CLASS 11 CHAPTER-11
LINES

Excercise 10.2

Q18.$P(a, b)$ is the mid-point of the line segment between axes. Show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$
Solution: Let

$$\mathbf{A} = x\mathbf{e_1} \tag{1}$$
$$\mathbf{B} = y\mathbf{e_2} \tag{2}$$
$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{3}$$

where

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

as shown in Figure 1

Now we know

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{x\mathbf{e_1} + y\mathbf{e_2}}{2} \tag{5}$$
$$2\mathbf{P} = x\mathbf{e_1} + y\mathbf{e_2} \tag{6}$$
$$\mathbf{e_1}^\top (2\mathbf{P}) = \mathbf{e_1}^\top (x\mathbf{e_1} + y\mathbf{e_2}) \tag{7}$$
$$= x \tag{8}$$
$$\mathbf{e_2}^\top (2\mathbf{P}) = \mathbf{e_2}^\top (x\mathbf{e_1} + y\mathbf{e_2}) \tag{9}$$
$$= y \tag{10}$$

So, from here we can say that

$$x = 2\mathbf{e_1}^\top\mathbf{P} = 2a \tag{11}$$
$$y = 2\mathbf{e_2}^\top\mathbf{P} = 2b \tag{12}$$
$$\mathbf{A} = \begin{pmatrix} 2\mathbf{e_1}^\top\mathbf{P} \\ 0 \end{pmatrix} \tag{13}$$
$$\mathbf{B} = \begin{pmatrix} 0 \\ 2\mathbf{e_2}^\top\mathbf{P} \end{pmatrix} \tag{14}$$

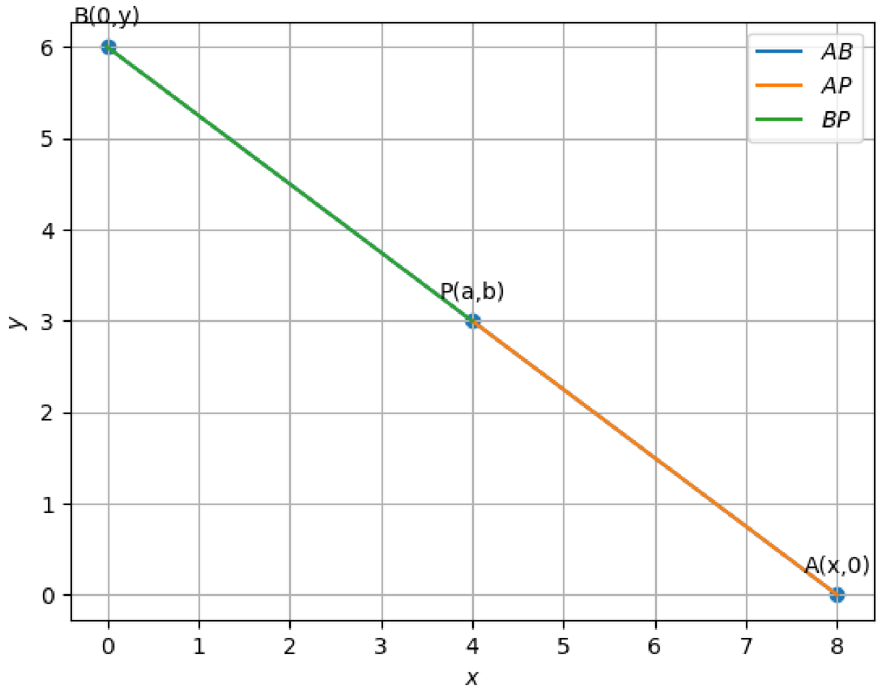

Figure 1:

Now direction vector is

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \tag{15}$$

$$= \begin{pmatrix} 2\mathbf{e_1}^\top \mathbf{P} \\ -2\mathbf{e_2}^\top \mathbf{P} \end{pmatrix} \tag{16}$$

so normal vector is

$$\mathbf{n} = \begin{pmatrix} 2\mathbf{e_2}^\top \mathbf{P} \\ 2\mathbf{e_1}^\top \mathbf{P} \end{pmatrix} \tag{17}$$

So, the equation of line passing through $\mathbf{P}$

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{P}\right) = 0 \tag{18}$$

$$\begin{pmatrix} 2\mathbf{e_2}^\top\mathbf{P} & 2\mathbf{e_1}^\top\mathbf{P} \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} a \\ b \end{pmatrix}\right) = 0 \tag{19}$$

$$\begin{pmatrix} 2\mathbf{e_2}^\top\mathbf{P} & 2\mathbf{e_1}^\top\mathbf{P} \end{pmatrix} \mathbf{x} = 4ab \tag{20}$$

$$\begin{pmatrix} \mathbf{e_2}^\top\mathbf{P} & \mathbf{e_1}^\top\mathbf{P} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2ab \tag{21}$$

$$bx + ay = 2ab \tag{22}$$

$$\frac{x}{a} + \frac{y}{b} = 2 \tag{23}$$

Hence proved.