VECTOR ALGEBRA

January 26, 2023

1. **Problem statement :** Evaluate the product $\left(3\vec{a} - 5\vec{b}\right)\cdot\left(2\vec{a} + 7\vec{b}\right)$
   Solution:

   $$(3\mathbf{a} - 5\mathbf{b})^\top (2\mathbf{a} + 7\mathbf{b}) = \left(3\mathbf{a}^\top\right)(2\mathbf{a}) + \left(3\mathbf{a}^\top\right)(7\mathbf{b}) - \left(5\mathbf{b}^\top\right)(2\mathbf{a}) - \left(5\mathbf{b}^\top\right)(7\mathbf{b}) \tag{1}$$

   Properties of Vector

   $$\mathbf{a}^\top\mathbf{a} = \|\mathbf{a}\|^2 \tag{2}$$
   $$\mathbf{a}^\top\mathbf{b} = \mathbf{b}^\top\mathbf{a} \tag{3}$$

   By using (2) and (3)

   $$(3\mathbf{a} - 5\mathbf{b})^\top (2\mathbf{a} + 7\mathbf{b}) = 6\|\mathbf{a}\|^2 + 21\mathbf{a}^\top\mathbf{b} - 10\mathbf{b}^\top\mathbf{a} - 35\|\mathbf{b}\|^2 \tag{4}$$
   $$= 6\|\mathbf{a}\|^2 - 35\|\mathbf{b}\|^2 + 11\mathbf{a}^\top\mathbf{b} \tag{5}$$