Three Dimensional Geometry

12^{th} Maths - Chapter 11

This is Problem-3 from Exercise 11.1

1. A tangent PQ at a point of a circle of radius 5cm meets a line through the centre O at a point Q so that OQ=12cm then length of PQ is

   Solution: The input parameters for this problem are available in Table (1)

Symbol	**Value**	**Description**
r	5	Radius
$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Centre $\mathbf{O}$
$\mathbf{P}$	$r\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	Point $\mathbf{P}$
d	12	Lenght of OQ

Table 1

The distance form origin to point $\mathbf{Q}$ is given by

$$\|\mathbf{Q}\|^2 = d^2 \tag{1}$$

Then equation of line is given as

$$(\mathbf{Q} - \mathbf{P})^\top \mathbf{P} = 0 \tag{2}$$
$$\mathbf{P}^\top \mathbf{Q} = \|\mathbf{P}\|^2 = r^2 \tag{3}$$
$$\mathbf{P}^\top \mathbf{Q} = 25 \tag{4}$$

For $\theta = 0^\circ$ The point $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{5}$$

Now substituting the value of $\mathbf{P}$ in (4) gives

$$\begin{pmatrix} 5 & 0 \end{pmatrix} \mathbf{Q} = 25 \tag{6}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{Q} = 5 \tag{7}$$

$$\mathbf{Q} = \begin{pmatrix} 5 \\ \mu \end{pmatrix} \tag{8}$$

$$\mathbf{Q} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{9}$$

The (9) can be expressed in the form of parametric equation

$$\mathbf{Q} = \mathbf{A} + \mu\mathbf{m} \tag{10}$$

Then substituting (10) in (1) yeilds,

$$\implies (\mathbf{A} + \mu\mathbf{m})^\top (\mathbf{A} + \mu\mathbf{m}) = d^2 \tag{11}$$

$$\implies \mathbf{A}^\top\mathbf{A} + (\mu\mathbf{m})^\top \mu\mathbf{m} + \mathbf{A}^\top\mu\mathbf{m} + (\mu\mathbf{m})^\top \mathbf{A} = d^2 \tag{12}$$

$$\implies \|\mathbf{A}\|^2 + \mu^2 \|\mathbf{m}\|^2 + 2\mu\mathbf{A}^\top\mathbf{m} = d^2 \tag{13}$$

$$\implies \mu^2 \|\mathbf{m}\|^2 + 2\mu\mathbf{A}^\top\mathbf{m} + \|\mathbf{A}\|^2 = d^2 \tag{14}$$

where

$$\mathbf{A} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \text{ and } \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{15}$$

substituting the values of $\mathbf{A}$ and $\mathbf{m}$ in (14) gives

$$\implies \mu^2(1) + 2\mu \begin{pmatrix} 5 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 25 = 144 \tag{16}$$

$$\implies \mu^2 = 119 \tag{17}$$

$$\implies \mu = \pm\sqrt{119} \tag{18}$$

substituting the value of μ in (9) yeilds

$$\mathbf{Q_1} = \begin{pmatrix} 5 \\ \sqrt{119} \end{pmatrix}, \mathbf{Q_2} = \begin{pmatrix} 5 \\ -\sqrt{119} \end{pmatrix} \tag{19}$$

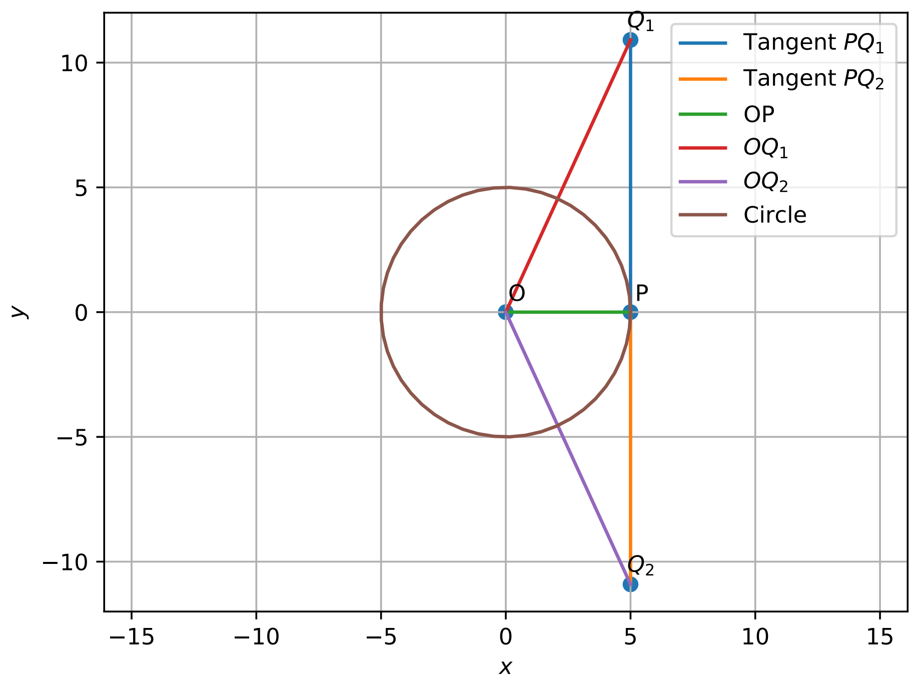
Q1
Q2
O
P
Tangent PQ1
Tangent PQ2
OP
OQ1
OQ2
Circle
10
5
0
−5
−10
−15
−10
−5
0
5
10
15
x
y

Figure 1