

# Straight Lines

## 11<sup>th</sup> Maths - Chapter 10

This is Problem-10 from Exercise 10.4

1. If three lines whose equations are  $y = m_1x + c_1$ ,  $y = m_2x + c_2$  and  $y = m_3x + c_3$  are concurrent, then show that  $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$ .

**Solution:** Given lines can be written as

$$m_1x - y + c_1 = 0 \quad (1)$$

$$m_2x - y + c_2 = 0 \quad (2)$$

$$m_3x - y + c_3 = 0 \quad (3)$$

The above lines can be written in the form of

$$\mathbf{n}^\top \mathbf{x} = c \quad (4)$$

Therefore,

$$\begin{pmatrix} m_1 & -1 \end{pmatrix} \mathbf{x} = c_1 \quad (5)$$

$$\begin{pmatrix} m_2 & -1 \end{pmatrix} \mathbf{x} = c_2 \quad (6)$$

$$\begin{pmatrix} m_3 & -1 \end{pmatrix} \mathbf{x} = c_3 \quad (7)$$

Solving equations (5), (6) and (7) augmented matrix is

$$\begin{pmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{pmatrix} \quad (8)$$

$$\xleftrightarrow{R_2 \leftarrow m_1 R_2 - m_2 R_1} \begin{pmatrix} m_1 & -1 & c_1 \\ 0 & m_2 - m_1 & m_1 c_2 - m_2 c_1 \\ m_3 & -1 & c_3 \end{pmatrix} \quad (9)$$

$$\xleftrightarrow{R_3 \leftarrow m_1 R_3 - m_3 R_1} \begin{pmatrix} m_1 & -1 & c_1 \\ 0 & m_2 - m_1 & m_1 c_2 - m_2 c_1 \\ 0 & m_3 - m_1 & m_1 c_3 - m_3 c_1 \end{pmatrix} \quad (10)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 \frac{m_2 - m_1}{m_3 - m_1} - R_2} \begin{pmatrix} m_1 & -1 & c_1 \\ 0 & m_2 - m_1 & m_1 c_2 - m_2 c_1 \\ 0 & 0 & (m_1 c_3 - m_3 c_1) \left( \frac{m_2 - m_1}{m_3 - m_1} \right) - (m_1 c_2 - m_2 c_1) \end{pmatrix} \quad (11)$$

Now, for lines to be concurrent, then the third row should be equal to zero.

Therefore,

$$(m_1c_3 - m_3c_1) \left( \frac{m_2 - m_1}{m_3 - m_1} \right) - (m_1c_2 - m_2c_1) = 0 \quad (12)$$

$$\frac{(m_1c_3 - m_3c_1)(m_2 - m_1) - (m_1c_2 - m_2c_1)(m_3 - m_1)}{m_3 - m_1} = 0 \quad (13)$$

$$(m_1c_3 - m_3c_1)(m_2 - m_1) - (m_1c_2 - m_2c_1)(m_3 - m_1) = 0 \quad (14)$$

$$m_2c_3 - m_1c_3 + m_3c_1 - m_3c_2 + m_1c_2 - m_2c_1 = 0 \quad (15)$$

$$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0 \quad (16)$$

Hence proved

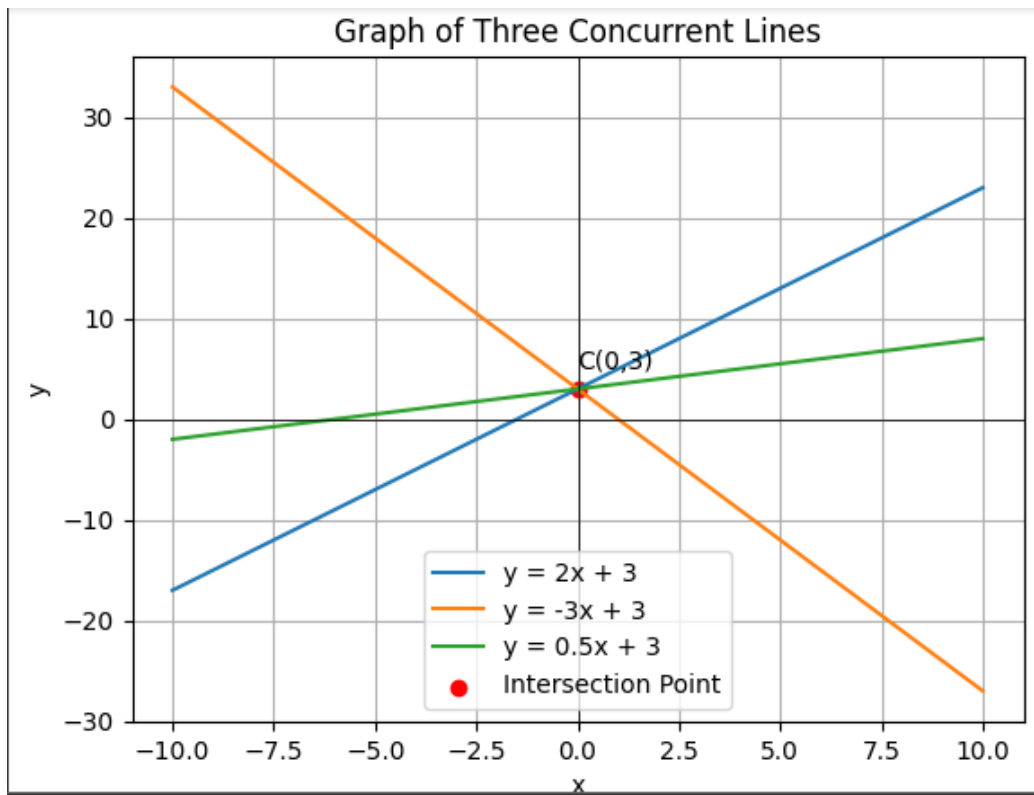


Figure 1: Straight Lines