CHAPTER-7
COORDINATE GEOMETRY

Excercise 7.1

Q6.Name the type of quadilateral formed,if any, by the following points, and give reasons for your answer:

1. $(-1,-2),(1,0),(-1,2),(-3,0)$
2. $(-3,5),(3,1),(0,3),(-1,-4)$
3. $(4,5),(7,6),(4,3),(1,2)$

Solution:

1. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix}-1\\-2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}1\\0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-1\\2\end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix}-3\\0\end{pmatrix} \tag{1}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}-1\\-2\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix} \tag{2}$$

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix}-1\\2\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-2\\2\end{pmatrix} \tag{3}$$

$$\mathbf{C}-\mathbf{D} = \begin{pmatrix}-1\\2\end{pmatrix} - \begin{pmatrix}-3\\0\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix} \tag{4}$$

$$\mathbf{D}-\mathbf{A} = \begin{pmatrix}-3\\0\end{pmatrix} - \begin{pmatrix}-1\\-2\end{pmatrix} = \begin{pmatrix}-2\\2\end{pmatrix} \tag{5}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix}-1\\2\end{pmatrix} - \begin{pmatrix}-1\\-2\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix} \tag{6}$$

$$\mathbf{D}-\mathbf{B} = \begin{pmatrix}-3\\0\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-4\\0\end{pmatrix} \tag{7}$$

$$\mathbf{B}-\mathbf{A} = \mathbf{C}-\mathbf{D} \text{ and } \mathbf{C}-\mathbf{B} = \mathbf{D}-\mathbf{A}. \tag{8}$$

Hence, $ABCD$ is a parallelogram.

(a) Now checking if the adjacent sides are orthogonal to each other

$$(\mathbf{B}-\mathbf{A})^{\top}(\mathbf{C}-\mathbf{B}) = \begin{pmatrix} 2 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = -4+4=0 \tag{9}$$

(b) Now checking if the diagonals are also orthogonal then it is a square else a rectangle.

$$(\mathbf{C}-\mathbf{A})^{\top}(\mathbf{D}-\mathbf{B}) = \begin{pmatrix} 0 & 4 \end{pmatrix} \begin{pmatrix} -4 \\ 0 \end{pmatrix} = 0 \tag{10}$$

Hence the diagonals are orthogonal to each other.

So, we can conclude that $ABCD$ is a square.

As shown in Figure 1 we can see that $ABCD$ is a square hence we can conclude that our theoritical result is verified.

2. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \tag{11}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \tag{12}$$

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \tag{13}$$

$$\mathbf{C}-\mathbf{D} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \tag{14}$$

$$\mathbf{D}-\mathbf{A} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -9 \end{pmatrix} \tag{15}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{16}$$

$$\mathbf{D}-\mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \tag{17}$$

$$\mathbf{B}-\mathbf{A} \neq \mathbf{C}-\mathbf{D} \text{ and } \mathbf{C}-\mathbf{B} \neq \mathbf{D}-\mathbf{A}, \tag{18}$$

Hence, $ABCD$ is not a parallelogram, it can be a irregular quadilateral.

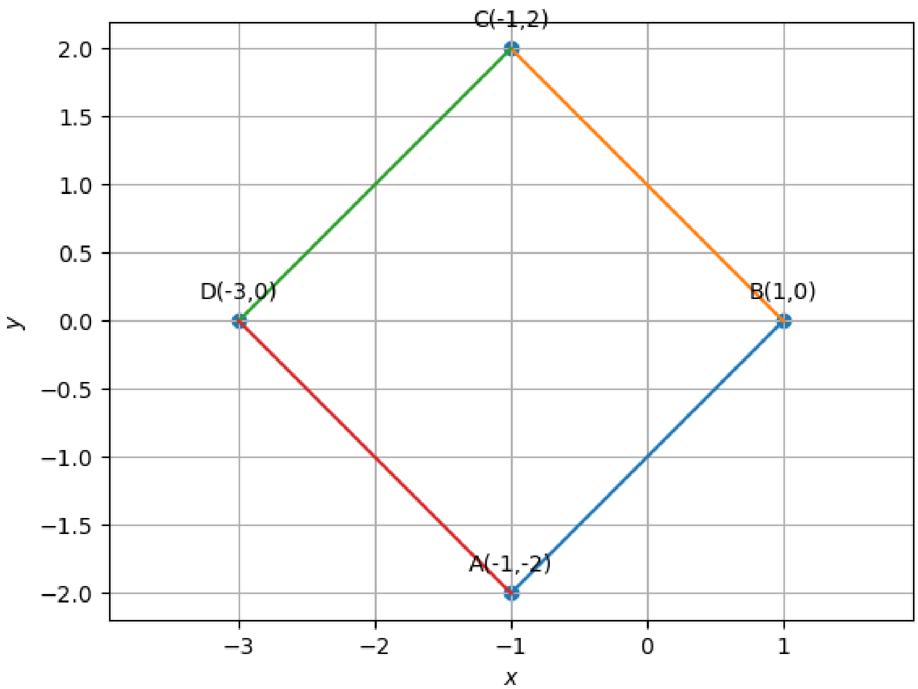

Figure 1:

(a) Now to check if any three points are collinear,
if rank of $\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{B}\end{pmatrix} = 1$ then points are collinear
Forming the collinearity matrix

$$\begin{pmatrix} 6 & -3 \\ -4 & 2 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 + \frac{2}{3}R_1} = \begin{pmatrix} 6 & -3 \\ 0 & 0 \end{pmatrix} \tag{19}$$

Hence, rank = 1

Since none of the opposite sides are parallel to each other and three points are collinear so these does not form a quadilateral.

As shown in Figure 2 we can see that $ABCD$ does not form a quadilateral and three points are collinear hence, our theoritical result is verified.

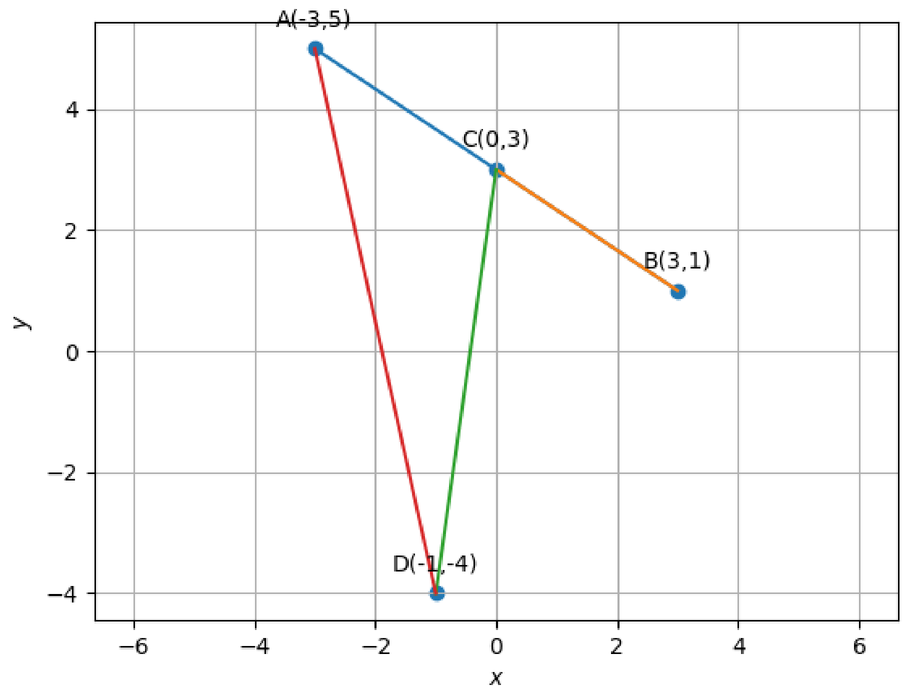

Figure 2:

3. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{20}$$

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}7\\6\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix} \tag{21}$$

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}7\\6\end{pmatrix}=\begin{pmatrix}-3\\-3\end{pmatrix} \tag{22}$$

$$\mathbf{C}-\mathbf{D}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix} \tag{23}$$

$$\mathbf{D}-\mathbf{A}=\begin{pmatrix}1\\2\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}-3\\-3\end{pmatrix} \tag{24}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}0\\-2\end{pmatrix} \tag{25}$$

$$\mathbf{D}-\mathbf{B}=\begin{pmatrix}1\\2\end{pmatrix}-\begin{pmatrix}7\\6\end{pmatrix}=\begin{pmatrix}-6\\-4\end{pmatrix} \tag{26}$$

$$\tag{27}$$

$$\mathbf{B}-\mathbf{A}=\mathbf{C}-\mathbf{D} \text{ and } \mathbf{C}-\mathbf{B}=\mathbf{D}-\mathbf{A}, \tag{28}$$

Hence, $ABCD$ is a parallelogram.

(a) Now checking if the adjacent sides are orthogonal to each other

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{B})=\begin{pmatrix}3 & 1\end{pmatrix}\begin{pmatrix}-3\\-3\end{pmatrix}=-9-3=-12 \tag{29}$$

Since inner product is not zero so adjacent sides are not orthogonal.

Hence, we can say that $ABCD$ is neither a rectangle nor a square.

(b) Now checking if the diagonals are orthogonal then it is a Rhombus.

$$(\mathbf{C}-\mathbf{A})^\top(\mathbf{D}-\mathbf{B})=\begin{pmatrix}0 & -2\end{pmatrix}\begin{pmatrix}-6\\-4\end{pmatrix}=0+8=8 \tag{30}$$

Hence the diagonals are also not orthogonal so we conclude that $ABCD$ is a parallelogram.

As shown in Figure 3 we can see that $ABCD$ forms a parallelogram hence, our theoritical result is verified.

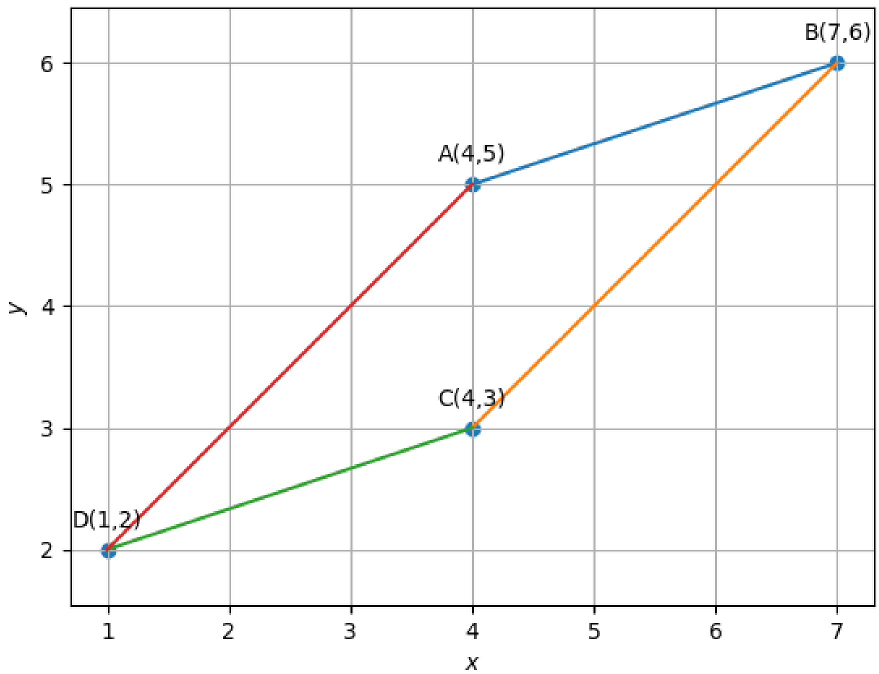

Figure 3: