

CONIC SECTIONS

Exercise 11.4

Q2. Find the coordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of a hyperbola whose equation is given by $\frac{y^2}{9} - \frac{x^2}{27} = 1$.

1. **Solution:** The equation of the hyperbola can be rearranged as

$$-x^2 + 3y^2 - 27 = 0 \quad (1)$$

The above equation can be equated to the generic equation of conic sections

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (2)$$

Comparing coefficients of both equations (1) and (2)

$$\mathbf{V} = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix} \quad (3)$$

$$\mathbf{u} = \mathbf{0} \quad (4)$$

$$f = -27 \quad (5)$$

From equation (3), since \mathbf{V} is already diagonalized, the eigen values λ_1 and λ_2 are given as

$$\lambda_1 = -1 \quad (6)$$

$$\lambda_2 = 3 \quad (7)$$

1. The eccentricity of the hyperbola is given as

$$e = \sqrt{1 - \frac{\lambda_2}{\lambda_1}} = \sqrt{1 + \frac{3}{1}} \quad (8)$$

$$= 2 \quad (9)$$

2. For the standard hyperbola, the coordinates of Foci are given as

$$\mathbf{F} = \pm \frac{\left(\frac{1}{e\sqrt{1-e^2}} \right) (e^2) \sqrt{\frac{\lambda_1}{f_0}}}{\frac{\lambda_1}{f_0}} \mathbf{e}_2 \quad (10)$$

where

$$f_0 = -f \quad (11)$$

$$(10) \implies = \pm \frac{\left(\frac{1}{2\sqrt{1-4}}\right) (4) \sqrt{\frac{-1}{27}}}{\frac{-1}{27}} \mathbf{e}_2 \quad (12)$$

$$= \pm \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (13)$$

3. The vertices of the hyperbola are given by

$$\pm \begin{pmatrix} 0 \\ \sqrt{\left|\frac{f_0}{\lambda_2}\right|} \end{pmatrix} = \pm \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (14)$$

4. The length of latus rectum is given as

$$2 \frac{\sqrt{|f_0 \lambda_2|}}{\lambda_1} = 2 \frac{\sqrt{|27(3)|}}{-1} \quad (15)$$

$$= 18 \quad (16)$$

as length cannot be negative

The corresponding is shown in Figure 1

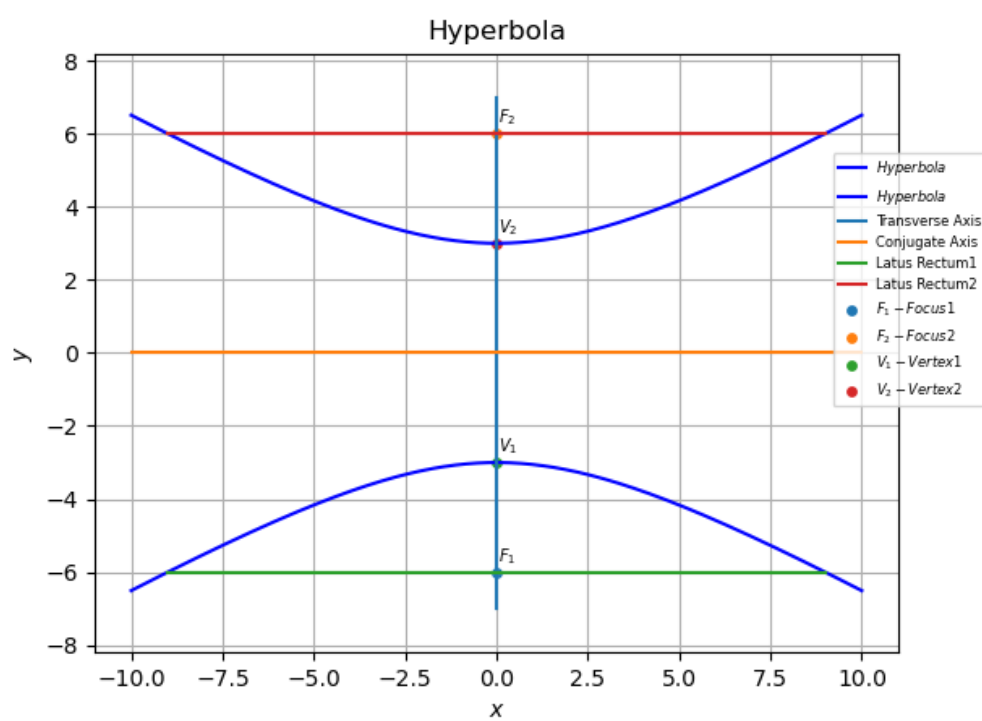


Figure 1: