Excercise 6.3

Q19.Find the points on the curve $x^2+y^2-2x-3=0$ at which the tangents are parallel to the x-axis.

Solution: The equation of circle is given as

$$x^2+y^2-2x-3=0 \tag{1}$$

The standard equation of circle is given by

$$\|\mathbf{x}\|^2+2\mathbf{x}^\top\mathbf{u}+f=0 \tag{2}$$

comparing (1) and (2) we get

$$\mathbf{u}=\begin{pmatrix}-1\\0\end{pmatrix} \tag{3}$$

$$f=-3 \tag{4}$$

Hence, the centre and radius are given as

$$\mathbf{c}=-\mathbf{u}=\begin{pmatrix}1\\0\end{pmatrix} \tag{5}$$

$$r=\sqrt{\|\mathbf{u}\|^2-f} \tag{6}$$

$$=2 \tag{7}$$

For a circle the point of contact of tangent are given by

$$\mathbf{q}_{ij}=\left(\pm r\frac{\mathbf{n}_j}{\|\mathbf{n}_j\|}-\mathbf{u}\right)\ \text{i,j}=1,2 \tag{8}$$

Since, tangents are parallel to x-axis, the normal is given as

$$\mathbf{n}=\begin{pmatrix}0\\1\end{pmatrix} \tag{9}$$

Substituting in (8) we get

$$\mathbf{q}_{11} = \left(\pm 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right) \tag{10}$$

$$= \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{11}$$

Hence, the two points of contact are

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{12}$$

See Fig.1.

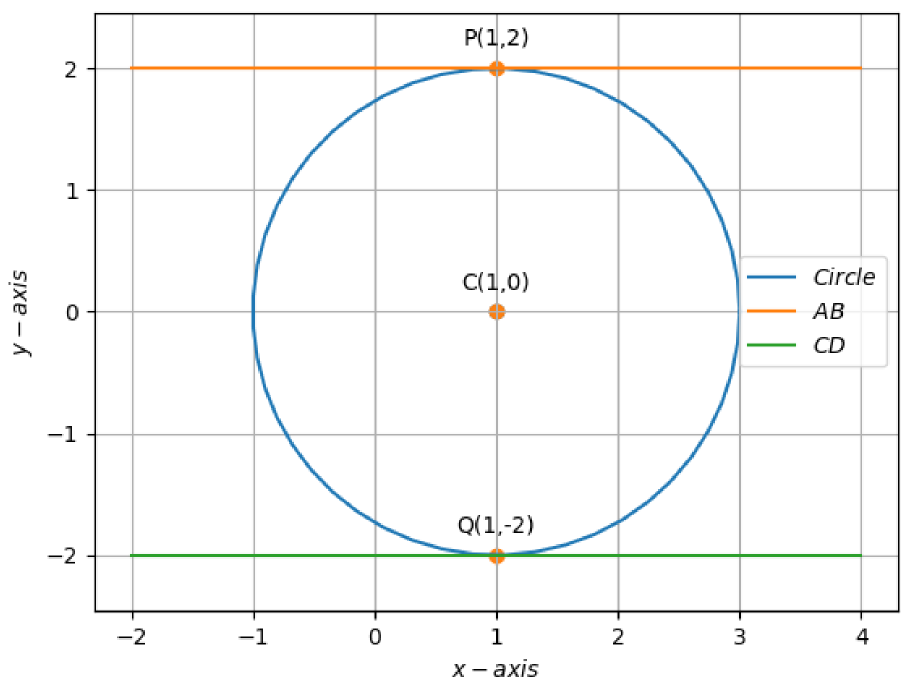

Figure 1: