VECTORS

12^{th}Math - Chapter 10

This is Problem-8 from Exercise 10.3

Find the magnitude of two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, having the same magnitude and such that the angle between them is 60 ° and their scalar product is $\frac{1}{2}$.

Solution: Given

$$\mathbf{a}^\top \mathbf{b} = \frac{1}{2} \tag{1}$$

$$\|\mathbf{a}\| = \|\mathbf{b}\| \tag{2}$$

$$\theta = 60^\circ \tag{3}$$

We know that (4)

$$\theta = \cos^{-1}\left(\frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}\right) \tag{5}$$

$$\|\mathbf{a}\|\|\mathbf{b}\| = \frac{\mathbf{a}^\top \mathbf{b}}{\cos\theta} \tag{6}$$

$$\|\mathbf{a}\|^2 = \frac{\mathbf{a}^\top \mathbf{b}}{\cos\theta} \tag{7}$$

$$\|\mathbf{a}\| = \sqrt{\frac{\mathbf{a}^\top \mathbf{b}}{\cos\theta}} \tag{8}$$

$$= \sqrt{\frac{\frac{1}{2}}{\cos(60^\circ)}} \tag{9}$$

$$= 1 \tag{10}$$

$$\implies \|\mathbf{a}\| = 1 \tag{11}$$

$$\implies \|\mathbf{b}\| = 1 \tag{12}$$