Properties of Triangles

1 10^{th} Maths - Chapter 7

This is Problem-3 from Exercise 7.1

1. Determine if the points $(1,5),(2,3),$ and $(-2,-11)$ are collinear.
   Solution: We know that points $\mathbf{A},\mathbf{B}$ and $\mathbf{C}$ are collinear, if

$$\text{rank}\begin{pmatrix}\mathbf{A}^\top\\ \mathbf{B}^\top\\ \mathbf{C}^\top\end{pmatrix} = 1 \tag{1}$$

$$\begin{pmatrix}\mathbf{A}^\top\\ \mathbf{B}^\top\\ \mathbf{C}^\top\end{pmatrix} = \begin{pmatrix}1 & 5\\ 2 & 3\\ -2 & -11\end{pmatrix} \tag{2}$$

   Performing a sequence of row reduction operations

$$\xleftrightarrow[R_3\to R_3+2R_1]{R_2\to R_2-2R_1} \begin{pmatrix}1 & 5\\ 0 & -7\\ 0 & -1\end{pmatrix} \tag{3}$$

$$\xleftrightarrow{R_3\to R_3-\frac{1}{7}R_2} \begin{pmatrix}1 & 5\\ 0 & -7\\ 0 & 0\end{pmatrix} \tag{4}$$

   Since the matrix has only 2 pivot points, the rank of the mareix is 2. Therefore, referring to equation 1, the points are not collinear as the rank of the matrix is not equalt to 1.
   Refer to the figure 1 depicting a triangle with points $(1,5),(2,3),$ and $(-2,-11)$.

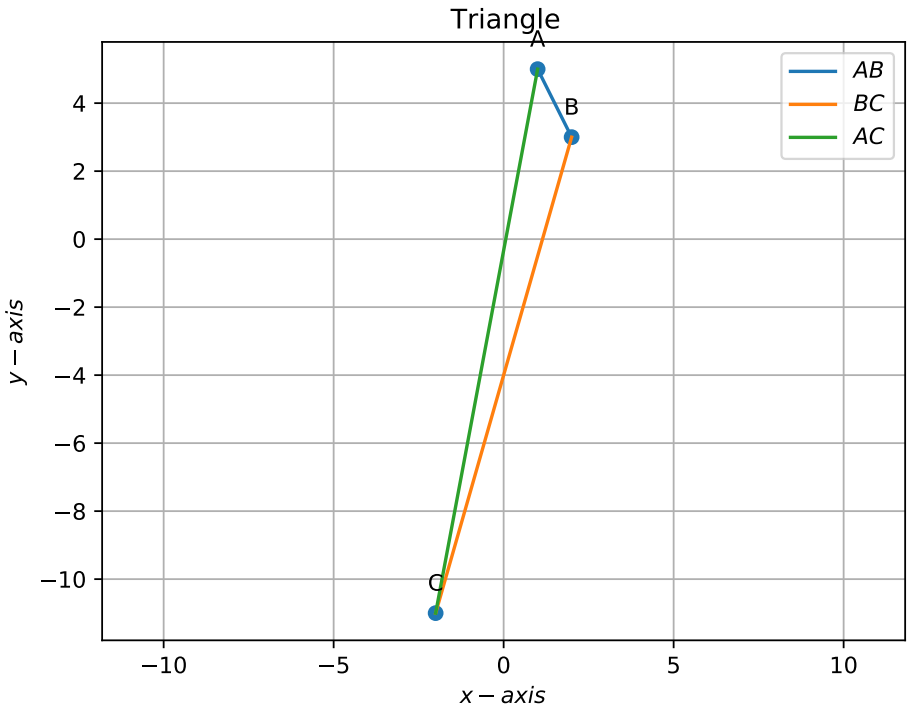

Figure 1