

8.4.24

AI24BTECH11031 - Shivram S

Question: The altitude of a right-angled triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

Variable	Description	Value
BC	Hypotenuse of the triangle	13 cm
AB	Base of the triangle	x cm
AC	Altitude of the triangle	$x - 7$ cm

TABLE 0: Variables Used

Let the length of the base be x cm. The altitude of the triangle is 7 cm less than its base, i.e., $x - 7$ cm. By Pythagoras' Theorem

$$AB^2 + AC^2 = BC^2 \quad (1.1)$$

$$x^2 + (x - 7)^2 = 13^2 \quad (1.2)$$

$$2x^2 - 14x - 120 = 0 \quad (1.3)$$

$$x^2 - 7x - 60 = 0 \quad (1.4)$$

$$(1.5)$$

The equation $y = x^2 - 7x - 60$ can be expressed as a conic

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (1.6)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{7}{2} \\ -\frac{1}{2} \end{pmatrix}, f = -60 \quad (1.7)$$

To find the roots of the equation, we find the points of intersection of the conic with the x -axis

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \quad (1.8)$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (1.9)$$

The values of k are given by

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (1.10)$$

$$= \frac{1}{1} \left(\frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 + 60} \right) \quad (1.11)$$

$$k_1 = -5, k_1 = 12 \quad (1.12)$$

Hence the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}, \begin{pmatrix} 12 \\ 0 \end{pmatrix} \quad (1.13)$$

Hence the solutions of the equation are $x = -5$ and $x = 12$. We reject $x = -5$ as the length of the side can't be negative. Hence, the lengths of the sides are

$$AB = 12 \text{ cm} \quad (1.14)$$

$$AC = 7 \text{ cm} \quad (1.15)$$

$$BC = 13 \text{ cm} \quad (1.16)$$

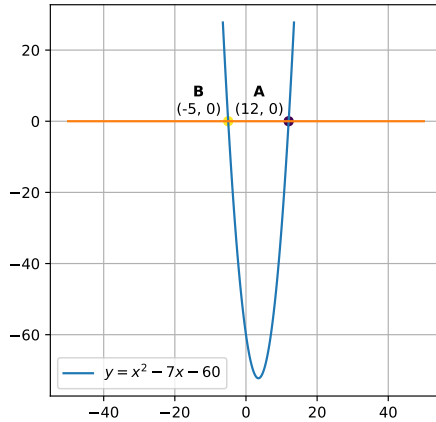


Fig. 1.1: Points of intersection of $y = x^2 - 7x - 60$ with x -axis

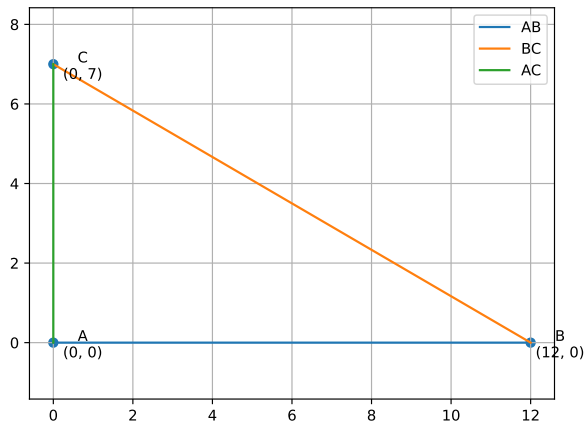


Fig. 1.2: Triangle with sides $AB = 12$ cm, $AC = 7$ cm, and $BC = 13$ cm