

VECTORS

1 10th Maths - EXERCISE-7.3

1. That a median of a triangle divides it into two triangles of equal areas.
verify this result for $\triangle ABC$ whose vertices are $\mathbf{A}(4, -6)$, $\mathbf{B}(3, -2)$ and $\mathbf{C}(5, 2)$.

2 SOLUTION

Given points are

$$\mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad (1)$$

The median of the triangle

$$\mathbf{D} = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (2)$$

$$= \frac{\begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}}{2} \quad (3)$$

$$= \frac{\begin{pmatrix} 8 \\ 0 \end{pmatrix}}{2} \quad (4)$$

$$\mathbf{D} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (5)$$

The ar(ABD) can be expressed as

$$ar(ABD) = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{D})\| \quad (6)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} \quad (7)$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} \quad (8)$$

Substituting the values of (7) and (8) in (6),

$$ar(ABD) = \frac{1}{2} \begin{vmatrix} 1 & 0 \\ -4 & -6 \end{vmatrix} \quad (9)$$

$$= \frac{6}{2} \quad (10)$$

$$ar(ABD) = 3 \quad (11)$$

Also, the $ar(ACD)$ can be expressed as

$$ar(ACD) = \frac{1}{2} \|(\mathbf{A} - \mathbf{C}) \times (\mathbf{A} - \mathbf{D})\| \quad (12)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -8 \end{pmatrix} \quad (13)$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} \quad (14)$$

Substituting the values of (13) and (14) in (12),

$$ar(ACD) = \frac{1}{2} \begin{vmatrix} -1 & 0 \\ -8 & -6 \end{vmatrix} \quad (15)$$

$$= \frac{6}{2} \quad (16)$$

$$ar(ACD) = 3 \quad (17)$$

The median of the triangle is both side areas are equal $\triangle ABD = \triangle ACD$

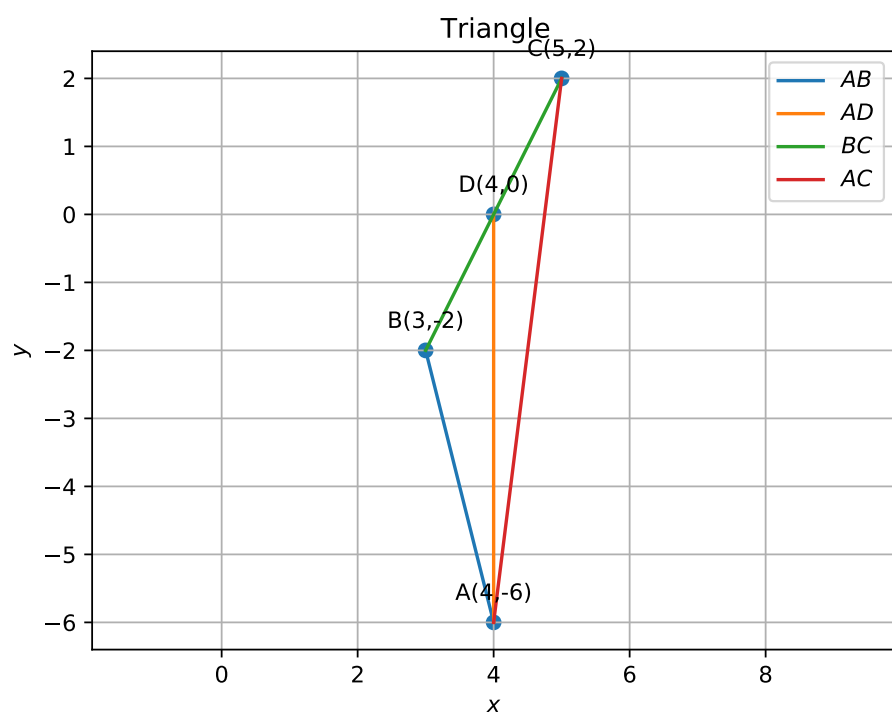


Figure 1: Triangle