Equation of Line

1 11^{th} Maths - Chapter 10

This is Problem-12 from Exercise 10.2

1. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point $(2, 3)$.
   Solution: Let $\mathbf{P}(a, 0)$, and $\mathbf{Q}(0, a)$ be the 2 points on x and y-axes respectively having a as the intercept on both the axes. We know that the the direction vector $\mathbf{m}$ of the line joining two points $\mathbf{P}, \mathbf{Q}$ is given by

$$\mathbf{m} = \mathbf{P} - \mathbf{Q} \tag{1}$$

$$= \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ a \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2}$$

   $\implies$ The normal vector $\mathbf{n}$ to the line is given as

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3}$$

   The equation of a line with normal vector $\mathbf{n}$ and passing through a point $\mathbf{A}(2, 3)$ is given by

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{4}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) = 0 \tag{5}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - 5 = 0 \tag{6}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 5 \tag{7}$$

To find the intercepts, we know that, since **P** and **Q** lie on the straight line, they should satisfy the equation 7.

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{P} = 5 \tag{8}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = 5 \tag{9}$$

$$a + 0 = 5 \tag{10}$$

$$a = 5 \tag{11}$$

Both **P** and **Q** have the same intercept value, hence the intercept on both x and y-axes is 5 units. The line segment is as shown in Figure 1.

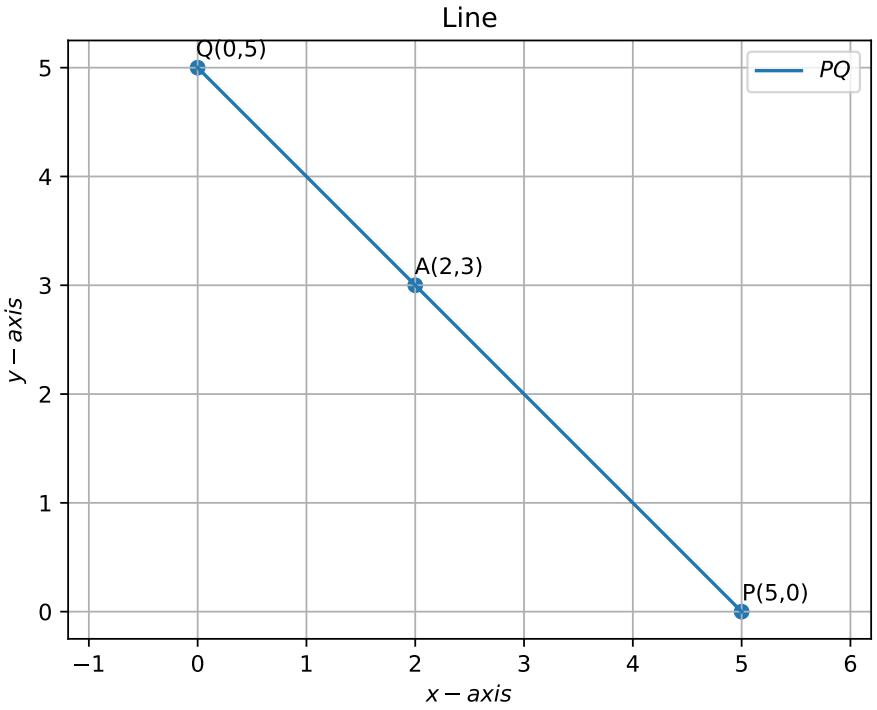

Figure 1