

# Conic Sections - Hyperbola

## 1 11<sup>th</sup> Maths - Chapter 11

This is Problem-1 from Exercise 11.4

1. Find the coordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of a hyperbola whose equation is given by  $\frac{x^2}{16} - \frac{y^2}{9} = 1$ .

**Solution:** From the given equation of hyperbola, we can get values of

$$a = 4 \quad (1)$$

$$b = 3 \quad (2)$$

The given equation of the hyperbola can be rearranged as

$$9x^2 - 16y^2 - 144 = 0 \quad (3)$$

The above equation can be equated to the generic equation of conic sections

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (4)$$

Comparing coefficients of both equations (3) and (4)

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & -16 \end{pmatrix} \quad (5)$$

$$\mathbf{u} = 0 \quad (6)$$

$$f = -144 \quad (7)$$

From equation (5), since  $\mathbf{V}$  is already diagonalized, the Eigen values  $\lambda_1$  and  $\lambda_2$  are given as

$$\lambda_1 = 9 \quad (8)$$

$$\lambda_2 = -16 \quad (9)$$

(a) The eccentricity of the hyperbola is given as

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \quad (10)$$

$$= \sqrt{1 - \frac{9}{-16}} \quad (11)$$

$$= \frac{5}{4} \quad (12)$$

(b) For the standard hyperbola, the coordinates of Foci are given as:

$$\mathbf{F} = \pm \frac{\left(\frac{1}{e\sqrt{1-e^2}}\right) (e^2) \sqrt{\frac{\lambda_2}{f_0}}}{\frac{\lambda_2}{f_0}} \mathbf{e}_1 \quad (13)$$

where

$$f_0 = -f \quad (14)$$

$$(13) \implies = \pm \frac{\left(\frac{1}{\frac{5}{4}\sqrt{1-\frac{25}{16}}}\right) \left(\frac{25}{16}\right) \sqrt{\frac{-16}{144}} \mathbf{e}_1}{\frac{-16}{144}} \quad (15)$$

$$= \pm \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (16)$$

(c) The vertices of the hyperbola are given by

$$\pm \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (17)$$

$$= \pm \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (18)$$

(d) The length of the latus rectum is given as

$$2 \frac{\sqrt{|f_0 \lambda_1|}}{\lambda_2} \quad (19)$$

$$= 2 \frac{\sqrt{|144(9)|}}{-16} \quad (20)$$

$$= \frac{9}{2} \quad (21)$$

as length can't be negative.

The relevant diagram is shown in Figure 1

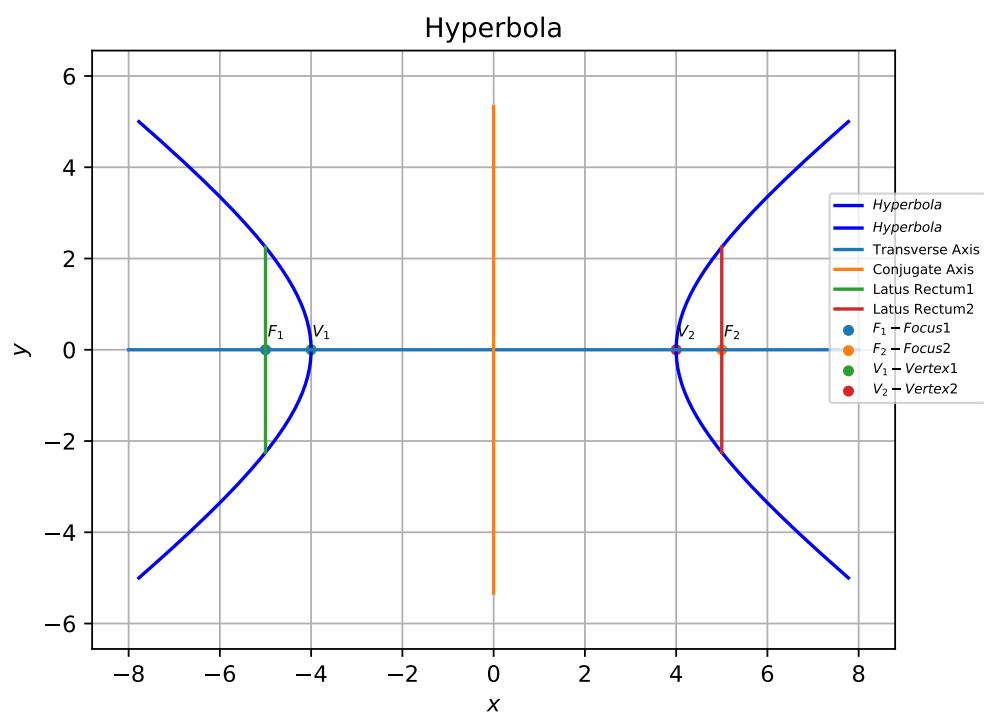


Figure 1