Assignment 1

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1) Find the area of the triangle with vertices **A**(1,1,2), **B** (2,3,5), **C** (1,5,5).
Solution: The area of the triangle ABC is given by

$$ar(ABC) = \frac{1}{2}\left\|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A})\right\| \tag{0.0.1}$$

given points are

$$\mathbf{A} = \begin{pmatrix}1\\1\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}2\\3\\5\end{pmatrix}, \mathbf{C} = \begin{pmatrix}1\\5\\5\end{pmatrix} \tag{0.0.2}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}1\\2\\3\end{pmatrix} \tag{0.0.3}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix}0\\4\\3\end{pmatrix} \tag{0.0.4}$$

$$\text{area} = \left\|\begin{pmatrix}1\\2\\3\end{pmatrix}\times\begin{pmatrix}0\\4\\3\end{pmatrix}\right\| \tag{0.0.5}$$

$$\tag{0.0.6}$$

The cross product or vector product of $\mathbf{P}, \mathbf{Q}$ is defined as

$$\mathbf{P}\times\mathbf{Q} = \begin{pmatrix}\begin{vmatrix}\mathbf{P}_{23} & \mathbf{Q}_{23}\end{vmatrix}\\ \begin{vmatrix}\mathbf{P}_{31} & \mathbf{Q}_{31}\end{vmatrix}\\ \begin{vmatrix}\mathbf{P}_{12} & \mathbf{Q}_{12}\end{vmatrix}\end{pmatrix} \tag{0.0.7}$$

$$\begin{vmatrix}\mathbf{P}_{23} & \mathbf{Q}_{23}\end{vmatrix} = \begin{vmatrix}2 & 4\\3 & 3\end{vmatrix} = -6 \tag{0.0.8}$$

$$\begin{vmatrix}\mathbf{P}_{31} & \mathbf{Q}_{31}\end{vmatrix} = \begin{vmatrix}1 & 0\\3 & 3\end{vmatrix} = 3 \tag{0.0.9}$$

$$\begin{vmatrix}\mathbf{P}_{12} & \mathbf{Q}_{12}\end{vmatrix} = \begin{vmatrix}1 & 0\\2 & 4\end{vmatrix} = 4 \tag{0.0.10}$$

Hence

$$\begin{pmatrix}1\\2\\3\end{pmatrix}\times\begin{pmatrix}0\\4\\3\end{pmatrix} = \begin{pmatrix}-6\\3\\4\end{pmatrix} \tag{0.0.11}$$

The area of the triangle is given by,

$$ar(ABC) = \frac{1}{2}\left\|\begin{pmatrix}-6\\3\\4\end{pmatrix}\right\| \tag{0.0.12}$$

$$= \frac{1}{2}\sqrt{36+9+16} \tag{0.0.13}$$

$$= \frac{\sqrt{61}}{2} \tag{0.0.14}$$

The area of the triangle is $\frac{\sqrt{61}}{2}$ square units.