Vector Algebra

1 12^{th} Maths - Chapter 10

This is Problem 7 from Exercise-10.5

1. If $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\overrightarrow{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\overrightarrow{a} - \overrightarrow{b} + 3\overrightarrow{c}$.

   Solution: Given Vectors

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{1}$$

$$2\mathbf{a} - \mathbf{b} + 3\mathbf{c} = \mathbf{u} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} \tag{2}$$

   the unit vector parallel to $\mathbf{u}$ is

$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\mathbf{u}}{\sqrt{22}} \tag{3}$$

$$\implies \hat{\mathbf{u}} = \frac{1}{\sqrt{22}} \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{22}} \\ \frac{-3}{\sqrt{22}} \\ \frac{2}{\sqrt{22}} \end{pmatrix} \tag{4}$$