

Coordinate Geometry

1. **Problem statement :** Find the area of a rhombus of its vertices are $(3, 0)$, $(4, 5)$, $(-1, 4)$ and $(-2, -1)$ taken in order

Solution:

The input vertices for this problem are given as

$$\mathbf{A} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad (1)$$

for the given vertices of the rhombus, length of the adjacent side its as follows:

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad (2)$$

the length of the another adjacent side can be obtained by

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (3)$$

To find the area of the rhombus, we can take the cross product of two adjacent sides are given by

$$\|\mathbf{a} \times \mathbf{b}\| = \left\| \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \right\| = |a_1 b_2 - a_2 b_1| \quad (4)$$

Thus, the area of rhombus can be calculated as

$$\|(\mathbf{A} - \mathbf{D}) \times (\mathbf{B} - \mathbf{A})\| = \begin{vmatrix} 5 & 1 \\ 1 & 5 \end{vmatrix} = 5 \times 5 - 1 \times 1 = 24 \quad (5)$$

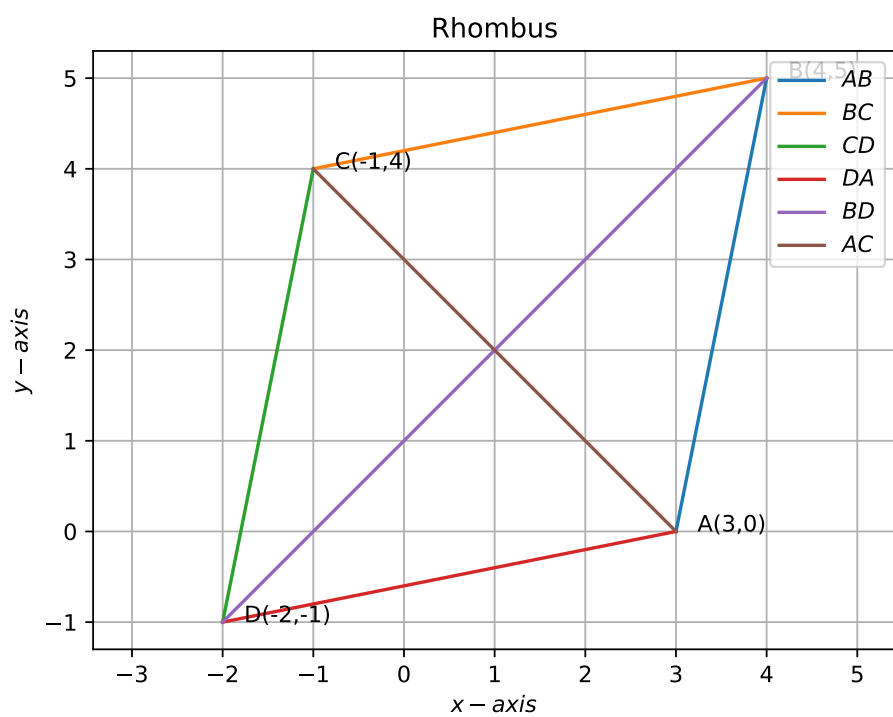


Figure 1