11.11.4.5

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Class 11, Chapter 11, Exercise 4.5

Q. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas, whose equation is given by $5y^2 - 9x^2 = 36$.

The equation of the hyperbola can be rearranged as

$$-x^2 + \frac{5}{9}y^2 - 4 = 0 \tag{1}$$

The above equation can be equaded to the generic equation of conic sections

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

Comparing coefficients of both equations (1) and (2)

$$\mathbf{V} = \begin{pmatrix} -1 & 0 \\ 0 & \frac{5}{9} \end{pmatrix} \tag{3}$$

$$\mathbf{u} = \mathbf{0} \tag{4}$$

$$f = -4 \tag{5}$$

From equation (3), since $\mathbf{V}$ is already diagonalized, the eigen values λ_1 and λ_2 are given as

$$\lambda_1 = -1 \tag{6}$$

$$\lambda_2 = \frac{5}{9} \tag{7}$$

1) The eccentricity of the hyperbola is given as

$$e = \sqrt{1 - \frac{\lambda_2}{\lambda_1}} = \sqrt{1 + \frac{5}{9}} \tag{8}$$

$$= \frac{\sqrt{14}}{3} \tag{9}$$

2) For the standard hyperbola, the coordinates of Focii are given as

$$\mathbf{F} = \pm \frac{\left(\frac{1}{e\sqrt{1-e^2}}\right)\left(e^2\right)\sqrt{\frac{\lambda_1}{f_0}}}{\frac{\lambda_1}{f_0}}\mathbf{e}_2 \tag{10}$$

where

$$f_0 = -f \tag{11}$$

$$(10) \implies = \pm \frac{\left(\frac{1}{\frac{\sqrt{14}}{3}\sqrt{1-\frac{14}{9}}}\right)\left(\frac{14}{9}\right)\sqrt{\frac{-1}{4}}}{\frac{-1}{4}}\mathbf{e}_2 \tag{12}$$

$$= \pm \begin{pmatrix} 0 \\ \frac{6}{2\sqrt{\frac{14}{5}}} \end{pmatrix} \tag{13}$$

3) The vertices of the hyperbola are given by

$$\pm \begin{pmatrix} 0 \\ \sqrt{\left|\frac{f_0}{\lambda_2}\right|} \end{pmatrix} = \pm \begin{pmatrix} 0 \\ \frac{6}{\sqrt{5}} \end{pmatrix} \tag{14}$$

4) The length of latus rectum is given as

$$2\frac{\sqrt{|f_0 \lambda_2|}}{\lambda_1} = 2\frac{\sqrt{\left|14\left(\frac{5}{9}\right)\right|}}{-1} \tag{15}$$

$$= 4\frac{\sqrt{5}}{3} \tag{16}$$

as length cannot be negative.

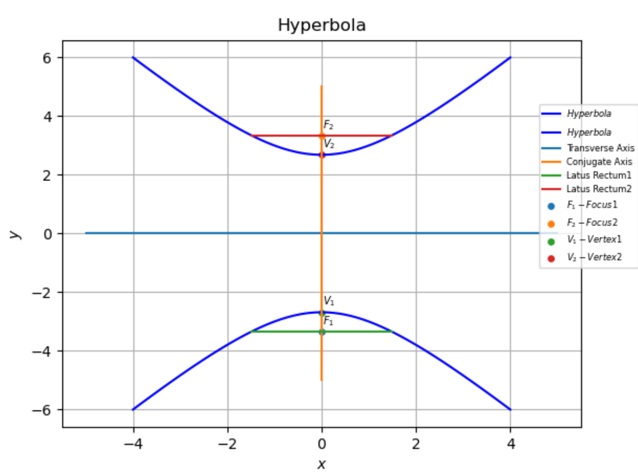

Fig. 1