11.10.4.21

Lokesh Surana

Class 11, Chapter 10, Exercise 4.21

Q.21. Find equation of the line which is equidistant from parallel lines 9x + 6y - 7 = 0 and 3x + 2y + 6 = 0.

Solution: Equation of lines are

$$L_1: (9 \quad 6) \mathbf{x} - 7 = 0 \tag{1}$$

$$\implies L_1: \left(1 \quad \frac{2}{3}\right)\mathbf{x} - \frac{7}{9} = 0 \tag{2}$$

$$\implies$$
 $\mathbf{n}_1 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix}$ and $c_1 = \frac{7}{9}$ (3)

$$L_2: (3 \ 2)\mathbf{x} + 6 = 0$$
 (4)

$$\implies L_2: \left(1 \quad \frac{2}{3}\right)\mathbf{x} + 2 = 0 \tag{5}$$

$$\implies$$
 $\mathbf{n}_2 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix}$ and $c_2 = -2$ (6)

The equation of desired line will be $\mathbf{n}^{\mathsf{T}}\mathbf{x} = c$. As it is given that lines are parallel, \mathbf{n} must be same as \mathbf{n}_1 and \mathbf{n}_2 .

$$L: \left(1 \quad \frac{2}{3}\right)\mathbf{x} - c = 0 \tag{7}$$

The distance between two parallel lines $\mathbf{n}^{\mathsf{T}}\mathbf{x} = c_1$ and $\mathbf{n}^{\mathsf{T}}\mathbf{x} = c_2$ is given as

$$d = \frac{|c_1 - c_2|}{\|\mathbf{n}\|} \tag{8}$$

So for given lines, we need to solve

$$\frac{|c_1 - c|}{\|\mathbf{n}\|} = \frac{|c - c_2|}{\|\mathbf{n}\|} \tag{9}$$

$$\implies \frac{\left|\frac{7}{9} - c\right|}{\sqrt{1^2 + \left(\frac{2}{3}\right)^2}} = \frac{|c + 2|}{\sqrt{1^2 + \left(\frac{2}{3}\right)^2}} \tag{10}$$

Case 1.

$$\frac{7}{9} - c = -c - 2 \tag{11}$$

$$\implies \frac{7}{9} = -2 \tag{12}$$

Case 2.

$$\frac{7}{9} - c = c + 2 \tag{14}$$

$$\implies 2c = -\frac{11}{9} \tag{15}$$

$$\implies c = -\frac{11}{18} \tag{16}$$

The equation of line is

$$L: \left(1 \quad \frac{2}{3}\right)\mathbf{x} - c = 0 \tag{17}$$

$$L: \left(1 \quad \frac{2}{3}\right)\mathbf{x} + \frac{11}{18} = 0 \tag{18}$$

$$\implies L: (18 \ 12) \mathbf{x} + 11 = 0$$
 (19)

(20)

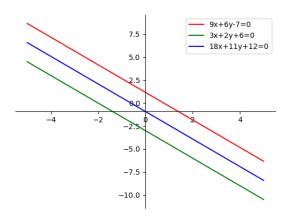


Fig. 1: Given lines and equidistant line