CHAPTER-7
COORDINATE GEOMETRY

Excercise 7.2

Q3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are $(0,-1),(2,1)$ and $(0,3)$. Find the ratio of this area to the area of the given triangle

Solution:

The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{1}$$

Calculating midpoints:

$$\mathbf{P} = \frac{1}{2}(\mathbf{A}+\mathbf{B}) = \frac{1}{2}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{Q} = \frac{1}{2}(\mathbf{B}+\mathbf{C}) = \frac{1}{2}\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{3}$$

$$\mathbf{R} = \frac{1}{2}(\mathbf{A}+\mathbf{C}) = \frac{1}{2}\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

Calculating the area of the midpoints:

$$ar(PQR) = \frac{1}{2}\|(\mathbf{P}-\mathbf{Q}) \times (\mathbf{Q}-\mathbf{R})\| \tag{5}$$

$$\mathbf{P}-\mathbf{Q} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{6}$$

$$\mathbf{Q}-\mathbf{R} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{7}$$

$$= \frac{1}{2}\begin{vmatrix} 0 & 1 \\ -2 & 1 \end{vmatrix} \tag{8}$$

$$ar(PQR) = 1 \tag{9}$$

Calculating the area of given triangle:

$$ar(ABC) = \frac{1}{2}\|(\mathbf{A}-\mathbf{B}) \times (\mathbf{A}-\mathbf{C})\| \tag{10}$$

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \tag{11}$$

$$\mathbf{A}-\mathbf{C} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{12}$$

$$= \frac{1}{2}\begin{vmatrix} -2 & 0 \\ -2 & -4 \end{vmatrix} \tag{13}$$

$$ar(ABC) = 4 \tag{14}$$

Resultant ratio of two areas is 1:4.

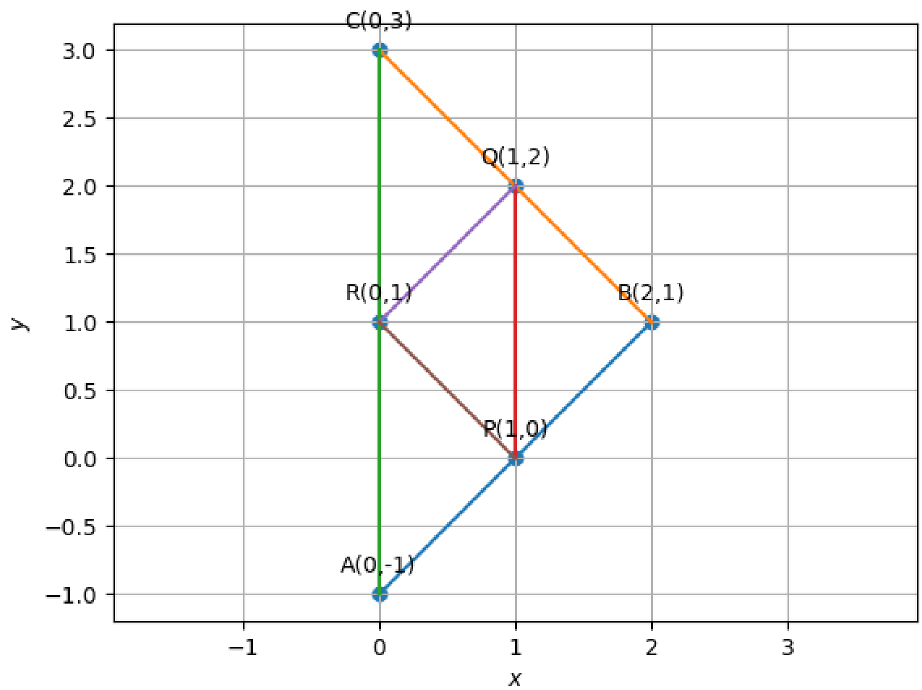

Figure 1: