# Assignment 2

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***Abstract*—This document contains the solution of NCERT class 12 chapter 10 exercise 10.3 question number 11.**

## 1 Problem

Show that $\|\mathbf{a}\|\mathbf{b}+\|\mathbf{b}\|\mathbf{a}$ is perpendicular to $\|\mathbf{a}\|\mathbf{b}-\|\mathbf{b}\|\mathbf{a}$, for any two non zero vectors $\mathbf{a}$ and $\mathbf{b}$.

## 2 Solution

1) Theory
   We need to show that vectors, $\|\mathbf{a}\|\mathbf{b}+\|\mathbf{b}\|\mathbf{a}$ and $\|\mathbf{a}\|\mathbf{b}-\|\mathbf{b}\|\mathbf{a}$ are perpendicular to each other.
   Two vectors are perpendicular if and only if the inner product between them is zero. The inner product between the two given vectors is,

$$\left(\|\mathbf{a}\|\mathbf{b}+\|\mathbf{b}\|\mathbf{a}\right)^\top\left(\|\mathbf{a}\|\mathbf{b}-\|\mathbf{b}\|\mathbf{a}\right)=0 \tag{2.0.1}$$

   Expanding the LHS gives,

$$\|\mathbf{a}\|^2\mathbf{b}^\top\mathbf{b}+\|\mathbf{a}\|\|\mathbf{b}\|\mathbf{a}^\top\mathbf{b}-\|\mathbf{a}\|\|\mathbf{b}\|\mathbf{b}^\top\mathbf{a}-\|\mathbf{b}\|^2\mathbf{a}^\top\mathbf{a} \tag{2.0.2}$$

$$\|\mathbf{a}\|^2\|\mathbf{b}\|^2+\|\mathbf{a}\|\|\mathbf{b}\|\mathbf{a}^\top\mathbf{b}-\|\mathbf{a}\|\|\mathbf{b}\|\mathbf{a}^\top\mathbf{b}-\|\mathbf{b}\|^2\|\mathbf{a}\|^2=0 \tag{2.0.3}$$

   Hence,

$$\left(\|\mathbf{a}\|\mathbf{b}+\|\mathbf{b}\|\mathbf{a}\right)^\top\left(\|\mathbf{a}\|\mathbf{b}-\|\mathbf{b}\|\mathbf{a}\right)=0 \tag{2.0.4}$$

   As the inner products between the two vectors is zero, we can say that $(\|\mathbf{a}\|\mathbf{b}+\|\mathbf{b}\|\mathbf{a})$ and $(\|\mathbf{a}\|\mathbf{b}-\|\mathbf{b}\|\mathbf{a})$ are perpendicular to each other.
   Hence Proved.
2) Example
   Let us take,

$$\mathbf{a}=\begin{pmatrix}3\\4\end{pmatrix} \tag{2.0.5}$$

$$\mathbf{b}=\begin{pmatrix}5\\12\end{pmatrix} \tag{2.0.6}$$

   Then,

$$\|\mathbf{a}\|=\sqrt{3^2+4^2}=5 \tag{2.0.7}$$

$$\|\mathbf{b}\|=\sqrt{5^2+12^2}=13 \tag{2.0.8}$$

$$\|\mathbf{a}\|\mathbf{b}+\|\mathbf{b}\|\mathbf{a}=5\begin{pmatrix}5\\12\end{pmatrix}+13\begin{pmatrix}3\\4\end{pmatrix} \tag{2.0.9}$$

$$=\begin{pmatrix}25+39\\60+52\end{pmatrix}=\begin{pmatrix}64\\112\end{pmatrix} \tag{2.0.10}$$

$$\|\mathbf{a}\|\mathbf{b}-\|\mathbf{b}\|\mathbf{a}=5\begin{pmatrix}5\\12\end{pmatrix}-13\begin{pmatrix}3\\4\end{pmatrix} \tag{2.0.11}$$

$$=\begin{pmatrix}25-39\\60-52\end{pmatrix}=\begin{pmatrix}-14\\8\end{pmatrix} \tag{2.0.12}$$

$$\left(\|\mathbf{a}\|\mathbf{b}+\|\mathbf{b}\|\mathbf{a}\right)^\top\left(\|\mathbf{a}\|\mathbf{b}-\|\mathbf{b}\|\mathbf{a}\right)=\begin{pmatrix}64 & 112\end{pmatrix}\begin{pmatrix}-14\\8\end{pmatrix} \tag{2.0.13}$$

$$=-896+896 \tag{2.0.14}$$

$$=0 \tag{2.0.15}$$

Hence these two vectors are perpendicular.