Circles

1 11^{th} Maths - Chapter 11

This is Problem-6 from Exercise 11.1

Q2. Find the centre and radius of the given circle $(\mathbf{x}+5)^2+(\mathbf{y}–3)^2=36$.

Solution:

Given circle equation is

$$(\mathbf{x}+5)^2+(\mathbf{y}–3)^2=36 \tag{1}$$

The general equation of the circle is

$$\|\mathbf{x}\|^2+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{2}$$

Where,

$$\mathbf{u}=-\mathbf{c} \text{ and } f=\|\mathbf{u}\|^2-r^2 \tag{3}$$

by expanding (1)

$$\mathbf{x}^2+10\mathbf{x}+25+\mathbf{y}^2-6\mathbf{y}+9-36=0 \tag{4}$$

$$\|\mathbf{x}\|^2+2\begin{pmatrix}5 & -3\end{pmatrix}\mathbf{x}-2=0 \tag{5}$$

by comparing (3) to (5) we get

$$\mathbf{u}=\begin{pmatrix}5\\-3\end{pmatrix} \tag{6}$$

$$f=-2 \tag{7}$$

$$\mathbf{c}=\begin{pmatrix}-5\\3\end{pmatrix} \tag{8}$$

$$\|\mathbf{u}\|^2=34 \tag{9}$$

$$r^2=\|\mathbf{u}\|^2-f \tag{10}$$

$$r^2=36, r=\pm6 \tag{11}$$

radius of circle is positive so the centre and radius of $(\mathbf{x}+5)^2+(\mathbf{y}–3)^2=36$. is $\begin{pmatrix}-5\\3\end{pmatrix}$ and 6 respectively

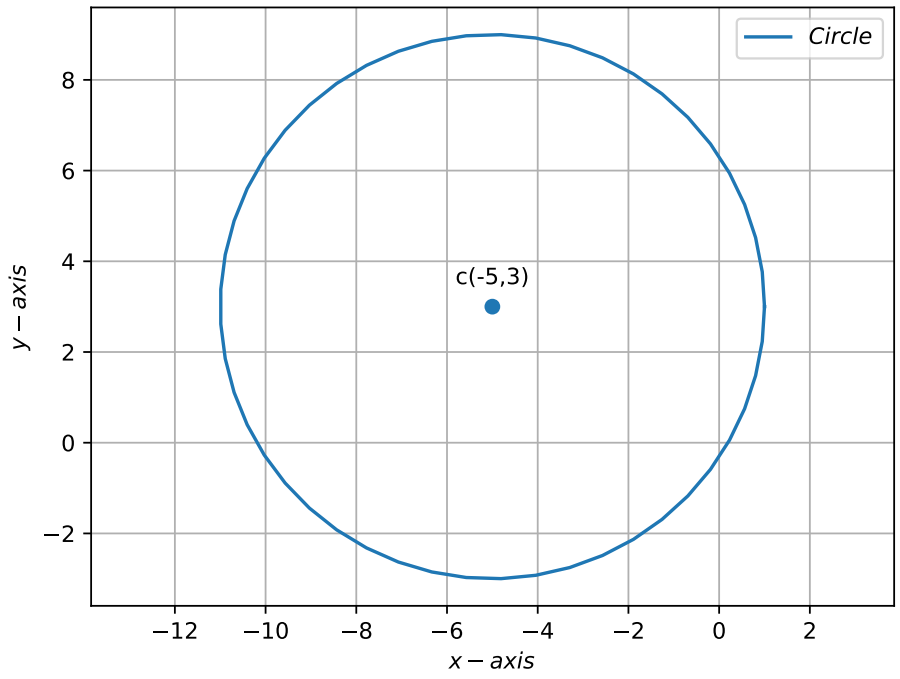

Figure 1