

CHAPTER-7
TRIANGLES

1 Exercise 7.1

Q1. In quadrilateral $CBAD$, $CA = AD$ and BA bisect $\angle A$ shown in figure 1. Show that $\triangle CAB \cong \triangle DAB$. What can you say about BC and BD ?

Construction

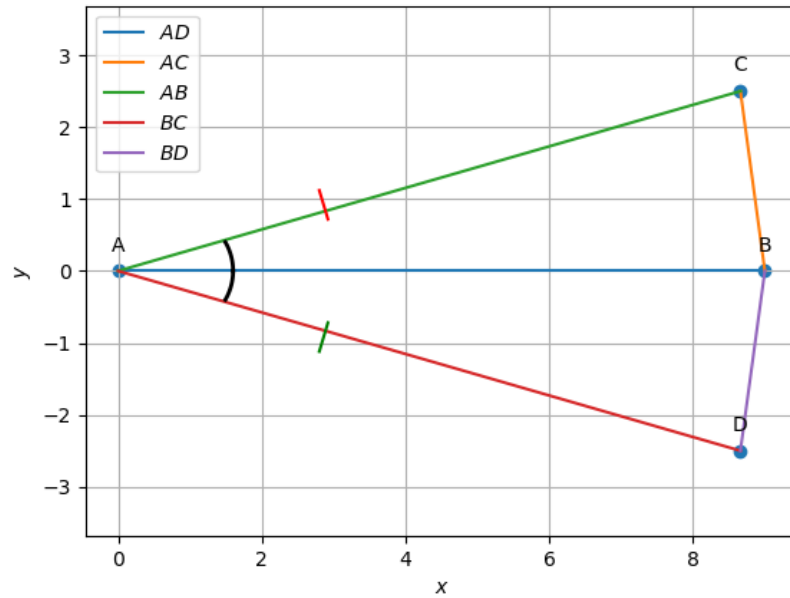


Figure 1: Quadrilateral CBAD

The input parameters for construction are shown in 1:

Symbol	Values	Description
θ	30°	$\angle BAD = \angle BAC$
a	9	AB
c	5	AC
\mathbf{e}_1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	basis vector

Table 1: Parameters

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = a\mathbf{e}_1, \mathbf{C} = \begin{pmatrix} c \cos \theta \\ c \sin \theta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} c \cos \theta \\ -c \sin \theta \end{pmatrix} \quad (1)$$

Solution:

$$\mathbf{C} - \mathbf{A} = \mathbf{A} - \mathbf{D} \quad (2)$$

$$\angle CAB = \angle DAB \quad (3)$$

To Prove:

$$\triangle CAB \cong \triangle DAB \quad (4)$$

Proof:

In $\triangle CAB$ and $\triangle CAD$

Let equation of AB be $y = 0$, which can be written as:

$$\mathbf{n}^\top \mathbf{x} = 0, \quad (5)$$

$$(6)$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (7)$$

Finding the angles(according to assumptions):

$$\text{Let } \theta_1 = \angle CBA \quad (8)$$

$$\mathbf{m}_1 = \mathbf{B} - \mathbf{C} = \begin{pmatrix} 4.7 \\ -2.5 \end{pmatrix}, \mathbf{m}_2 = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} \quad (9)$$

$$\theta_1 = \cos^{-1} \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (10)$$

$$\Rightarrow \theta_1 = \cos^{-1} \frac{\begin{pmatrix} 4.7 & -2.5 \end{pmatrix} \begin{pmatrix} 9 \\ 0 \end{pmatrix}}{(9.2)(9)} = 59.3^\circ \quad (11)$$

$$\text{Let } \theta_2 = \angle ABD \quad (12)$$

$$\mathbf{n}_1 = \mathbf{D} - \mathbf{B} = \begin{pmatrix} -4.7 \\ 2.5 \end{pmatrix}, \mathbf{n}_2 = \mathbf{A} - \mathbf{B} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} \quad (13)$$

$$\theta_2 = \cos^{-1} \frac{\mathbf{n}_1^\top \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (14)$$

$$\Rightarrow \theta_2 = \cos^{-1} \frac{\begin{pmatrix} -4.7 & 2.5 \end{pmatrix} \begin{pmatrix} -9 \\ 0 \end{pmatrix}}{(9.2)(9)} = 59.3^\circ \quad (15)$$

$$(16)$$

from (11) and (15)

$$\angle BAC = \angle BAD \quad (\text{Sum of the angles in a triangle} = 180^\circ)$$

Since all the angles and sides of triangles CAB and CAD are equal , from the definition of congruency both the triangles are said to be congruent to each other.

$$\triangle CAB \cong \triangle DAB \quad (17)$$

$$AB = AD \quad (18)$$