

EE5600 Assignment 1

Perabhattula Venkatesh
AI20MTECH01004

Abstract—This document contains the solution to a Area of triangle problem.

Download all python and latex codes from

https://github.com/venky-p/EE5600/Assignment_1

1 PROBLEM

Problem Set: Vector2, Example II, Problem 5

1.1. Find the area of the triangle formed by the points $\mathbf{P} \begin{pmatrix} a \\ c+a \end{pmatrix}$, $\mathbf{Q} \begin{pmatrix} a \\ c \end{pmatrix}$ and $\mathbf{R} \begin{pmatrix} -a \\ c-a \end{pmatrix}$

2 SOLUTION

We are going to solve this problem using vectors

$$\mathbf{P} = \begin{pmatrix} a \\ c+a \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} a \\ c \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -a \\ c-a \end{pmatrix} \quad (2.1.1)$$

Rewriting P, Q and R as product of a matrix and a vector

$$\mathbf{P} = \mathbf{A}\mathbf{u} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \quad (2.1.2)$$

$$\mathbf{R} = \mathbf{C}\mathbf{u} = \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \quad (2.1.3)$$

$$\mathbf{Q} = \mathbf{B}\mathbf{u} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \quad (2.1.4)$$

$$\text{Area of given } \triangle = \frac{1}{2} \|\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R}\| \quad (2.1.5)$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = (\mathbf{A}\mathbf{u} - \mathbf{B}\mathbf{u}) \times (\mathbf{A}\mathbf{u} - \mathbf{C}\mathbf{u}) \quad (2.1.6)$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = \mathbf{A}\mathbf{u} \times \mathbf{A}\mathbf{u} - \mathbf{A}\mathbf{u} \times \mathbf{C}\mathbf{u} - \mathbf{B}\mathbf{u} \times \mathbf{A}\mathbf{u} + \mathbf{B}\mathbf{u} \times \mathbf{C}\mathbf{u} \quad (2.1.7)$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = \mathbf{A}\mathbf{u} \times \mathbf{B}\mathbf{u} + \mathbf{B}\mathbf{u} \times \mathbf{C}\mathbf{u} + \mathbf{C}\mathbf{u} \times \mathbf{A}\mathbf{u}$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = \begin{pmatrix} a \\ c+a \end{pmatrix} \times \begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} a \\ c \end{pmatrix} \times \begin{pmatrix} -a \\ c-a \end{pmatrix} + \begin{pmatrix} -a \\ c-a \end{pmatrix} \times \begin{pmatrix} a \\ c+a \end{pmatrix}$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = \begin{pmatrix} 0 \\ 0 \\ -a^2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a^2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.1.8)$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = \begin{pmatrix} 0 \\ 0 \\ -2a^2 \end{pmatrix} \quad (2.1.9)$$

By Substituting (2.1.9) in (2.1.5), We get

$$\text{Area of given } \triangle = \frac{1}{2} \left\| \begin{pmatrix} 0 \\ 0 \\ -2a^2 \end{pmatrix} \right\| \quad (2.1.10)$$

$$= \frac{1}{2} \sqrt{0^2 + 0^2 + (-2a^2)^2} \quad (2.1.11)$$

$$= \frac{1}{2} 2a^2 \quad (2.1.12)$$

$$= a^2 \quad (2.1.13)$$

$$\therefore \text{Area of given } \triangle = a^2 \text{ units}^2 \quad (2.1.14)$$

3 NUMERICAL EXAMPLE

Let,

$$\mathbf{u} = \begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (3.1.15)$$

Solution: By substituting the given values in (2.1.1), We get

$$\mathbf{P} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad (3.1.16)$$

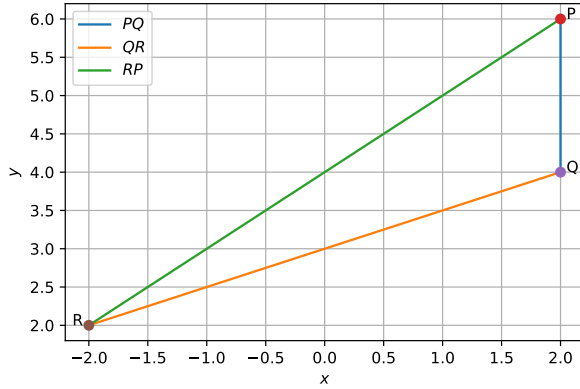


Fig. 3.1: Plot obtained from Python code

Using (2.1.14),

$$\text{Area of given } \triangle = a^2 \quad (3.1.17)$$

$$= 2^2 \quad (3.1.18)$$

$$= 4 \text{ units}^2 \quad (3.1.19)$$