CHAPTER-10
VECTOR ALGEBRA

Excercise 10.4

Q5.Find λ and μ if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\mathbf{0}$
Solution:

$$\text{Let } \mathbf{A}=\begin{pmatrix}2\\6\\27\end{pmatrix} \text{ and } \mathbf{B}=\begin{pmatrix}1\\\lambda\\\mu\end{pmatrix} \tag{1}$$

$$\tag{2}$$

The cross product or vector product of $\mathbf{A},\mathbf{B}$ is defined as

$$\mathbf{A}\times\mathbf{B}=\begin{pmatrix}\begin{vmatrix}\mathbf{A}_{23} & \mathbf{B}_{23}\\\mathbf{A}_{31} & \mathbf{B}_{31}\\\mathbf{A}_{12} & \mathbf{B}_{12}\end{vmatrix}\end{pmatrix} \tag{3}$$

Hence

$$\begin{vmatrix}\mathbf{A}_{23} & \mathbf{B}_{23}\end{vmatrix}=\begin{vmatrix}6 & \lambda\\27 & \mu\end{vmatrix}=6\mu-27\lambda \tag{4}$$

$$\begin{vmatrix}\mathbf{A}_{31} & \mathbf{B}_{31}\end{vmatrix}=\begin{vmatrix}27 & \mu\\2 & 1\end{vmatrix}=27-2\mu \tag{5}$$

$$\begin{vmatrix}\mathbf{A}_{12} & \mathbf{B}_{12}\end{vmatrix}=\begin{vmatrix}2 & 1\\6 & \lambda\end{vmatrix}=2\lambda-6 \tag{6}$$

Substituting the values

$$\mathbf{A}\times\mathbf{B}=\begin{pmatrix}6\mu-27\lambda\\27-2\mu\\2\lambda-6\end{pmatrix} \tag{7}$$

Now we know

$$\mathbf{A}\times\mathbf{B}=\mathbf{0} \tag{8}$$

So,

$$\begin{pmatrix} 6\mu - 27\lambda \\ 27 - 2\mu \\ 2\lambda - 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

So we have three equations.

$$2\mu = 27 \tag{10}$$
$$2\lambda = 6 \tag{11}$$
$$6\mu - 27\lambda = 0 \tag{12}$$

The above equations can be represented in matrix form as

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \\ 6 & -27 \end{pmatrix} \begin{pmatrix} \mu \\ \lambda \end{pmatrix} = \begin{pmatrix} 27 \\ 6 \\ 0 \end{pmatrix} \tag{13}$$

The augmented matrix is given as

$$\left(\begin{array}{cc|c} 2 & 0 & 27 \\ 0 & 2 & 6 \\ 6 & -27 & 0 \end{array}\right) \tag{14}$$

Applying sequence of row operations

$$\xleftrightarrow{R_3 \to R_3 - 3R_1} \left(\begin{array}{cc|c} 2 & 0 & 27 \\ 0 & 2 & 6 \\ 0 & -27 & -81 \end{array}\right) \tag{15}$$

$$\xleftrightarrow{R_3 \to R_3 + \frac{27}{2} R_2} \left(\begin{array}{cc|c} 2 & 0 & 27 \\ 0 & 2 & 6 \\ 0 & 0 & 0 \end{array}\right) \tag{16}$$

From here we conclude that

$$2\mu = 27 \tag{17}$$
$$\mu = 13.5 \tag{18}$$
$$2\lambda = 6 \tag{19}$$
$$\lambda = 3 \tag{20}$$

Hence, the values are $\lambda = 3$ and $\mu = 13.5$