

TRIANGLES

9th Math - Chapter 7

Problem

In the given Figure 1, $AC=AE$, $AB=AD$ and $\angle BAD = \angle EAC$. Show that $BC=DE$.

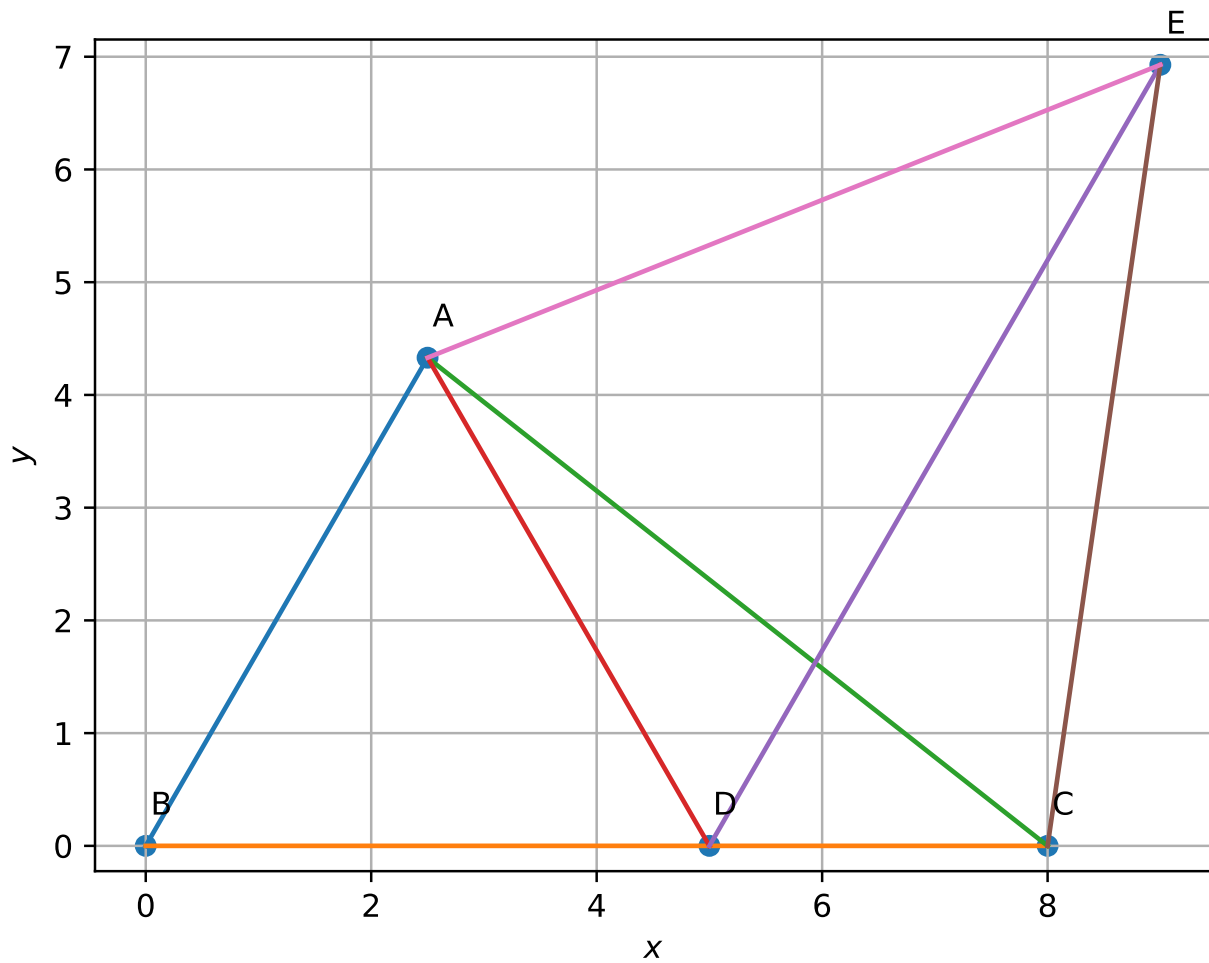


Figure 1

Construction

The input parameters for construction.

Symbol	Values	Description
θ	60°	$\angle BAD = \angle EAC$
a	8	BC
c	5	AB
\mathbf{e}_1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	basis vector

$$b = \sqrt{a^2 + c^2 - 2ac \cos \theta} \quad (1)$$

$$\mathbf{A} = c \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{D} = \left(2c \sin \frac{\theta}{2} \right) \mathbf{e}_1 \quad (2)$$

$$\angle BCA = \cos^{-1} \left(\frac{a^2 + b^2 - c^2}{2ab} \right) \quad (3)$$

$$\angle ACE = 90^\circ - \frac{\theta}{2} \quad (4)$$

$$\phi = 180^\circ - (\angle BCA + \angle ACE) \quad (5)$$

$$\mathbf{E} = \mathbf{C} + \left(2b \sin \frac{\theta}{2} \right) \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} \quad (6)$$

$$(7)$$

Solution: Given:

$$\mathbf{A} - \mathbf{C} = \mathbf{A} - \mathbf{E} \quad (8)$$

$$\mathbf{A} - \mathbf{B} = \mathbf{A} - \mathbf{D} \quad (9)$$

$$\angle BAD = \angle EAC \quad (10)$$

To prove :

$$\mathbf{B} - \mathbf{C} = \mathbf{D} - \mathbf{E} \quad (11)$$

Proof

In $\triangle ABC$ and in $\triangle ADE$

$$\|\mathbf{A} - \mathbf{B}\| = \left\| \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix} \right\| = 5 \quad (12)$$

$$\|\mathbf{A} - \mathbf{D}\| = \left\| \begin{pmatrix} -2.6 \\ 4.33 \end{pmatrix} \right\| = 5 \quad (13)$$

$$\Rightarrow \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{D}\| \quad (14)$$

$$\|\mathbf{A} - \mathbf{C}\| = \left\| \begin{pmatrix} -5.5 \\ 4.33 \end{pmatrix} \right\| = 7 \quad (15)$$

$$\|\mathbf{A} - \mathbf{E}\| = \left\| \begin{pmatrix} -6.5 \\ -2.5 \end{pmatrix} \right\| = 7 \quad (16)$$

$$\Rightarrow \|\mathbf{A} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{E}\| \quad (17)$$

$$\angle BAC = \cos^{-1} \frac{(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{C}\|} = 82^\circ \quad (18)$$

$$\angle DAE = \cos^{-1} \frac{(\mathbf{A} - \mathbf{D})^\top (\mathbf{A} - \mathbf{E})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{A} - \mathbf{E}\|} = 82^\circ \quad (19)$$

from (18) and (19)

$$\angle BAC = \angle DAE \quad (20)$$

from (14),(17) and (20)

$$\triangle ABC \cong \triangle ADE \tag{21}$$

from (21)

$$\mathbf{B - C = D - E} \tag{22}$$