CLASS-9
CHAPTER-10
CIRCLES

Excercise 10.5

Q1. In Figure 1. $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are the three points with centre $\mathbf{O}$ such that $\angle$BOC=30°and $\angle$AOB=60°. If $\mathbf{D}$ is a point on the circle other than the arc ABC, find $\angle$ADC

Solution

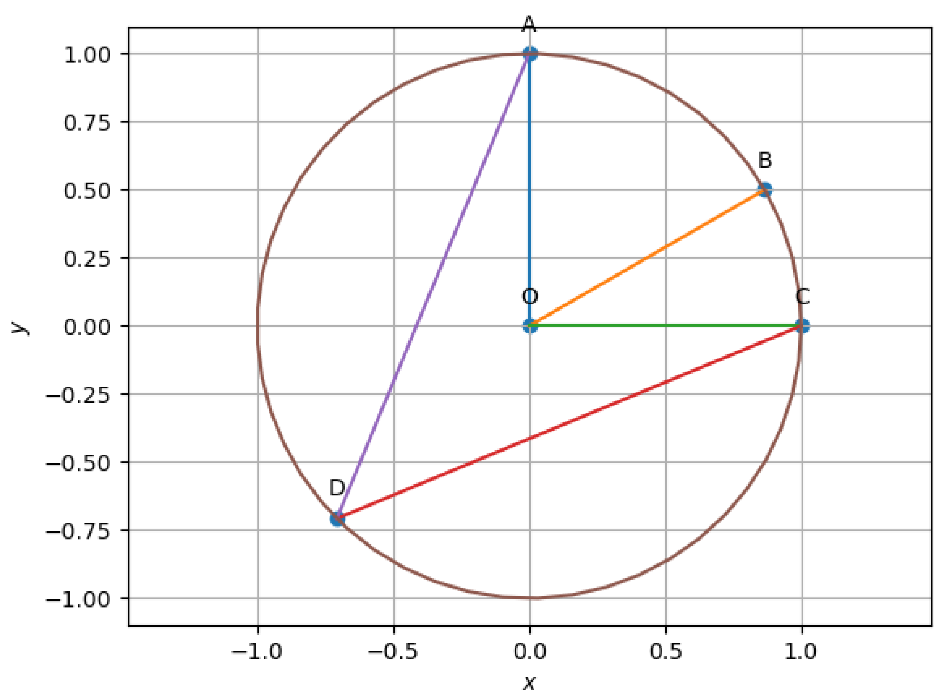

Figure 1:

Construction

Symbol	Values	Description
r	1 unit	Radius of OA and OB
$\mathbf{O}$	$\begin{pmatrix}0\\0\end{pmatrix}$	Center of the circle
$\mathbf{C}$	$\begin{pmatrix}1\\0\end{pmatrix}$	Standard basis vector $\mathbf{e}_1$
α	30°	∠BOC
β	60°	∠AOB
γ	??	∠ADC

Verification:

From assumptions the vector points $\mathbf{C}$,$\mathbf{A}$,$\mathbf{D}$ be

$$\mathbf{C} = \mathbf{e}_1 = \begin{pmatrix}1\\0\end{pmatrix}, \mathbf{A} = \begin{pmatrix}\cos(\alpha+\beta)\\ \sin(\alpha+\beta)\end{pmatrix}, \mathbf{D} = \begin{pmatrix}\cos\gamma\\ \sin\gamma\end{pmatrix} \tag{1}$$

The cosine of the angle subtended at point $\mathbf{D}$ is given by

$$\cos(\angle ADC) = \frac{(\mathbf{A}-\mathbf{D})^\top(\mathbf{C}-\mathbf{D})}{\|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{C}-\mathbf{D}\|} \tag{2}$$

Where

$$\mathbf{A}-\mathbf{D} = \begin{pmatrix}\cos(\alpha+\beta)-\cos\gamma\\ \sin(\alpha+\beta)-\sin\gamma\end{pmatrix}, \mathbf{C}-\mathbf{D} = \begin{pmatrix}1-\cos\gamma\\ -\sin\gamma\end{pmatrix} \tag{3}$$

$$(\mathbf{A}-\mathbf{D})^\top(\mathbf{C}-\mathbf{D}) = 4\sin\frac{\alpha+\gamma}{2}\sin\frac{\beta+\gamma}{2}\cos\frac{\alpha+\beta}{2} \tag{4}$$

$$\|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{C}-\mathbf{D}\| = 4\sin\frac{\alpha+\gamma}{2}\sin\frac{\beta+\gamma}{2} \tag{5}$$

Substituting (4) and (5) in (2),

$$\cos(\angle ADC) = \frac{4\sin\frac{\alpha+\gamma}{2}\sin\frac{\beta+\gamma}{2}\cos\frac{\alpha+\beta}{2}}{4\sin\frac{\alpha+\gamma}{2}\sin\frac{\beta+\gamma}{2}} \tag{6}$$

$$\cos(\angle ADC) = \cos\frac{\alpha+\beta}{2} \tag{7}$$

By substituting α and β values in (7)

$$\angle ADC = \frac{\alpha+\beta}{2} = \frac{(30^\circ + 60^\circ)}{2} = 45^\circ \tag{8}$$