Question: 12.11.1.5

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1 Problem

Find the direction cosines of the sides of a triangle whose vertices are $\begin{pmatrix}3\\5\\-4\end{pmatrix}$, $\begin{pmatrix}-1\\1\\2\end{pmatrix}$ and $\begin{pmatrix}-5\\-5\\-2\end{pmatrix}$.

2 Solution

Vertices are given by

$$\mathbf{A} = \begin{pmatrix}3\\5\\-4\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix}-1\\1\\2\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{C} = \begin{pmatrix}-5\\-5\\-2\end{pmatrix} \tag{2.0.3}$$

The sides are,

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix}4\\4\\-6\end{pmatrix} = \mathbf{m_1} \tag{2.0.4}$$

$$\mathbf{B}-\mathbf{C} = \begin{pmatrix}4\\6\\4\end{pmatrix} = \mathbf{m_2} \tag{2.0.5}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix}-8\\-10\\2\end{pmatrix} = \mathbf{m_3} \tag{2.0.6}$$

$$\tag{2.0.7}$$

The axes are,

$$\mathbf{e}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \tag{2.0.8}$$

$$\mathbf{e}_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \tag{2.0.9}$$

$$\mathbf{e}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix} \tag{2.0.10}$$

Direction cosines of $\mathbf{m}$,

$$\begin{pmatrix}\cos\theta_1\\\cos\theta_2\\\cos\theta_3\end{pmatrix} = \begin{pmatrix}\frac{\mathbf{m}^\top\mathbf{e}_1}{\|\mathbf{m}\|\|\mathbf{e}_1\|}\\\frac{\mathbf{m}^\top\mathbf{e}_1}{\|\mathbf{m}\|\|\mathbf{e}_1\|}\\\frac{\mathbf{m}^\top\mathbf{e}_1}{\|\mathbf{m}\|\|\mathbf{e}_1\|}\end{pmatrix} \tag{2.0.11}$$

$$= \frac{1}{\|\mathbf{m}\|}\begin{pmatrix}\mathbf{m}^\top\mathbf{e}_1\\\mathbf{m}^\top\mathbf{e}_2\\\mathbf{m}^\top\mathbf{e}_3\end{pmatrix} \tag{2.0.12}$$

$$= \frac{\mathbf{m}}{\|\mathbf{m}\|} \tag{2.0.13}$$

Direction cosines of side $\mathbf{m_1}$,

$$\begin{pmatrix}\cos\theta_1\\\cos\theta_2\\\cos\theta_3\end{pmatrix} = \frac{\mathbf{m_1}}{\|\mathbf{m_1}\|} \tag{2.0.14}$$

$$= \begin{pmatrix}\frac{2}{\sqrt{17}}\\\frac{2}{\sqrt{17}}\\\frac{-3}{\sqrt{17}}\end{pmatrix} \tag{2.0.15}$$

Direction cosines of side $\mathbf{m_2}$,

$$\begin{pmatrix}\cos\theta_1\\\cos\theta_2\\\cos\theta_3\end{pmatrix} = \frac{\mathbf{m_2}}{\|\mathbf{m_2}\|} \tag{2.0.16}$$

$$= \begin{pmatrix}\frac{2}{\sqrt{17}}\\\frac{3}{\sqrt{17}}\\\frac{2}{\sqrt{17}}\end{pmatrix} \tag{2.0.17}$$

Direction cosines of side $\mathbf{m_3}$,

$$\begin{pmatrix}\cos\theta_1\\\cos\theta_2\\\cos\theta_3\end{pmatrix} = \frac{\mathbf{m_3}}{\|\mathbf{m_3}\|} \tag{2.0.18}$$

$$= \begin{pmatrix}\frac{-4}{\sqrt{42}}\\\frac{-5}{\sqrt{42}}\\\frac{1}{\sqrt{42}}\end{pmatrix} \tag{2.0.19}$$

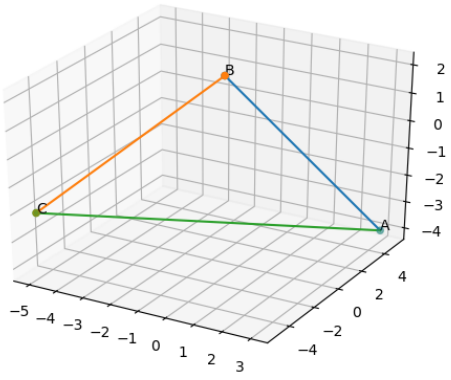

Fig. 0: Triangle ABC