

Vector Dot Product

1 12th Maths - Chapter 10

This is Problem-9 from Exercise 10.3

1. Find $\|\mathbf{x}\|$, if for a unit vector \mathbf{a} , $(\mathbf{x} - \mathbf{a}) \cdot (\mathbf{x} + \mathbf{a}) = 12$.

Solution:

$$(\mathbf{x} - \mathbf{a})^\top (\mathbf{x} + \mathbf{a}) = 12 \quad (1)$$

$$\mathbf{x}^\top \mathbf{x} - \mathbf{a}^\top \mathbf{x} + \mathbf{x}^\top \mathbf{a} - \mathbf{a}^\top \mathbf{a} = 12 \quad (2)$$

$$\implies \|\mathbf{x}\|^2 - \|\mathbf{a}\|^2 = 12 \quad (3)$$

$$\|\mathbf{x}\|^2 - 1 = 12 \quad (4)$$

$$\|\mathbf{x}\|^2 = 13 \quad (5)$$

$$\|\mathbf{x}\| = \sqrt{13} \quad (6)$$