

# Quiz 8

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**Abstract**—This document contains the solution of the question from NCERT 12th standard chapter 11 exercise 11.4 problem 20

## 1 EXERCISE 11.4

- 1) Find the equation of the line passing through the point  $(1, 2, -4)$  and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

The direction vector of the line

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad (1.0.1)$$

is

$$\mathbf{m}_1 = \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix} \quad (1.0.2)$$

The direction vector of the line

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \quad (1.0.3)$$

is

$$\mathbf{m}_2 = \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} \quad (1.0.4)$$

Let  $\mathbf{n}$  denote the direction vector of the line perpendicular to the given two lines.

$$\mathbf{m}_1^\top \mathbf{n} = 0 \implies \begin{pmatrix} 3 & -16 & 7 \end{pmatrix} \mathbf{n} = 0 \quad (1.0.5)$$

$$\mathbf{m}_2^\top \mathbf{n} = 0 \implies \begin{pmatrix} 3 & 8 & -5 \end{pmatrix} \mathbf{n} = 0 \quad (1.0.6)$$

$$(1.0.7)$$

Let us find the  $\mathbf{n}$  that satisfies these equations.

$$\begin{pmatrix} 3 & -16 & 7 \\ 3 & 8 & -5 \end{pmatrix} \mathbf{n} = 0 \quad (1.0.8)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \quad (1.0.9)$$

$$\begin{pmatrix} 3 & -16 & 7 \\ 0 & 24 & -12 \end{pmatrix} \mathbf{n} = 0 \quad (1.0.10)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{2}{3}R_2} \quad (1.0.11)$$

$$\begin{pmatrix} 3 & 0 & -1 \\ 0 & 24 & -12 \end{pmatrix} \mathbf{n} = 0 \quad (1.0.12)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 / 12} \quad (1.0.13)$$

$$\begin{pmatrix} 3 & 0 & -1 \\ 0 & 2 & -1 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 0 \quad (1.0.14)$$

$$3n_1 - n_3 = 0 \implies n_1 = \frac{n_3}{3} \quad (1.0.15)$$

$$2n_2 - n_3 = 0 \implies n_2 = \frac{n_3}{2} \quad (1.0.16)$$

$$\implies \mathbf{n} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \quad (1.0.17)$$

Hence the vector equation of the line passing through  $(1, 2, -4)$  is,

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \quad (1.0.18)$$