

Properties of Quadrilaterals

1 10th Maths - Chapter 7

This is Problem-8 from Exercise 7.4

1. ABCD is a rectangle formed by the points **A**(-1, -1), **B**(-1, 4), **C**(5, 4) and **D**(5, -1). **P**, **Q**, **R** and **S** are the mid-points of **AB**, **BC**, **CD** and **DA** respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Solution: Refer figure 1

$$\mathbf{P} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) = \frac{1}{2} \left(\begin{pmatrix} -1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} \right) = \begin{pmatrix} -1 \\ \frac{3}{2} \end{pmatrix} \quad (1)$$

$$\mathbf{Q} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) = \frac{1}{2} \left(\begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (2)$$

$$\mathbf{R} = \frac{1}{2}(\mathbf{C} + \mathbf{D}) = \frac{1}{2} \left(\begin{pmatrix} 5 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix} \right) = \begin{pmatrix} 5 \\ \frac{3}{2} \end{pmatrix} \quad (3)$$

$$\mathbf{S} = \frac{1}{2}(\mathbf{D} + \mathbf{A}) = \frac{1}{2} \left(\begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad (4)$$

$$(5)$$

We know that PQRS is a parallelogram. To know, if it is a rectangle, we need to ascertain whether any of the two adjacent sides are perpendicular. That means $(\mathbf{Q} - \mathbf{P})^\top (\mathbf{R} - \mathbf{Q})$ should be equal to zero.

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 3 \\ \frac{5}{2} \end{pmatrix} \quad (6)$$

$$\mathbf{R} - \mathbf{Q} = \begin{pmatrix} 5 \\ \frac{3}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -\frac{5}{2} \end{pmatrix} \quad (7)$$

$$(\mathbf{Q} - \mathbf{P})^\top (\mathbf{R} - \mathbf{Q}) = \begin{pmatrix} 3 & \frac{5}{2} \end{pmatrix} \begin{pmatrix} 3 \\ -\frac{5}{2} \end{pmatrix} \neq 0 \quad (8)$$

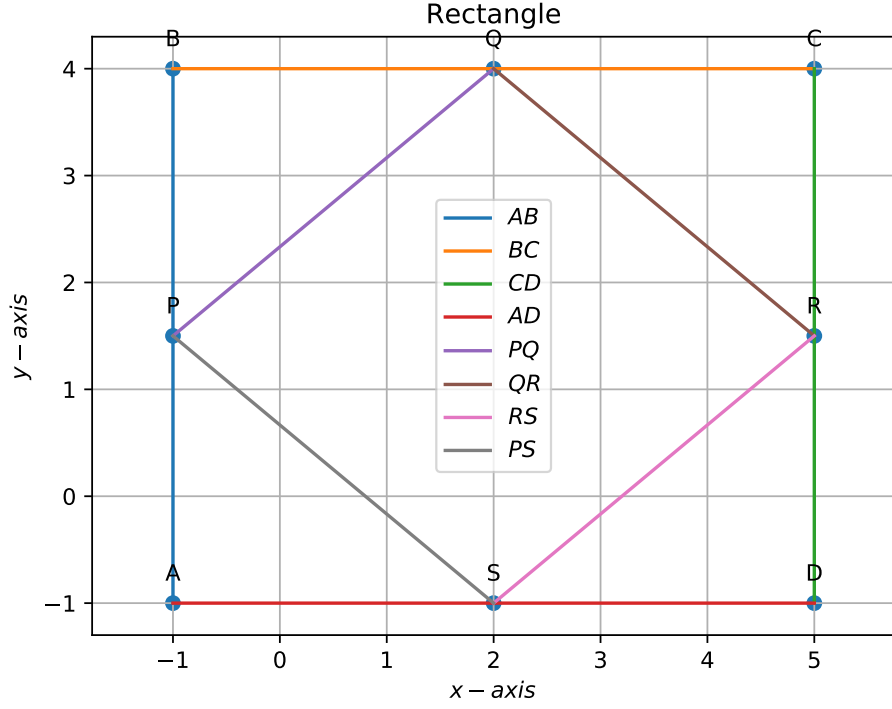


Figure 1

Therefore PQRS is not a rectangle. Let us check if it is a rhombus. For a rhombus, the diagonals bisect perpendicularly. That means $(\mathbf{R} - \mathbf{P})^\top (\mathbf{S} - \mathbf{Q})$ should be equal to zero.

$$\mathbf{R} - \mathbf{P} = \begin{pmatrix} 5 \\ \frac{3}{2} \end{pmatrix} - \begin{pmatrix} -1 \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (9)$$

$$\mathbf{S} - \mathbf{Q} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \quad (10)$$

$$(\mathbf{R} - \mathbf{P})^\top (\mathbf{S} - \mathbf{Q}) = (6 \ 0) \begin{pmatrix} 0 \\ -5 \end{pmatrix} = 0 \quad (11)$$

Therefore PQRS is a rhombus.