Vector Assignment

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Abstract—This document contains the solution to Question 17 of Exercise 2 in Chapter 10 of the class 12 NCERT textbook.

1) Show that the points
$$\mathbf{A}, \mathbf{B}, \mathbf{C}$$
 with position vectors $\mathbf{A} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ form

the vertices of a right angled triangle.

Solution: We write the direction vectors of the three sides as

$$\mathbf{c} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -1\\3\\5 \end{pmatrix} \tag{1}$$

$$\mathbf{a} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} -1 \\ -2 \\ -6 \end{pmatrix} \tag{2}$$

$$\mathbf{b} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} -2\\1\\-1 \end{pmatrix} \tag{3}$$

Taking the inner product of each pair of vectors,

$$\mathbf{c}^{\mathsf{T}}\mathbf{a} = -35\tag{4}$$

$$\mathbf{a}^{\mathsf{T}}\mathbf{b} = 6 \tag{5}$$

$$\mathbf{b}^{\mathsf{T}}\mathbf{c} = 0 \tag{6}$$

From (6), $\mathbf{b}^{\mathsf{T}}\mathbf{c} = 0$, which implies that $\mathbf{b} \perp \mathbf{c}$. Hence, $\triangle ABC$ is right angled at A. The Python code codes/perp.py verifies the answer.