

(c) 2

(d) $\sqrt[2]{2}$

44. The equation of the plane passing through the line of intersection of the plane $x+2y+3z=2$ and $x-y+z=3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point(3,1-1) is (2012)

(a) $5x-11y+z=17$

(b) $\sqrt{2}x+y=\sqrt[3]{2}-1$

(c) $x+y+z=\sqrt{3}$

(d) $x-\sqrt{2}y=1-\sqrt{2}$

45. If $\mathbf{a}$ and $\mathbf{b}$ are vectors such that $|\ \mathbf{a}+\mathbf{b}\ |=\sqrt{29}$ and $\mathbf{a}\times\left[2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right]=\left[2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right]\times\mathbf{b}$, then a possible value of $[\mathbf{a}+\mathbf{b}]\cdot\left[-7\hat{\imath}+2\hat{\jmath}+3\hat{k}\right]$ is (2012)

(a) 0

(b) 3

(c) 4

(d) 8

46. Let P be the image of the point(3,1,7) with respect to the plane $x-y+z=3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{z}=\frac{z}{1}$ is (2016)

(a) $x + y - 3z = 0$

(b) $3x + z = 0$

(c) $x - 4y + 7z = 0$

(d) $2x - y = 0$

47. The equation of the plane passing through the point(1,1,1) and perpendicular to the plane $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$,is (2017)

(a) $14x + 2y2y - 15z = 1$

(b) $14x - 2y + 15z = 27$

(c) $14x + 2y + 15z = 31$

(d) $-14x + 2y - 15z = 3$

48. L et O be the origin and let PQR be an arbitary triangle.The point S is such that $\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OP} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$. then the triangle PQR has S as its (2017)

(a) Centroid

(b) Circumcenter

(c) Incenter

(d) Orthocenter

D : MCQ'S with One or More Than One Correct Answer

1. Let $\mathbf{a} = a_1 i + a_2 j + a_3 k$, $\mathbf{b} = b_1 i + b_2 j + b_3 k$ and $\mathbf{c} = c_1 i + c_2 j + c_3 k$ be three non-zero vectors such that $\mathbf{c}$ is a unit vector peerpendicular to both the vectors $\mathbf{a}$ and $\mathbf{b}$. If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ is equal to (1986)
   (a) 0
   (b) 1
   (c) $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$
   (d) $\frac{3}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)\left(c_1^2 + c_2^2 + c_3^2\right)$

2. The number of vectors of unit length perpendicular to vectors $\mathbf{a} = (1, 1, 0)$ $\mathbf{b} = (0, 1, 1)$ is (1987)
   (a) one
   (b) two
   (c) three
   (d) infinite

3. Let $\mathbf{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} + 2\hat{j} + \hat{k}$ and $\mathbf{a} = \hat{i} - \hat{j} - 2\hat{k} - 2\hat{k}$ be three vectors. A vector in the plane of $\mathbf{b}$ and $\mathbf{c}$ whose projection on $\mathbf{a}$ is of magnitude, $\sqrt{\frac{2}{3}}$, is (1993)

(a) $2\hat{i}+3\hat{j}-3\hat{k}$

(b) $2\hat{i}+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}-hatj-5\hat{k}$

(d) $2\hat{i}+\hat{j}+5\hat{k}$

4. The vector $\frac{1}{3}\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ is

(a) a unit vector

(b) makes an angle $\frac{\pi}{3}$ with the vector $\left(2\hat{i}-4\hat{j}+3\hat{k}\right)$

(c) parallel to the vector $\left(-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}\right)$

(d) perpendicular to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$

5. If $a = i + j + k$,$b = 4i + 3j + 4k$ and $c = i + \alpha j + \beta k$ are linearly dependent vectors and $\mid c \mid = \sqrt{3}$, then (1998)

(a) $\alpha = 1, \beta = -1$

(b) $\alpha = 1, \beta = \pm 1$

(c) $\alpha = -1, \beta = \pm 1$

(d) $\alpha = \pm 1, \beta = 1$

6. For three vectors u,v,w which of the following expression is not equal to any of the remaining three? (1998)

(a) $u \cdot (v \times w)$

(b) $(v \times w) \cdot u$

(c) $v \cdot (u \times w)$

(d) $(u \times v) \cdot w$

7. Which of the following expressions are meaningful? (1998)

   (a) $u\,(v \times w)$

   (b) $u \cdot (v \cdot w)$

   (c) $(u \cdot v)\,w$

   (d) $u \times (v \cdot w)$

8. Let a and b be two be non-collinear unit vectors. If $u = a - (a \cdot b)\,b$ and $v = a \times b$, then $\mid v \mid$ is (1999)

   (a) $\mid u \mid$

   (b) $\mid u \mid + \mid u \cdot a \mid$

   (c) $\mid u \mid + \mid u \cdot b \mid$

   (d) $\mid u \mid + u \cdot (a + b)$

9. Let $\mathbf{A}$ be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{\jmath} + 3\hat{k}$ and $4\hat{\jmath} - 3\hat{k}$ and that P_2 is parallel to $\hat{\jmath} - \hat{k}$ and $3\hat{\jmath} + 3\hat{k}$ then the angle between the vector $\mathbf{A}$ and a given vector $2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ is (2006)

   (a) $\frac{\pi}{2}$

   (b) $\frac{\pi}{4}$

   (c) $\frac{\pi}{6}$

   (d) $\frac{3\pi}{4}$

10. The vector(s) which is/are coplanar with vectors $\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and perpendicular to the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ is/are (2011)

(a) $\hat{j} - \hat{k}$

(b) $\hat{i} + \hat{j}$

(c) $\hat{i} - \hat{j}$

(d) $\hat{j} + \hat{k}$

11. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are) (2012)

    (a) $y + 2z = -1$

    (b) $y + z = -1$

    (c) $y - z = -1$

    (d) $y - 2z = -1$

12. A line l is passing through the origin is perpendicular to the lines
    $l_1 : (3+t)\,\hat{i} + (1+2t)\,\hat{j} + (4+2t)\,\hat{k}, \infty < t < \infty$
    $l_2 : (3+2s)\,\hat{i} + (3+2s)\,\hat{j} + (2+s)\,\hat{k}, \infty < s < \infty$
    Then the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of $landl_1$ is(are) (2013)

    (a) $\left[\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right]$

    (b) $(1, 1, 0)$

    (c) $(1, 1, 1)$

    (d) $\left[\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right]$

13. Two lines $l_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $l_2 : x = \alpha, \frac{y}{4} = \frac{z}{2-\alpha}$ are coplanar , then α can take value(s) (2013)

(a) 1

(b) 2

(c) 3

(d) 4

14. Let $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each air of them is $\frac{\pi}{3}$. If $\mathbf{a}$ is a non-zero vector perpendicular to $\mathbf{x}$ and $\mathbf{y} \times \mathbf{z}$ and $\mathbf{b}$ is a non-zero vector perpendicular to $\mathbf{y}$ and $\mathbf{z} \times \mathbf{x}$, then (2014)

(a) $\mathbf{b} = [\mathbf{b} \cdot \mathbf{z}]\ [\mathbf{z} - \mathbf{x}]$

(b) $\mathbf{a} = [\mathbf{a} \cdot \mathbf{y}]\ [\mathbf{y} - \mathbf{z}]$

(c) $\mathbf{a} \cdot \mathbf{b} = [\mathbf{a} \cdot \mathbf{y}]\ [\mathbf{b} \cdot \mathbf{z}]$

(d) $\mathbf{a} = -\,[\mathbf{a} \cdot \mathbf{y}]\ [\mathbf{z} - \mathbf{y}]$

15. From a point $P(\lambda, \lambda, \lambda)$, perpendicular PQ and PR arec drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle PQR$ is a right angle, then the possible value(s) of λ is/(are) (2014)

(a) $\sqrt{2}$

(b) 1

(c) -1

(d) -$\sqrt{2}$

16. In R^3 consider the planes $P_1 : y = 0$ and $P_2 : x + z - 1$.Let P_3 be the plane different from P_1 and P_2 which passes through the intersection

of P_1 and P_2. If the distance of the point(0,1,0) from P_3 is 1 and the distance of point $(\alpha, \beta, 0)$ from P_3 is 2, then which of the following relation is(are) true (2015)

(a) $2\alpha + \beta + 2y + 2 = 0$

(b) $2\alpha - \beta + 2y + 4 = 0$

(c) $2\alpha + \beta + 2y - 10 = 0$

(d) $2\alpha - \beta + 2y - 8 = 0$

17. In R^3, let L be astraight line passing through the origin suppose that all the points on L are at a costant distance from two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - 2y + z - 1 = 0$. Let M be the ocus of the foot of the perpendicular drawn from the points on L to plane P_1. Which of the following points lie(s) on M ?(2015)

(a) $0, \frac{5}{6}, \frac{2}{3}$

(b) $\frac{1}{6}, \frac{1}{3}, \frac{1}{6}$

(c) $\frac{5}{6}, 0, \frac{2}{3}$

(d) $\frac{1}{3}, 0, \frac{2}{3}$

18. Let $\triangle PQR$ be a triangle. Let $\mathbf{a} = \overrightarrow{QR}$,$\mathbf{a} = \overrightarrow{RP}$ and $\mathbf{a} = \overrightarrow{PQ}$. If $\mid \mathbf{a} \mid= 12$,$\mid \mathbf{b} \mid= \sqrt[4]{3}$ and $\mid \mathbf{c} \mid= 24$, then which of the following is(are)true? (2015)

(a) $\frac{|\mathbf{c}|}{2} - \mid \mathbf{a} \mid= 2$

(b) $\frac{|\mathbf{c}|}{2} + \mid \mathbf{a} \mid= 30$

(c) $\mid \mathbf{a} \times \mathbf{b} \mid= \sqrt[48]{3}$

(d) $\mathbf{a} \cdot \mathbf{b} = -42$

19. Consider a pyramid OPQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin, OP and OR along the x-axis and the y-axis respectively. The base OPQR of the pyramid is a squarq with OP=3. The point S is directly above the mid-point, T of diagonal OQ such that TS=3. Then (2016)

    (a) the acute angle between OQ and OS is $\frac{\pi}{3}$

    (b) the equation of the plane containg the triangle OQS is $x - y = 0$

    (c) the length of the perpendicular from P to the plane containg the triangle OQS is $\frac{3}{\sqrt{2}}$

    (d) the perpendicular distance from O to the staright line containing RS is $\sqrt{\frac{15}{2}}$

20. Let $\hat{u} = u_1\hat{i} + u_2\hat{j}$ be a unit vector in R^3 and $\hat{w} = \frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + 2\hat{k}\right)$. Given that there exists a vector $\mathbf{v}$ in R^3 such that $mid\hat{u} \times \mathbf{v} \mid= 1$ and $\hat{w}\left[\hat{u} \times \mathbf{v}\right] = 1$. Which of the following statement(s) is(are) correct? (2016)

    (a) there is exactly one choice for such $\mathbf{v}$

    (b) There are infinitely many choices for such $\mathbf{v}$

    (c) If $\hat{u}$ lies in the xy-plane then $\mid u_1 \mid=\mid u_2 \mid$

    (d) If $\hat{u}$ lies in the xz-plane then $2 \mid u_1 \mid=\mid u_2 \mid$

21. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then,which of the following statement(s) is(are) TRUE? (2018)

(a) The lines of intersection ofP_1 and P_2 has direction ratios 1,2,-1

(b) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(c) The acute angle between P_1 and P_2 is 60○.

(d) If P_3 is the plane passing through the point (4,2,2) and perpendicular to the line of intersection of P_1 and P_2,then the distance of the point (2,1,1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

22. Let L_1 and L_2 denote the lines
$\mathbf{r} = \hat{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right), \lambda \in R$ and $\mathbf{r} = \mu\left(2\hat{i} = \hat{j} + 2\hat{k}\right), \mu \in R$
respectively. If L_3 is a line which is perendicular to both L_1 and L_2 and cuts boyh of them, then which of the following option describe(s) L_3? (2019)

(a) $\mathbf{r} = \frac{2}{9} + \left(4\hat{i} + \hat{j} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

(b) $\mathbf{r} = \frac{2}{9}\left(2\hat{i} - \hat{j} + 2\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

(c) $\mathbf{r} = t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

(d) $\mathbf{r} = \frac{1}{3}\left(2\hat{i} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

23. Three lines $L_1 : \mathbf{r} = \lambda\hat{i}, \lambda \in R$
$L_2 : \mathbf{r} = \hat{k} + \mu\hat{j}, \mu \in R$ and
$L_3 : \mathbf{r} = \hat{i} + \hat{j} + \nu\hat{k}, \nu \in R$
are given. For which point(s) Q on L_2 can find a point P on L_1 and R on L_3 so that P,Q and R ae collinear? (2019)

(a) $\hat{k} - \frac{1}{2}\hat{j}$

(b) $\hat{k}$

(c) $\hat{k}+\hat{j}$

(d) $\hat{k}+\frac{1}{2}\hat{j}$

E : Subjective Problems

1. From a point O inside the triangle ABC, perpendiculars OD,OE,OF are drawn to the sides BC,CA,AB respectively. Prove that the perpendiculars from A,B,C to the sides EF,FD,DE are concurrent. (1978)

2. $A_1, A_2, ...A_n$ are the vectors of a regular plane polygon with n sides and O is it's center. Show that $\sum_{i=1}^{n-1}\left(\overrightarrow{OA_i}\times\overrightarrow{OA_i+1}\right)=(1-n)\left(\overrightarrow{OA_2}\times\overrightarrow{OA_1}\right)$ (1982)

3. Find all values of λ such that $x,y,z\neq(0,0,0)$ and $(\mathbf{i}+\mathbf{j}+3\mathbf{k})\,x+(3\mathbf{i}-3\mathbf{j}+\mathbf{k})\,y+(-4\mathbf{i}+5\mathbf{j})\,z=\lambda\,(x\mathbf{i}\times\mathbf{j}y+\mathbf{k})\,z$ where $\mathbf{i},\mathbf{j},\mathbf{k}$ are unit vectors along the coordinate axes. (1982)

4. A vector $\mathbf{A}$ has components A_1, A_2, A_3 in a right-handed rectangular cartesian coordinate system oxyz. The coordinate system is rotated about the x-axis throughh an angle $\frac{\pi}{2}$. Find the components of A in the new coordinate system in terms of A_1, A_2, A_3. (1983)

5. The position vectors of the points A,B,C and D are $3\hat{\imath}-2\hat{\jmath}-\hat{k}, 2\hat{\imath}+3\hat{\jmath}-4\hat{k}, -\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $4\hat{\imath}+5\hat{\jmath}+\lambda\hat{k}$,respectively. If the points A,B,C and D lies in a plane,find the value of λ.(1986)

6. If A,B,C,D are any four points in space,prove that-$|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}| = 4$(area of triangle ABC)(1987)

7. Let OABC be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA.Using vector methods prove that BD and CO intersects in the same ratio. (1988)

8. If vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ are coplanar,show that (1989)
$$\begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \\ \mathbf{a}\cdot\mathbf{a}, & \mathbf{a}\cdot\mathbf{b}, & \mathbf{a}\cdot\mathbf{c} \\ \mathbf{b}\cdot\mathbf{a}, & \mathbf{b}\cdot\mathbf{b}, & \mathbf{b}\cdot\mathbf{c} \end{vmatrix} = 0$$

9. In a triangle OAB,E is the midpoint of BO and D is a point on AB such that $AD : DB = 2 : 1$. If OD and AE intersects at P,determine the ratio $OP : PD$ using vector methods. (1989)

10. Let $\mathbf{A} = 2\mathbf{i} = \mathbf{k}, \mathbf{B} = \mathbf{i} = \mathbf{j} + \mathbf{k}$ and $\mathbf{C} = -4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$. Determine a vector $\mathbf{R}$ satisfying $\mathbf{R}\times\mathbf{B} = \mathbf{C}\times\mathbf{B} and \mathbf{R}\cdot\mathbf{A} = 0$ (1990)

11. Determine the value of 'c' so that for all real values x, the vector $cx\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and $x\hat{\imath} + 2\hat{\jmath} + 2cx\hat{k}$ make an obtuse angle with each other.(1991)

12. In a triangle ABC, D and E are points on BC and AC respectively, such that BD=2DC and AE=3EC. Let P be the point of intersection of AD and BE. Find BP:PE using vector methods.(1993)

13. If the $\mathbf{b},\mathbf{c},\mathbf{d}$ are not coplanar,then prove that the vector $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d}) + (\mathbf{a}\times\mathbf{c})\times(\mathbf{d}\times\mathbf{b}) + (\mathbf{a}\times\mathbf{d})\times(\mathbf{b}\times\mathbf{c})$ is parallel to $\mathbf{a}$ (1994)

14. The position vectors of the vertices A,B and C of a tetrahedron ABCD are $\hat{i}+\hat{j}+\hat{k}$ and $3\hat{i}$ respectively. The altitude fro vertex D to the opposite face ABC meets the median line through A of the triangle ABC at the point E. If the length of the side AD is 4 and tthe volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$ find the position vector of the point E for all it's possible positions. (1996)

15. If A,B and C are vectors such that $\mid B \mid = \mid C \mid$. Prove that $[(A+B)\times(A+C)]\times(B\times C)\,(B+C)=0$ (1997)

16. Prove, by vector methods or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the mid-points of the parallel sides.(You may assume that the trapezium is not a parallelogram.)(1998)

17. For any two vectors u and v, prove that (1998)

    (a) $(u\cdot v)^2+\mid u\times v\mid^2=\mid u\mid^2\mid v\mid^2$ and

    (b) $\left(1+\mid u\mid^2\right)\left(1+\mid v\mid^2\right)=(1-u\cdot v)^2+\mid u+v+(u\times v)\mid^2$

18. Let u and v be unit vectors. If w is a vector such that $w+(w\times u)=v$ then prove that $\mid (u\times v)\cdot w\mid\leq\frac{1}{2}$ and that the equality holdes if and only if u is perpendicular to v. (1999)

19. Show, by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices.(2001)

20. Find 3-dimensional vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ satisfying $\mathbf{v_1} \cdot \mathbf{v_1} = 4, \mathbf{v_1} \cdot \mathbf{v_2} = 2, \mathbf{v_1} \cdot \mathbf{v_3} = 6, \mathbf{v_2} \cdot \mathbf{v_2} = \mathbf{v_2} \cdot \mathbf{v_3} = -5\mathbf{v_3} \cdot \mathbf{v_3} = 29$ (2001)

21. Let $\mathbf{A}(t) = f_1(t)\hat{\imath} + f_2(t)\hat{\jmath}$ and

    $\mathbf{B}(t) = g_1(t)\hat{\imath} + g_2(t)\hat{\jmath}, t < [0,1]$

    where f_1, f_2, g_1, g_2 are continuous functions. If $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are non zero vectors for all t and $\mathbf{A}(0) = 2\hat{\imath} + 3\hat{\jmath}, \mathbf{A}(1) = 6\hat{\imath} + 2\hat{\jmath}, \mathbf{B}(0) = 3\hat{\imath} + 2\hat{\jmath} and \mathbf{B}(1) = 2\hat{\imath} + 6\hat{\jmath}$.Then show that $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are parallel for some t. (2001)

22. Let V be the volume of the parallelopiped formed by the vectors $\mathbf{a} = a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$,$\mathbf{b} = b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ and $\mathbf{c} = c_1\hat{\imath}+c_2\hat{\jmath}+c_3\hat{k}$. If a_r, b_r, c_r where $r = 1, 2, 3$ are non negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$. Show that $V \geq L^3$ (2002)

23. (a) Find the equation of the plane passing through the points $(2, 1, 0), (5, 0, 1) and (4, 11)$.

    (b) If P is the point $(2, 1, 6)$ then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it.(2003)

24. If $\mathbf{u}, \mathbf{v}, \mathbf{w}$, are three non-coplanar unitvectors and α, β are the angles between $\mathbf{u}$ and $\mathbf{v}$ and $\mathbf{w}$. $\mathbf{w}$ and $\mathbf{u}$ respectively and $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that $\begin{bmatrix} \mathbf{x}\times\mathbf{y} & \mathbf{y}\times\mathbf{z} & \mathbf{z}\times\mathbf{x} \end{bmatrix} \frac{1}{16} \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{bmatrix}^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$ (2003)

25. If $\mathbf{a}, \mathbf{b}, \mathbf{c} and \mathbf{d}$ are distinct vectors such that $\mathbf{a}\times\mathbf{c} = \mathbf{b}\times\mathbf{d}$ and $\mathbf{a}\times\mathbf{b} = \mathbf{c}\times\mathbf{d}$. Prove that $(\mathbf{a}-\mathbf{d})\cdot(\mathbf{b}-\mathbf{c}) \neq 0$ i.e.$\mathbf{a}\cdot\mathbf{b}+\mathbf{d}\cdot\mathbf{c} \neq \mathbf{d}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$ (2004)

26. Find the equation of the line passing through $(1, 1, 1)$ & parallel to the lines L_1, L_2 having direction ratios$(1, 0, -1), (1, -1, 0)$. Find the volume of tetrahedron formed by origin and the points where thses planes intersect the coordinate axes.(2004)

27. A paeallelopiped $'S'$ has base points A,B,C and D and upper face points $A', B', C' and D'$. This parallelopiped is compressed by upper face A', B', C', D' to form a new parallelopiped 'T' having upper face points A'', B'', C'', D''. Volume of parallelopiped $'T'$ is 90 percent of both volume of parallelopioed S. Prove that the locus of A'', is a plane (2004)

28. $P_1 and P_2$ are planes passing through origin. $L_1 and L_2$ are two lines on $P_1 and P_2$ respectively such that their intersection is origin. Show that their exists points A, B, C whose permutation A', B', C' can be choosen such that (i) A is on L_1,B on P_1 but not on L_1 and C not on P_1 (ii) A' is on L_2 B' on P_2 but not on L_2 and C not on P_2(2004)

29. Find the equation of the plane containing the line $2x - y + z - 3 = 0$,$3x+y+z = 5$ and at a distance og $\frac{1}{\sqrt{6}}$ from the point $(2, 1, -1)$.(2005)

30. If the incident ray on a surface is along the unit vector **w**, the reflected ray is along the unit vector **w** and the normal is along unit vactor **a** ourwards. Express **w** in terms of **a** and **v**. (2005)

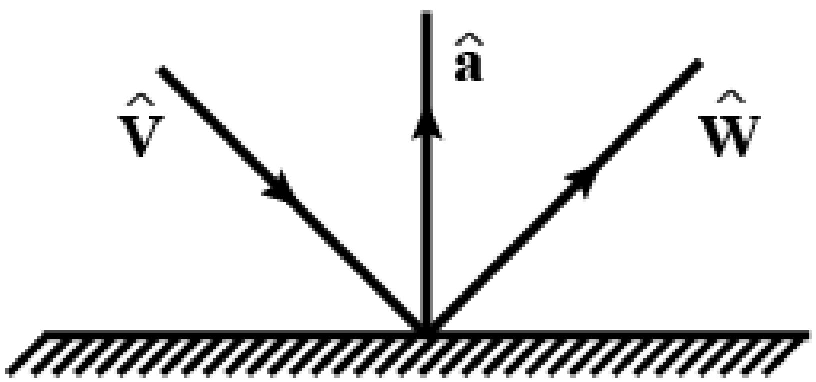

Figure 2.1:

F : Match The Following

DIRECTIONS (Q. 1-6): Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled 1, 2, 3 and 4. while the statements in Columa-II are labelled as a,b,c, d and e. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answer to these questions have to be darkened as illustrated in the following example: If the correct matches are 1-a. s and e: 2-b and c: 3-1 and 2: and 4-d

1. Match the following.(2006)

column-I	column-II
1. Two rays $x+y=\mid a\mid$ and $ax-y=1$ intersects eachother in the first quadrant in the interval $a\in(a_0,\infty)$, the value of a_0 is	a) 2
2. Point (α,β,γ) lies on the plane $x+y+z=2$. Let $\mathbf{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$,$\hat{k}\times\left(\hat{k}\times\mathbf{a}\right)=0$, then $\gamma=$	b) $\frac{4}{3}$
3. $\left\|\int_0^1\left(1-y^2\right)dy\right\|+\left\|\int_1^0\left(y^2-1\right)dy\right\|$	c) $\left\|\int_0^1\sqrt{1-x}dx\right\|+\left\|\int_{-1}^0\sqrt{1-x}dx\right\|$
4. If $\sin A\sin B\sin C+\cos A\cos B=1$,then the value of $\sin C=$	d) 1

2. Consider the following linear equations

$ax+by+cz=0$;$bx+cy+az=0$;$cx+ay+bz=0$

Match the coditions/expressions in Column I with statements in Column II.(2007)

column-I column-II

1. $a+b+c \neq 0$and$a^2+b^2+c^2 = ab+bc+ca$	a) the equation represent planes meeting only at a single point
2. $a+b+c = 0$and$a^2+b^2+c^2 \neq ab+bc+ca$	b) the equation represent the line $x=y=z$
3. $a+b+c \neq 0$and$a^2+b^2+c^2 \neq ab+bc+ca$	c) the equation representidentical planes.
4. $a+b+c = 0$and$a^2+b^2+c^2 = ab+bc+ca$	d) the equation represent the whole of the three dimensional space.

3. Match the statements/expressions given in Column I with the values given in Column II.(2009)

column-I column-II

1. Root(s)of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	a) $\frac{\pi}{6}$
2. Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\cos\frac{3x}{\pi}\right]$,f where [y] denotes the largest integer less than or equal to y	b) $\frac{\pi}{4}$
3. Volume of the parallelopiped with it's edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	c) $\frac{\pi}{2}$
4. Angle beteen vector $\mathbf{a} and \mathbf{b}$ where $\mathbf{a}, \mathbf{b} and \mathbf{c}$ are unit vectors satisfying $\mathbf{a}+\mathbf{b}+\sqrt{3}\mathbf{c}=0$	d) π

4. Match the statements/expressions given in Column I with the values given in Column II.(2009)

column-I column-II

1. The number of solution of the given $xe^{\sin x} - \cos x = 0$ in the interval $\left[0, \frac{\pi}{2}\right]$	a) 1
2. Value(s) of k for which the planes $kx+4y+z = 0, 4x+ky+2z = 0$ and $2x + 2y + z = 0$ intersects in a straight line.	b) 2
3. Value(s) of k for which $\mid x - 1 \mid + \mid x-2 \mid + \mid x+1 \mid + \mid x+2 \mid = 4k$ has integer solution(s)	c) 4
4. If $y' = y + 1$ and $y(0) = 1$,then value(s) of y(1 and 2)	d) 5

5. Match the statements/expressions given in Column I with the values given in Column II.(2009)

column-I column-II

1. A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively.If length PQ=d,thej d^2 is	a) -4
2. The value of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left[\frac{3}{5}\right]$ are	b) 0
3. Non-zero vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ satisfy $\mathbf{a}\cdot\mathbf{b} = 0.(\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}+\mathbf{c}) = 0$ and $2 \mid \mathbf{b}+\mathbf{c} \mid = \mid \mathbf{b}-\mathbf{a} \mid$. If $\mathbf{a} = \mu\mathbf{b}+4\mathbf{c}$,then the possible values of μ are	c) 4
4. Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is	d) 5
5.	e) 6

6. Match the statements/expressions given in Column I with the values given in Column II.(2010)

column-I column-II

1. If $\mathbf{a} = \hat{j} + \sqrt{3}\hat{k}, \mathbf{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\mathbf{c} = \sqrt[2]{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\mathbf{a}$ and $\mathbf{b}$ is	a) $\frac{\pi}{6}$
2. If $\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$, then the value of $f\left[\frac{\pi}{6}\right]$ is	b) $\frac{2\pi}{3}$
3. The value of $\frac{\pi^2}{ln^3} \int_{\frac{5}{6}}^{\frac{7}{6}} \sec(\pi x)dx$ is	c) $\frac{\pi}{3}$
4. The maximum vaalue of $\left\| arg\left[\frac{1}{1-z}\right] \right\| for \mid z \mid= 1, z \neq 1$ is given by	d) π
5.	e) $\frac{\pi}{2}$

DIRECTIONS (Q. 7-9): Each question has matching lists have chances (p),(q),(r) and (s) out of which ONLY ONE is correct.

7. Match List I with List II and select the answer using the code given below the list

List-I List-II

1. Volume of parallelopiped determined by vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ is 2.Then the volume of parallelopiped determined by vectors $2\left(\mathbf{a} \times \mathbf{b}\right), 3\left(\mathbf{b} \times \mathbf{c}\right)$ and $2\left(\mathbf{c} \times \mathbf{a}\right)$ is	a) 100
2. Volume of parallelopiped determined by vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ is 5.Then the volume of parallelopiped determined by vectors $3\left(\mathbf{a} + \mathbf{b}\right), 3\left(\mathbf{b} + \mathbf{c}\right)$ and $2\left(\mathbf{c} + \mathbf{a}\right)$ is	b) 30
3. Area of triangle with adjcent sides determined by the vectors $\mathbf{a}$ and $\mathbf{b}$ is 20. Then the area of triangle with adjcent sides determined by the vectors $(3\mathbf{a} + 2\mathbf{b})$ and $(\mathbf{a} - \mathbf{b})$ is	c) 24
4. Area of parallelogram with adjcent sides determined by the vectors $\mathbf{a}$ and $\mathbf{b}$ is 30. Then the area of parallelogram with adjcent sides determined by the vectors $(\mathbf{a} + \mathbf{b})$ and $\mathbf{a}$ is	d) 60

Codes:

	1	2	3	4
(p)	d	b	c	a
(q)	b	c	a	d
(r)	c	d	a	b
(s)	a	d	c	b

8. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$,$L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7xy + 2z = 3,\ P_2 = 3x + 5y - 6z = 4$.Let $ax + by + cz = d$ be the equation of the plane pasing through the point of intersection of lines L_1 and L_2 and erpedicular to plane P_1 and P_2.

   (2013)

   Match List I with List II and select the answer using the code given below the list

List-I	List-II
1. a =	a) 13
2. b =	b) -3
3. c =	c) 1
4. d =	d) -2

   Codes:

	1	2	3	4
(p)	c	b	d	a
(q)	a	c	d	b
(r)	c	b	a	d
(s)	b	d	a	c

9. Match List I with List II and select the answer using the code given below the list (2014)

List-I 7

List-II

1. Let $y(x) = \cos\left(2\cos^{-1} x\right), x \in [-1,1], x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{\left(x^2-1\right)\frac{d^2y(x)}{dx^2} + \frac{dy(x)}{dx}\right\}$ equals	a) 1
2. Let $A_1, A_2....A_n (n > 2)$ be the vertices of a regular polygon of n sides with it's center at the origin. Let $\mathbf{a_k}$ be the position vector of the points $A_k, k = 1, 2, ..., n$. If $\left\lvert\sum_{k=1}^{n-1}\left[\mathbf{a_k}\times\mathbf{a_k+1}\right]\right\rvert = \left\lvert\sum_{k=1}^{n-1}\left[\mathbf{a_k}\cdot\mathbf{a_k+1}\right]\right\rvert$,then the minimum value of n is	b) 2
3. If the normal from the point $p(h,1)$ on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line $x+y=8$, then the value of h	c) 8
4. Number of positive solution satisfying the equation $\tan^{-}1\frac{1}{2x+1} + \tan^{-}1\frac{1}{4x+1} = \tan^{-}1\frac{2}{x^2}$ is	d) 9

Codes:

	1	2	3	4
(p)	d	c	b	a
(q)	b	d	c	a
(r)	d	c	a	b
(s)	b	d	a	c

DIRECTIONS (Q.10-11): Refer to directions (1-6).

10. Match the following: (2015)

column-I column-II

1. In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i}+\beta\hat{j}$ on $\sqrt{3}\hat{i}+\hat{j}$ is $\sqrt{3}$ and if $\alpha = 2+\sqrt{3}\beta$, the possible value of $\mid \alpha \mid$ is/are	a) 1
2. Let a and b be real numbers such that the function $f(x) = \begin{cases} -3ax^2-2, & x<1 \\ bx+a^2, & x\geq 1 \end{cases}$ is differentiable for all $x \in R$ Then possible value of a is (are)	b) 2
3. Let $\mu \neq 1$ be a complex cube root of unity. If $\left(3-3\mu+2\mu^2\right)^{4n+3} + \left(2+3\mu-3\mu^2\right)^{4n+3} + \left(-3+2\mu+3\mu^2\right)^{4n+3} = 0$ then possible value(s) of n is (are)	c) 3
4. Let the harmonic mean of two possitive real numbers a and b be 4. If q is a positive real number such that a,5,q,b is an arithmetic progression, then the value(s) of $\mid q-a \mid$ is (are)	d) 4
5.	e) 5

11. match the following: (2015)

column-I	column-II

1. In a triangle $\triangle XYZ$, let a,b and c be the length of the sides opposite to the angles X,Y and Z respectively. If $2\left(a^2 - b^2\right) = 1$ and $\lambda = \frac{\sin X - Y}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda) = 0$ is(are)

2. In a triange $\triangle XYZ$, let a,b and c be length of the sides opposite to the angles X,Y and Z respectively. If $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$, then possible value(s) of $\frac{a}{b}$ is (are)

3. In R^2, let $\sqrt{3}\hat{\imath} + \hat{\jmath}, \hat{\imath} + \sqrt{3}\hat{\jmath}$ and $\beta\hat{i} + ([)\, 1 - \beta\hat{j}$ be position vectors of X,Y and Z with respect to origin O,respectively . If the distance of Z from the bisector of the acute angle of **OX** with **OY**is $\frac{3}{\sqrt{2}}$, then possible value(s) of $\mid \beta \mid$ is (are)

4. Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0, x = 2, y^2 = 4x$ and $y = \mid \alpha x - 1 \mid + \mid \alpha x - 2 \mid + \alpha x$ where $\alpha \in 0, 1$. Then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$, when $\alpha = 0$ and $\alpha = 1$ is(are)

5.

a) 1

b) 2

c) 3

d) 5

e) 6

F : Comprehension Based Questions

Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

1. The unit vector perpendicular to both L_1 and L_2 is (2008)

   (a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$

   (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{\sqrt[5]{3}}$

   (c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{\sqrt[5]{3}}$

   (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

2. The shortest distance between L_1 and L_2 is (2008)

   (a) $\frac{17}{\sqrt{3}}$

   (b) 0

   (c) $\frac{41}{\sqrt[5]{3}}$

   (d) $\frac{17}{\sqrt[5]{3}}$

3. The distance of the point(1,1,1) from the plane passing through the point(-1,-2,-1) wwhose normal is perpendicular to both the lines L_1 and L_2 is (2008)

   (a) $\frac{2}{\sqrt{75}}$

   (b) $\frac{7}{\sqrt{75}}$

   (c) $\frac{13}{\sqrt{75}}$

   (d) $\frac{22}{\sqrt{75}}$

H : Assertion And Reason Type Questions

1. Consider the plane $3x-6y-2z = 15$ and $2x+y-2z = 5$. STATEMENT-1: The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$,$y = 1 + 2t$ and $z = 15t$ because STATEMENT-2: The vector $14i + 2j + 15k$ is parallel to the line of intersection of given planes. (2007)

   (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

   (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

   (c) Statement-1 is True, Statement-2 is False

   (d) Statement-1 is False, Statement-2 is True.

2. Let the vectors $\mathbf{PO}, \mathbf{OR}, \mathbf{RS}, \mathbf{ST}, \mathbf{TU}$ and $\mathbf{JP}$ represent the sides ofa regular hexagon. STATEMENT-1: $\mathbf{PQ} \times (\mathbf{RS} + \mathbf{ST}) \neq \mathbf{0}$.because STATEMENT-2: $\mathbf{PQ} \times \mathbf{RS} = \mathbf{0}$ and $\mathbf{PQ} \times \mathbf{ST} \neq \mathbf{0}$ (2007)

   (a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.

   (b) Statement-1 is True, Statement-2 is True, Statement2 is NOT a correct explanation for Statement-1

   (c) statement-1 is True, Statement-2 is False

   (d) Statement-1 is False, Statement-2 is True.

3. Consider three planes $P_1 : x - y + z = 1$ $x + y - z = 1$ $x - 3y + 3z = 2$ let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3,P_3 and P_1 , P_2 and P_1 respectively.

STATEMENT-1Z: At least two of the lines L_1, L_2 and L_3 are non-parallel and

STATEMENT-2: The three planes dose not have a common point. (2008)

(a) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT - 2 is a correct explanation for STATEMENT- 1

(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation for STATEMENT- 1

(c) STATEMENT-1 is True, STATEMENT -2 is False

(d) STATEMENT - 1 is False, STATEMENT-2 is True

I : Integer Value Correct Type

1. If $\mathbf{a}$ and $\mathbf{b}$ are vectors in space given by $\mathbf{a} = \frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\mathbf{b} = \frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$,then find the value of $(2\mathbf{a}+2\mathbf{b}) \cdot [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} - 2\mathbf{b})]$.(2010)

2. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$ then find $\mid d \mid$.(2010)

3. Let $\mathbf{a} = -\hat{i} - \hat{k}, \mathbf{b} = -\hat{i} + \hat{j}$ and $\mathbf{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$, then the value

of $\mathbf{r} \cdot \mathbf{b}$ is (2011)

4. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit vectors satisfying $\mid \mathbf{a} - vecb \mid^2 + \mid \mathbf{b} - vecc \mid^2 + \mid$ $+ \mid \mathbf{c} - veca \mid^2 + \mid = 9$, then $2\mathbf{a} + 5\mathbf{b} + 5\mathbf{c}$ is (2012)

5. Consider the set of eight vectors $V = a\hat{i} + b\hat{j} + c\hat{k}; a, b, c \in -1, 1$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is (2013)

6. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $K - 20 =$ is (2013)

7. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} = \overrightarrow{p\vec{a}} + \overrightarrow{q\vec{b}} + \overrightarrow{r\vec{c}}$, where p,q and r are scalars then the value of $\frac{p^2+2q^2+r^2}{q^2}$is (2014)

8. Suppose that $\mathbf{p}, \mathbf{q}$ and $\mathbf{r}$ are three non-coplanar unit vectors i R^3.Let the components of vector $\mathbf{s}$ along $\mathbf{p}, \mathbf{q}$ and $\mathbf{r}$ be 4,3 and 5 respectively. If the components of this vector $\mathbf{s}$ along $(-\mathbf{p} + \mathbf{q} + \mathbf{r})$,$(\mathbf{p} - \mathbf{q} + \mathbf{r})$ and $(-\mathbf{p} - \mathbf{q} + \mathbf{r})$ are x,y and z respectively, then the value of $2x + y + z$ is (2015)

9. Let $\mathbf{a}$ and $\mathbf{b}$ be two unit vectors such that $\mathbf{a} \cdot \mathbf{b} = 0$. For some $x, y \in R$, let $\mathbf{c} = x\mathbf{a} + y\mathbf{b} + (\mathbf{a} \times \mathbf{b})$. If $\mathbf{c} = 2$ and the vector $\mathbf{c}$ is inclined at the same angle α to both $\mathbf{a}$ and $\mathbf{b}$ then the value of $8\cos^2\alpha$ is ....(2018)

10. Let P be a point in the first octant, whose image Qin the plane $x + y =$ 3(that is,the line segment PQ is perpendicular to the plane $x + y = 3$

end the mid-point of PQ lies in the planr $x = y = 3$)lies on the z-axis.Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane,then the length of PR is.....(2018)

11. Consider the cube in the first octant with sides OP,OQ and OR of length 1, along the a-axis and z-axis,respectively,where $O(0,0,0)$ is the origin. Let $S\left[\frac{1}{2},\frac{1}{2},\frac{1}{2}\right]$ be the center of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\mathbf{p} = \overrightarrow{SP}, \mathbf{q} = \overrightarrow{SQ}, \mathbf{r} = \overrightarrow{SR}$ and $\mathbf{t} = \overrightarrow{ST}$, then the value of $\mid (\mathbf{p} \times \mathbf{q}) \times (\mathbf{r} \times \mathbf{t}) \mid$ is ......(2018)

12. Three lines are given by $\mathbf{r} = \lambda\hat{i}, \lambda \in R; \mathbf{r} = \mu\left(\hat{i}+\hat{j}\right), \mu \in R$ and $\mathbf{r} = \gamma\left(\hat{i}+\hat{j}+\hat{k}, \gamma \in R\right)$. Let the lines cuts the plane $x+y+z=1$ at the points A,B and C respectively. If the area of the triangle ABC is $\triangle$ then the value of $\left(6\triangle^2\right)$ equals ......(2019)

13. Let $\mathbf{a} = 2\hat{i}+\hat{j}-\hat{k}$ and $\mathbf{b} = \hat{i}+2\hat{j}-\hat{k}$ be two vectors. Consider a vector $\mathbf{c} = \alpha\mathbf{a}+\beta\mathbf{b}, \alpha, \beta \in R$. If the projection of $\mathbf{c}$ on the vector $(\mathbf{a}-\mathbf{b})$ is $\sqrt[3]{2}$, then the minimum vakue of $\left[\mathbf{c}-(\mathbf{a}\times\mathbf{b})\right]\mathbf{c}$ equals.........(2019)

Section-B [JEE Advanced/IIT-JEE]

A : Fill in the Blanks

1. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$ is (2002)

(a) $x - y + z = 1$

(b) $x + y + z = 5$

(c) $x + 2y - z = 1$

(d) $2x - y + z = 5$

2. If $|\,\mathbf{a}\,| = 4, |\,\mathbf{b}\,| = 2$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$ then $(\mathbf{a} \times \mathbf{b})^2$ is equal to (2002)

(a) 48

(b) 16

(c) $\mathbf{a}$

(d) None of these

3. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors such that $\begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} = 4 \end{bmatrix}$ then $\begin{bmatrix} \mathbf{a} \times \mathbf{b} & \mathbf{b} \times \mathbf{c} & \mathbf{c} \times \mathbf{a} \end{bmatrix} =$

(a) 16

(b) 64

(c) 4

(d) 8

4. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors show that $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ and $|\,\mathbf{a}\,| = 7, |\,\mathbf{b}\,| = 5, |\,\mathbf{c}\,| = 3$ then angle between vector $\mathbf{b}$ and $\mathbf{c}$ is (2002)

(a) $60°$

(b) $30°$

(c) $45°$

(d) $90°$

5. If $| \mathbf{a} |= 5, | \mathbf{b} |= 4, | \mathbf{c} |= 3$ thus what will be the value of $| \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} |$, given that $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$

   (a) 25

   (b) 50

   (c) -25

   (d) -50

6. If the vector $\mathbf{c}, \mathbf{a} = x\hat{i} + y\hat{j} + z\hat{k}$,$\mathbf{b} = \hat{j}$ are such that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ form a right handed system then $\mathbf{c}$ is

   (a) $z\hat{i} - x\hat{k}$

   (b) $\mathbf{0}$

   (c) $y\hat{j}$

   (d) $-z\hat{i} + x\hat{k}$

7. $\mathbf{a} = 3\hat{i} - 5\hat{j}$ and $\mathbf{b} = 6\hat{i} + 3\hat{j}$ are two vectors and $\mathbf{c}$ is a vector such that $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ then $| \mathbf{a} |:| \mathbf{b} |:| \mathbf{c} |$(2002)

   (a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$

   (b) $\sqrt{34} : \sqrt{45} : 39$

   (c) 34:39:45

   (d) 39:35:34

8. IF $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}$ then $\mathbf{a} + \mathbf{b} + \mathbf{c} =$

   (a) abc

(b) -1

(c) 0

(d) 2

9. The d.r. of normal to the plane through $(1,0,0),(0,1,0)$ which makes an angle$\frac{\pi}{4}$ with plane $x+y=3$ are (2002)

   (a) 1,$\sqrt{2}$,1

   (b) 1,1,$\sqrt{2}$

   (c) 1,1,2

   (d) $\sqrt{2}$,1,1

10. Let $\mathbf{u}=\hat{\imath}+\hat{\jmath}$,$\mathbf{v}=\hat{\imath}-\hat{\jmath}$ and $\mathbf{w}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\mathbf{u}\cdot\hat{n}=0$ and $\mathbf{v}\cdot\hat{n}=0$ then $\mid \mathbf{w}\cdot\hat{n} \mid$ is equals to (2003)

    (a) 3

    (b) 0

    (c) 1

    (d) 2

11. A particle acted on by constant forces $4\hat{\imath}+\hat{\jmath}-3\hat{k}$ and $3\hat{\imath}+\hat{\jmath}-\hat{k}$ is displaced from the point $\hat{\imath}+2\hat{\jmath}-3\hat{k}$to the point $5\hat{\imath}+4\hat{\jmath}+\hat{k}$. The total work done by the force is (2003)

    (a) 50 units

    (b) 20 units

    (c) 30 units

(d) 40 units

12. The vector $\overrightarrow{AB} = 3\hat{i} + 4\hat{k} \& \overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of triangle ABC. The length of the median through A is (2003)

(a) $\sqrt{288}$

(b) $\sqrt{18}$

(c) $\sqrt{72}$

(d) $\sqrt{33}$

13. The shortest distance from the plane $12x+4y+3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(a) 39

(b) 26

(c) $11\frac{4}{13}$

(d) 13

14. The two lines $x = ay + b$, $z = cy + d$ and $x = ay + b$, $z = cy + a$ will be perpendicular, if and only if (2003)

(a) $aa^{'} + cc^{'} + 1 = 0$

(b) $aa^{'} + bb^{'} + cc^{'} + 1 = 0$

(c) $aa^{'} + bb^{'} + cc^{'} = 0$

(d) $\left(a + a^{'}\right)\left(b + b^{'}\right) + \left(c + c^{'}\right)$

15. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{1} = \frac{z-5}{1}$ are coplanar if (2003)

(a) k=3 or -2

(b) k= 0 or -1

(c) k= 1 or -1

(d) k=0 or -3

16. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three vectors such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=0, \mid \mathbf{a} \mid = 1, \mid \mathbf{b} \mid = 2$ and $\mid \mathbf{c} \mid = 3$ then $\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}$ is equal to (2003)

    (a) 1

    (b) 0

    (c) -7

    (d) 7

17. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ (2003)

    (a) 4

    (b) 1

    (c) 2

    (d) 3

18. A tetrahedron has vertices at$O(0,0,0)$, $A(1,2,1)B(2,1,3)$ and $C(-1,1,2)$. Then the angle between the faces OAB and ABC will be (2003)

    (a) $90°$

    (b) $\cos^{-} 1\frac{19}{35}$

    (c) $\cos^{-} 1\frac{17}{31}$

(d) 30°

19. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$
and vectors $(1,a,a^2)$,$(1,b,b^2)$ and $(1,c,c^2)$ are non-coplanar, then the product abc equals (2003)

(a) 0

(b) 2

(c) -1

(d) 1

20. Consider points A,,B,C and D with position vectors $7\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$,$\hat{\imath} - 6\hat{\jmath} + 10\hat{k}$,$-\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and $5\hat{\imath} - \hat{\jmath} + \hat{k}$ respectively. Then ABCD is a (2003)

(a) parllelogram but not a rhombus

(b) square

(c) rhombus

(d) rectangle

21. If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are three non-planar vectors then $(\mathbf{u}+\mathbf{v}-\mathbf{w})\cdot(\mathbf{u}-\mathbf{w})\times(\mathbf{v}-\mathbf{w})$ equals (2003)

(a) $3\mathbf{u}\cdot\mathbf{v}\times\mathbf{w}$

(b) 0

(c) $\mathbf{u}\cdot\mathbf{v}\times\mathbf{w}$

(d) $3\mathbf{u} \cdot \mathbf{w} \times \mathbf{v}$

22. Two system of rectangular axes have the same origin. If a plane cuts them at distances a,b,c and $a^{'}, b^{'}, c^{'}$ from the origin then (2003)

    (a) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

    (b) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = 0$

    (c) $\frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

    (d) $\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

23. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is (2004)

    (a) $\frac{5}{2}$

    (b) $\frac{9}{2}$

    (c) $\frac{7}{2}$

    (d) $\frac{3}{2}$

24. A line with dircction cosines proportional to 2, 1,2 meets each of the lines each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of each of the points of intersection is given by (2004)

    (a) $(2a, 3a, 3a), (2a, a, a)$

    (b) $(3a, 2a, 2a), (a, a, a)$

    (c) $(3a, 2a, 3a), (a, a, 2a)$

    (d) $(3a, 3a, 3a), (a, a, a)$

25. If the straight lines $x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$ and $x = \frac{t}{2}, y = 1+t, z = 2-t$ with parameters s and t respectively, are co-planar then λ is (2004)

(a) 0

(b) -1

(c) $-\frac{1}{2}$

(d) -2

26. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of the sphere and the plane (2004)

(a) $2x - y - z = 1$

(b) $x - 2y - z = 1$

(c) $x - y - 2z = 1$

(d) $x - y - z = 1$

27. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are three non-zero coplanar vectors such that no two of these are collinear. I the vector $\mathbf{a} + 2\mathbf{b}$ is collinear with $\mathbf{c}, \mathbf{b} + 3\mathbf{c}$ is collinear with $\mathbf{a}$(λ being some non-zero scalar then $\mathbf{a} + 2\mathbf{b} + 6\mathbf{c}$ is equals (2004)

(a) 0

(b) $\lambda\mathbf{a}$

(c) $\lambda\mathbf{b}$

(d) $\lambda \mathbf{c}$

28. A particles is acted upon by constant force $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$.The work done in standard units by the forces is given by (2004)

    (a) 15

    (b) 30

    (c) 25

    (d) 40

29. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non coplanar vectors and λ is a real number then the vectors $\mathbf{a}+2\mathbf{b}+3\mathbf{c}$ $\lambda\mathbf{b}+4\mathbf{c}$ and $(2\lambda-1)\,\mathbf{c}$ are non-coplanar for(2004)

    (a) No value for λ

    (b) All expect one value for λ

    (c) All expect two value for λ

    (d) ALL values of λ

30. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be such that $\mid \mathbf{u} \mid = 1, \mid \mathbf{v} \mid = 2$ and $\mid \mathbf{w} \mid = 3$. If the projection $\mathbf{v}$ along $\mathbf{u}$ is equal to that of $\mathbf{w}$ along $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{w}$ are perpendicular to each other then $\mathbf{u}-\mathbf{v}+\mathbf{w}$ equals(2004)

    (a) 14

    (b) $\sqrt{7}$

    (c) $\sqrt{14}$If the pane

    (d) 2

31. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-zero vectors such that $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3} \mid \mathbf{b} \mid\mid \mathbf{c} \mid \mathbf{a}$. If θ is the acute angle between vectors $\mathbf{b}$ and $\mathbf{c}$ then $\sin\theta$ equals (2004)

(a) $\frac{2\sqrt{2}}{3}$

(b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$

32. If C is the mid-point of AB and P is any point outside AB, then (2005)

(a) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$

(b) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

(c) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = 0$

(d) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 0$

33. If the angle θ between the lines $\frac{x+1}{2} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ then the value of λ is (2005)

(a) $\frac{5}{3}$

(b) $\frac{-3}{5}$

(c) $\frac{3}{4}$

(d) $\frac{-4}{3}$

34. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is (2005)

(a) $0°$

(b) $90°$

(c) $45°$

(d) $30°$

35. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid-point of line joining the center of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals (2005)

(a) -1

(b) 1

(c) -2

(d) 2

36. The distance between the line $\mathbf{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and the plane $\mathbf{r} \cdot \left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5(2005)$

(a) $\frac{10}{9}$

(b) $\frac{10}{3\sqrt{3}}$

(c) $\frac{3}{10}$

(d) $\frac{10}{3}$

37. For any vector $\mathbf{a}$, the value of $\left(\mathbf{a} \times \hat{i}^2\right) + \left(\mathbf{a} \times \hat{j}^2\right) + \left(\mathbf{a} \times \hat{k}^2\right)$ is equal to (2005)

(a) $3a^-2$

(b) a^-2

(c) $2a^-2$

(d) $4a^-2$

38. If non-zero numbers a, b, c are in H.P., then the straight line. $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is (2005)

(a) $(-1,2)$

(b) $(-1,-2)$

(c) $(-1,-2)$

(d) $(1,-\frac{1}{2})$

39. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}, c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is (2005)

(a) the Geometric Mean of a and b

(b) the Arithmetic Mean of a and b

(c) equal to zero

(d) the Harmonic Mean of a and b

40. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar vectors and λ is real number then $\left[\lambda\left(\mathbf{a}+\mathbf{b}\right) \quad \lambda^2\mathbf{b} \quad \lambda\mathbf{c}=\right]$ $\left[\mathbf{a} \quad \mathbf{b}+\mathbf{c} \quad \mathbf{b}\right]$ for (2005)

(a) exactly one value of λ

(b) no value λ

(c) exactly three value of λ

(d) exactly two value of λ

41. Let $\mathbf{a}=\hat{i}-\hat{k}, \mathbf{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $\mathbf{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$. Then $[\mathbf{a},\mathbf{b},\mathbf{c}]$ depends on (2005)

(a) only y

(b) only x

(c) both x and y

(d) neither x nor y

42. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius (2005)

(a) 3

(b) 1

(c) 2

(d) $\sqrt{2}$

43. If $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are any three vectors such that $\mathbf{a} \cdot \mathbf{b} \neq =, \mathbf{b} \cdot \mathbf{c} \neq =$ then $\mathbf{a}$ and $\mathbf{c}$ are (2005)

(a) inclined at an angle of $\frac{\pi}{3}$ between them

(b) inclined at an angle of $\frac{\pi}{6}$ between them

(c) perpendicular

(d) parallel

44. The values of a, for which points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a rightangled triangle with $c = \frac{\pi}{2}$ are (2005)

(a) 2 and 1

(b) -2 and -1

(c) -2 and 1

(d) 2 and -1

45. The two lines $x = ay + b$,$z = cy + d$ and $x = a'y + b', z = c'y + d'$ are perpendicular to each other if (2006)

(a) $aa' + cc' = -1$

(b) $aa' + cc' = 1$

(c) $\frac{a}{a'} + \frac{c}{c'} = -1$

(d) $\frac{a}{a'} + \frac{c}{c'} = 1$

46. The image of the point (-1, 3,4) in the plane $x - 2y = 1$ is (2006)

(a) $\left(\frac{-17}{3}, \frac{-19}{3}, 4\right)$

(b) (15,11,4)

(c) $\left(\frac{-17}{3}, \frac{-19}{3}, 1\right)$

(d) None of these

47. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y- axis, then the angle that the line makes with the positive direction of the z-axis is (2006)

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{3}$

48. If $\mathbf{u}$ and $\mathbf{v}$ are unit vectors and θ is the acute angle between them, then $2\mathbf{u} \times 3\mathbf{v}$ is a unit vector for (2007)

   (a) no value of θ

   (b) exactly one value of θ

   (c) exactly two value of θ

   (d) more than two values of θ

49. If (2.3, 5) is one end of a diameter of the sphere $x^2+y^2+z^2-6x-12y-2z+20=0$ then the cooordinates of the other end of the diameter are (2007)

   (a) (4,3,5)

   (b) (4,3,-3)

   (c) (4,9,-3)

   (d) (4,-3,3)

50. Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}, \mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}, \mathbf{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vectors lies in the plane of $\mathbf{a}$ and $\mathbf{b}$, then x equals (2007)

   (a) -4

   (b) -2

   (c) 0

   (d) 1

51. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals (2007)

(a) 1

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$

(d) $\frac{1}{2}$

52. The vector $\mathbf{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\mathbf{b} = \hat{i} + \hat{j}$ and $\mathbf{c} = \hat{j} + \hat{k}$ and bisects the angle between $\mathbf{b}$ and $\mathbf{c}$. Then which one of the following gives possible values of α and β ? (2008)

(a) $\alpha = 2, \beta = 2$

(b) $\alpha = 1, \beta = 2$

(c) $\alpha = 2, \beta = 1$

(d) $\alpha = 1, \beta = 1$

53. The non-zero Vectors are $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are related by $\mathbf{a} = 8\mathbf{b}$ and $\mathbf{c} = -7\mathbf{b}$. Then the angle between $\mathbf{a}$ and $\mathbf{c}$ is (2008)

(a) 0

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) π

54. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left[0, \frac{17}{2}, \frac{-13}{2}\right]$. Then (2008)

(a) a=2,b=8

(b) a=4,b=6

(c) a=6,b=4

(d) a=8,b=2

55. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to (2008)

(a) -5

(b) 5

(c) 2

(d) -2

56. Let the line $\frac{x-2}{3}=\frac{y-2}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$ then (α,β) equals (2009)

(a) (-6,7)

(b) (5,-15)

(c) (-5,5)

(d) (6,-17)

57. The projections of a vector on the three coordinate axis are 6,-3, 2 respectively. The direction cosines of the vector are:(2009)

(a) $\frac{6}{5},\frac{-3}{5},\frac{2}{5}$

(b) $\frac{6}{7},\frac{-3}{7},\frac{2}{7}$

(c) $\frac{-6}{7},\frac{-3}{7},\frac{2}{7}$

(d) -6,-3,2

58. If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are non-coplanar vectors and p, q are real numbers,then the equality $\begin{bmatrix} 3\mathbf{u} & p\mathbf{v} & p\mathbf{w} \end{bmatrix}$- $\begin{bmatrix} p\mathbf{v} & w\mathbf{v} & q\mathbf{u} \end{bmatrix}$- $\begin{bmatrix} 2\mathbf{w} & q\mathbf{u} & q\mathbf{v} \end{bmatrix}$=0 holds for (2009)

(a) exactly two values of (p,q)

(b) more than two but, not all values of (p,q)

(c) all values of (p,q)

(d) exactly one values of (p,q)

59. Let $\mathbf{a} = \hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\mathbf{b}$ satisfying $\mathbf{a} \times \mathbf{b} + \mathbf{c} = 0$ and $\mathbf{a} \cdot \mathbf{c} = 3$ (2010)

(a) $2\hat{i} - \hat{j} + 2\hat{k}$

(b) $\hat{i} - \hat{j} - 2\hat{k}$

(c) $\hat{i} + \hat{j} - 2\hat{k}$

(d) $-\hat{i} + \hat{j} - 2\hat{k}$

60. If the vectors $\mathbf{a} = \hat{i} - \hat{j} + 2\hat{k}$,$\mathbf{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\mathbf{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutualy orthogonal, then (λ, μ) =(2010)

(a) (2,-3)

(b) (-2,3)

(c) (3,-2)

(d) (-3,2)

61. Statement-1:The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$. (2010)

(a) Statement -1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is false.

(c) Statement -1 is false, Statement-2 is true.

(d) Statement - 1 is true, Statement 2 is true; Statement-2 is a correct explanation for Statement-1.

62. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive Z-axis, then θ equals (2010)

(a) 45°

(b) 60°

(c) 75°

(d) 30°

63. Ifthe angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x+2y+z = 4$ is $\cos^{-} 1\sqrt[5]{14}$, then λ equals (2011)

(a) $\frac{3}{2}$

(b) $\frac{2}{5}$

(c) $\frac{5}{3}$

(d) $\frac{2}{3}$

64. If $\mathbf{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\mathbf{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $(2\mathbf{a} - \mathbf{b})\ [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} + 2\mathbf{b})]$ is (2011)

(a) -3

(b) 5

(c) 3

(d) -5

65. The vectors $\mathbf{a}$ and $\mathbf{b}$ are not perpendicular and $\mathbf{c}$ and $\mathbf{d}$ are two vectors satisfying $\mathbf{b} \times \mathbf{c} = \mathbf{b} \times \mathbf{d}$ and $\mathbf{a} \cdot \mathbf{d} = 0$. Then the vector $\mathbf{d}$ is equal to (2011)

(a) $\mathbf{c} + \left[\frac{\mathbf{a \cdot c}}{\mathbf{a \cdot b}}\right] \mathbf{b}$

(b) $\mathbf{b} + \left[\frac{\mathbf{b \cdot c}}{\mathbf{a \cdot b}}\right] \mathbf{c}$

(c) $\mathbf{c} - \left[\frac{\mathbf{a \cdot c}}{\mathbf{a \cdot b}}\right] \mathbf{b}$

(d) $\mathbf{b} - \left[\frac{\mathbf{b \cdot c}}{\mathbf{a \cdot b}}\right] \mathbf{c}$

66. Statement-1: The point A(1,0, 7)) is the mirror image of the point B(1, 6, 3) in the line : $\frac{x}{2} = \frac{y-1}{2} = \frac{z-2}{3}$
Statement-2: The line $\frac{x}{2} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A(1, 0,7) and B(1, 6, 3). (2011)

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

67. Let **a** and **b** be two unit vectors. If the vectors $\mathbf{c} = \hat{a} + 2\hat{b}$ and $\mathbf{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is: (2012)

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{4}$

68. A equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is: (2012)

(a) $x - 2y + 2z - 3 = 0$

(b) $x - 2y + 2z + 1 = 0$

(c) $x - 2y + 2z - 1 = 0$

(d) $x - 2y + 2z + 5 = 0$

69. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to: (2012)

(a) -1

(b) $\frac{2}{9}$

(c) $\frac{9}{2}$

(d) 0

70. Let ABCD be a parallelogram such that $\overrightarrow{AB} = q$ and $\overrightarrow{AD} = p$and $\angle BAD$ an acute angle. If **r** is the vector that coincides with the altitude directed from the vertex B to the side AD, then **r** is given by : (2012)

(a) $\mathbf{r} = 3\mathbf{q} - \frac{3(\mathbf{q}\cdot\mathbf{p})}{(\mathbf{p}\cdot\mathbf{p})}\mathbf{p}$

(b) $\mathbf{r} = -\mathbf{q} + \frac{3(\mathbf{q}\cdot\mathbf{p})}{(\mathbf{p}\cdot\mathbf{p})}\mathbf{p}$

(c) $\mathbf{r} = \mathbf{q} - \frac{3(\mathbf{q}\cdot\mathbf{p})}{(\mathbf{p}\cdot\mathbf{p})}\mathbf{p}$

(d) $\mathbf{r} = -3\mathbf{q} - \frac{3(\mathbf{q}\cdot\mathbf{p})}{(\mathbf{p}\cdot\mathbf{p})}\mathbf{p}$

71. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is (2013)

(a) $\frac{3}{2}$

(b) $\frac{5}{2}$

(c) $\frac{7}{2}$

(d) $\frac{9}{2}$

72. f the line $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have (2013)

(a) any value

(b) exactly one value

(c) exactly two values

(d) exactly three values

73. If the vectors $\overrightarrow{AB} = 3\hat{\imath} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is 2013)

(a) $\sqrt{18}$

(b) $\sqrt{72}$

(c) $\sqrt{33}$

(d) $\sqrt{45}$

74. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line : (2014)

(a) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

(b) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

(c) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(d) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

75. The angle between the lines whose direction cosines satisfy the equation $l + m + n = 0$ and $l^2 = m^2 + n^2$ is (2014)

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{4}$

76. If $\begin{bmatrix} \mathbf{a}\times\mathbf{b} & \mathbf{b}\times\mathbf{c} & \mathbf{c}\times\mathbf{a} \end{bmatrix} = \lambda \begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{bmatrix}$ then λ is equal to (2014)

(a) 0

(b) 1

(c) 2

(d) 3

77. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be three non-zero vectors such that no two ofthem are collinear and $(\mathbf{a}\times\mathbf{b})\times\mathbf{c} = \frac{1}{3} \mid \mathbf{b} \mid\mid \mathbf{c} \mid \mathbf{a}$.If θ is angle between vectors $\mathbf{b}$ and $\mathbf{c}$, then a value of$\sin\theta$ is: (2015)

(a) $\frac{2}{3}$

(b) $\frac{-\sqrt[2]{3}}{3}$

(c) $\frac{\sqrt[2]{2}}{3}$

(d) $\frac{-\sqrt{2}}{3}$

78. The equation of the plane containing the line $2x - y + z = 3$ and $x + y + 4z = 5$, and parallel to the plane,$x + 3y + 6z = 1$ is :(2015)

(a) $x + 3y + 6z = 7$

(b) $2x + 6y + 12z = -13$

(c) $2x + 6y + 12z = 13$

(d) $x + 3y + 6z = -7$

79. The distance of the point (1, 0, 2) from the point of intesection of the lines $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + x = 16$ is :(2015)

(a) $\sqrt[3]{21}$

(b) 13

(c) $\sqrt[2]{14}$

(d) 8

80. If the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-4}{3}$ lies in the plane $lx + my - z = 9$ then $l^2 + m^2$ is equal to (2016)

(a) 5

(b) 2

(c) 12

(d) 18

81. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be three unit vectors such that $\mathbf{a}\times\ [\mathbf{b}\times]\ \mathbf{c} = \frac{\sqrt{3}}{2}\ [\mathbf{b}+\mathbf{c}]$. If $\mathbf{b}$ is not parallel to $\mathbf{c}$ then the angle between $\mathbf{a}$ and $\mathbf{c}$ is (2016)

(a) $\frac{2\pi}{3}$

(b) $\frac{5\pi}{6}$

(c) $\frac{3\pi}{4}$

(d) $\frac{\pi}{2}$

82. The distance of the point (1, -5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is: (2016)

(a) $\frac{10}{\sqrt{3}}$

(b) $\frac{20}{3}$

(c) $\sqrt[3]{10}$

(d) $\sqrt[10]{3}$

83. Let $\mathbf{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\mathbf{b} = \hat{\imath} + \hat{\jmath}$ and $\mathbf{c}$ be a vectors such that $\mid \mathbf{c} - \mathbf{a} \mid = 3, \mid\ [\mathbf{a}\times\mathbf{b}]\times\mathbf{c}\mid = 3$ and the angle between $\mathbf{c}$ and $\mathbf{a}\times\mathbf{b}$ is $30°$. Then $\mathbf{a}\cdot\mathbf{c}$ is equal to :(2017)

(a) $\frac{1}{8}$

(b) $\frac{25}{8}$

(c) 2

(d) 5

84. If the image of the point P(1, -2, 3) in the plane,$2x+3y-4z+22=0$ measured parallel to line,$\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to:(2017)

(a) $\sqrt[6]{5}$

(b) $\sqrt[3]{5}$

(c) $\sqrt[2]{42}$

(d) $\sqrt{42}$

85. The distance of the point (1, 3, -7) from the plane passing through the point (1, -1, -1), having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$ is :(2017)

(a) $\frac{10}{\sqrt{74}}$

(b) $\frac{20}{\sqrt{74}}$

(c) $\frac{10}{\sqrt{83}}$

(d) $\frac{5}{\sqrt{83}}$

86. let $\mathbf{u}$ be a vector coplanar with the vectors $\mathbf{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}$ and $\mathbf{b}=\hat{\jmath}+\hat{k}$. If $\mathbf{u}$ is perpendicular to $\mathbf{a}$ and $\mathbf{u}\cdot\mathbf{b}=24$, then $|\ \mathbf{u}\ |^2$ is equal to: (2018)

(a) 315

(b) 250

(c) 84

(d) 336

87. The length of the projection of the line segment joining the points (5, 1, 4) and (4, 1,3)on the plane,$x+y+z=7$ in: (2018)

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\sqrt{\frac{2}{3}}$

(d) $\frac{2}{\sqrt{3}}$

88. If Ł$_1$ is the line of intersetion of the planes $2x - 2y + 3z - 2 = 0$ and $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0,3x - y + 2z = 0$, then the distance of the origin from the plane,containing the lines Ł$_1$ and L_2 is : (2018)

(a) $\frac{1}{\sqrt[3]{2}}$

(b) $\frac{1}{\sqrt[2]{2}}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{1}{\sqrt[4]{2}}$

89. Let $\mathbf{a} = \hat{\imath} - \hat{\jmath}$,$\mathbf{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\mathbf{c}$ be a vector such that $\mathbf{a} \times \mathbf{c} + \mathbf{b} = \mathbf{0}$ and $\mathbf{a} \cdot \mathbf{c} = 4$, then $\mid \mathbf{c} \mid^2$ is equal to :(2018)

(a) $\frac{19}{2}$

(b) 9

(c) 8

(d) $\frac{17}{2}$

90. The equation of the line passing through (-4,3, 1), parallel to the plane $x + 2y - z - 5 - 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:(2018)

(a) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

(b) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(c) $\frac{x+4}{3} = \frac{y+3}{1} = \frac{z-1}{1}$

(d) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

91. The plane through the intersection of the planes $x + y + z - 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point: (2019)

    (a) (-3,0,-1)

    (b) (-3,1,-1)

    (c) (3,3,-1)

    (d) (3,2,-1)

92. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane,$x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is: (2019)

    (a) $\frac{\sqrt{5}}{2}$

    (b) $\sqrt[2]{5}$

    (c) $\frac{9}{2}$

    (d) $\frac{7}{2}$

93. A plane passing through the points (0, -1,0) and (9, 0, 1) and making an angle $\frac{\pi}{2}$ with the plane $y - z + 5 = 0$, also passes through the point: (2019)

    (a) (-$\sqrt{2}$,1,-4)

    (b) ($\sqrt{2}$,-1,4)

(c) (-$\sqrt{2}$,-1,-4)

(d) ($\sqrt{2}$,1,4)

94. Let $\alpha = 3\hat{i} + \hat{j}$ and $\beta = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\beta = \beta_{\mathbf{1}} - \beta_{\mathbf{2}}$, where β_1 is parallel to α and β_2 is perpendicular to α, then $\beta_1 \times \beta_2$ is equal to :(2019)

(a) $-3\hat{i} + 9\hat{j} + 5\hat{k}$

(b) $3\hat{i} - 9\hat{j} - 5\hat{k}$

(c) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

(d) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} - 5\hat{k}\right)$

Chapter 3

Circle

Section-A [JEE Advanced/IIT-JEE]

A : Fill in the Blanks

1. If A and B are points in the plane such that $\frac{PA}{PB} = K$(constant) for all P on a given circle, then the value of k cannot be equal to.......(1982)

2. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are..... and.....(1983)

3. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle. The radius of this circle is......(1984)

4. Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a circle. A pair of tangents from the point (4, 5) with a pair of radii form a quadrilaterall of area ....(1985)

5. From the origin chords are drawm to the circle$(x - 1)^2 + y^2 = 1$. The equation of the locus of the mid-points of these chords is ....(1985)

6. The equation of the line passing through the points of intersection of

the circles $3x^2+3y^2-2x+12y-9=0$ and $x^2+y^2+6x+2y-15=0$ is ....(1986)

7. From the point A(0, 3) on the circle $x^2+4x+(y-3)^2=0$, a chord AB is drawn and extended to a point M such that AM=2AB. The equation of the locus of M is.......(1986)

8. The area of the triangle formed by the tangents from the point (4, 3) to the the circle $x^2+y^2=9$ and the line joining their points of contact is.......(1987)

9. If the circle C:$x^2+y^2=16$ intersects anothere circle C_2 of radius 5 in such manner that common chord is of maximum length and has a slope equal to $\frac{3}{4}$,then the coordinates of the center of C_2 are........(1988)

10. The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2+y^2=4$ at $(1, \sqrt{3})$ is....... (1989)

11. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x-y+1=0$ and $x-2y+1=0$, then the value of λ=........(1991)

12. The equation of the locus of the mid-points of the circle $4x^2+4y^2-2x+4y+1=0$ that subtend an angle of $\frac{2\pi}{3}$ at its centre is......(1993)

13. The intercept on the line $y=xby$ the circle $x^2+y^2-2x=0$ is AB. Equation of the circle with AB as a diameter is......(1996)

14. For each natural number k, let C_k, denote the circle with radius k centimetres and centre at the origin. On the circle C_k, α-particle moves

k centimetres in the counter-clockwise direction. After completing its motion on C_k the particle moves to C_k+1 in the radial direction. The motion of the particle continues in this manner. The particle starts at (1,0). If the particle crosses the positive direction of the x-axis for the first time on the circle C_n. then n=.......(1997)

15. The chords of contact of the pair of tangents drawn from each point on the line $2x+y=4$ to circle$x^2+y^2=1$ pass through the point.....(1997)

B : True/False

1. No tangent can be drawn from the point $(\frac{5}{2},1)$ to the circumcircle of the triangle with vertices $(1,\sqrt{3})$,$(1,-\sqrt{3})$, $(3,-\sqrt{3})$.(1985)

2. The line $x+3y=0$ is a diameter of the circle $x^2+y^2-6x+2y=0$. (1989)

C : MCQ'S with One Correct Answer

1. A square is inscribed in the circle $x^2+y^2-2x+4y+3=0$. Its sides are parallel to the coordinate axes. The one vertex of the square is (1980)

   (a) $(1+\sqrt{2},-2)$

   (b) $(1-\sqrt{2},-2)$

   (c) $(1,-2+\sqrt{2})$

   (d) none of these

2. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1,1) is 1980)

   (a) $x^2 + y^2 - 6x + 4 = 0$

   (b) $x^2 + y^2 - 3x + 1 = 0$

   (c) $x^2 + y^2 - 4y + 2 = 0$

   (d) none of these

3. The centre ofthe circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is (1983)

   (a) $\left[\frac{-16}{5}, \frac{27}{10}\right]$

   (b) $\left[\frac{-16}{7}, \frac{53}{10}\right]$

   (c) $\left[\frac{-16}{5}, \frac{53}{10}\right]$

   (d) none of these

4. The equation of the circle passing through (1, 1) and the points of intersection of $x^2+y^2+13x-3y = 0$ and $2x^2+2y^2+4x-7y-25 = 0$ is (1983)

   (a) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$

   (b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

   (c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$

   (d) none of these

5. The locus of the mid-point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is (1984)

(a) $x + y = 2$

(b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = 2$

(d) $x + y = 1$

6. Ifa circle passes through the point (a, b) and cuts the circle $x^2+y^2 = k$ orthogonally, then the equation of the locus of ()

   (a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$

   (b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$

   (c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$

   (d) $x^2 + y^2 - 2ax - 3by + (a^2 + b^2 - k^2) = 0$

7. Ifthe two circles $(x-1)^2+(y-3)^2 = r^2$ and $x^2+y^2-8x+2y+8 = 0$ intersect in two distinct points, then (1989)

   (a) $2 < r < 8$

   (b) $r < 2$

   (c) $r = 2$

   (d) $r > 2$

8. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of acircleof area 154 sq. units. Then the equation of this circle is (1989)

   (a) $x^2 + y^2 + 2x - 2y = 62$

   (b) $x^2 + y^2 + 2x - 2y = 47$

   (c) $x^2 + y^2 - 2x + 2y = 47$

(d) $x^2 + y^2 - 2x + 2y = 62$

9. The centre of a circle passing through the points (0, 0),(1,0) and touching the circle $x2 + y2 = 9$ is (1992)

   (a) $\left[\frac{3}{2}, \frac{1}{2}\right]$

   (b) $\left[\frac{1}{2}, \frac{3}{2}\right]$

   (c) $\left[\frac{1}{2}, \frac{1}{2}\right]$

   (d) $\left[\frac{3}{2}, -2^{\frac{1}{2}}\right]$

10. The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation: (1993)

    (a) $x^2 - 6x - 10y + 14 = 0$

    (b) $x^2 - 10x - 6y + 14 = 0$

    (c) $y^2 - 6x - 10y + 14 = 0$

    (d) $y^2 - 10x - 6y + 14 = 0$

11. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if (1994)

    (a) $r < 2$

    (b) $r > 8$

    (c) $2 < r < 8$

    (d) $2 \leq r \leq 8$

12. The angle between a pair of tangents drawn from a point p to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is (1996)

(a) $x^2 + y^2 + 4x - 6y + 4 = 0$

(b) $x^2 + y^2 + 4x - 6y - 9 = 0$

(c) $x^2 + y^2 + 4x - 6y + 9 = 0$

(d) $x^2 + y^2 + 4x - 6y - 4 = 0$

13. If two distinct chords, drawn from the point (p,q) on the circle $x^2+y^2 = px + qy$ (where $pq \neq 0$) are bisected by the X-axis, then (1999)

(a) $p^2 = q^2$

(b) $p^2 = 8q^2$

(c) $p^2 < 8q^2$

(d) $p^2 > 8q^2$

14. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) and (4,3) respectively, then $\angle PQR$ is equal to (2000)

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{6}$

15. If the circles $x^2+y^2+2x+2khy+6 = 0, x^2+y^2+2ky+k = 0$ intersect orthogonally, then k is

(a) 2 or -$\frac{3}{2}$

(b) -2 or -$\frac{3}{2}$

(c) 2 or $\frac{3}{2}$

(d) -2 or $\frac{3}{2}$

16. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle PAB as P moves on the circle is (2001)

    (a) a parabola

    (b) a circle

    (c) a ellipse

    (d) a pair of straight lines

17. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals (2001)

    (a) $\sqrt{PQRS}$

    (b) $\frac{PQ+RS}{2}$

    (c) $\frac{2PQ \cdot RS}{PQ+RS}$

    (d) $\frac{\sqrt{PQ^2+RS^2}}{2}$

18. Ifthe tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets a straight line $5x - 2y + 6 = 0$ at a point Q on the y- axis,then the length of PQ is (2002)

(a) 4

(b) $\sqrt[2]{5}$

(c) 5

(d) $\sqrt[3]{5}$

19. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and$y^2 - 14y + 45 = 0$, is (2003)

    (a) (4,7)

    (b) (7,4)

    (c) (9,4)

    (d) (4,9)

20. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1),then the radius of the circle is (2004)

    (a) $\sqrt{3}$

    (b) $\sqrt{2}$

    (c) 3

    (d) 2

21. A circle is given by $x^2 + (y-1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is (2005)

    (a) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \leq 0\}$

    (b) $\{(x, y) : x^2 + (y-1)^2 = 4\} \cup \{(x, y) : y \leq 0\}$

    (c) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \leq 0\}$

(d) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \leq 0\}$

22. Tangents drawn from the point P(1,8) to the circle $x^2+y^2-6x-4y-11=0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is (2009)

    (a) $x^2 + y^2 + 4x - 6y + 19 = 0$

    (b) $x^2 + y^2 - 4x - 10y + 19 = 0$

    (c) $x^2 + y^2 - 2x + 6y - 29 = 0$

    (d) $x^2 + y^2 - 6x - 4y + 19 = 0$

23. The circle passing through the point (-1,0) and touchingthe y-axis at (0, 2) also passes through the point (2011)

    (a) $\left[-\frac{3}{2}, 0\right]$

    (b) $\left[-\frac{5}{2}, 2\right]$

    (c) $\left[-\frac{3}{2}, \frac{5}{2}\right]$

    (d) $[-4, 0]$

24. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x-5y = 20$ to the circle $x^2+y^2 = 9$ is (2012)

    (a) $20(x^2 + y^2) - 36x + 45y = 0$

    (b) $20(x^2 + y^2) + 36x - 45y = 0$

    (c) $36(x^2 + y^2) - 20x + 45y = 0$

    (d) $36(x^2 + y^2) + 20x - 45y = 0$

25. A line $y = mx + 1$ intersectrs the circle $(x-3)^2 + (y+2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct ? (2019)

   (a) $2 \leq m < 4$

   (b) $-3 \leq m < -1$

   (c) $4 \leq m < 6$

   (d) $6 \leq m < 8$

D : MCQ'S with One or More Than One Correct Answer

1. The equations of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are (1988)

   (a) x=0

   (b) y=0

   (c) $(h^2 - r^2)x - 2rhy = 0$

   (d) $(h^2 - r^2)x + 2rhy = 0$

2. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is (1998)

   (a) 0

(b) 1

(c) 3

(d) 4

3. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1), Q(x_2Y_2), R(x_3, y_3), S(x_4, y_4)$ then (1998)

   (a) $x_1 + x_2 + x_3 + x_4 = 0$

   (b) $y_1 + y_2 + y_3 + y_4 = 0$

   (c) $x_1x_2x_3x_4 = c^4$

   (d) $y_1y_2y_3y_4 = c^4$

4. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $\sqrt[2]{7}$ on y-axis is (are) (2013)

   (a) $x^2 + y^2 - 6x + 8y + 9 = 0$

   (b) $x^2 + y^2 - 6x + 7y + 9 = 0$

   (c) $x^2 + y^2 - 6x - 8y + 9 = 0$

   (d) $x^2 + y^2 - 6x - 7y + 9 = 0$

5. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then (2014)

   (a) radius of S is 8

   (b) radius of S is 7

   (c) Center of S is (-7,1)

   (d) Center of S is (-8,1)

6. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) (2016)

   (a) $\frac{1}{3}, \frac{1}{\sqrt{3}}$

   (b) $\frac{1}{4}, \frac{1}{2}$

   (c) $\frac{1}{3}, -\frac{1}{\sqrt{3}}$

   (d) $\frac{1}{4}, \frac{1}{2}$

7. Let Tbe the line passing through the points P(-2, 7) and Q(2.-5). Let F_1, be the set of allpairs of circles (S_1, S_2) such that T is tangent to S, at P and tangent to S_2, at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1, be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1, and passing through the point R(1, 1) be F_2, Let E_2, be the set of the mid-points of the line segments in the set F_2 Then, which of the following statement(s) is (are) TRUE? (2018)

   (a) The point (2,7) lies in E_1

   (b) The point $(\frac{4}{5}, \frac{7}{5})$ does NOT lies in E_2

   (c) The point $(\frac{1}{2}, 1)$ lies in E_2

   (d) The point $(0, \frac{3}{2})$ does NOT lies in E_1

E : Subjective Problems

1. Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point (5, 5).(1978)

2. Let A be the centre of the circlex $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that B(1,7) and D(4,-2) on the circle meet at the point C.(1981)

3. Find1 the area of the quadrilateral ABCD. Find the equations of the circle passing through 4,3) and touching the lines x+y=2 and x-y =2. (1982)

4. Through a fixed point (h, k) secants are drawn to the circle $x^2+y^2 = P$. Show that the locus of the mid-points of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$. (1983)

5. The abscissa of the two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with AB as diameter. (1984)

6. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_2, lies in the first quadrant, find the equation of the circle C_1, which is concentric with C_1 and cuts intercepts of length 8 on these lines the tangents at the points (1986)

7. Let a given line L_1, intersects the x and y axes at P and Q, respectively. Let another line L_2, perpendicular to L_1, cut the x and y axes at R

and S,respectively. Show that the locus of the point of intersection of the lines PS and OR is a circle passing through the origin. (1989)

8. The circlec $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{\frac{1}{2}} = 0$. Find k. (1987)

9. If $\left[m_i \frac{1}{m_i}\right], m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle ,then show that $m_1 m_2 m_3 m_4 = 1$ (1989)

10. A circle touches the line $y = x$ at a point P such that $OP = \sqrt[4]{2}$,, where O is the origin. The circle contains the point(-10,2) in its interior and the length of its chord on the line $x + y = 0$ is $\sqrt[6]{2}$. Determine the equation of the circle. (1990)

11. Two circles, cach of radius 5 units, touch each other at (1,2).If the equation of their common tangent is $4x + 3y = 10$, find the cquation of the circles. (1991)

12. Let a circle be given by $2x(x - a) + y(2y - b) = 0, (a \neq 0, b \neq 0))$.Find the condition on a and b if two chords,each biscected by the x- axis, can be drawn to the circle from $\left[a, \frac{b}{2}\right]$. (1992)

13. Consider a family of circles passing through two fixed points A(3,7) and B(6,5). Show that the chords in which the circle $x^2+y^2-4x-6y-3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinate of this point.

14. Find the coordinates of the point at which the circles $x^2+y^2-4x-2y=-4$ and $x^2+-12x-8y=-36$ touch each other. Also find equations common tangents touching the circles in the distinct points.

15. Find the intervals of values of a for which the line $y+x=0$ bisects two chords drawn from a point $\left[\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right]$ to the circle $2x^2+2y^2-(1+\sqrt{2}a)x+(1-\sqrt{2}a)y=0$. (1996)

16. A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively and the distance betwcen the point A and the mid-point of the line segment DC is d, prove that the area of the circle is $\frac{\pi d^2 \cos^2\alpha}{\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta\cos(\beta-\alpha)}$ (1996)

17. Let C be any circle with centre $(0,\sqrt{2})$. Prove that at the most two rational points can be there on C. (A rational point is a point both of whose coordinates are rational numbers) (1997)

18. C_1 and C_2 are two concentric circles, the radius of C_2 being twice of that C_1. From a point P on C_2, tangents PA and PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1.(1998)

19. Let T_1,T_2 be two tangents drawn from (-2,0) on to the circle C : $x^2+y^2=1$. Determine the circle touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time (1999)

20. Let $2x^2+y^2-3xy=0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA.(2001)

21. Let C_1 and C_2 be two circles with C_2 , lying inside C_1 A circle Clying inside C_1, touches C_1 internally and C_2 externally. Identify the locus of the centre of C . (2001)

22. For the circle $x^2 + y^2 = r^2$, find the value of r for which the area enclosed by the tangents drawn from the point P (6, 8) to the circle and the chord of contact is maximum.(2003)

23. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at (1,- 1) and cutting orthogonally the circle having line segment joining (0,3) and (-2,-1) as diameter.(2004)

24. Circles with radii 3,4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact (2005)

F : Match The Following

DIRECTIONS (Q.1): Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled 1, 2, 3 and 4. while the statements in Columa-II are labelled as a,b,c, d and e. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answer to these questions have to be darkened as illustrated in the following example: If the correct matches are 1-a. s and e: 2-b and c: 3-1 and 2: and 4-d

1. Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circie $C_3 : (x-h)^2 + (y-ky)^2 = r^2$ satisfies the following conditions:

   (a) Centre of C_3, is collinear with the centres of C_1, and C_2

   (b) C_1 and C_2 both lie inside C_3 and

   (c) C_3 touches C_1 at M and C_2 at N

   (d) Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_2 be a tangent to the parabola $x^2 = 8\alpha y$. There are some expressions given in the List - I whose values are given in List - II below

column-I	column-II
1. $(2h+k)$	a) 6
2. $\frac{Length of ZW}{Length of XY}$	b) $\sqrt{6}$
3. $\frac{Area of triangle MZN}{Area of triangle ZMW}$	c) $\frac{5}{4}$
4.	d) $\frac{21}{5}$
5.	e) $\sqrt[2]{6}$
6.	f) $\frac{10}{3}$

Which of the following is the only CORRECT combination?

(a) (i)(f)

(b) (i)(d)

(c) (ii)(e)

(d) (ii)(b)

2. Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circie $C_3 : (x-h)^2 + (y-ky)^2 = r^2$ satisfies the following conditions:

   (a) Centre of C_3, is collinear with the centres of C_1, and C_2

   (b) C_1 and C_2 both lie inside C_3 and

   (c) C_3 touches C_1 at M and C_2 at N

   (d) Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_2 be a tangent to the parabola $x^2 = 8\alpha y$. There are some expressions given in the List - I whose values are given in List - II below

column-I	column-II
1. $(2h+k)$	a) 6
2. $\frac{Length of ZW}{Length of XY}$	b) $\sqrt{6}$
3. $\frac{Area of triangle MZN}{Area of triangle ZMW}$	c) $\frac{5}{4}$
4.	d) $\frac{21}{5}$
5.	e) $\sqrt[2]{6}$
6.	f) $\frac{10}{3}$

Which of the following is the only INCORRECT combination?

(a) (iv)(e)

(b) (i)(a)

(c) (iii)(c)

(d) (iv)(f)

G : Comprehension Based Questions

PASSAGE-1 ABCD is a square of side length 2 units. C_1 is the circle touching all the sides of the square ABCD and C_2, is the circumcircle of square ABCD. L is a fixed line in the same plane and R is a fixed point.

1. P is any point of C_1, and is another point on C_2, then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ is equal to (2006)

   (a) 0.75

   (b) 1.25

   (c) 1

   (d) 0.5

2. Ifa circle is such that it touches the line L and the circle C_1 externally, such that both the circles are on the same side of the line, then the locus of centre of the circleis (2006)

   (a) ellipse

(b) parabola

(c) hyperbola

(d) pair of straight line

3. A line L' through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. 1f locus of S cuts L' at T_2 and T_3 and AC at T_1 then area of $\triangle T_1T_2T_3$ is (2006)

(a) $\frac{1}{2}$ sq.units

(b) $\frac{2}{3}$ sq.units

(c) 1 sq. units

(d) 2 sq. units

PASSAGE-2

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ. QR, RP are D, E, F,respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left[\frac{\sqrt[3]{3}}{2}, \frac{3}{2}\right]$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

4. The equation of circle C is (2008)

(a) $(x - \sqrt[2]{3})^2 + (y - 1)^2 = 1$

(b) $(x - \sqrt[2]{3})^2 + (y + \frac{1}{2})^2 = 1$

(c) $(x - \sqrt{3}^2 + (y + 1)^2 = 1$

(d) $(x - \sqrt{3}^2 + (y - 1)^2 = 1$

5. Points E and F are given by (2008)

(a) $\left[\frac{\sqrt{3}}{2}, \frac{3}{2}\right]$, $[\sqrt{3}, 0]$

(b) $\left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$, $[\sqrt{3}, 0]$

(c) $\left[\frac{\sqrt{3}}{2}, \frac{3}{2}\right]$, $\left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$

(d) $\left[\frac{3}{2}, \frac{\sqrt{3}}{2}\right]$, $\left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$

6. Equations of the sides QR, RP are (2008)

(a) $y = \frac{2}{\sqrt{3}}x + 1$,$y = \frac{2}{\sqrt{3}}x - 1$

(b) $y = \frac{1}{\sqrt{3}}x, y = 0$

(c) $y = \frac{\sqrt{3}}{2}x + 1$,$y = \frac{\sqrt{3}}{2}x - 1$

(d) $y = \sqrt{3}x, y = 0$

PASSAGE-3 A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PTis atangent to the circle $(x-3)^2 + y^2 = 1$.

7. A possible equation of L is (2012)

(a) $x - \sqrt{3}y = 1$

(b) $x + \sqrt{3}y = -1$

(c) $x - \sqrt{3}y = 1$

(d) $x + \sqrt{3}y = 5$

8. A common tangent of the two circles is

(a) x=4

(b) y=2

(c) $x + \sqrt{3}y = 4$

(d) $x + \sqrt[2]{2}y = 6$

PASSAGE-4 Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$

9. Let E_1, E_2 and F_1, F_2, be the chords of S passing through thepoint (P_1, P_2) and parallel to the x-axis and the y-axis respectively. Let G_1, G_2, be the chord of S passing through P_0 and having slope -1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3 and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3, and G_3 lie on the curve (2018)

   (a) $x + y = 4$

   (b) $(x-4)^2 - (y-4)^2 = 16$

   (c) $(x-4)(y-4) = 4$

   (d) $xy = 4$

10. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve (2018)

   (a) $(x+y)^2 = 3xy$

   (b) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2^{\frac{4}{3}}$

   (c) $x^2 + y^2 = 2xy$

(d) $x^2 + y^2 = x^2y^2$

H : Assertion And Reason Type Questions

1. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$ STATEMENT-1: Thetangents are mutually perpendicular because STATEMENT-2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$ (2007)

   (a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.

   (b) Statement-1 is True, Statement-2 is True;, Statement-2 is NOT a correct explanation for Statement-1.

   (c) Statement-1 is True, Statement-2 is False.

   (d) Statement-1 is False, Statement-2 is True.

2. Consider $L_1 : 2x + 3y + p - 3 = 0$
   $L_2 : 2x + 3y + p + 3 = 0$
   where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$ STATEMENT-1:If line L_1, is a chord of circle C, then line L_2 is not always a diameter of circle C and STATEMENT-2:If line L_1, is a diameter of circle C, then line L_1, is not a chord of circle C. (2008)

(a) Statement-1 is True, Statement-2 is True; Statement-2 is acorrect explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

I : Integer Value Correct Type

1. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segement joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1, and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C (2009)

2. The straight line $2x-3y=1$ divides the circular region $x^2+y^2 \leq 6$ into two parts. If $S = \left[\left[2, \frac{3}{4}\right], \left[\frac{5}{2}, \frac{3}{4}\right], \left[\frac{1}{4}, -\frac{1}{4}\right], \left[\frac{1}{8}, \frac{1}{4}\right]\right]$ then the number of points (s) in S lying inside the smaller part is (2011)

3. For how many values of p, the circle $x^2+y^2+2x+4y-p=0$ and the coordinate axes have exactly three common points? (2017)

4. Let the point B be the reflection of the point A(2,3) with respect to the line $8x-6y-23=0$. Let T_A and T_B be circles of radii 2 and 1 with centers Aand B respectively. Let T be a common tangent to the

circles T_A and T_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing throughA and B, then the length of the line segment AC is (2019)

Section-B [JEE Mains /AIEEE]

1. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure $45°$ at the major segment of the circle then value of m is (2002)

   (a) $2 \pm \sqrt{2}$

   (b) $-2 \pm \sqrt{2}$

   (c) $-1 \pm \sqrt{2}$

   (d) none of these

2. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2 = 25$. The locus of any point in the set is (2002)

   (a) $4 \leq x^2 + y^2 \leq 64$

   (b) $x^2 + y^2 \leq 25$

   (c) $x^2 + y^2 \geq 25$

   (d) $3 \leq x^2 + y^2 \leq 9$

3. he centre of the circle passing through (0,0) and (1,0) and touching the circle $x^2 + y^2 = 9$ is (2002)

   (a) $\left[\frac{1}{2}, \frac{1}{2}\right]$

(b) $\left[\frac{1}{2}, -\sqrt{2}\right]$

(c) $\left[\frac{3}{2}, \frac{1}{2}\right]$

(d) $\left[\frac{1}{2}, \frac{3}{2}\right]$

4. The equation of a circle with origin as a centre and passing through equilater al triangle whose median is of length 3a is (2002)

    (a) $x^2 + y^2 = 9a^2$

    (b) $x^2 + y^2 = 16a^2$

    (c) $x^2 + y^2 = 4a^2$

    (d) $x^2 + y^2 = a^2$

5. If the two circles $(x-1)^2(y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct point, then (2003)

    (a) $r > 2$

    (b) $2 < r < 8$

    (c) $r < 2$

    (d) $r = 2$

6. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq.units.Then the equation of the circle is (2003)

    (a) $x^2 + y^2 - 2x + 2y = 62$

    (b) $x^2 + y^2 + 2x - 2y = 62$

    (c) $x^2 + y^2 + 2x - 2y = 47$

    (d) $x^2 + y^2 - 2x + 2y = 47$

7. If a circle passes through the point (a,b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is (2004)

   (a) $2ax - 2by - (a^2 + b^2 + 4) = 0$

   (b) $2ax + 2by - (a^2 + b^2 + 4) = 0$

   (c) $2ax - 2by + (a^2 + b^2 + 4) = 0$

   (d) $2ax + 2by + (a^2 + b^2 + 4) = 0$

8. A variable circle passes through the fixed point A(p,q) and touches x-axis. The locus of the other end of the diameter through A is (2004)

   (a) $(y - q)^2 = 4px$

   (b) $(x - q)^2 = 4py$

   (c) $(y - p)^2 = 4qx$

   (d) $(x - p)^2 = 4qy$

9. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameter of a circle of circumference 10π, then the equationofthe circle is (2004)

   (a) $x^2 + y^2 + 2x - 2y - 23 = 0$

   (b) $x^2 + y^2 + 2x + 2y - 23 = 0$

   (c) $x^2 + y^2 + -x - 2y - 23 = 0$

   (d) $x^2 + y^2 - 2x + 2y - 23 = 0$

10. Intercept on the line y=x by the circle $x^2+y^2-2x = 0$ is AB. Equation of the circle on AB as a diameter is (2004)

   (a) $x^2 + y^2 + x - y = 0$

(b) $x^2 + y^2 + x + y = 0$

(c) $x^2 + y^2 - x + y = 0$

(d) $x^2 + y^2 - x - y = 0$

11. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line 5x +by-a=0 passes through P and Q for (2005)

    (a) exactly one value ofa

    (b) no value of a

    (c) infinitely many values of a

    (d) exactly two values of a

12. A circle touches the x-axis and also touches the circle with centre at (0,3) and radius 2. The locus of the centre of the circle is (2005)

    (a) an ellipse

    (b) a circle

    (c) a hyperbola

    (d) a parabola

13. If a circle passes through the point (a,b) and cuts the circle $x^2+y^2 = p^2$ orthogonally, then the equation of the locus of its centre is (2005)

    (a) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$

    (b) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

    (c) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$

(d) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

14. If the pair of lines $ax^2 + 2(a+b)xy + by^2 = 0$ lie along diameters ofa circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then (2005)

    (a) $3a^2 - 10ab + 3b^2 = 0$

    (b) $3a^2 + 10ab + 3b^2 = 0$

    (c) $3a^2 - 2ab + 3b^2 = 0$

    (d) $3a^2 + 2ab + 3b^2 = 0$

15. If the lines 3x-4y -7 = 0 and 2x-3y-5 =0 are two diameters of a circle of area 49π square units, the equation ofthe circle is (2006)

    (a) $x^2 + y^2 + 2x - 2y - 47 = 0$

    (b) $x^2 + y^2 + 2x - 2y - 62 = 0$

    (c) $x^2 + y^2 - 2x + 2y - 47 = 0$

    (d) $x^2 + y^2 - 2x + 2y - 62 = 0$

16. Let C be the circle with centre (0,0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is (2006)

    (a) $x^2 + y^2 = \frac{3}{2}$

    (b) $x^2 + y^2 = 1$

    (c) $x^2 + y^2 = \frac{27}{4}$

    (d) $x^2 + y^2 = \frac{9}{4}$

17. Consider a family of circles which are passing through the point(-1,1) and are tangent to x-axis. If (h,k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval (2006)

    (a) $-\frac{1}{2} \leq k \leq \frac{1}{2}$

    (b) $k \leq \frac{1}{2}$

    (c) $0 \leq k \leq \frac{1}{2}$

    (d) $k \geq \frac{1}{2}$

18. The point diametrically opposite to the point P(1,0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is (2007)

    (a) (3,-4)

    (b) (-3,4)

    (c) (-3,-4)

    (d) (3,4)

19. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is (2008)

    (a) $(x-2)y'^2 = 25 - (y-2)^2$

    (b) $(y-2)y'^2 = 25 - (y-2)^2$

    (c) $(y-2)^2y'^2 = 25 - (y-2)^2$

    (d) $(x-2)^2y'^2 = 25 - (y-2)^2$

20. If P and Q are the points of intersection of the circle $x^2 + y^2 + 33x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P,Q and (1,1) for: (2009)

(a) all except one value of p

(b) all except two values of p

(c) exactly one value of p

(d) all values of p

21. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if (2010)

(a) $-35 < m < 15$

(b) $15 < m < 65$

(c) $35 < m < 85$

(d) $-85 < m < -35$

22. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if (2011)

(a) $\mid a \mid = c$

(b) $a = 2c$

(c) $\mid a \mid = 2c$

(d) $2 \mid a \mid = c$

23. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3) is (2012)

(a) $\frac{10}{3}$

(b) $\frac{3}{5}$

(c) $\frac{6}{5}$

(d) $\frac{5}{3}$

24. The circle passing through (1,-2) and touching the axis ofx at (3,0) also passes through the point (2013)

    (a) (-5,2)

    (b) (5,-2)

    (c) (-5,-2)

    (d) (5,2)

25. Let C be the circle with centre at (1,1) and radius =1. If T is the circle centred at (0,y), passing through origin and touching the circle C externally, then the radius of T is equal (2014)

    (a) $\frac{1}{2}$

    (b) $\frac{1}{4}$

    (c) $\frac{\sqrt{3}}{\sqrt{2}}$

    (d) $\frac{\sqrt{3}}{2}$

26. Locus of the image of the point (2,3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a: (2015)

    (a) circle of radius $\sqrt{2}$.

    (b) circle of radius $\sqrt{3}$.

    (c) straight line parallel tox-axis.

    (d) straight line parallel to y-axis.

27. The number of common tangents to the circles $x^2+y^2-4x-6x-12=0$ and $x^2+y^2+6x+18y+26=0$, is: (2015)

(a) 3

(b) 4

(c) 1

(d) 2

28. The centres of those circles which touch the circle, $x^2+y^2-8x-8y-4=0$, externally and also touch the x-axis,lie on: (2016)

(a) a hyerbola

(b) a parabola

(c) a circle

(d) an ellipse, which is not a circle

29. If one of the diameters of the circle, given by the equation, $x^2+y^2-4x+6y-12=0$, is a chord of a circle S, whose centre is at (-3,2),then the radius of S is: (2016)

(a) 5

(b) 10

(c) $\sqrt[5]{2}$

(d) $\sqrt[5]{3}$

30. Ifa tangent to the circle $x^2+y^2=l$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is: (2019)

(a) $x^2 + y^2 - 4x^2y^2 = 0$

(b) $x^2 + y^2 - 2xy = 0$

(c) $x^2 + y^2 - 16x^2y^2 = 0$

(d) $x^2 + y^2 - 2x^2y^2 = 0$

Chapter 4

Conic Sections

Section-A [JEE Advanced/IIT-JEE]

A : Fill in the Blanks

1. The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is...............(1994)

2. An ellipse has eccentricityand one focus at the point $P\left[\frac{1}{2}, 1\right]$. Its one directrix is the common tangent, nearer to the point P, to the circle $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$. The equation of the ellipse, in the standard form is......(1996)

C : MCQ'S with One Correct Answer

1. The equation $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1, r > 1$ represents (1981)

   (a) an ellipse

   (b) a hyperbola

(c) a circle

(d) none of these

2. Each of the four inequalties given below defines a region in the xy plane. One of these four regions does not have the following property. For any two points (x_1, y_1) and (x_2, y_2) in the region, the point $\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$ is also in the region. The inequality defining this region is (1981)

(a) $x^2 + y^2 \leq 1$

(b) Max$\{| x |, | y |\} \leq 1$

(c) $x^2 - y^2 \leq 1$

(d) $y^2 - x$

$eq0$

3. The equation $x^2 + y^2 + 2x + 3y - 8x - 18y + 35 = k$ represents (1994)

(a) no locus if $k > 0$

(b) no ellipse if $k < 0$

(c) no point if $k = 0$

(d) no hyperbola if $k > 0$

4. Let E be the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and C be the circle $x^2 + y^2 = 9$. Let P and Q be the points (1,2) and (2,1) respectively. Then (1994)

(a) Q lies inside C but outside E

(b) Q lies outside both C and E

(c) P lies inside both C and E

(d) P lies inside C but outside E

5. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2Px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and parabola is (1995)

   (a) $\left[\frac{P}{2}, P\right]$ or $\left[\frac{P}{2}, -P\right]$

   (b) $\left[\frac{P}{2}, -\frac{P}{2}\right]$

   (c) $\left[-\frac{P}{2}, P\right]$

   (d) $\left[-\frac{P}{2}, -\frac{P}{2}\right]$

6. The radius of the circle passing through the foei of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having its centre at (0,3) is (1995)

   (a) 4

   (b) 3

   (c) $\sqrt{\frac{1}{2}}$

   (d) $\frac{7}{2}$

7. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\theta, b\tan\theta)$, where $\theta + \phi = \frac{\pi}{2}$ be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If(h,k) is the point of intersection of the normals at P and Q then k is equal to (1999)

   (a) $\frac{a^2+b^2}{a}$

   (b) $-\frac{a^2+b^2}{a}$

   (c) $\frac{a^2+b^2}{b}$

(d) $-\frac{a^2+b^2}{b}$

8. If x=9 is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents is (1999)

   (a) $9x^2 + 8y^2 + 18x - 9 = 0$

   (b) $9x^2 + 8y^2 + 18x + 9 = 0$

   (c) $9x^2 + 8y^2 - 18x - 9 = 0$

   (d) $9x^2 + 8y^2 - 18x + 9 = 0$

9. The curve described parametrically by $x = t^2 + t + 1, y = t^2 - t + 1$ represents (1999)

   (a) a pair of straight lines

   (b) an ellippse

   (c) a parabola

   (d) a hyperbola

10. If $x + y = k$ is normal to $y^2 = 12x$, then k is (2000)

    (a) 3

    (b) 9

    (c) -9

    (d) -3

11. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of k is (2000)

(a) $\frac{1}{8}$

(b) 8

(c) 4

(d) $\frac{1}{4}$

12. The equation of the common tangent touching the circle $(x-3)^2+y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is (2001)

    (a) $\sqrt{3}y = 3x + 1$

    (b) $\sqrt{3}y = -(x + 3)$

    (c) $\sqrt{3}y = (x + 3)$

    (d) $\sqrt{3}y = -(3x + 1)$

13. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is (2001)

    (a) $x = -1$

    (b) $x = 1$

    (c) $x = -\frac{3}{2}$

    (d) $x = \frac{3}{2}$

14. If $a > 2b > 0$ then the positive value of m for which $y = mx-b\sqrt{1 + m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is (2002)

    (a) $\frac{2b}{\sqrt{a^2-4b^2}}$

    (b) $\frac{\sqrt{a^2-4b^2}}{2b}$

    (c) $\frac{2b}{a-2b}$

(d) $\frac{b}{a-2b}$

15. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y = 4ax$ is another parabola with directrix (2002)

 (a) $x = -a$

 (b) $-\frac{a}{2}$

 (c) $x = 0$

 (d) $\frac{a}{2}$

16. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is (2002)

 (a) $3y = 9x + 2$

 (b) $y = 2x + 1$

 (c) $2y = x + 8$

 (d) $y = x + 2$

17. The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is (2003)

 (a) $\frac{27}{4} sq.units$

 (b) 9 sq.units

 (c) $\frac{27}{2} sq.units$

 (d) 27 sq.units

18. The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are (2003)

(a) $\{-1, 1\}$

(b) $\{-2, 2\}$

(c) $\{-2, -\frac{1}{2}\}$

(d) $\{2, -\frac{1}{2}\}$

19. For hyperbola $\frac{x^2}{\sin^2\alpha} - \frac{y^2}{cos^2\alpha} = 1$ which of the following remains constant with change in $'\alpha'$ (2003)

(a) abscissae of vertices

(b) abscissae of foci

(c) eccentricity

(d) directrix

20. Iftangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the mid-point ofthe intercept made by the tangents between the coordinate axes is (2004)

(a) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

(b) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

(c) $\frac{x^2}{2} + \frac{y^2}{4} = 1$

(d) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

21. The angle between the tangents drawn from the point (1,4) to the parabola $y^2 = 4x$ is (2004)

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

22. Ifthe line $2x + \sqrt{6}y = 2$ touches the hyperbola $x^2 - y^2 = 4$ then the point of contact is (2004)

    (a) $(-2\sqrt{6})$

    (b) $(-5, \sqrt[2]{6})$

    (c) $(\frac{1}{2}, \frac{1}{\sqrt{6}})$

    (d) $(4, -\sqrt{6})$

23. The minimum area of triangle formed by the tangent to the $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$& coordinate axes is (2005)

    (a) ab sq.units

    (b) $\frac{a^2+b^2}{2}$sq.units

    (c) $\frac{(a+b)^2}{2}$sq.units

    (d) $\frac{a^2+ab+b^2}{3}$sq.units

24. Tangent to the curve $y = x^2 + 6$ at a point (1,7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the coordinates of Q are (2005)

    (a) (-6,-11)

    (b) (-9,-13)

(c) (-10,-15)

(d) (-7,-5)

25. The axis ofa parabola is along the line $y = x$ and the distances of its vertex and focus from origin are $\sqrt{2}$ and $\sqrt[2]{2}$ respectively. If vertex and focus both lie in the first quadrant, then the equation of the parabola is (2006)

(a) $(x + y)^2 = (x - y - 2)$

(b) $(x - y)^2 = (x + y - 2)$

(c) $(x - y)^2 = 4(x + y - 2)$

(d) $(x - y)^2 = 8(x + y - 2)$

26. A hyperbola, having the transverse axis of length $2\sin\theta$,is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is (2007)

(a) $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$

(b) $x^2\sec^2\theta - y^2\text{cosec}^2\theta = 1$

(c) $x^2\sin^2\theta - y^2\cos^2\theta = 1$

(d) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

27. Leta and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(-5xy + 6y) = 0$ represents (2008)

(a) four straight lines, when c=0 and a, b are ofthe same sign.

(b) two straight lines and a circle, when a=b, and cis of sign opposite to that of a

(c) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(d) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

28. Consider a branch of the hyperbola$x^2 - y^2 - \sqrt[2]{2}x - \sqrt[4]{2}y - 6 = 0$. with vertex at the point A. Let B be one of the end point so its latus rectum. IfC is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is (2008)

(a) $1 - \sqrt{\frac{2}{3}}$

(b) $\sqrt{\frac{3}{2}} - 1$

(c) $1 + \sqrt{\frac{2}{3}}$

(d) $\sqrt{\frac{3}{2}} + 1$

29. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $X^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is (2008)

(a) $\frac{31}{10}$

(b) $\frac{29}{10}$

(c) $\frac{21}{10}$

(d) $\frac{27}{10}$

30. The normal at a point P on the elipse $x^2 + 4y^2 = 16$ mets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of

M intersects the latus rectums of the given ellipse at the points (2009)

(a) $\left[\pm\frac{\sqrt[3]{5}}{2}, \pm\frac{2}{7}\right]$

(b) $\left[\pm\frac{\sqrt[3]{5}}{2}, \pm\sqrt{\frac{19}{4}}\right]$

(c) $[\pm\sqrt[2]{3}, \pm\frac{1}{7}]$

(d) $\left[\pm\sqrt[2]{3}, \pm\frac{\sqrt[4]{3}}{7}\right]$

31. The locus of the orthocentre of the triangle formed by the lines $(1+p)x - PY + p(1+p) = 0$, $(1+q)x - qy + q(1+q) = 0$, and $y = 0$, where $p \neq q$, is (2009)

(a) a hyperbola

(b) a parabola

(c) an ellipse

(d) a straight line

32. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.If the normal at the point P intersects the x-axis at (9,0), then the eccentricity of the hyperbola is (2011)

(a) $\sqrt{\frac{5}{2}}$

(b) $\sqrt{\frac{3}{2}}$

(c) $\sqrt{2}$

(d) $\sqrt{3}$

33. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be thepoint that divides the line segment from (0,0) to (x,y) in the ratio $1:3$. Then the locus of P is (2011)

(a) $x^2 = y$

(b) $y^2 = 2x$

(c) $y^2 = x$

(d) $x^2 = 2y$

34. The ellipse E: $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E, passing through the point (0,4) circumscribes the rectangleR. The eccentricity of the ellipse E, is (2012)

(a) $\frac{\sqrt{2}}{2}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

35. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is (2014)

(a) 3

(b) 6

(c) 9

(d) 15

D : MCQ'S with One or More Than One Correct Answer

1. The number of values of c such that the straight line $y = 4x+c$ touches the curve $(x^2/4) + y2 = 1$ is (1998)

   (a) 0

   (b) 1

   (c) 2

   (d) infinity

2. If P=(x,y), $F_1 = (3,0), F_2 = (-3,0)$ and $16x^2 + 25y^2 = 400$,then $PF_1 + PF_2$ equals (1998)

   (a) 8

   (b) 6

   (c) 10

   (d) 12

3. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are (1999)

   (a) $\frac{2}{5}, \frac{1}{5}$

   (b) $-\frac{2}{5}, \frac{1}{5}$

   (c) $-\frac{2}{5}, -\frac{1}{5}$

   (d) $\frac{2}{5}, -\frac{1}{5}$

4. he equations of the common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are (2006)

   (a) $y = 4(x-1)$

   (b) $y = 0$

   (c) $y = -4(x-1)$

   (d) $y = -30x - 50$

5. Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then (2006)

   (a) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

   (b) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

   (c) focus of hyperbola is (5,0)

   (d) vertex of hyperbola is $(\sqrt[5]{3}, 0)$

6. Let $P(x_1, y_1)$ and $Q(x_2, y_2), y_1 < 0, y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are (2008)

   (a) $x^2 + \sqrt[2]{3}y = 3 + \sqrt{3}$

   (b) $x^2 - \sqrt[2]{3}y = 3 + \sqrt{3}$

   (c) $x^2 + \sqrt[2]{3}y = 3 - \sqrt{3}$

   (d) $x^2 - \sqrt[2]{3}y = 3 - \sqrt{3}$

7. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2\frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, then (2009)

   (a) $b + c = 4a$

   (b) $b + c = 2a$

   (c) locus of point A is an ellipse

   (d) locus of point A is a pair of straight lines

8. The tangent PTand the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively The locus of the centroid of the triangle PTN is a parabola whose (2009)

   (a) Vertex is $\left[\frac{2a}{3}, 0\right]$

   (b) directrix is $x = 0$

   (c) latus rectum is $\frac{2a}{3}$

   (d) focus is $(a, 0)$

9. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then (2009)

   (a) equation of ellipse is $2x^2 + 2y^2 = 2$

   (b) the foci of ellipse are $(\pm 1, 0)$

   (c) equation of ellipse is $2x^2 + 2y^2 = 4$

(d) the foci of ellipse are $(\pm\sqrt{2}, 0)$

10. Let A and B be two distinct points on the parabola $y = 4x$. If the axis of the parabola touchesa circle of radius r having AB as its diameter, then the slope of the line joining A and B can be (2010)

    (a) $-\frac{1}{r}$

    (b) $\frac{1}{r}$

    (c) $-\frac{2}{r}$

    (d) $\frac{2}{r}$

11. Let the eccentricity of the hyperbola $\frac{2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then (2011)

    (a) the equation of the hyperbola is $\frac{2}{3} - \frac{y^2}{2} = 1$

    (b) a focus ofthe hyperbola is (2, 0)

    (c) the eccentricity of the hyperbola is $\frac{\sqrt{5}}{2}$

    (d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

12. Let L be a normal to the parabola $y = 4x$. If L passes throughthe point (9,6) then L is given by (2011)

    (a) $y - x + 3 = 0$

    (b) $y + 3x - 33 = 0$

    (c) $y + x - 15 = 0$

    (d) $y - 2x + 12 = 0$

13. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4=1}$, parallel to the straight line $2x-y=1$. The points of contact of thetangents on the hyperbola are (2012)

(a) $\left[\frac{9}{\sqrt[2]{2}},\frac{1}{\sqrt{2}}\right]$

(b) $\left[-\frac{9}{\sqrt[2]{2}},-\frac{1}{\sqrt{2}}\right]$

(c) $\sqrt[3]{3},-\sqrt[2]{2}$

(d) $-\sqrt[3]{3},\sqrt[2]{2}$

14. Let Pand Qbe distinct points on the parabola $y^2=2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the ares ofthe triangle $\triangle OPQ$ is $\sqrt[3]{2}$,then which of the following is (are) the coordinates of P? (2015)

(a) $(4,\sqrt[2]{2})$

(b) $(9,\sqrt[3]{2})$

(c) $\left[\frac{1}{4},\frac{1}{\sqrt{2}}\right]$

(d) $(1\sqrt{2})$

15. Let E_1 and E_2 be two ellipses whose centers are at the origin The major axes of E_1 and E_2, lie along the x-axis and the y-axis, respectively. Let S E_1 and E_2 be the circle $x^2+(y-1)^2=2$.The straight line $x+y=3$ touches the curves.S at P,Q and R respectively. Suppose that PO $=$ PR $=\frac{\sqrt[2]{2}}{3}$. If e_1, and e_2 are the eccentricities of E_1 and E_2 respectively, then the correct expression(s) is (are) (2015)

(a) $e_1^2+e_2=\frac{43}{40}$

(b) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{7}}$

(c) $\mid e_1^2 - e_2^2 \mid = \frac{5}{8}$

(d) $e_1 e_2 = \frac{\sqrt{3}}{4}$

16. Consider the hyperbola $H : x^2 - y^2 = 1$ and a circle S with center N$(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x > 1$ and $y > 0$. The common tangent to H and S at P intersects the x-axis at point M. If(l, m) is the centroid of the triangle PMN, then the correct expressions is(are) (2015)

(a) $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2} for x_1 > 1$

(b) $\frac{dm}{dx_1} = \frac{x_1}{3\left[\sqrt{x_1^2 - 1}\right]} for x_1 > 1$

(c) $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2} for x_1 > 1$

(d) $\frac{dm}{dy_1} = 1 - \frac{1}{3} for y_1 > 1$

17. he circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects theparabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1, at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $\sqrt[2]{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lies on the y-axis, then (2016)

(a) $Q_2 Q_3 = 12$

(b) $R_2 R_3 = \sqrt[4]{6}$

(c) area of the triangle $OR_2R_3 = \sqrt[6]{2}$

(d) area of the triangle $OQ_2Q_3 = \sqrt[4]{2}$

18. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2+y^2-4x-16y+64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then (2016)

   (a) $SP = \sqrt[2]{5}$

   (b) $SQ : QP = (\sqrt{5+1}) : 4$

   (c) the x-intercept of the normal to the parabola at P is 6

   (d) the slope of the tangent to the circle at Q is $\frac{1}{2}$

19. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$,then which of the following cannot be sides ofa right angled triangle? (2017)

   (a) a,4,1

   (b) a,4,2

   (c) 2a,8,1

   (d) 2a,4,1

20. Ifa chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h,k), then which of the following is(are) possible value(s) of p,h and k? (2017)

   (a) p=-2, h=2, k=4

   (b) p=-1, h=1, k=-3

   (c) p= 2, h=3, k=4

   (d) p=-5, h=4, k=-3

21. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O(0,0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE? (2018)

    (a) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length ofthe latus rectum is 1

    (b) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

    (c) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = l$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

    (d) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

E : Subjective Problems

1. Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point (h,k). Show that $h > 2$. (1981)

2. A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B. If AB subtends aright angle at the vertex of the parabola. find the slope of AB (1982)

3. Three normals are drawn from the point (c,0) to the curve $y = x$. Show

that c must be greater than $\frac{1}{2}$. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other.(1991)

4. Through the vertex O of parabola $y = 4x$, chords OP and OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis ofthe parabola at a fixed point. Also find the locus of the middle point of PQ.(1994)

5. Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1 : 2 is a parabola. Find the vertex of this parabola.(1995)

6. Let 'd' be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ to the tangent drawn at a point P on the ellipse. If F_1 and F_2 are the two foci of the ellipse, then $(PF_1 - PF_2) = 4a^2\left[1 - \frac{b^2}{d^2}\right]$.(1995)

7. Point A,B and C lie on the parabola $y^2 = 4ax$. The tangents to the parabola A,B and C taken in pairs, intersects with points P,Q and R. Determine the ratio of the areas of the triangle ABC and PQR (1996)

8. From a point A common tangents are drawn to the circle $x^2 + y^2 = a^2$ and parabola $y^2 = 4ax$. Find the area of thequadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola (1996)

9. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. Prove that the tangents at P and Q of the ellipse $x^2+2y^2 = 6$ are at right angles. (1997)

10. The angle between a pair of tangents drawn from a point P to the parabola $y^2 = 4ax$ is 45○. Show that the locus of the point P is a hyperbola.(1998)

11. Consider the family of circles $x^2 + y^2 = r^2, 2 < r < 5$. If in the first quadrant, the common tangent to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A and B, then find the equation of the locus of the mid-point of AB.(1999)

12. Find the co-ordinates of all the points P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, or which the area of the triangle PON is a maximum, where O denotes the origin and N, the foot of the perpendicular from O to the tangent at P.(1999)

13. Let ABC be an equilateral triangle inscribed in the circle $x^2+y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ meets the ellipse respecti vely, at P, O, R. so that P,Q,R lie on the same side of the major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P, Q and R are concurrent. (2000)

14. Let C, and C, be respectively, the parabolas x =y- 1 and y=x- 1. Let P be any point on C_1 and Q be any point on C_2. Let P_1 and Q_1, be the reflections of P and Q,respectively with respect to the line $y = x$. Prove that P_1, lies on C_2, Q_1 lies on C_1, and $PQ > min\{PP_1, QQ_1$. Hence or otherwise determine points P_0 and Q_0, on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \geq PQ$ for all pairs of points (P,Q)

with P on C_1, and Q on C_2. (2000)

15. Let Pbe a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, 0 < b < a$. Let the line parallel to y-axis passing through P meet the circle $x^2 + y^2 = a^2$ at the point Q such that P and Q are on the same side of x-axis. For two positive real numbers r and s, find the locus of the point R on PQ such that $PR : RQ = r : s$ as P varies over the ellipse.(2000)

16. Prove that,in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. (2002)

17. Normals are drawn from the point P with slopes $m_1 m_2, m_3$ to the parabola $y = 4x$. If locus of P with $m_1 m_2 = \alpha$ is a part of the parabola itself then find α. (2003)

18. Tangent is drawn to parabola $y^2 - 2y - 4x + 5 = 0$ at a point P which cuts the directrix at the point Q. A point R is such that it divides QP externally in the ratio $\frac{1}{2} : 1$. Find the locus of point R. (2004)

19. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9} + \frac{y^2}{4} = 1$ to the circle $x2 + y^2 = 9$. Find the locus of mid-point of the chord of contact. (2005)

20. Find the equation of the common tangent in $1^s t$ quadrant to he circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Also find the length of the intercept of the tangent between the coordinate axes (2005)

F : Match The Following

DIRECTIONS (Q.1): Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled 1, 2, 3 and 4. while the statements in Columa-II are labelled as a,b,c, d and e. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answer to these questions have to be darkened as illustrated in the following example: If the correct matches are 1-a. s and e: 2-b and c: 3-1 and 2: and 4-d

1. Match the following : (3,0) is the pt. from which three normals are drawn to the parabola $y^2 = 4x$ which meet the parabola in the points P, Q and R. Then (2006)

column-I	column-II
1. Area of triange $\triangle PQR$	a) 2
2. radius of circumcircle $\triangle PQR$	b) $\frac{5}{2}$
3. centroide of $\triangle PQR$	c) $(\frac{5}{2}, 0)$
4. circumcenter of $\triangle PQR$	d) $(\frac{2}{3}, 0)$

2. Match the statements in Column I with the properties in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix givén in the ORS. (2007)

   column-I column-II

1. Two intersecting circles — a) have a common tangent
2. Twomutually external circles — b) have a common normal
3. Two circles, one strictly inside the other — c) do not have a common tangent
4. Two branches of a hyperbola — d) do not have a common normal

3. Match the conics in Column I with the statements/expressions in Column II. . (2009)

column-I	column-II
1. circle	a) The locus of the point (h,k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
2. parabola	b) Points z in the complex plane satisfying $\mid z + 2 \mid - \mid z - 2 \mid = \pm 3$
3. ellipse	c) Points of the conic have parametric representation $x = \sqrt{3}\left[\frac{1-t^2}{1+t^2}\right], y = \left[\frac{2t}{1-t^2}\right]$
4. hyperbola	d) The accentricity of the conic lies in the interval $1 < x < \infty$
5.	e) Points z in the complex plane satisfying $Re(z+1)^2 = \mid z \mid^2 = 1$

DIRECTIONS (Q-4): Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

4. A line L:$y = mx+3$ meets y-axis at E(0, 3) and the arc of the parabola $y^2 = 16x, 0 \leq 6 \leq 0$ at the point F(x_0, Y_0). The tangent to the parabola at $F(x_0, Y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the arca of the triangle EFG has a local maximum. (2013)

Match List I with List II and select the correct answer using the code given below the lists:

List-I	List-II
1. m=	a) $\frac{1}{2}$
2. Maximum area of $\triangle EFG$ is	b) 4
3. $y_0 =$	c) 2
4. $y_1 =$	d) 1

Codes:

	1	2	3	4
(p)	d	b	c	a
(q)	b	c	a	d
(r)	c	d	a	b
(s)	a	d	c	b

Q(5-7) By appropriately matching the in formation given in the three columns of the following table. Column 1, 2, and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

column-I	column-II	column-III
(I) $x^2+y^2=a^2$	(i) $my=m^2x+a$	(P) $\left[\frac{a}{m^2}\frac{2a}{m}\right]$
(II) $x^2+a^2y^2=a^2$	(ii) $y=mx+a\sqrt{m^2+1}$	(Q) $\left[\frac{-ma}{\sqrt{m^2+1}},\frac{a}{\sqrt{m^2+1}}\right]$
(III) $y^2=4ax$	(iii) $y=mx+\sqrt{a^2m^2+1}$	(R) $\left[\frac{-a^2m}{\sqrt{a^2m^2+1}},\frac{1}{\sqrt{a^2m^2+1}}\right]$
(IV) $x^2-a^2y^2=a^2$	(iv) $y=mx+\sqrt{a^2m^2-1}$	(S) $\left[\frac{-a^2m}{\sqrt{a^2m^2-1}},\frac{1}{\sqrt{a^2m^2-1}}\right]$

5. For $a=\sqrt{2}$, if a tangent is drawn to a suitable conic (Column-I) at the point of contact (-1,1), then which of the following options is the only correct combination for obtaining its equation? (2017)

   (a) (I)(i)(P)

   (b) (I)(ii)(Q)

   (c) (II)(ii)(Q)

   (d) (III)(ii)(P)

6. Tangent to a suitable conic (column I) is found to be $y=x+8$ and its point of contact is (8,16), then which of the following options is the only correct combination? (2018)

   (a) (I)(ii)(Q)

   (b) (II)(iv)(R)

   (c) (III)(i)(P)

   (d) (III)(ii)(Q)

7. The tangent to a suitable conic (Column I) at $\left[\sqrt{3},\frac{1}{2}\right]$ is found to be $\sqrt{3}x+2y=4$, then which of the following options is the only correct combination? (2018)

(a) (IV)(iii)(S)

(b) (IV)(iv)(S)

(c) (II)(iii)(R)

(d) (II)(iv)(R)

8. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of $60°$ at one of its vertices N. Let the area of the triangle LMN be $\frac{4}{\sqrt{3}}$ (2018)

List-I	List-II
1. The length of the conjugate axis of H is	a) 8
2. The eccentricity of H is	b) $\frac{4}{\sqrt{3}}$
3. The distance between the foci of H is	c) $\frac{2}{\sqrt{3}}$
4. The length of the latus rectum of H is	d) 4

The correct option is:

(a) $1 \to d; 2 \to b; 3 \to a; 4 \to c$

(b) $1 \to d; 2 \to c; 3 \to a; 4 \to b$

(c) $1 \to d; 2 \to a; 3 \to c; 4 \to b$

(d) $1 \to c; 2 \to d; 3 \to b; 4 \to a$

G : Comprehension Based Questions

PASSAGE-1 Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants,respectively Tangents to the curcle at Pand Q intersect the x-axis at R and tangents to the parabola at Pand Q intersect the x-axis at S.

1. The ratio of the areas of the triangles PQS and POR is (2007)
   (a) $1 : \sqrt{2}$
   (b) $1 : 2$
   (c) $1 : 4$
   (d) $1 : 8$
2. The radius of the circumcircle of the triangle PRS is (2007)
   (a) 5
   (b) $\sqrt[5]{3}$
   (c) $\sqrt[3]{2}$
   (d) $\sqrt[2]{3}$
3. The radius of the incircle of the triangle PQR is (2007)
   (a) 4
   (b) 3
   (c) $\frac{8}{3}$
   (d) 2

PASSAGE-2 The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the point A and B (2010)

4. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

   (a) $2x - \sqrt{5}y - 20x = 0$

   (b) $2x - \sqrt{5}y + x = 0$

   (c) $3x - 4y + 8 = 0$

   (d) $4x - 3y + 4 = 0$

5. Equation of the circle with AB as its diameter is

   (a) $x^2 + y^2 - 12x + 24 = 0$

   (b) $x^2 + y^2 + 12x + 24 = 0$

   (c) $x^2 + y^2 - 24x + 12 = 0$

   (d) $x^2 + y^2 - 24x - 12 = 0$

PASSAGE-3 Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B. (2010)

6. The coordinates of A and B are

   (a) (3,0) and (0,2)

   (b) $\left[\frac{-8}{5}, \frac{\sqrt[2]{161}}{15}\right]$ and $\left[\frac{-9}{5}, \frac{8}{5}\right]$

   (c) $\left[\frac{-8}{5}, \frac{\sqrt[2]{161}}{15}\right]$ and (0,2)

   (d) $\left[\frac{-9}{5}, \frac{8}{5}\right]$ and (3,0)

7. The orthocenter of the triangle PAB is

   (a) $\left[5, \frac{8}{7}\right]$

   (b) $\left[\frac{7}{5}, \frac{25}{8}\right]$

   (c) $\left[\frac{11}{5}, \frac{8}{5}\right]$

   (d) $\left[\frac{8}{25}, \frac{7}{5}\right]$

8. The equation of the locus of the point whose distances from the point P and the line AB are equal,is

   (a) $x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

   (b) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

   (c) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

   (d) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

   PASSAGE-4 Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a, a > 0$.

9. Length of chord PQ is (2013)

   (a) 7

   (b) 5

   (c) 2

   (d) 3

10. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$,then $\tan\theta$ (2013)

(a) $\frac{2}{3}\sqrt{7}$

(b) $-\frac{2}{3}\sqrt{7}$

(c) $\frac{2}{3}\sqrt{5}$

(d) $-\frac{2}{3}\sqrt{5}$

PASSAGE-5 Let a, r, s, t be nonzero real numbers. Let $P(at^2, 2at)$, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point (2a,0). (2014)

11. The value of r is

   (a) $-\frac{1}{t}$

   (b) $\frac{t^2+1}{t}$

   (c) $\frac{1}{t}$

   (d) $\frac{t^2-1}{t}$

12. If $st = 1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

   (a) $\frac{(t^2+1)^2}{2t^3}$

   (b) $a\frac{(t^2+1)^2}{2t^3}$

   (c) $a\frac{(t^2+1)^2}{t^3}$

   (d) $a\frac{(t^2+2)^2}{t^3}$

PASSAGE-6 Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse Suppose $\frac{x^2}{9} + \frac{y^2}{4} = 1$ a parabola having vertex at the origin and focus at F_2, intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

13. The orthocentre ofthe triangle F_1MN is (2016)

    (a) $\left[-\frac{9}{10}, 0\right]$

    (b) $\left[\frac{2}{3}, 0\right]$

    (c) $\left[\frac{9}{10}, 0\right]$

    (d) $\left[\frac{2}{3}, \sqrt{6}\right]$

14. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MOR to area of the quadrilateral MF_1NF_2, is

    (a) $3:4$

    (b) $4:5$

    (c) $5:8$

    (d) $2:3$

H : Assertion And Reason Type Questions

1. STATEMENT-1: The curve $y = -\frac{x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$. because STATEMENT-2: A parabola is symmetric about its axis.(2007)

(a) Statement-1 is True, Statement-2 is Tue; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

I : Integer Value Correct Type

1. The line $2x^2 + y^2 = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is (2010)

2. Consider the parabola $y^2 = 8x$. Let A, be the area of the triangle formed by the end points of its latus rectum and the point $P\left[\frac{1}{2}, 2\right]$ on the parabola and $\triangle_2$, be the area of the triangle formed by drawing tangents at P and at the end the points of the latus rectum. Then $\frac{\triangle_1}{\triangle_2}$ is (2011)

3. Let S be the focus of the parabola $y = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is (2012)

4. A vertical line passing through the point (h,0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\triangle(h)$ = area of the triangle

PQR,$\triangle_1$=$max_{\frac{1}{2}\leq h\leq 1}\triangle(h)$ and $\triangle_2$=$min_{\frac{1}{2}\leq h\leq 1}\triangle(h)$ then, $\frac{8}{\sqrt{5}}\triangle_1-8\triangle_2$ (2013)

(a) g(x) is continuous but not differentiable at a

(b) g(r) is differentiable on R

(c) g(X) is continuous but not differentiable at b

(d) g(x) is continuous and differentiable at either (a) or (b) but not both

5. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is (2015)

6. Let the curve Cbe the mirror image of the parabola $y^2 = 4x$ with respect to the line $x+y+4 = 0$. If A and B are the pointsof intersection of C with the line $y = -5$, then the distance between A and B is (2015)

7. Suppose that the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ are, $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0,0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$ respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through G,0). If m_1, is the slope of T_1, and m_2 be slope of T_2, then the value of $\left[\frac{1}{m_1^2} + m_2^2\right]$ is (2015)

Section-B [JEE Mains /AIEEE]

1. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are (2002)

   (a) $x = \pm(y + 2a)$

   (b) $y = \pm(x + 2a)$

   (c) $x = \pm(y + a)$

   (d) $y = \pm(x + a)$

2. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$ then (2003)

   (a) $t_2 = t_1 + \frac{1}{t_1}$

   (b) $t_2 = -t_1 - \frac{1}{t_1}$

   (c) $t_2 = -t_1 + \frac{1}{t_1}$

   (d) $t_2 = t_1 - \frac{1}{t_1}$

3. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is (2003)

   (a) 9

   (b) 1

   (c) 5

   (d) 7

4. If $a0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then (2004)

(a) $d^2 + (3b - 2c)^2 = 0$

(b) $d^2 + (3b + 2c)^2 = 0$

(c) $d^2 + (2b + 3c)^2 = 0$

(d) $d^2 + (2b - 3c)^2 = 0$

5. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x^2 = 4$, then the equation of the ellipse: (2004)

   (a) $4x^2 + 3y^2 = 1$

   (b) $3x^2 + 4y^2 = 12$

   (c) $4x^2 + 3y^2 = 12$

   (d) $3x^2 + 4y^2 = 1$

6. Let Pbe the point (1,0) and Qa point on the locus $y^2 = 8x$ The locus of mid point of PQ is (2005)

   (a) $y^2 - 4x + 2 = 0$

   (b) $y^2 + 4x + 2 = 0$

   (c) $x^2 + 4y + 2 = 0$

   (d) $x^2 - 4y + 2 = 0$

7. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is (2005)

   (a) an ellipse

   (b) a circle

   (c) a parabola

(d) a hyperbola

8. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is (2005)

   (a) $\frac{1}{\sqrt{2}}$

   (b) $\frac{1}{2}$

   (c) $\frac{1}{4}$

   (d) $\frac{1}{\sqrt{3}}$

9. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is (2006)

   (a) $xy = \frac{105}{64}$

   (b) $xy = \frac{3}{4}$

   (c) $xy = \frac{35}{16}$

   (d) $xy = \frac{64}{105}$

10. In an ellipse, the distance between its foci is 6 and minoraxis is 8. Then its eccentricity is (2006)

    (a) $\frac{3}{5}$

    (b) $\frac{1}{2}$

    (c) $\frac{4}{5}$

    (d) $\frac{1}{\sqrt{5}}$

11. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) is (2006)

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{4}$

12. For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies=? (2007)

    (a) abscissae of vertices

    (b) abscissae of foci

    (c) eccentricity

    (d) directrix.

13. The equation of a tangent to the parabola $y = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is (2007)

    (a) (2,4)

    (b) (-2,0)

    (c) (1,1)

    (d) (0,2)

14. The normal to a curve at P(x,y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a (2007)

    (a) circle

(b) hyperbola

(c) ellipse

(d) parabola

15. Afocus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is Then the length of the semi-major axis is (2007)

(a) $\frac{8}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{5}{3}$

16. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at (2008)

(a) (0,2)

(b) (1,0)

(c) (2,0)

(d) (0,1)

17. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the co-ordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is (2009)

(a) $x^2 + 4y^2 = 4$

(b) $x^2 + 12y^2 = 16$

(c) $4x^2 + 48y^2 = 48$

(d) $x^2 + 16y^2 = 16$

18. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is (2010)

    (a) $2x + 1 = 0$

    (b) $x = -1$

    (c) $2x - 1 = 0$

    (d) $x = 1$

19. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (-3,1) and has eccentricity $\sqrt{\frac{2}{5}}$ is (2011)

    (a) $5x^2 + 3y^2 - 48 = 0$

    (b) $3x^2 + 5y^2 - 15 = 0$

    (c) $5x^2 + 3y^2 - 32 = 0$

    (d) $3x^2 + 5y^2 - 32 = 0$

20. Statement-1: An equation of a common tangent to the parabola $y = \sqrt[16]{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + \sqrt[2]{3}$

    Statement-2:Iftheline $y = mx + \frac{\sqrt[4]{3}}{m}(m \neq 0)$ is a common tangent to the parabola $y = \sqrt[16]{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$ (2012)

    (a) Statement-1 is false, Statement-2 is true.

    (b) Statement-1 is true, statement-2 is true; statement-2is a correct explanation for Statement-1.

(c) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for Statement-1.

(d) Statement-1 is true, statement-2 is false.

21. An ellipse is drawn by takinga diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ is semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is: (2012)

(a) $4x^2+y^2=4$

(b) $x^2+4y^2=8$

(c) $4x^2+y^2=8$

(d) $x^2+4y^2=16$

22. The equation of the circle passing through the foci of the elipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centreat (0,3) is (2013)

(a) $x^2+y^2-6y-7=0$

(b) $x^2+y^2-6y+7=0$

(c) $x^2+y^2-6y-5=0$

(d) $x^2+y^2-6y+5=0$

23. Given :A circle, $2x^2+2y^2=5$ and a parabola, $y^2=\sqrt[4]{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y=x+\sqrt{5}$

Statement-2 : If the line, $y=mx+\frac{\sqrt{5}}{m}(m\neq 0)$ is their common tangent, then m satisfies $m^4-3m^2+2=0$ (2013)

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

24. The locus of the foot of perpendicular drawn from the centre ofthe ellipse $x^2 + 3y^2 = 6$ on any tangent to it is (2014)

(a) $\left(x^2 + y^2\right)^2 = 6x^2 + 2y^2$

(b) $\left(x^2 + y^2\right)^2 = 6x^2 - 2y^2$

(c) $\left(x^2 - y^2\right)^2 = 6x^2 + 2y^2$

(d) $\left(x^2 - y^2\right)^2 = 6x^2 - 2y^2$

25. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is (2014)

(a) $\frac{1}{8}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

26. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. Ifthe point P divides the line segment OQ internally in the ratio 1 : 3, then locus of P is: (2015)

(a) $y^2 = 2x$

(b) $x^2 = 2y$

(c) $x^2 = y$

(d) $y^2 = x$

27. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1,1) (2015)

(a) meets the curve again in the third quadrant.

(b) meets the curve again in the fourth quadrant.

(c) does not meet the curve again.

(d) mects the curve again in the second quadrant.

28. The area(in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$. is : (2015)

(a) $\frac{27}{2}$

(b) 27

(c) $\frac{27}{3}$

(d) 18

29. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the eentre C of the circle, $x^2 + (y+6)^2 = 1$. Then the oquation of the circle, passing through C and having ts centre at P is: (2016)

(a) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(b) $x^2 + y^2 - 4x + 9y + 18 = 0$

(c) $x^2 + y^2 - 4x + 8y + 18 = 0$

(d) $x^2 + y^2 - x + 4y - 12 = 0$

30. The eccentricity of the hyperbola whose length of the latus rectum is cqual to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is: (2016)

(a) $\frac{2}{\sqrt{3}}$

(b) $\sqrt{3}$

(c) $\frac{4}{3}$

(d) $\frac{4}{\sqrt{3}}$

31. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point: (2017)

(a) $(-\sqrt{2}, -\sqrt{3})$

(b) $(\sqrt[3]{2}, \sqrt[2]{3})$

(c) $(\sqrt[2]{2}, \sqrt[3]{3})$

(d) $(\sqrt{3}, \sqrt{2})$

32. The radius ofa circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = | x |$ is (2018)

(a) $4(\sqrt{2} + 1)$

(b) $2(\sqrt{2} + 1)$

(c) $2(\sqrt{2} - 1)$

(d) $4(\sqrt{2}-1)$

33. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0,3) then the area(in sq. units) of $\triangle APTQ$ is: (2018)

(a) $\sqrt[54]{3}$

(b) $\sqrt[66]{3}$

(c) $\sqrt[36]{5}$

(d) $\sqrt[45]{5}$

34. Tangent and normal are drawn at P(16,16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B respectively. If C is the centre of the circle through the points P,A and B and $\angle CPB = \theta$,then a value of $\tan\theta$ is (2018)

(a) 2

(b) 3

(c) $\frac{4}{3}$

(d) $\frac{1}{2}$

35. Two sets A and B are as under:

$A = \{(a,b) \in R \times R :\mid a-5 \mid < 1 and \mid b-5 \mid < 1\};$

$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \leq 36\}$. Then: (2018)

(a) $A \subset B$

(b) $A \cap B = \phi(anemptyset)$

(c) neither $A \subset B$ or $B \subset A$

(d) $B \subset A$

36. If the tangent at (1,7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is : (2018)

(a) 185

(b) 85

(c) 195

(d) 95

37. Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it? (2019)

(a) $(5, \sqrt[2]{6})$

(b) (8,6)

(c) $(6, \sqrt[4]{2})$

(d) (4,-4)

38. Let $0 < \theta, \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval: (2019)

(a) $(3, \infty)$

(b) $(\frac{3}{2}, 2)$

(c) $(2, 3)$

(d) $(1, \frac{3}{2})$

39. Equation of a common tangent to the circle ,$x^2 + y^2 - 6x = 0$ and the parabola $y^2 = 4x$, is: (2019)

    (a) $\sqrt[2]{3}y = 12x + 1$

    (b) $\sqrt{3}y = x + 3$

    (c) $\sqrt[2]{3}y = -x - 12$

    (d) $\sqrt{3}y = 3x + 1$

40. If the line $y = mx + \sqrt[7]{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is :(2019)

    (a) $\frac{\sqrt{5}}{2}$

    (b) $\frac{\sqrt{15}}{2}$

    (c) $\frac{2}{\sqrt{5}}$

    (d) $\frac{3}{\sqrt{5}}$

41. If one end of a focal chord of the parabola, $y^2 = 8x$ is at (1,4), then the length of this focal chord is : (2019)

    (a) 25

    (b) 22

    (c) 24

    (d) 20

Chapter 5

abcd