CIRCLES

9^{th}Math - Chapter 10

This is Problem-7 from Exercise 10.5

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of quadrilateral,prove that it is a rectangle.

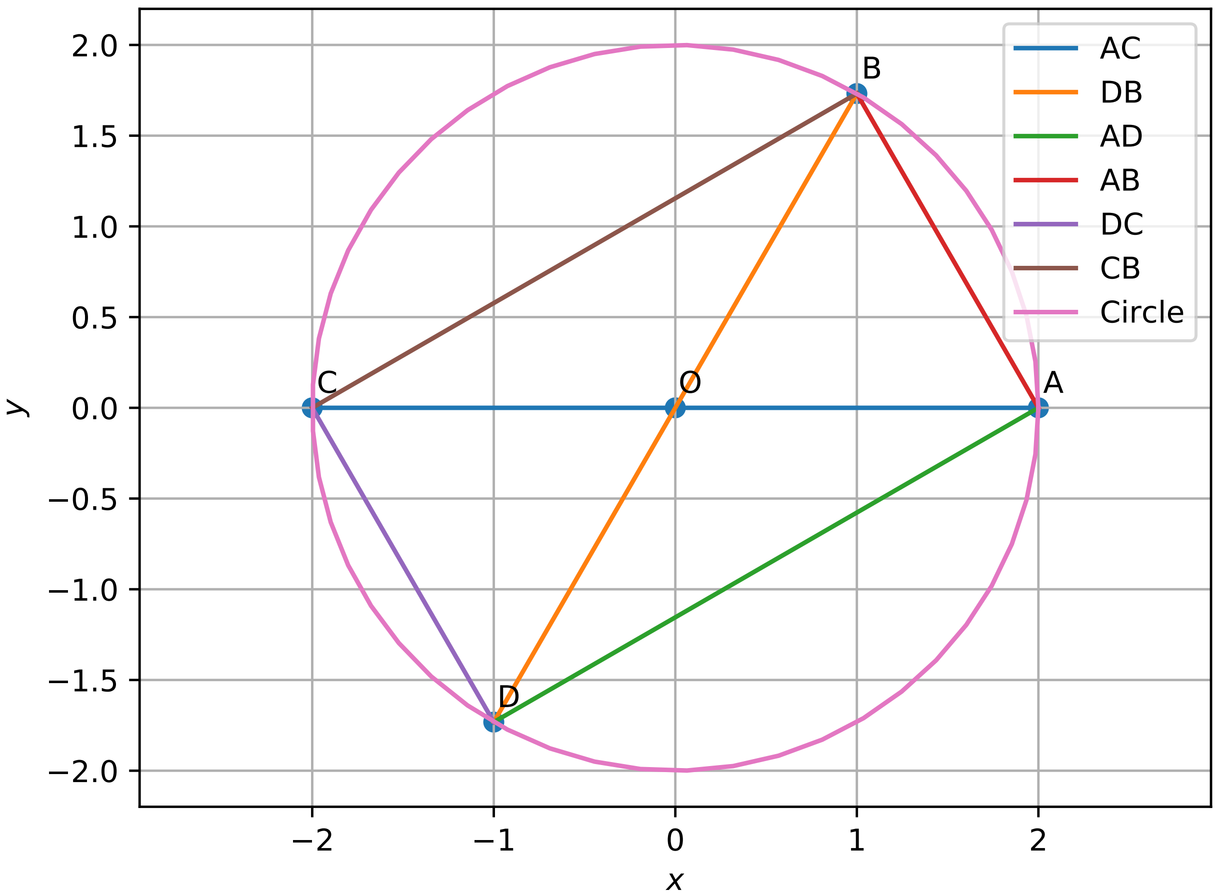

Figure 1

Construction:

The input parameters for construction

Symbol	Values	Description
r	2	radius
$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	center

$$\mathbf{A} = r\begin{pmatrix} \cos 0 \\ \sin 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = r\begin{pmatrix} \cos \frac{\pi}{3} \\ \sin \frac{\pi}{3} \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \tag{2}$$

$$\mathbf{C} = 2\mathbf{O} - \mathbf{A} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{D} = 2\mathbf{O} - \mathbf{B} = \begin{pmatrix} -1 \\ -\sqrt{3} \end{pmatrix} \tag{4}$$

Solution: Consider a circle of radius 2 units.Let AC and DB are diameters of circle which are diagonals of cyclic quadrilateral.

$$\mathbf{A} - \mathbf{C} = \mathbf{D} - \mathbf{B} \tag{5}$$

To prove $ABCD$ is rectangle

(i) AB and DC are parellel to each other

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{7}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} -1 \\ -\sqrt{3} \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{9}$$

$$\implies \quad ABCD \text{ is parallelogram} \tag{10}$$

(ii) Let's check the angle between adjacent sides of this quadrilateral,AB and BC

$$\left(\mathbf{A} - \mathbf{B}\right)^\top \left(\mathbf{B} - \mathbf{C}\right) = \begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \begin{pmatrix} 3 \\ \sqrt{3} \end{pmatrix} \tag{11}$$

$$= 0 \tag{12}$$

$$\implies \angle ABC = 90^\circ \tag{13}$$

from (i) and (ii) , Hence the quadrilateral $ABCD$ is rectangle.