12.11.3.9

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CLASS 12, CHAPTER 11, EXERCISE 3.9

Q.9. Find the equation of the plane through the intersection of the planes $3x–y + 2z–4 = 0$ and $x + y + z–2 = 0$ and the point $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$.

Solution: The equation of given are given by

$$P_1 : \mathbf{n}_1^\top \mathbf{x} = c_1 \tag{1}$$
$$P_1 : \mathbf{n}_1^\top \mathbf{x} = c_2 \tag{2}$$

The intersection of the planes is given by the solution of the system of equations

$$P : P_1 + \lambda P_2 = 0 \tag{3}$$
$$P : \mathbf{n}_1^\top \mathbf{x} - c_1 + \lambda \left(\mathbf{n}_2^\top \mathbf{x} - c_2\right) = 0 \tag{4}$$

If this plane is passing through a point $\mathbf{P}$, then following will be satisfied

$$P : \left(\mathbf{n}_1^\top + \lambda \mathbf{n}_2^\top\right) \mathbf{P} - (c_1 + \lambda c_2) = 0 \tag{5}$$
$$\implies \lambda = \frac{c_1 - n_1^\top \mathbf{P}}{n_2^\top \mathbf{P} - c_2} \tag{6}$$

To get the value of λ, we need to substitute the value of $\mathbf{P}, \mathbf{n}_1, \mathbf{n}_2, c_1$ and c_2 in the above equation. The equation of given planes are given by

$$P_1 : \begin{pmatrix} 3 & -1 & 2 \end{pmatrix} \mathbf{x} = 4 \tag{7}$$
$$P_2 : \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = 2 \tag{8}$$

The intersection of the planes is given by the solution of the system of equations

$$P : P_1 + \lambda P_2 = 0 \tag{9}$$
$$P : \begin{pmatrix} 3+\lambda & -1+\lambda & 2+\lambda \end{pmatrix} \mathbf{x} - (4 + 2\lambda) = 0 \tag{10}$$

These plane shall pass through point $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$, which means that

$$\lambda = \frac{4 - \begin{pmatrix} 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - 2} \tag{11}$$
$$\lambda = -\frac{2}{3} \tag{12}$$

The equation of plane is as follows:

$$\frac{1}{3}\begin{pmatrix} 7 & -5 & 4 \end{pmatrix} \mathbf{x} = \frac{8}{3} \tag{13}$$
$$\implies P : \begin{pmatrix} 7 & -5 & 4 \end{pmatrix} \mathbf{x} = 8 \tag{14}$$