VECTORS

12^{th}Math - Chapter 10

This is Problem-16 from Exercise 10.5

If θ is the angle between two vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$,then $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} \geq 0$ only when

(a) $0 < \theta < \frac{\pi}{2}$

(b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$

(d) $0 \leq \theta \leq \pi$

Solution: Given $\mathbf{a}, \mathbf{b}$ are two vectors

$$\text{We know that} \tag{1}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}\right) \tag{2}$$

$$\implies \mathbf{a}^\top \mathbf{b} = \cos\theta \|\mathbf{a}\|\|\mathbf{b}\| \tag{3}$$

$$\mathbf{a}^\top \mathbf{b} \geq 0 \tag{4}$$

$$\cos\theta \|\mathbf{a}\|\|\mathbf{b}\| \geq 0 \tag{5}$$

$$\implies \cos\theta \geq 0 \tag{6}$$

$$\therefore \theta \text{ lies between } 0 \leq \theta \leq \frac{\pi}{2} \text{ and } \frac{3\pi}{2} \leq \theta \leq 2\pi \tag{7}$$

(a) $0 < \theta < \frac{\pi}{2}$
Comparing with (7), option (a) is incorrect.

(b) $0 \leq \theta \leq \frac{\pi}{2}$
Comparing with (7), option (b) is correct.

(c) $0 < \theta < \pi$
Comparing with (7), option (c) is incorrect.

(d) $0 \leq \theta \leq \pi$
Comparing with (7), option (d) is incorrect.