

STRAIGHT LINES

1 11th Maths - Chapter 10

This is Problem 5 from Exercise-10.4

1. Find perpendicular distance from the origin to the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} \quad (1)$$

$$\mathbf{m} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} \cos \phi - \cos \theta \\ \sin \phi - \sin \theta \end{pmatrix} \quad (2)$$

The normal vector is,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos \phi - \cos \theta \\ \sin \phi - \sin \theta \end{pmatrix} = \begin{pmatrix} \sin \phi - \sin \theta \\ \cos \theta - \cos \phi \end{pmatrix} \quad (3)$$

$$\|\mathbf{n}\| = \sqrt{(\sin \phi - \sin \theta)^2 + (\cos \theta - \cos \phi)^2} \quad (4)$$

$$= \sqrt{2 - 2(\sin \phi \sin \theta + \cos \phi \cos \theta)} \quad (5)$$

$$= \sqrt{2 - 2(\cos(\phi - \theta))} \quad (6)$$

$$= \sqrt{2(1 - \cos(\phi - \theta))} \quad (7)$$

$$= \sqrt{2 \left(2 \sin^2 \left(\frac{\phi - \theta}{2} \right) \right)} \quad (8)$$

$$\Rightarrow \|\mathbf{n}\| = 2 \sin \left(\frac{\phi - \theta}{2} \right) \quad (9)$$

The line equation is,

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \quad (10)$$

$$\Rightarrow (\sin \phi - \sin \theta \quad \cos \theta - \cos \phi) \left(\mathbf{x} - \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \right) = 0 \quad (11)$$

$$\Rightarrow (\sin \phi - \sin \theta \quad \cos \theta - \cos \phi) \mathbf{x} = (\sin \phi - \sin \theta \quad \cos \theta - \cos \phi) \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (12)$$

$$= (\sin \phi - \sin \theta) \cos \theta + (\cos \theta - \cos \phi) \sin \theta \quad (13)$$

$$= \sin \phi \cos \theta - \sin \theta \cos \theta + \sin \theta \cos \theta - \sin \theta \cos \phi \quad (14)$$

$$\Rightarrow (\sin \phi - \sin \theta \quad \cos \theta - \cos \phi) \mathbf{x} = \sin(\phi - \theta) \quad (15)$$

from (15)

$$c = \sin(\phi - \theta) \quad (16)$$

The perpendicular distance from the origin to the line is

$$d = \frac{|c|}{\|\mathbf{n}\|} \quad (17)$$

$$\Rightarrow d = \frac{\sin(\phi - \theta)}{2 \sin\left(\frac{\phi - \theta}{2}\right)} \quad (18)$$

The input parameters are

Symbol	Value	Description
θ	0°	assumed angle
ϕ	90°	assumed angle
A	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	point A
B	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	point B
m	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	directional vector
n	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	normal vector

Table 1

substituting the input parameters in (18)

$$\Rightarrow d = \frac{\sin 90^\circ}{2 \sin 45^\circ} \quad (19)$$

$$\Rightarrow d = \frac{1}{\sqrt{2}} \quad (20)$$

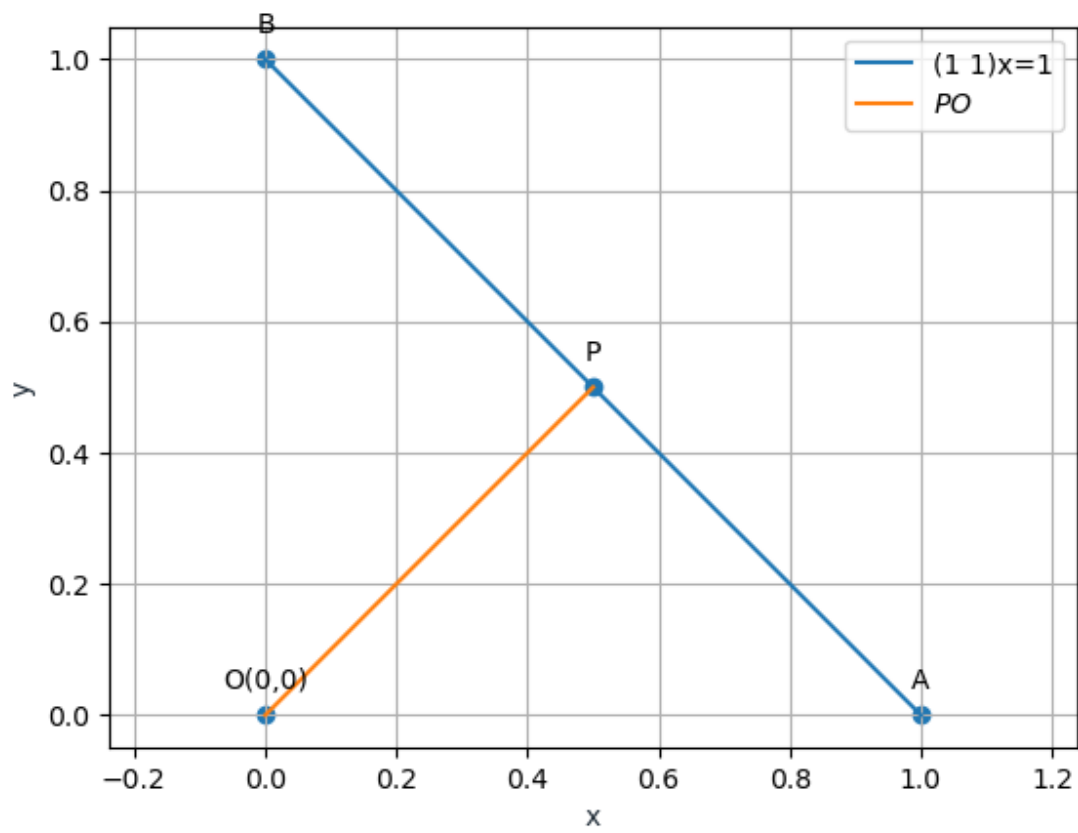


Figure 1