

Coordinate Geometry

10th Maths - Chapter 7

This is Problem-7 from Exercise 7.1

1. The point on the x -axis which is equidistant from $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$

Solution:

The input parameters for this problem are available in Table (1)

Symbol	Value	Description
A	$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$	First point
B	$\begin{pmatrix} -2 \\ 9 \end{pmatrix}$	Second point
O	?	Desired point

Table 1

If **O** lies on the x -axis and is equidistant from the points **A** and **B**,

$$\|\mathbf{O} - \mathbf{A}\| = \|\mathbf{A} - \mathbf{B}\| \quad (1)$$

$$\implies \|\mathbf{O} - \mathbf{A}\|^2 = \|\mathbf{O} - \mathbf{B}\|^2 \quad (2)$$

which can be expressed as

$$\begin{aligned} (\mathbf{O} - \mathbf{A})^\top (\mathbf{O} - \mathbf{A}) &= (\mathbf{O} - \mathbf{B})^\top (\mathbf{O} - \mathbf{B}) \\ \implies \|\mathbf{O}\|^2 - 2\mathbf{O}^\top \mathbf{A} + \|\mathbf{A}\|^2 &= \|\mathbf{O}\|^2 - 2\mathbf{O}^\top \mathbf{B} + \|\mathbf{B}\|^2 \quad (3) \end{aligned}$$

which can be simplified to obtain

$$\mathbf{O} = x\mathbf{e}_1 \quad (4)$$

where

$$x = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top \mathbf{e}_1} \quad (5)$$

now substituting the A and B values in (5)

$$(\mathbf{A} - \mathbf{B})^\top = \left(\begin{pmatrix} 2 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ 9 \end{pmatrix} \right)^\top = (4 \quad -14) \quad (6)$$

$$\|\mathbf{A}\|^2 = 21 \quad (7)$$

$$\|\mathbf{B}\|^2 = 85 \quad (8)$$

upon substituting the values in (5) the value of $x = -7$

Hence, the desired point is \mathbf{O} is $\begin{pmatrix} -7 \\ 0 \end{pmatrix}$.

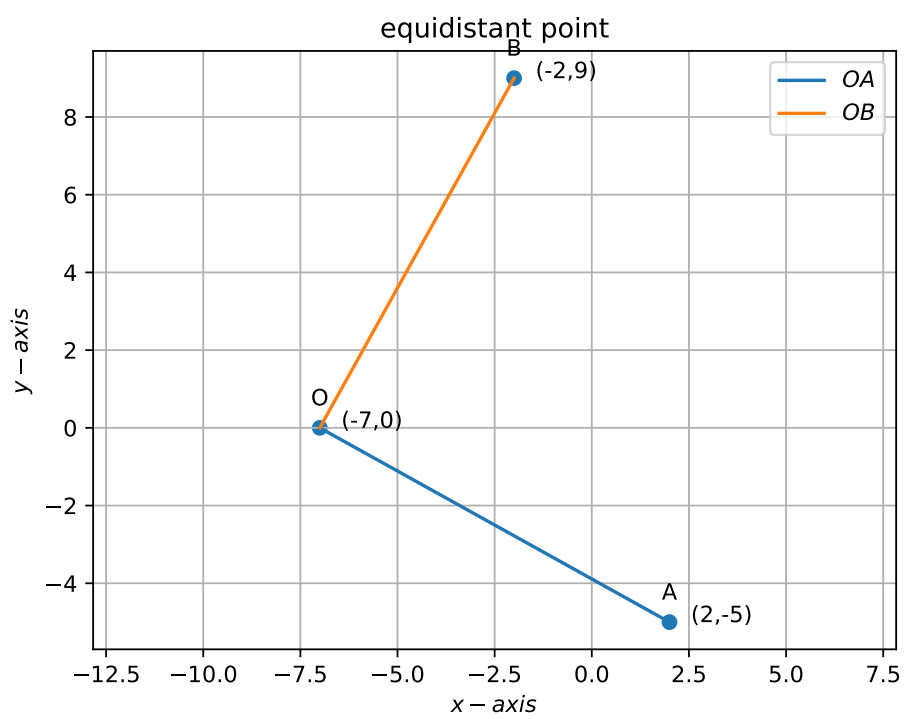


Figure 1