VECTOR ALGEBRA

January 27, 2023

1. **Problem statement :** Find the area of a triangle with vertices $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}$
   Solution: From the given information,
   $$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{1}$$
   $$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} \tag{2}$$
   The area of a triangle using the vector product is then obtained as
   $$\frac{1}{2} \left\| (\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) \right\| \tag{3}$$
   $$= \frac{1}{2} \left\| \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} \right\| \tag{4}$$
   Cross product of two vectors are
   $$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 & -A_3 & A_2 \\ A_3 & 0 & -A_1 \\ -A_2 & A_1 & 0 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B3 \end{pmatrix} \tag{5}$$

By using (5)

$$\frac{1}{2}\left\|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A})\right\| = \frac{1}{2}\left\|\begin{pmatrix}0 & -3 & 2\\ 3 & 0 & -1\\ -2 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\4\\3\end{pmatrix}\right\| \tag{6}$$

$$= \frac{1}{2}\left\|\begin{pmatrix}-6\\-3\\4\end{pmatrix}\right\| \tag{7}$$

$$= \frac{1}{2}\sqrt{(-6)^2+(-3)^2+(4)^2} \tag{8}$$

$$= \frac{\sqrt{61}}{2} \tag{9}$$

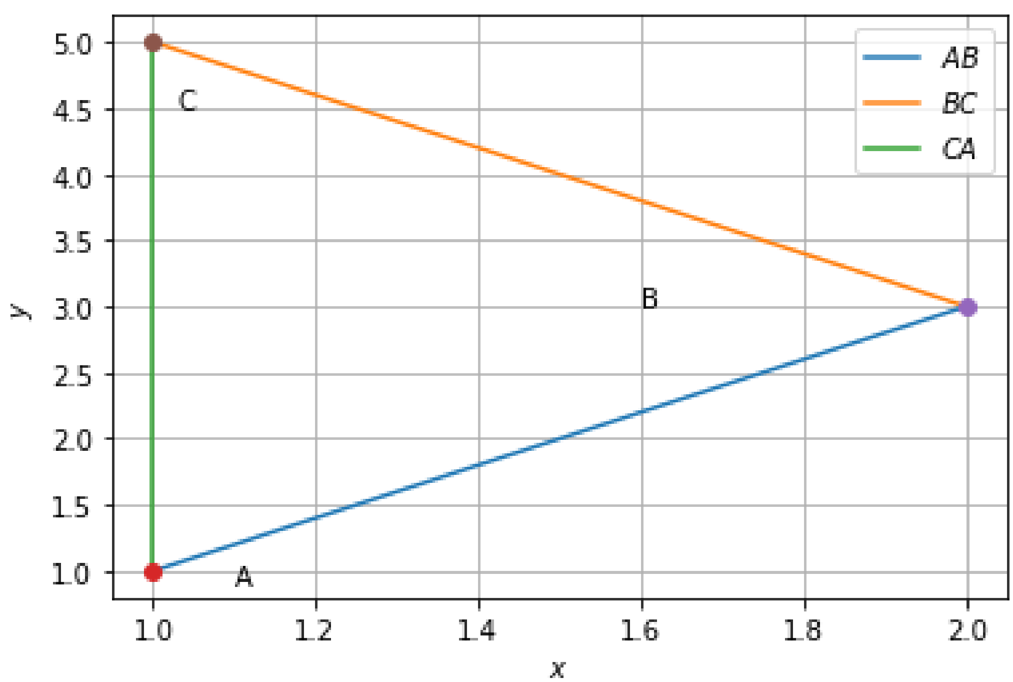

Figure 1