12.10.3.17

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Class 12, Chapter 10, Exercise 3.17

17) Show that the vectors form the vertices $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$ of a right angled triangle.

Solution: Let's first check if points are not collinear and hence forming a triangle, using row reduction.

$$\begin{pmatrix} 2 & 1 & 3 \\ -1 & -3 & -4 \\ 1 & -5 & -4 \\ 1 & 1 & 1 \end{pmatrix} \xleftrightarrow{R_2 \to \frac{R_1}{2} + R_2} \begin{pmatrix} 2 & 1 & 3 \\ 0 & -\frac{5}{2} & -\frac{5}{2} \\ 1 & -5 & -4 \\ 1 & 1 & 1 \end{pmatrix} \tag{1}$$

$$\xleftarrow{R_3 \to -\frac{R_1}{2} + R_3} \begin{pmatrix} 2 & 1 & 3 \\ 0 & -\frac{5}{2} & -\frac{5}{2} \\ 0 & -\frac{11}{2} & -\frac{11}{2} \\ 1 & 1 & 1 \end{pmatrix} \tag{2}$$

$$\xleftarrow{R_4 \to -\frac{R_1}{2} + R_4} \begin{pmatrix} 2 & 1 & 3 \\ 0 & -\frac{5}{2} & -\frac{5}{2} \\ 0 & -\frac{11}{2} & -\frac{11}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \tag{3}$$

$$\xleftarrow{R_3 \to -\frac{11R_2}{5} + R_3} \begin{pmatrix} 2 & 1 & 3 \\ 0 & -\frac{5}{2} & -\frac{5}{2} \\ 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \tag{4}$$

$$\xleftarrow{R_4 \to \frac{R_2}{5} + R_4} \begin{pmatrix} 2 & 1 & 3 \\ 0 & -\frac{5}{2} & -\frac{5}{2} \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{5}$$

Hence Rank of matrix is 3, which means that points are not collinear. Now, let's check if they form a right angled triangle.

Let's name the triangle ABC, where $\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$. So sides of triangle are:

$$\mathbf{a} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \tag{6}$$

$$\mathbf{b} = \mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \tag{7}$$

$$\mathbf{c} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ -2 \\ -6 \end{pmatrix} \tag{8}$$

If inner product of two vectors is zero, then they are perpendicular. So, we have:

$$\mathbf{a}^\top \mathbf{b} = 2 \times (-1) + (-1) \times 3 + 1 \times 5 = 0 \tag{9}$$

$$\mathbf{b}^\top \mathbf{c} = (-1) \times (-1) + 3 \times (-2) + 5 \times (-6) = -35 \tag{10}$$

$$\mathbf{c}^\top \mathbf{a} = (-1) \times 2 + (-2) \times (-1) + (-6) \times 1 = -6 \tag{11}$$

So, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular and therefore triangle ABC is right angled triangle.