CLASS-11
CHAPTER-10
STRAIGHT LINES

Excercise 10.3

Q3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

1. $x - \sqrt{3}y + 8 = 0$
2. $y - 2 = 0$
3. $x - y = 4$

Solution:

1. From the given equation:

$$\mathbf{m} = \frac{1}{\sqrt{3}} \tag{1}$$

$$c = \frac{8}{\sqrt{3}} \tag{2}$$

The directional vector is given by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{3}} \end{pmatrix} \tag{3}$$

The normal vector is given by:

$$\mathbf{n} = \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 1 \end{pmatrix} \tag{4}$$

$$\mathbf{n}^\top = \begin{pmatrix} -\frac{1}{\sqrt{3}} & 1 \end{pmatrix} \tag{5}$$

Angle between perpendicular and the positive x-axis is given by:

$$\cos\theta = \frac{\mathbf{e}_1^\top \mathbf{n}}{\|\mathbf{e}_1\| \, \|\mathbf{n}\|} \tag{6}$$

$$= \frac{\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 1 \end{pmatrix}}{\frac{2}{\sqrt{3}}} \tag{7}$$

$$= -\frac{1}{2} \tag{8}$$

$$\implies \theta = 120^\circ \tag{9}$$

The perpendicular distance from the origin to the line is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{8}{2} = 4 \tag{10}$$

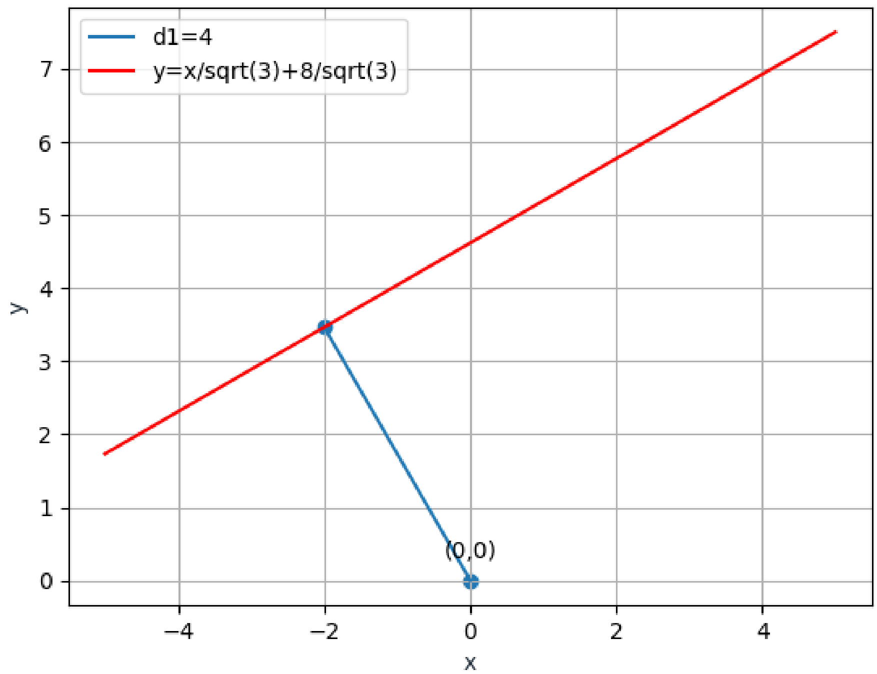

Figure 1:

2. From the given equation:

$$\mathbf{m} = 0 \tag{11}$$
$$c = 2 \tag{12}$$

The directional vector is given by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{13}$$

The normal vector is given by:

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{14}$$
$$\mathbf{n}^\top = \begin{pmatrix} 0 & 1 \end{pmatrix} \tag{15}$$

Angle between perpendicular and the positive x-axis is given by:

$$\cos\theta = \frac{\mathbf{e}_1^\top \mathbf{n}}{\|\mathbf{e}_1\| \, \|\mathbf{n}\|} \tag{16}$$

$$= \frac{\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{1} \tag{17}$$

$$= 0 \tag{18}$$

$$\implies \theta = 90^\circ \tag{19}$$

The perpendicular distance from the origin to the line is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{2}{1} = 2 \tag{20}$$

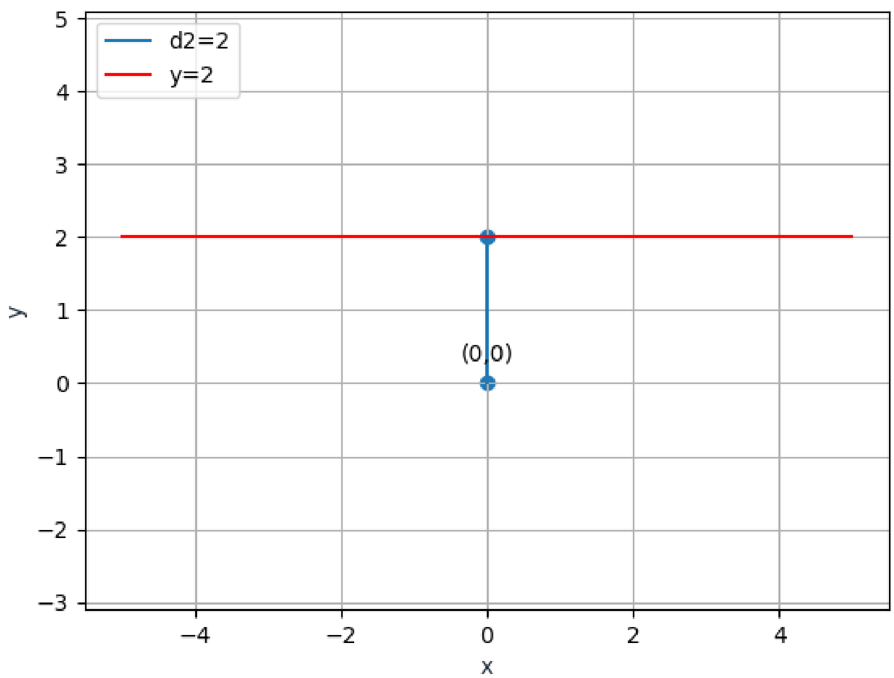

Figure 2:

3. From the given equation:

$$\mathbf{m} = 1 \tag{21}$$

$$c = -4 \tag{22}$$

The directional vector is given by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{23}$$

The normal vector is given by:

$$\mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{24}$$

$$\mathbf{n}^\top = \begin{pmatrix} -1 & 1 \end{pmatrix} \tag{25}$$

Angle between perpendicular and the positive x-axis is given by:

$$\cos\theta = \frac{\mathbf{e}_1^\top \mathbf{n}}{\|\mathbf{e}_1\| \, \|\mathbf{n}\|} \tag{26}$$

$$= \frac{\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix}}{\sqrt{2}} \tag{27}$$

$$= -\frac{1}{2} \tag{28}$$

$$\implies \theta = 315^\circ \tag{29}$$

The perpendicular distance from the origin to the line is given by:

$$d = \frac{|c|}{\|\mathbf{n}\|} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \tag{30}$$

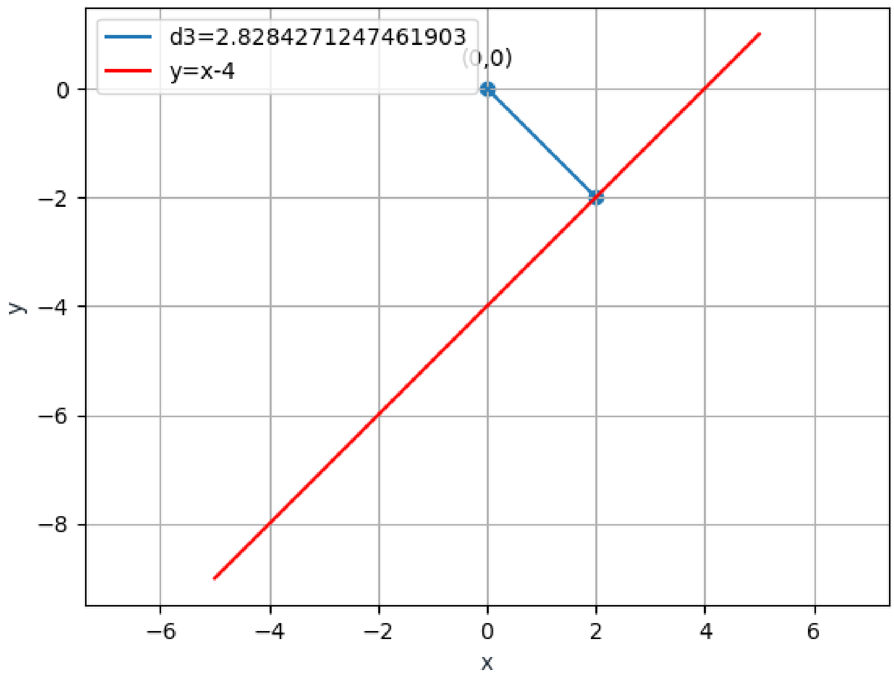

Figure 3: