Assignment 1

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1) In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.
   a) $7x + 5y + 6z + 30 = 0$ and $3x–y–10z + 4 = 0$
   b) $2x + y + 3z–2 = 0$ and $x–2y + 5 = 0$
   c) $2x–2y + 4z + 5 = 0$ and $3x–3y + 6z–1 = 0$
   d) $2x–y + 3z–1 = 0$ and $2x–y + 3z + 3 = 0$
   e) $4x + 8y + z–8 = 0$ and $y + z–4 = 0$

   Solution: The angle between the planes is the angle between the normals of the given planes.

$$\mathbf{n_1}^\top \mathbf{x} = c_1,\ \mathbf{n_2}^\top \mathbf{x} = c_2 \tag{0.0.1}$$

The angle θ between the planes is given by,

$$\cos\theta = \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{0.0.2}$$

a)

$$\mathbf{n_1} = \begin{pmatrix}7\\5\\6\end{pmatrix},\ \mathbf{n_2} = \begin{pmatrix}3\\-1\\-10\end{pmatrix} \tag{0.0.3}$$

$$\mathbf{n_1}^\top \mathbf{n_2} = \begin{pmatrix}7 & 5 & 6\end{pmatrix}\begin{pmatrix}3\\-1\\-10\end{pmatrix} \tag{0.0.4}$$

$$= -44 \tag{0.0.5}$$

$$\|\mathbf{n_1}\| = \sqrt{7^2 + 5^2 + 6^2} \tag{0.0.6}$$

$$= \sqrt{110} \tag{0.0.7}$$

$$\|\mathbf{n_2}\| = \sqrt{3^2 + (-1)^2 + (-10)^2} \tag{0.0.8}$$

$$= \sqrt{110} \tag{0.0.9}$$

$$\cos\theta = -\frac{44}{\sqrt{110}\sqrt{110}} \tag{0.0.10}$$

$$= -\frac{2}{5} \tag{0.0.11}$$

The planes are inclined at an angle of $\arccos\left(-\frac{2}{5}\right)$ degrees.

b)

$$\mathbf{n_1} = \begin{pmatrix}2\\1\\3\end{pmatrix},\ \mathbf{n_2} = \begin{pmatrix}1\\-2\\0\end{pmatrix} \tag{0.0.12}$$

$$\mathbf{n_1}^\top \mathbf{n_2} = \begin{pmatrix}2 & 1 & 3\end{pmatrix}\begin{pmatrix}1\\-2\\0\end{pmatrix} \tag{0.0.13}$$

$$= 0 \tag{0.0.14}$$

$$\cos\theta = 0 \tag{0.0.15}$$

The planes are perpendicular.

c)

$$\mathbf{n_1} = \begin{pmatrix}2\\-2\\4\end{pmatrix},\ \mathbf{n_2} = \begin{pmatrix}3\\-3\\6\end{pmatrix} \tag{0.0.16}$$

$$\mathbf{n_1}^\top \mathbf{n_2} = \begin{pmatrix}2 & -2 & 4\end{pmatrix}\begin{pmatrix}3\\-3\\6\end{pmatrix} \tag{0.0.17}$$

$$= 36 \tag{0.0.18}$$

$$\|\mathbf{n_1}\| = \sqrt{2^2 + (-2)^2 + 4^2} \tag{0.0.19}$$

$$= \sqrt{24} \tag{0.0.20}$$

$$\|\mathbf{n_2}\| = \sqrt{3^2 + (-3)^2 + 6^2} \tag{0.0.21}$$

$$= \sqrt{54} \tag{0.0.22}$$

$$\cos\theta = \frac{36}{\sqrt{24}\sqrt{54}} \tag{0.0.23}$$

$$= 1 \tag{0.0.24}$$

The planes are parallel.

d)

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \tag{0.0.25}$$

$$\mathbf{n_1}^\top \mathbf{n_2} = \begin{pmatrix} 2 & -1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \tag{0.0.26}$$

$$= 14 \tag{0.0.27}$$

$$\|\mathbf{n_1}\| = \sqrt{2^2 + (-1)^2 + 3^2} \tag{0.0.28}$$

$$= \sqrt{14} \tag{0.0.29}$$

$$\|\mathbf{n_2}\| = \sqrt{2^2 + (-1)^2 + 3^2} \tag{0.0.30}$$

$$= \sqrt{14} \tag{0.0.31}$$

$$\cos\theta = \frac{14}{\sqrt{14}\sqrt{14}} \tag{0.0.32}$$

$$= 1 \tag{0.0.33}$$

The planes are parallel.

e)

$$\mathbf{n_1} = \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \tag{0.0.34}$$

$$\mathbf{n_1}^\top \mathbf{n_2} = \begin{pmatrix} 4 & 8 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \tag{0.0.35}$$

$$= 9 \tag{0.0.36}$$

$$\|\mathbf{n_1}\| = \sqrt{4^2 + 8^2 + 1^2} \tag{0.0.37}$$

$$= 9 \tag{0.0.38}$$

$$\|\mathbf{n_2}\| = \sqrt{0^2 + 1^2 + 1^2} \tag{0.0.39}$$

$$= \sqrt{2} \tag{0.0.40}$$

$$\cos\theta = \frac{9}{9\sqrt{2}} \tag{0.0.41}$$

$$= \frac{1}{\sqrt{2}} \tag{0.0.42}$$

The planes are inclined at an angle of 45 degrees.