

Question: 9.10.4.5

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1 PROBLEM

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

2 SOLUTION

Consider Reshma, Salma and Mandip be standing at **A**, **B** and **C** respectively, and the center the of the circle **O**. Let

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{O} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (2.0.2)$$

Therefore, the equation of the circle is given by

$$\|\mathbf{x} - \mathbf{O}\|^2 = 25 \quad (2.0.3)$$

$$\|\mathbf{x}\|^2 - 2\mathbf{O}^\top \mathbf{x} + \|\mathbf{O}\|^2 - 25 = 0 \quad (2.0.4)$$

$$\|\mathbf{x}\|^2 - 2 \begin{pmatrix} 5 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.5)$$

Also, **A** and **C** are equidistant (6m) from **B**, we can say that they lie on the circle having **B** as center and radius 6m. Equation of this circle is given by,

$$\|\mathbf{x}\|^2 - 2\mathbf{B}^\top \mathbf{x} + \|\mathbf{B}\|^2 - 36 = 0 \quad (2.0.6)$$

$$\|\mathbf{x}\|^2 = 36 \quad (2.0.7)$$

$$i.e., \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -36 \quad (2.0.8)$$

line passing through **A** and **C** from (2.0.5) and (2.0.8),

$$\begin{pmatrix} 5 & 0 \end{pmatrix} \mathbf{x} = 18 \quad (2.0.9)$$

$$\mathbf{x} = \begin{pmatrix} \frac{18}{5} \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.10)$$

$$i.e., \mathbf{h} = \begin{pmatrix} \frac{18}{5} \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.11)$$

For circle, $\mathbf{V} = \mathbf{I}$

$$\mu_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{(\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}))^2 - \mathbf{g}(\mathbf{h}) (\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right)$$

where,

$$\mathbf{g}(\mathbf{h}) = \mathbf{h}^\top \mathbf{V} \mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \quad (2.0.13)$$

here,

$$\mu_i = \pm \frac{24}{5} \quad (2.0.14)$$

Therefore,

$$\mathbf{A} = \begin{pmatrix} \frac{18}{5} \\ \frac{24}{5} \end{pmatrix} \quad (2.0.15)$$

$$\mathbf{C} = \begin{pmatrix} \frac{18}{5} \\ -\frac{24}{5} \end{pmatrix} \quad (2.0.16)$$

Distance between Reshma and Mandip,

$$\|\mathbf{A} - \mathbf{C}\| = \left\| \begin{pmatrix} 0 \\ \frac{48}{5} \end{pmatrix} \right\| \quad (2.0.17)$$

$$= \frac{48}{5} \quad (2.0.18)$$

O	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$	Center of the given circle
B	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Point where Salma is standing
<i>r</i>	5	radius of given circle
<i>d</i>	6	distance AB and BC

TABLE 0: Table1

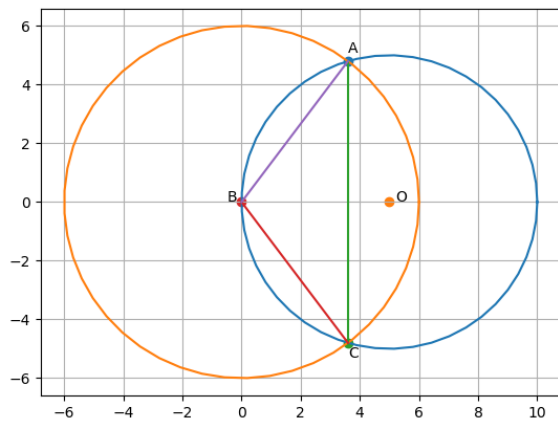


Fig. 0: Figure 1