VECTOR ALGEBRA

12^{th} Maths - Chapter 10

This is Problem-4 from Exercise 10.3

1. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

   Solution: let $\mathbf{A}$ and $\mathbf{B}$ be the given vectors

   $$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix} \tag{1}$$

   Let $\mathbf{C}$ be the projection of $\mathbf{A}$ on $\mathbf{B}$ then $\mathbf{C}$ is given as

   $$\mathbf{C} = \frac{\mathbf{A}^\top \mathbf{B}}{\|\mathbf{B}\|^2}\mathbf{B} \tag{2}$$

   Where

   $$\|\mathbf{B}\|^2 = \mathbf{B}^\top \mathbf{B} = 114 \tag{3}$$

   Then substituting the values of $\mathbf{A}$ and $\mathbf{B}$ in (2) gives

   $$\mathbf{C} = \frac{\begin{pmatrix} 1 & 3 & 7 \end{pmatrix}\begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix}}{2}\begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix} \tag{4}$$

   $$= \frac{10}{19}\begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix} \tag{5}$$

   $$= \begin{pmatrix} \frac{70}{19} \\ \frac{-10}{19} \\ \frac{80}{19} \end{pmatrix} \tag{6}$$