Quiz 4

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Abstract—**This document contains the solution of the question from NCERT 11th standard chapter 10 exercise 10.1 problem 6**

1 EXERCISE 10.1

1) Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right angled triangle.
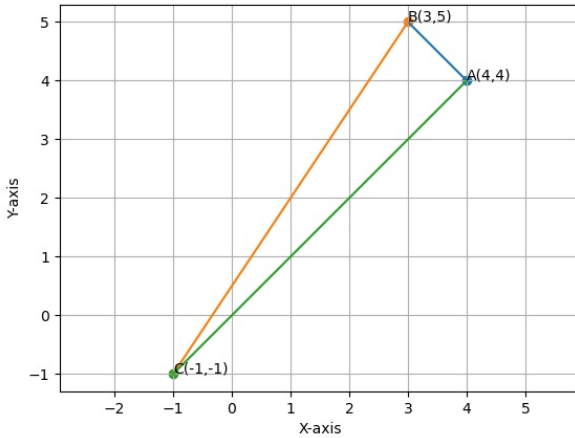

Fig. 1: Triangle ABC

We need to show that points $\begin{pmatrix}4\\4\end{pmatrix}$, $\begin{pmatrix}3\\5\end{pmatrix}$ and $\begin{pmatrix}-1\\-1\end{pmatrix}$ are vertices of a right triangle.

Let,

$$\mathbf{A} = \begin{pmatrix}4\\4\end{pmatrix}, \mathbf{B} = \begin{pmatrix}3\\5\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-1\\-1\end{pmatrix} \quad (1.0.1)$$

To check whether the points A, B, C form a triangle we find the rank of the matrix $\begin{pmatrix}\mathbf{A} & \mathbf{B} & \mathbf{C}\end{pmatrix}$

$$\begin{pmatrix}4 & 3 & -1\\4 & 5 & -1\end{pmatrix} \quad (1.0.2)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \quad (1.0.3)$$

$$\begin{pmatrix}4 & 3 & -1\\0 & 2 & 0\end{pmatrix} \quad (1.0.4)$$

$$(1.0.5)$$

The rank of the matrix is 2 and the points are in 2-Dimensional space. So the points A, B, C form a triangle.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}3\\5\end{pmatrix} - \begin{pmatrix}4\\4\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix} \quad (1.0.6)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix}-1\\-1\end{pmatrix} - \begin{pmatrix}4\\4\end{pmatrix} = \begin{pmatrix}-5\\-5\end{pmatrix}$$

$$(1.0.7)$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{A} - \mathbf{C}) = \begin{pmatrix}-1 & 1\end{pmatrix}\begin{pmatrix}-5\\-5\end{pmatrix} = 5 - 5 = 0$$

$$(1.0.8)$$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{C}) = 0 \quad (1.0.9)$$

Hence, the angle between $\mathbf{A} - \mathbf{B}$ and $\mathbf{B} - \mathbf{C}$ is 90 degrees.

Hence, $\triangle ABC$ is a right triangle with right angle at vertex

$$\mathbf{A} = \begin{pmatrix}4\\4\end{pmatrix} \quad (1.0.10)$$