

Assignment 1

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- 1) In the triangle ABC with vertices $A(2, 3)$, $B(4, -1)$ and $C(1, 2)$, find the equation and length of altitude from the vertex A .

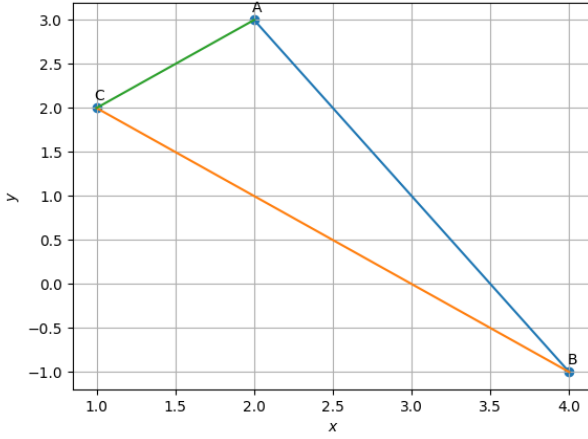


Fig. 1: Graph

Solution:

- a) Given points are,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.0.1)$$

The normal vector for the altitude from vertex A is the direction vector of the line BC . The direction vector of the line BC is given by,

$$\mathbf{m}_{BC} = \mathbf{B} - \mathbf{C} \quad (0.0.2)$$

$$= \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.0.3)$$

$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} \quad (0.0.4)$$

$$\mathbf{m}_{BC} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (0.0.5)$$

$$\mathbf{n}_{BC} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (0.0.6)$$

The equation of line BC is given by,

$$\mathbf{n}_{BC}^T \mathbf{x} = \mathbf{n}_{BC}^T \mathbf{B} \quad (0.0.7)$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -1 \end{pmatrix} \quad (0.0.8)$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 3 \quad (0.0.9)$$

The length of the altitude from a point A to the line $\mathbf{n}^T \mathbf{x} = c$ is given by

$$d = \frac{|\mathbf{n}^T \mathbf{A} - c|}{\|\mathbf{n}\|} \quad (0.0.10)$$

$$d = \frac{\left| \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3 \right|}{\left\| \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\|} \quad (0.0.11)$$

$$d = \sqrt{2} \quad (0.0.12)$$

The length of the altitude from the vertex A to BC is $\sqrt{2}$ units.

- b) The normal vector of the altitude from A is,

$$\mathbf{n} = \mathbf{m}_{BC} \quad (0.0.13)$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (0.0.14)$$

The equation of the altitude from vertex A is given by,

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} \quad (0.0.15)$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (0.0.16)$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1 \quad (0.0.17)$$

The equation of the altitude from vertex A to the line BC is given by,

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1 \quad (0.0.18)$$