

# Question : 11.10.4.23

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## 1 PROBLEM

Prove that the products of the lengths of the perpendiculars drawn from the points  $\begin{pmatrix} \sqrt{a^2 - b^2} \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} -\sqrt{a^2 - b^2} \\ 0 \end{pmatrix}$  to the line  $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$  is  $b^2$ .

$$d_1 d_2 = \frac{|(\mathbf{n}^\top \mathbf{P})^2 - c^2|}{\|\mathbf{n}\|} \quad (2.0.9)$$

$$= \frac{\left| \frac{\cos^2 \theta (a^2 - b^2)}{a^2} - 1 \right|}{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}} \quad (2.0.10)$$

$$= \frac{(b^2 \cos^2 \theta + a^2 \sin^2 \theta) a^2 b^2}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta) a^2} \quad (2.0.11)$$

$$= b^2 \quad (2.0.12)$$

Hence Proved.

## 2 SOLUTION

$$\mathbf{P} = \begin{pmatrix} \sqrt{a^2 - b^2} \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$(2.0.2)$$

Given line is,

$$\left( \frac{\cos \theta}{a} \quad \frac{-\sin \theta}{b} \right) \mathbf{x} = 1 \quad (2.0.3)$$

Comparing with

$$\mathbf{n}^\top \mathbf{x} = c \quad (2.0.4)$$

$$c = 1 \quad (2.0.5)$$

Distance from point  $\mathbf{P}$  to the line,

$$d_1 = \frac{|\mathbf{n}^\top \mathbf{P} - c|}{\|\mathbf{n}\|} \quad (2.0.6)$$

Distance from point  $-\mathbf{P}$  to the line,

$$d_2 = \frac{|-\mathbf{n}^\top \mathbf{P} - c|}{\|\mathbf{n}\|} \quad (2.0.7)$$

$$= \frac{|\mathbf{n}^\top \mathbf{P} + c|}{\|\mathbf{n}\|} \quad (2.0.8)$$

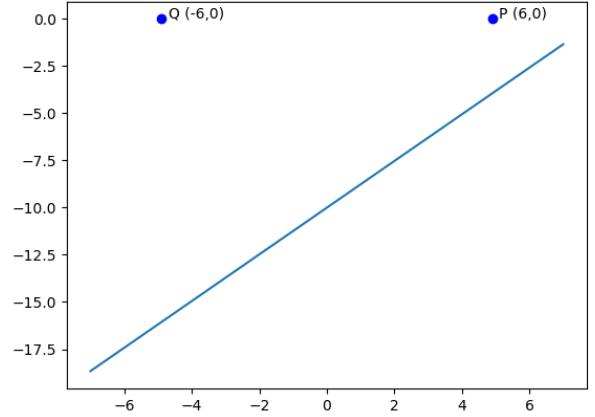


Fig. 0: Figure 1