Assignment 2

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vector

***Abstract*—This document contains the solution to find the vertex of parabola**

Download all python codes from

https://github.com/Anjalibagade/EE5600/tree/master/Assignment2

and latex codes from

https://github.com/Anjalibagade/EE5600/Assignment2

Problem

Vector-2, Example-4, Question-5

Sketch the loci of the following equation

$$y^2 = x \tag{0.0.1}$$

Solution: Given equation is

$$y^2 = x \tag{0.0.2}$$

We can write above equation as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.0.3}$$

where,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}, f = 0 \tag{0.0.4}$$

$$\left|V\right| = 0 \tag{0.0.5}$$

From above equation we can say that the curve is parabola.

The vertex of parabola can be given as $\mathbf{c}$

$$\begin{pmatrix} \mathbf{u}^T + \eta\mathbf{p_1}^T \\ V \end{pmatrix}\mathbf{c} = \begin{pmatrix} -f \\ \eta\mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{0.0.6}$$

where

$$\mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \eta = \mathbf{p_1}^T\mathbf{u} \tag{0.0.7}$$

$$\eta = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \end{pmatrix} \tag{0.0.8}$$

From 0.0.17 we can find out few values given below

$$\left(\mathbf{u}^T + \eta\mathbf{p_1}^T\right) = \begin{pmatrix} -\frac{1}{2} & 0 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \end{pmatrix} \tag{0.0.9}$$

$$\left(\eta\mathbf{p_1} - \mathbf{u}\right) = \begin{pmatrix} -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.10}$$

Substituting all the values in equation 0.0.17

$$\begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{0.0.11}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.12}$$

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.13}$$

The eccentricity of parabola is given by,

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \tag{0.0.14}$$

$$\lambda_1 = 0, \lambda_2 = 1 \tag{0.0.15}$$

$$e = 1 \tag{0.0.16}$$

Directrix of parabola is given by,

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2e^2\mathbf{u^T n}} \tag{0.0.17}$$

Substituting all the values in 0.0.17 equation we get,

$$c = \frac{-1}{4} \tag{0.0.18}$$

Foci of the parabola is given by,

$$\mathbf{F} = \frac{ce^2\mathbf{n} - \mathbf{u}}{\lambda_2} \tag{0.0.19}$$

Substituting all the values in 0.0.19 equation

$$\mathbf{F} = \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix} \tag{0.0.20}$$

Plot of parabola obtained from Python code is shown below.

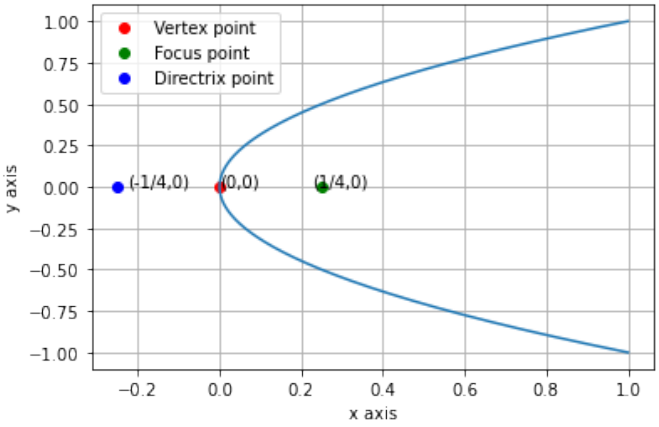

Fig. 0: Plot of parabola with vertex **c**