11.11.5.3

Lokesh Surana

CLASS 11, CHAPTER 11, EXERCISE 5.3

Q. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Solution: Uniformly loaded suspension bridge cable hangs in the form of a parabola facing upwards.

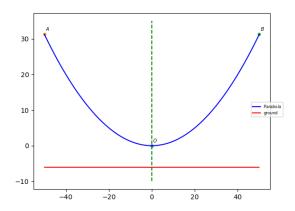
О	Lowest point of cable	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
d	Length of the cable	100 m
d_1	Length of longest wire	30 m
d_2	Length of shortest wire	6 m
A	End point of cable	$\begin{pmatrix} \frac{d}{2} \\ d_1 - d_2 \end{pmatrix}$
В	End point of cable	$\begin{pmatrix} -\frac{d}{2} \\ d_1 - d_2 \end{pmatrix}$

TABLE I: points

This will give us a setup similar to figure 1,

Here A and B are the points on the parabola where the cable is attached to the roadway, i.e. longest wire is attached at this points. And vertex of parabola O is point where shortest wire is attached, which is 6m from the ground.

With the assumption of point O being $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, we'll get



1

Fig. 1: Representation of parabola with vertex at origin.

$$\mathbf{A} = \begin{pmatrix} \frac{d}{2} \\ d_1 - d_2 \end{pmatrix} = \begin{pmatrix} 50 \\ 24 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -\frac{d}{2} \\ d_1 - d_2 \end{pmatrix} = \begin{pmatrix} -50 \\ 24 \end{pmatrix} \tag{2}$$

The generic equation of conic is

$$g(\mathbf{x}) = \mathbf{x}^{\mathsf{T}} \mathbf{V} \mathbf{x} + 2 \mathbf{u}^{\mathsf{T}} \mathbf{x} + f = 0 \tag{3}$$

As conic is upward facing parabola,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{4}$$

Points O, A, and B are on conic, so we have

$$\mathbf{O}^{\mathsf{T}}\mathbf{V}\mathbf{O} + 2\mathbf{u}^{\mathsf{T}}\mathbf{O} + f = 0 \tag{5}$$

$$\mathbf{A}^{\mathsf{T}}\mathbf{V}\mathbf{A} + 2\mathbf{u}^{\mathsf{T}}\mathbf{A} + f = 0 \tag{6}$$

$$\mathbf{B}^{\mathsf{T}}\mathbf{V}\mathbf{B} + 2\mathbf{u}^{\mathsf{T}}\mathbf{B} + f = 0 \tag{7}$$

Rewrite the equations as

$$2\mathbf{O}^{\mathsf{T}}\mathbf{u} + f = -\mathbf{O}^{\mathsf{T}}\mathbf{VO} \tag{8}$$

$$2\mathbf{A}^{\mathsf{T}}\mathbf{u} + f = -\mathbf{A}^{\mathsf{T}}\mathbf{V}\mathbf{A} \tag{9}$$

$$2\mathbf{B}^{\mathsf{T}}\mathbf{u} + f = -\mathbf{B}^{\mathsf{T}}\mathbf{V}\mathbf{B} \tag{10}$$

This can be formulated as matrix, as follows:

$$\begin{pmatrix} 2\mathbf{O}^{\top} & 1\\ 2\mathbf{A}^{\top} & 1\\ 2\mathbf{B}^{\top} & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u}\\ f \end{pmatrix} = -\begin{pmatrix} \mathbf{O}^{\top}\mathbf{V}\mathbf{O}\\ \mathbf{A}^{\top}\mathbf{V}\mathbf{A}\\ \mathbf{B}^{\top}\mathbf{V}\mathbf{B} \end{pmatrix}$$
(11)

Substituting the values of **O**, **A**, and **B** in the above equation, we get

$$\begin{pmatrix} 0 & 0 & 1 \\ 100 & 48 & 1 \\ -100 & 48 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\begin{pmatrix} 0 \\ -2500 \\ -2500 \end{pmatrix} \tag{12}$$

$$\implies f = 0 \text{ and } \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{625}{12} \end{pmatrix}$$
 (13)

So, equation of parabola is

$$\mathbf{x}^{\mathsf{T}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & -\frac{625}{12} \end{pmatrix} \mathbf{x} = 0 \tag{14}$$

At a point λ_1 m from middle, i.e.

$$\mathbf{x} = \lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2 \tag{15}$$

Substitute this in parabola equation, we get

$$(\lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2)^{\mathsf{T}} \mathbf{V} (\lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2) + 2\mathbf{u}^{\mathsf{T}} (\lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2) + f = 0$$
 (16)

$$\lambda_{1}^{2} \mathbf{e}_{1}^{\mathsf{T}} \mathbf{V} \mathbf{e}_{1} + \lambda_{2}^{2} \mathbf{e}_{2}^{\mathsf{T}} \mathbf{V} \mathbf{e}_{2} + \lambda_{1} \lambda_{2} \mathbf{e}_{1}^{\mathsf{T}} \mathbf{V} \mathbf{e}_{2} + \lambda_{1} \lambda_{2} \mathbf{e}_{2}^{\mathsf{T}} \mathbf{V} \mathbf{e}_{1} + 2 \lambda_{1} \mathbf{u}^{\mathsf{T}} \mathbf{e}_{1} + 2 \lambda_{2} \mathbf{u}^{\mathsf{T}} \mathbf{e}_{2} + f = 0$$

$$(17)$$

Substituting the values of V, u, e_1 , e_2 and f

$$\lambda_1^2 - \frac{6}{625}\lambda_2 = 0 \tag{18}$$

(19)

It is given that $\lambda_1 = 18$

$$\implies \lambda_2 = \frac{6}{625} (18)^2 = \frac{1944}{625} \qquad (20)$$

 \implies Length of a supporting wire attached to the roadway 18m from the middle is

$$= \lambda_2 + d_2 = \frac{1944}{625} + 6 = \frac{5694}{625}m \tag{21}$$

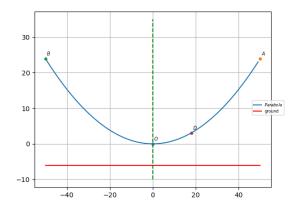


Fig. 2: Parabola