# Properties of Circle

## 1 $9^{th}$ Maths - Chapter 10

This is Problem-2 from Exercise 10.4

1. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of other chord.

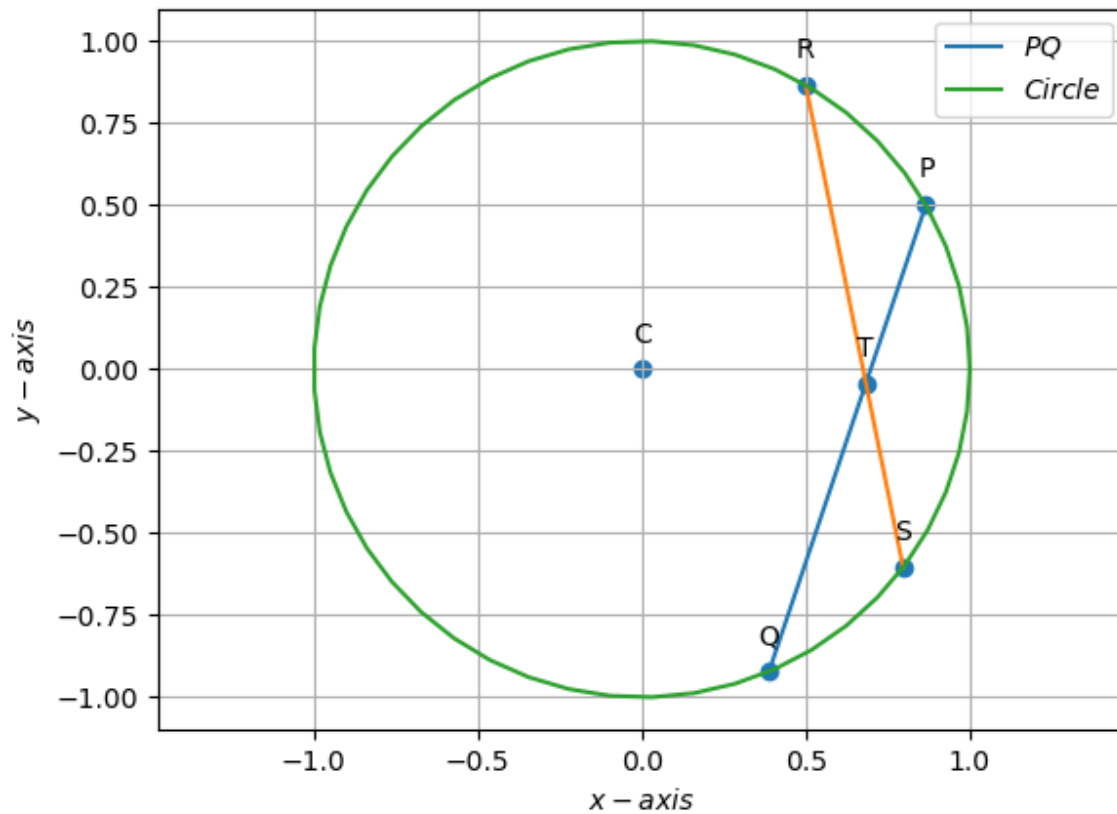   **Solution:** : Consider the circle of radius 1 and length of chord be 1.5



Figure 1: Two equal chords intersecting in a circle

| Symbol | Value | Description |
|:---:|:---:|:---:|
| **C** | $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ | Circle point |
| $d$ | 1.5 | Length of Chord |
| $r$ | 1 | Radius |
| $\theta_1$ | 30° | − |
| $\theta_3$ | 60° | − |
| $\theta_2$ | −67.1806° | − |
| $\theta_4$ | 37.1806° | − |

Table 1

## Construction

Consider

$$\mathbf{P} = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \tag{1}$$

$$\mathbf{Q} = \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} \tag{2}$$

$$\mathbf{R} = \begin{pmatrix} \cos\theta_3 \\ \sin\theta_3 \end{pmatrix} \tag{3}$$

$$\mathbf{S} = \begin{pmatrix} \cos\theta_4 \\ \sin\theta_4 \end{pmatrix} \tag{4}$$

So, here

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{5}$$

$$\|\mathbf{P} - \mathbf{Q}\|^2 = d^2 \tag{6}$$

$$\implies (\cos\theta_1 - \cos\theta_2)^2 + (\sin\theta_1 - \sin\theta_2)^2 = d^2 \tag{7}$$

$$\implies 2 - 2\left(\cos(\theta_1 - \theta_2)\right) = d^2 \tag{8}$$

$$\implies 2\left(2\left(\sin^2\left(\frac{\theta_1 - \theta_2}{2}\right)\right)\right) = d^2 \tag{9}$$

$$\implies \sin^2\left(\frac{\theta_1 - \theta_2}{2}\right) = \frac{d^2}{4} \tag{10}$$

$$\implies \sin\left(\frac{\theta_1 - \theta_2}{2}\right) = \frac{d}{2} \tag{11}$$

$$\implies \left(\frac{\theta_1 - \theta_2}{2}\right) = \sin^{-1}\left(\frac{d}{2}\right) \tag{12}$$

Substituting the values from table (1) in (12) So we get

$$(\theta_1 - \theta_2) = 97.1806° \tag{13}$$

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Simillary we can say that

$$(\theta_3 - \theta_4) = 97.1806° \tag{14}$$

Obtain the point of intersection of line $PQ, RS$

$$\mathbf{n_1^\top} (\mathbf{X} - \mathbf{P}) = 0 \tag{15}$$

$$\mathbf{n_2^\top} (\mathbf{X} - \mathbf{R}) = 0 \tag{16}$$

$$\mathbf{Omat} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{17}$$

$$\mathbf{n_1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ \sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{18}$$

$$= \begin{pmatrix} \sin\theta_1 - \sin\theta_2 \\ \cos\theta_2 - \cos\theta_1 \end{pmatrix} \tag{19}$$

$$\mathbf{n_2} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta_3 - \cos\theta_4 \\ \sin\theta_3 - \sin\theta_4 \end{pmatrix} \tag{20}$$

$$= \begin{pmatrix} \sin\theta_3 - \sin\theta_4 \\ \cos\theta_4 - \cos\theta_3 \end{pmatrix} \tag{21}$$

From the Result of python code by substituting $\theta_1, \theta_2, \theta_3, \theta_4$ in point $\mathbf{T}$ then we get the point of intersection as :

$$\begin{pmatrix} \sin\theta_2 - \sin\theta_1 & \cos\theta_2 - \cos\theta_1 \\ \sin\theta_4 - \sin\theta_3 & \cos\theta_4 - \cos\theta_3 \end{pmatrix} \mathbf{X} = \begin{pmatrix} \cos\theta_1 (\sin\theta_2 - \sin\theta_1) - \sin\theta_1 (\cos\theta_2 - \cos\theta_1) \\ \cos\theta_3 (\sin\theta_4 - \sin\theta_3) - \sin\theta_3 (\cos\theta_4 - \cos\theta_2) \end{pmatrix} \tag{22}$$

$$\mathbf{T} = \begin{pmatrix} \frac{\cos\theta_4 - \cos\theta_2 (\sin\theta_2 \cos\theta_1 - \sin\theta_1 \cos\theta_2)}{(\cos\theta_4 - \cos\theta_2)(\sin\theta_2 - \sin\theta_1) - (\cos\theta_2 - \cos\theta_1)(\sin\theta_4 - \sin\theta_3)} \\ \sin\theta_1 + \frac{\sin\theta_2 - \sin\theta_1}{\cos\theta_2 - \cos\theta_1} \left( \frac{\cos\theta_4 - \cos\theta_2 (\sin\theta_2 \cos\theta_1 - \sin\theta_1 \cos\theta_2)}{(\cos\theta_4 - \cos\theta_2)(\sin\theta_2 - \sin\theta_1) - (\cos\theta_2 - \cos\theta_1)(\sin\theta_4 - \sin\theta_3)} \right) - \cos\theta_1 \end{pmatrix} \tag{23}$$

$$= \begin{pmatrix} 0.68341409 \\ -0.04288508 \end{pmatrix} \tag{24}$$

From the Result of python code , we get

$$\|\mathbf{P} - \mathbf{T}\| = 0.5727 \tag{25}$$

$$\|\mathbf{R} - \mathbf{T}\| = 0.9272 \tag{26}$$

$$\|\mathbf{Q} - \mathbf{T}\| = 0.9272 \tag{27}$$

$$\|\mathbf{S} - \mathbf{T}\| = 0.5727 \tag{28}$$

Hence, we proved that

$$\|\mathbf{P} - \mathbf{T}\| = \|\mathbf{S} - \mathbf{T}\| \tag{29}$$

$$\|\mathbf{Q} - \mathbf{T}\| = \|\mathbf{R} - \mathbf{T}\| \tag{30}$$

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