11.11.2.5

Lokesh Surana

Class 11, Chapter 11, Exercise 2.5

Q. Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum $y^2 = 10x$

Solution: The given equation of the parabola can be rearranged as

$$y^2 - 10x = 0 \tag{1}$$

The above equation can be equated to the generic equation of conic sections

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2}$$

Comparing coefficients of (1) and (2),

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

$$\mathbf{u} = -\begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{4}$$

$$f = 0 \tag{5}$$

1) From (3), since $\mathbf{V}$ is already diagonalized, the Eigen values λ_1 and λ_2 are given as

$$\lambda_1 = 0 \tag{6}$$

$$\lambda_2 = 1 \tag{7}$$

and the eigenvector matrix

$$\mathbf{P} = \mathbf{I}. \tag{8}$$

$$\therefore \mathbf{n} = \sqrt{\lambda_2}\mathbf{p_1} \tag{9}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{10}$$

Since

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2\mathbf{u}^\top\mathbf{n}}, \tag{11}$$

Substituting values of $\mathbf{u}, \mathbf{n}, \lambda_2$ and f in (11), we get

$$c = \frac{5^2 - 1(0)}{-2\begin{pmatrix} 5 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}} = -\frac{5}{2} \tag{12}$$

$$\tag{13}$$

The focus $\mathbf{F}$ of parabola is expressed as

$$\mathbf{F} = \frac{ce^2\mathbf{n} - \mathbf{u}}{\lambda_2} \tag{14}$$

$$= \frac{-\frac{5}{2}(1)^2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix}}{1} \tag{15}$$

$$= \begin{pmatrix} \frac{5}{2} \\ 0 \end{pmatrix} \tag{16}$$

2) The directrix is given by

$$\mathbf{n}^\top\mathbf{x} = c \tag{17}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = -\frac{5}{2} \tag{18}$$

$$\tag{19}$$

3) The equation for the axis of parabola passing through $\mathbf{F}$ and orthogonal to the directrix is given as

$$\mathbf{m}^\top(\mathbf{x} - \mathbf{F}) = 0 \tag{20}$$

where $\mathbf{m}$ is the normal vector to the axis and also the slope of the directrix.

$$\because \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{21}$$

$$\implies \begin{pmatrix} 0 & 1 \end{pmatrix}\left(\mathbf{x} - \begin{pmatrix} \frac{5}{2} \\ 0 \end{pmatrix}\right) = 0 \tag{22}$$

$$\text{or, } \begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} = 0 \tag{23}$$

4) The latus rectum of a parabola is given by

$$l = \frac{\eta}{\lambda_2} = -\frac{2\mathbf{u}^\top \mathbf{p_1}}{\lambda_2} \tag{24}$$

$$= -\frac{2\begin{pmatrix}-5 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}}{1} \tag{25}$$

$$= 10 \text{ units} \tag{26}$$

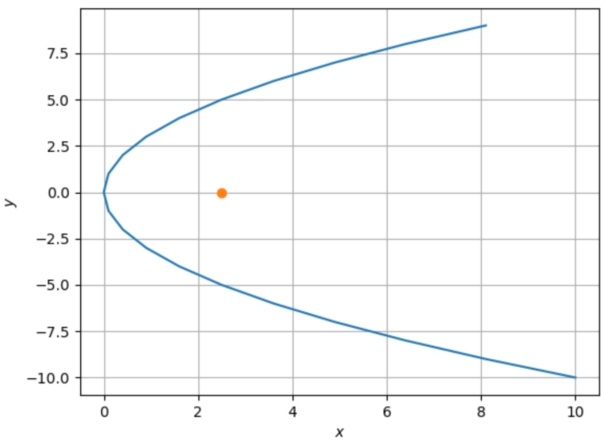

Fig. 1: Parabola $y^2 = 10x$