CHAPTER-7
COORDINATE GEOMETRY

Excercise 7.4

Q2. Find a relation between x and y if the points $(x, y), (1, 2)$ and $(7, 0)$ are collinear.
Solution:
The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} \tag{2}$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} x-7 \\ y \end{pmatrix} \tag{3}$$

If points on a line are collinear, rank of matrix is " 1 "then the vectors are in linearly dependent. For 2×2 matrix Rank = 1 means Determinant is 0. Through pivoting,we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D}^\top \\ \mathbf{E}^\top \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} x-1 & y-2 \\ x-7 & y \end{pmatrix} \xleftrightarrow{R_2 = R_2 - R_1} \begin{pmatrix} x-1 & y-2 \\ -6 & 2 \end{pmatrix} \tag{6}$$

$$\xleftrightarrow{R_2 = \frac{R_2}{-6}(x-1) - R_1} \begin{pmatrix} x-1 & y-2 \\ 0 & -\frac{1}{3}(x-1) - (y-2) \end{pmatrix} \tag{7}$$

If the rank of the matrix has to be 1 , then:

$$-\frac{1}{3}(x-1) - (y-2) = 0 \tag{8}$$

$$x + 3y = 7 \tag{9}$$

The above straight line equation is represented in vector form of a line as follows:

$$\mathbf{n}^\top \mathbf{X} = c \tag{10}$$

$$\begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{X} = 7 \tag{11}$$

Suppose, if $x = -2, y = 3$, then rank of F is equal to one which is collinear as shown in Figure:1

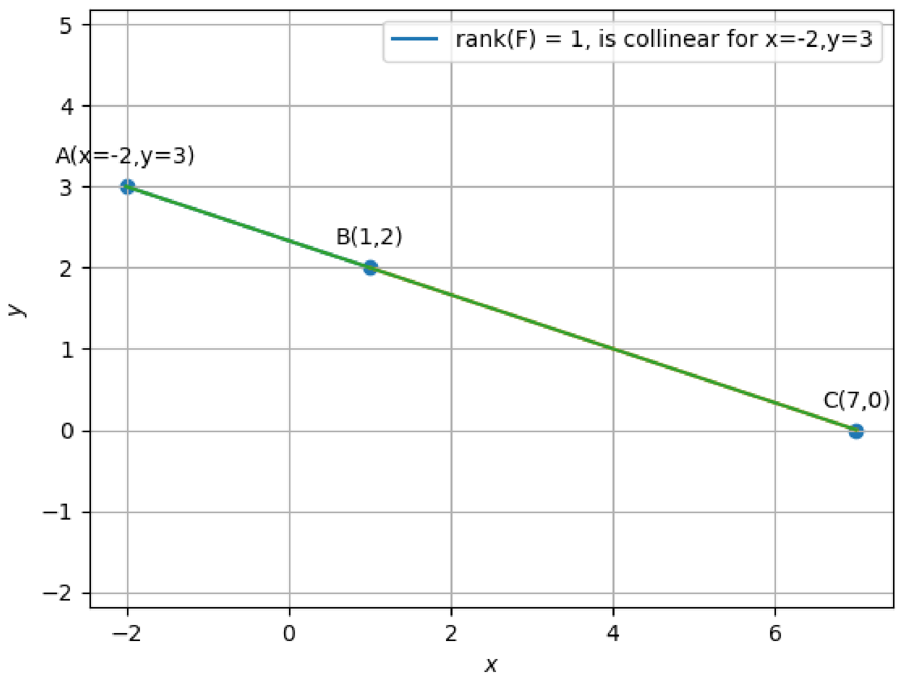

Figure 1: