

Conic Assignment

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Abstract—This document contains the solution to Question 6 of Exercise 5 in Chapter 11 of the class 11 NCERT textbook.

- 1) Find the area of the triangle formed by the lines joining the vertex of the parabola

$$x^2 = 12y \quad (1)$$

to the ends of its latus rectum.

Solution: Rewriting (1) in matrix form,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & -6 \end{pmatrix} \mathbf{x} = 0 \quad (2)$$

Since the parabola is clearly symmetric about the y -axis, we see that the directrix is parallel to the x -axis, thus

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (3)$$

Using the standard definition of the conic and equating \mathbf{u} and f ,

$$\begin{pmatrix} 0 \\ -6 \end{pmatrix} = c \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \mathbf{F} \quad (4)$$

$$0 = \|\mathbf{F}\|^2 - c^2 \quad (5)$$

From (4), we have

$$\mathbf{F} = \begin{pmatrix} 0 \\ c + 6 \end{pmatrix} \quad (6)$$

Using (6) in (5),

$$(c + 6)^2 = c^2 \quad (7)$$

$$\implies c = -3 \quad (8)$$

Thus,

$$\mathbf{F} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (9)$$

The latus rectum of the parabola is the chord passing through the focus parallel to the directrix. Its equation is given by

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \end{pmatrix} = 3 \quad (10)$$

Hence, for $\lambda \in \mathbb{R}$,

$$\mathbf{x} = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} \lambda \\ 3 \end{pmatrix} \quad (11)$$

Adding (2) to 12 times (10), and using (11)

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = 36 \quad (12)$$

$$\implies \lambda^2 = 36 \quad (13)$$

$$\implies \lambda = \pm 6 \quad (14)$$

Thus, the ends of the latus rectum are

$$\mathbf{x} = \begin{pmatrix} \pm 6 \\ 3 \end{pmatrix} \quad (15)$$

Since the vertex of the parabola is at $\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, we see that the area of the required triangle is

$$\text{ar}(\triangle PAB) = \frac{1}{2} \begin{vmatrix} 6 & 3 \\ -6 & 3 \end{vmatrix} = 18 \text{ sq. units} \quad (16)$$

The situation is illustrated in Fig. 1, plotted using the Python code `codes/parabola.py`.

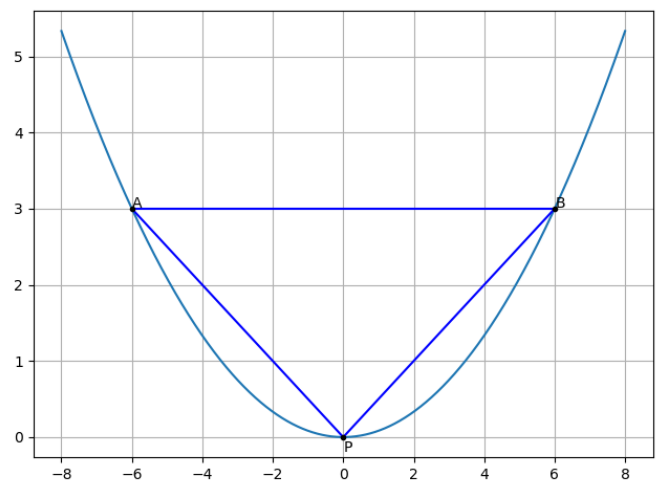


Fig. 1: PAB is the triangle whose area is to be found.