Coordinate Geometry

10^{th} Maths - Chapter 7

This is Problem-4 from Exercise 7.3

1. Find the area of quadrilateral whose vertices, taken in order, are $\begin{pmatrix}-4\\-2\end{pmatrix}$, $\begin{pmatrix}-3\\-5\end{pmatrix}$, $\begin{pmatrix}3\\-2\end{pmatrix}$ and $\begin{pmatrix}2\\3\end{pmatrix}$.

   Solution: The input parameters for this problem are available in Table (1)

Symbol	Value	Description
A	$\begin{pmatrix}-4\\-2\end{pmatrix}$	First point
B	$\begin{pmatrix}-3\\-5\end{pmatrix}$	Second point
C	$\begin{pmatrix}3\\-2\end{pmatrix}$	Third point
D	$\begin{pmatrix}2\\3\end{pmatrix}$	Fourth point

Table 1

By joining **B** to **D**, you will get two triangles **ABD** and **BCD**.
In general, the ar(ABD) can be expressed as

$$ar(ABD) = \frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{D})\right\| \tag{1}$$

$$\mathbf{A}-\mathbf{B}=\begin{pmatrix}-4\\-2\end{pmatrix}-\begin{pmatrix}-3\\-5\end{pmatrix}=\begin{pmatrix}-1\\3\end{pmatrix} \tag{2}$$

$$\mathbf{A}-\mathbf{D}=\begin{pmatrix}-4\\-2\end{pmatrix}-\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-6\\-5\end{pmatrix} \tag{3}$$

Substituting the values of (2) and (3) in (1),

$$\frac{1}{2}\begin{vmatrix}-1 & 3\\-6 & -5\end{vmatrix}=\frac{23}{2} \tag{4}$$

Also, the ar(BCD) can be expressed as

$$ar(BCD)=\frac{1}{2}\left\|(\mathbf{B}-\mathbf{C})\times(\mathbf{B}-\mathbf{D})\right\| \tag{5}$$

$$\mathbf{B}-\mathbf{C}=\begin{pmatrix}-3\\-5\end{pmatrix}-\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}-6\\-5\end{pmatrix} \tag{6}$$

$$\mathbf{B}-\mathbf{D}=\begin{pmatrix}-3\\-5\end{pmatrix}-\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-3\\-8\end{pmatrix} \tag{7}$$

Substituting the values of (6) and (7) in (5),

$$\frac{1}{2}\begin{vmatrix}-6 & -3\\-5 & -8\end{vmatrix}=\frac{33}{2} \tag{8}$$

Area of Quadrilateral **ABCD** = ar(ABD) + ar(BCD),

$$\frac{23}{2}+\frac{33}{2}=28 \tag{9}$$

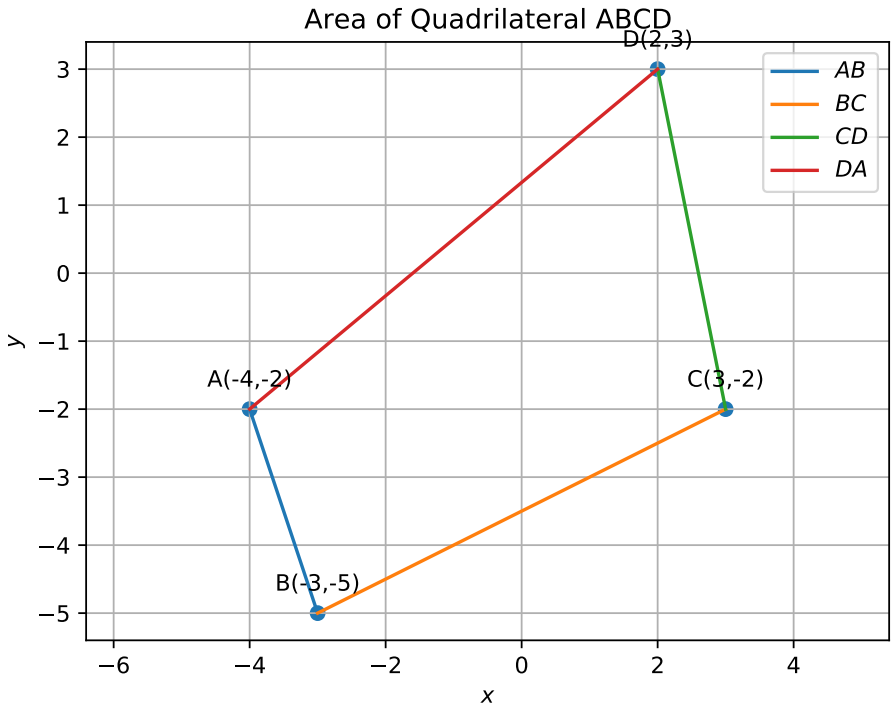

Figure 1