Line Assignment

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***Abstract*—This document contains the solution to Question 24 of Exercise 4 in Chapter 10 of the class 11 NCERT textbook.**

1) A person standing at the junction (crossing) of two straight paths represented by the equations

$$\begin{pmatrix}2 & -3\end{pmatrix}\mathbf{x} = -4 \tag{1}$$

and

$$\begin{pmatrix}3 & 4\end{pmatrix}\mathbf{x} = 5 \tag{2}$$

wants to reach the path whose equation is

$$\begin{pmatrix}6 & -7\end{pmatrix}\mathbf{x} = -8 \tag{3}$$

Find equation of the path that he should follow.
Solution: We first find the coordinates of the intersection of (1) and (2). Using the augmented matrix and row reduction methods,

$$\left(\begin{array}{cc|c}2 & -3 & -4\\ 3 & 4 & 5\end{array}\right) \xleftrightarrow{R_2 \to 2R_2 - 3R_1} \left(\begin{array}{cc|c}2 & -3 & -4\\ 0 & 17 & 22\end{array}\right) \tag{4}$$

$$\xleftrightarrow{R_1 \to 17R_1 + 3R_2} \left(\begin{array}{cc|c}17 & 0 & -1\\ 0 & 17 & 22\end{array}\right) \tag{5}$$

$$\xleftrightarrow[R_2 \to \frac{R_2}{17}]{R_1 \to \frac{R_1}{17}} \left(\begin{array}{cc|c}1 & 0 & -\frac{1}{17}\\ 0 & 1 & \frac{22}{17}\end{array}\right) \tag{6}$$

the intersection of the lines is

$$\mathbf{A} = \frac{1}{17}\begin{pmatrix}-1\\22\end{pmatrix} \tag{7}$$

Clearly, the man should follow the path perpendicular to (3) from $\mathbf{A}$ to reach it in the shortest time. The normal vector of (3) is

$$\mathbf{m} = \begin{pmatrix}6\\-7\end{pmatrix} \tag{8}$$

which is consequently the direction vector of the required line. Therefore, the required normal vector is given by

$$\mathbf{n} = \begin{pmatrix}7\\6\end{pmatrix} \tag{9}$$

and hence, the equation of the line is

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{A} \tag{10}$$

$$\implies \begin{pmatrix}7 & 6\end{pmatrix}\mathbf{x} = \frac{1}{17}\begin{pmatrix}7 & 6\end{pmatrix}\begin{pmatrix}-1\\22\end{pmatrix} = \frac{125}{17} \tag{11}$$

The Python code `codes/crossing.py` verifies the solution and depicts the situation in Fig. 1. In this figure $\mathbf{F}$ represents the foot of the prependicular drawn from $\mathbf{A}$ onto (3).

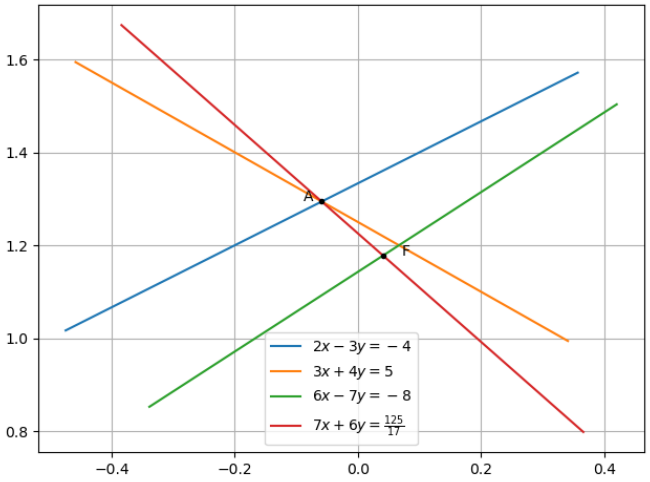

Fig. 1: AF is the required line.