Que: 11.11.5.5

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1 PROBLEM

A rod of length 12cm moves with its ends always touching the coordinate axes. Determine the equation of locus of a point P on the rod, which is 3cm from the end in contact with $x - axis$.

2 SOLUTION

let the angle made by the rod with x-axis be θ
The x-intecept and y-intercept of rod will be:

1) x-intercept:

$$\mathbf{A} = \begin{pmatrix} 12\cos\theta \\ 0 \end{pmatrix} \qquad (2.0.1)$$

2) y-intercept:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 12\sin\theta \end{pmatrix} \qquad (2.0.2)$$

3) direction vector of rod:

$$\mathbf{A} - \mathbf{B} = 12\begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} \qquad (2.0.3)$$

Unit vector along direction vector:

$$\mathbf{m} = \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} \qquad (2.0.4)$$

4) given point $\mathbf{P}$:

$$\mathbf{P} = \mathbf{A} - 3\mathbf{m} \qquad (2.0.5)$$

$$= \begin{pmatrix} 9\cos\theta \\ 3\sin\theta \end{pmatrix} \qquad (2.0.6)$$

5) parametric form of locus:

$$\mathbf{x} = \begin{pmatrix} 9\cos\theta \\ 3\sin\theta \end{pmatrix} \qquad (2.0.7)$$

$$(2.0.8)$$

Consider $\mathbf{Q} = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

$$\mathbf{Qx} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \qquad (2.0.9)$$

$$\|\mathbf{Qx}\|^2 = 1 \qquad (2.0.10)$$

$$(\mathbf{Qx})^\top (\mathbf{Qx}) = 1 \qquad (2.0.11)$$

$$\mathbf{x}^\top \mathbf{Q}^\top \mathbf{Qx} = 1 \qquad (2.0.12)$$

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{81} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 1 \qquad (2.0.13)$$

The locus of point $\mathbf{P}$ is a conic

$$\mathbf{x}^\top \mathbf{Vx} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \qquad (2.0.14)$$

where,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{81} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \qquad (2.0.15)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad (2.0.16)$$

$$f = -1 \qquad (2.0.17)$$

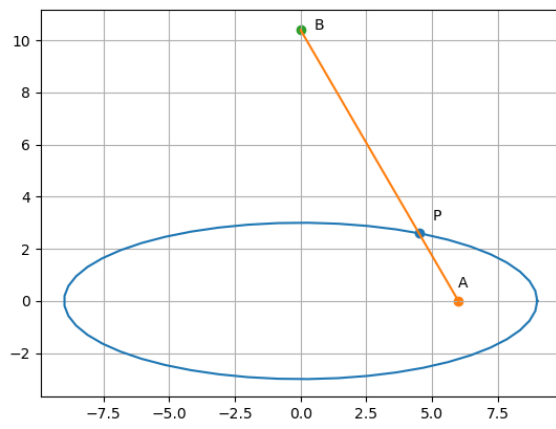| Parameter	Value	Description
$\mathbf{A}$	$\begin{pmatrix} 12\cos\theta \\ 0 \end{pmatrix}$	x-intercept of rod
$\mathbf{C}$	$\begin{pmatrix} 0 \\ 12\sin\theta \end{pmatrix}$	y-intercept of the rod
$\mathbf{P}$	$\begin{pmatrix} 9\cos\theta \\ 3\sin\theta \end{pmatrix}$	Point on rod, at given distance from $\mathbf{A}$
θ	$\frac{\pi}{3}$	parameter θ for $\mathbf{A}, \mathbf{B}, \mathbf{P}$
length	12	Length of the rod
dist	3	Distance between $\mathbf{A}, \mathbf{P}$

TABLE 5: Table 1

Fig. 5: Figure 1