Assignment 1

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The Question

The coordinates of the vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3). The line joining the first two is divided in the ratio $l : k$, and the line joining his point of division to the opposite angular point is then divided in the ratio $m : k + l$. Find the coordinates of the latter point of section.

The solution using scalar algebra

The problem figure is depicted in Fig. 1

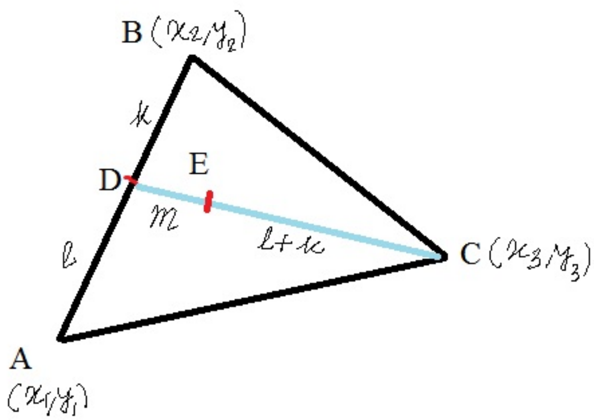

Figure 1: Problem figure

From elementary result of coordinate geometry and in consideration of Fig. 1, the Cartesian coordinates of point D which divides the line AB in the ratio $l : k$, is given by:

$$x_D = \frac{lx_2 + kx_1}{l + k}, y_D = \frac{ly_2 + ky_1}{l + k} \tag{1}$$

Clearly, since E divides the line DC in the ratio $m : l + k$, following the same procedure as above and by setting $l = m$, $k = l + k$,$x_1 = x_D$,$y_1 = y_D$,and $x_2 = x_3, y_2 = y_3$ in Eq.1, we will obtain the coordinates of E which are given by:

$$x_E = \frac{mx_3 + (l + k)x_D}{m + l + k}, y_E = \frac{my_3 + (l + k)y_D}{m + l + k} \tag{2}$$

Which after simplification reads:

$$x_E = \frac{mx_3 + lx_2 + kx_1}{m + l + k}, y_E = \frac{my_3 + ly_2 + ky_1}{m + l + k} \tag{3}$$

The solution using vector algebra

Here a bold faced letter designates the position vector of the point in context. From elementary analysis of coordinate geometry and in view of Fig.1, as $\mathbf{D}$ divides the line AB in the ratio $AD : DC = l : k$, we have:

$$\mathbf{D} = \frac{l\mathbf{B} + k\mathbf{A}}{l+k} \tag{4}$$

The position vector $\mathbf{E}$ which divides CD in the ratio $DE : EC = m : l+k$, is clearly obtained by setting $l = m, k = l+k, \mathbf{A} = \mathbf{D}, \mathbf{B} = \mathbf{C}$ and is given by:

$$\mathbf{E} = \frac{m\mathbf{C} + (l+k)\mathbf{D}}{m+l+k} \tag{5}$$

Using Eq.4 into Eq.5 and simplifying yields :

$$\mathbf{E} = \frac{m\mathbf{C} + l\mathbf{B} + k\mathbf{A}}{m+l+k} \tag{6}$$

Where, $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$

In Fig.2, the solution obtained from the Python code is depicted for a particular choice of input viz. $l = 1, m = 1, k = 1$ and $A(0,0), B(3,3) \,\&\, C(6,0)$. Using, Eq.6 and the above mentioned input, we have: $\mathbf{E} = \begin{pmatrix} x_E \\ y_E \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

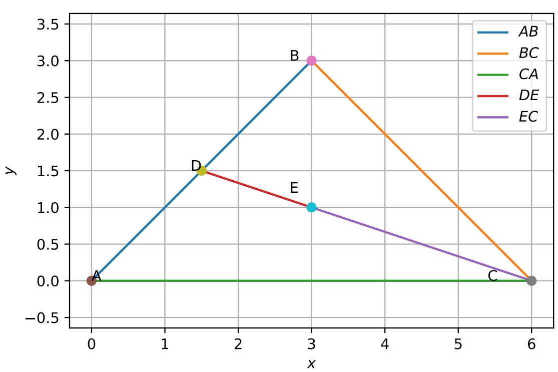

Figure 2: For $l = 1, m = 1, k = 1$ and $A(0,0), B(3,3) \,\&\, C(6,0)$, the solution $E(3,1)$ is obtained using Python