Straight Lines

11^{th} Maths - Chapter 10

This is Problem-8 from Exercise 2

1. Find the equation of line perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive x-axis is 30°

   Solution: The equation of a line is given by

   $$\mathbf{n}^\top \mathbf{x} = c \tag{1}$$

   Let the normal vector of the line is

   $$\mathbf{n} = \begin{pmatrix} \cos 30^\circ \\ \sin 30^\circ \end{pmatrix} \tag{2}$$

   The distance from the origin to the line is given by

   $$d = \frac{|c|}{\|\mathbf{n}\|} \tag{3}$$

   The magnitude for $\mathbf{n}$ is

   $$\|\mathbf{n}\| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \tag{4}$$

   From (3)

   $$c = d\,\|\mathbf{n}\| \tag{5}$$
   $$= \pm 5 \tag{6}$$

Then substituting them in (1) gives the equation of line

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \mathbf{x} = 5 \tag{7}$$

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \mathbf{x} = -5 \tag{8}$$

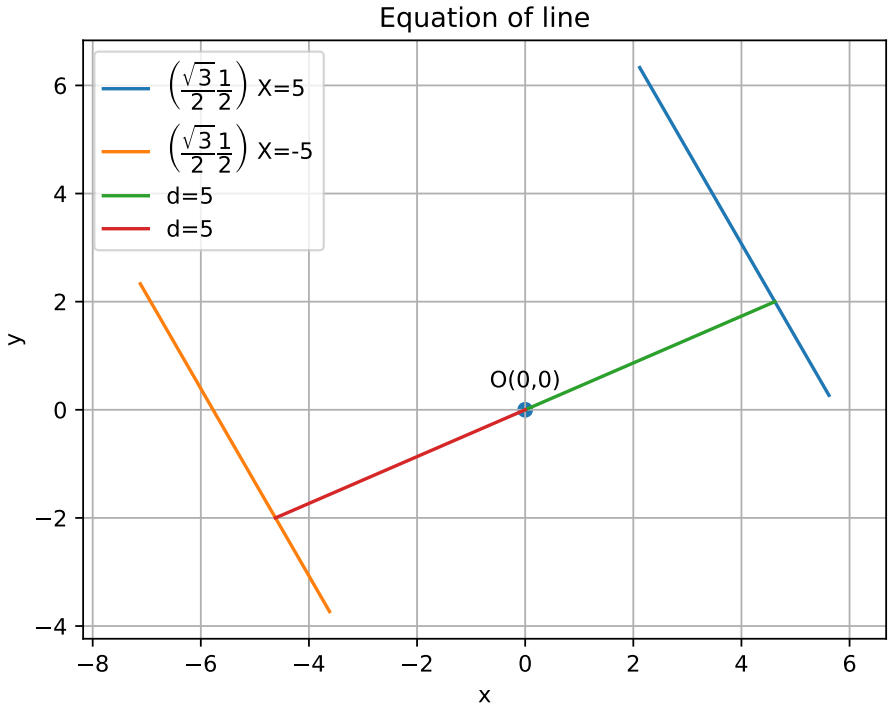

Figure 1