

Vector Algebra

1 12th Maths - Exercise 10.2.14

1. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ

2 Solution

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (1)$$

$$(2)$$

let \mathbf{A} be the given vector The Directional vectors of X, Y and Z axes are given respectively

$$\mathbf{B}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{B}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{B}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (3)$$

The magnitudes for \mathbf{A} and directional vectors $\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3$ are

$$\|\mathbf{A}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \quad (4)$$

$$\|\mathbf{B}_1\| = \sqrt{1^2 + 0^2 + 0^2} = \sqrt{1} \quad (5)$$

$$\|\mathbf{B}_2\| = \sqrt{0^2 + 1^2 + 0^2} = \sqrt{1} \quad (6)$$

$$\|\mathbf{B}_3\| = \sqrt{0^2 + 0^2 + 1^2} = \sqrt{1} \quad (7)$$

Let

$$\cos \theta_i = \frac{\mathbf{A}^T \mathbf{B}}{|\mathbf{A}| |\mathbf{B}|} \quad (8)$$

$$(9)$$

So for different values of $\cos \theta_i$ the direction cosines of vector \mathbf{A} are

$$\mathbf{a}_1 = \frac{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} \quad (10)$$

$$\mathbf{a}_2 = \frac{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} \quad (11)$$

$$\mathbf{a}_3 = \frac{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} \quad (12)$$

$$\mathbf{a}_1 = \mathbf{a}_2 = \mathbf{a}_3 = \frac{1}{\sqrt{3}} \quad (13)$$

the $\mathbf{a}_1 = \mathbf{a}_2 = \mathbf{a}_3 \implies$ that the vector is equally inclined to axes