

# Coordinate Geometry

1. **Problem statement :** Find a relation between  $x$  and  $y$  such that the point  $(x, y)$  is equidistant from the point  $(3, 6)$  and  $(-3, 4)$

**Solution:**

**Method I**

The input parameters for this problem are given as

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (1)$$

If  $\mathbf{P}$  equidistant from the points  $\mathbf{A}$  and  $\mathbf{B}$ ,

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \quad (2)$$

$$\implies \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \quad (3)$$

which can be expressed as

$$(\mathbf{P} - \mathbf{A})^\top (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B})^\top (\mathbf{P} - \mathbf{B}) \quad (4)$$

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{A} + \|\mathbf{A}\|^2 \quad (5)$$

$$= \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{B} + \|\mathbf{B}\|^2 \quad (6)$$

which can be simplified to obtain

$$\mathbf{P} = y\mathbf{e}_1 \quad (7)$$

where

$$y = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top \mathbf{e}_1} \quad (8)$$

now substituting the  $\mathbf{A}$  and  $\mathbf{B}$  values in (8)

$$(\mathbf{A} - \mathbf{B})^\top = \left( \begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right)^\top = (6 \ 2) \quad (9)$$

$$\|\mathbf{A}\|^2 = 45 \quad (10)$$

$$\|\mathbf{B}\|^2 = 25 \quad (11)$$

upon substituting the values in (8) the value of  $y = 5$

Hence, the desired point is  $\mathbf{P}$  is  $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ .

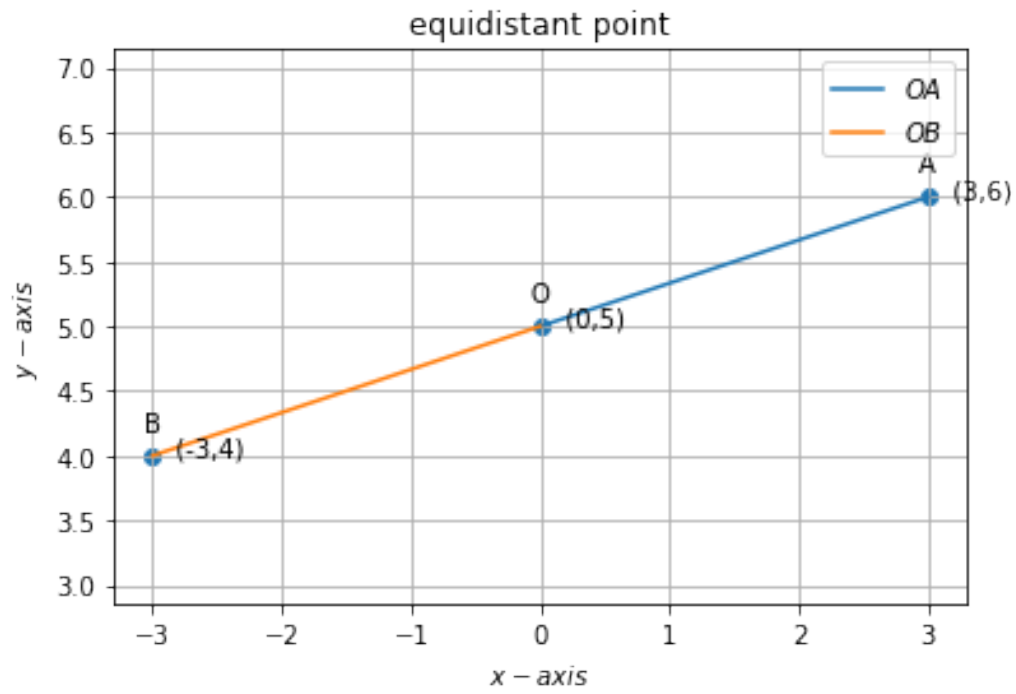


Figure 1