Problem: 10.10.2.3

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1 Problem

If the tangents PA and PB from a point $\mathbf{P}$ to a circle with center $\mathbf{O}$ are inclined to each other at $80°$, find $\angle POA$.

2 Solution

$$\angle APB = 80° \tag{2.0.1}$$

$$\angle APO = \frac{1}{2}\angle APB \tag{2.0.2}$$

$$= 40° \quad = \theta \ (say) \tag{2.0.3}$$

Therefore, it can be said that $\mathbf{P}$ lies on the line

$$\begin{pmatrix} -\sin\theta & \cos\theta \end{pmatrix} \mathbf{x} = 0 \tag{2.0.4}$$

Let the circle be $\|\mathbf{x}\|^2 = r^2$ and $\mathbf{A}$ be $\begin{pmatrix} 0 \\ r \end{pmatrix}$.

Therefore, the tangent that P lies on is given by:

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = r \tag{2.0.5}$$

The point $\mathbf{P}$ is given by:

$$\begin{pmatrix} -\cos\theta & \sin\theta \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ r \end{pmatrix} \tag{2.0.6}$$

Augmented Matrix:

$$\left(\begin{array}{cc|c} -\sin\theta & \cos\theta & 0 \\ 0 & 1 & r \end{array}\right) \tag{2.0.7}$$

$$\xleftrightarrow{R_1 \leftarrow -\frac{R_1}{-\sin\theta} + \cot\theta R_2} \tag{2.0.8}$$

$$\left(\begin{array}{cc|c} 1 & 0 & r\cot\theta \\ 0 & 1 & r \end{array}\right) \tag{2.0.9}$$

$$\mathbf{P} = \begin{pmatrix} r\cot\theta \\ r \end{pmatrix} \tag{2.0.10}$$

let $\angle POA = \phi$

$$\cos\phi = \frac{(\mathbf{P}-\mathbf{O})^\top (\mathbf{A}-\mathbf{O})}{\|\mathbf{P}-\mathbf{O}\| \, \|\mathbf{A}-\mathbf{O}\|} \tag{2.0.11}$$

$$= \frac{\begin{pmatrix} r\cot\theta & r \end{pmatrix} \begin{pmatrix} 0 \\ r \end{pmatrix}}{r^2 \operatorname{cosec}\theta} \tag{2.0.12}$$

$$\cos\phi = \sin\theta \tag{2.0.13}$$

$$\implies \phi = 90° - \theta \tag{2.0.14}$$

$$\phi = 50° \tag{2.0.15}$$

$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Center of the given circle
$\mathbf{A}$	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$	Point where tangent is taken
r	5	radius of given circle
$\angle APB$	$80°$	Angle between tangents

TABLE 0: Table 1

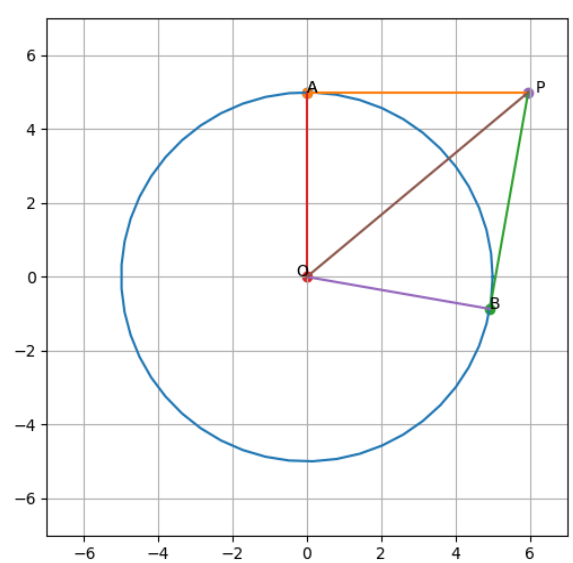

Fig. 0: Figure 1