ASSIGNMENT-4

Ojaswa Pandey

Download all python codes from

https://github.com/behappy0604/Summer−Internship−IITH/tree/main/Assignment−4

and latex-tikz codes from

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1 Question No. 2.43 (a)

Find the roots of the equation: x+$\frac{1}{x}$ = 3, x ≠=0

2 Solution

1) The given equation can be writen as:

$$x^2 + 1 = 3x \tag{2.0.1}$$

$$x^2 - 3x + 1 = 0 \tag{2.0.2}$$

2) The vector form of the equation is:

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -3 & -1 \end{pmatrix} \mathbf{x} + 1 = 0 \tag{2.0.3}$$

3) Comparing (2.0.2) with standard quadractic equation $ax^2 + bx + 1 = 0$ we get:

$$a = 1 \tag{2.0.4}$$

$$b = -3 \tag{2.0.5}$$

$$c = 1 \tag{2.0.6}$$

4) The discriminant is:

$$D = b^2 - 4ac \tag{2.0.7}$$

$$D = 5 \tag{2.0.8}$$

5) Now

$$a = \mathbf{e_1^T V e_1} \tag{2.0.9}$$

$$b = 2\mathbf{u^T e_1} \tag{2.0.10}$$

$$c = \mathbf{f} \tag{2.0.11}$$

6) Therefore,

$$zeroes = \frac{-2\mathbf{u^T e_1} \pm \sqrt{(2\mathbf{u^T e_1})^2 - (4\mathbf{e_1^T V e_1 f})^2}}{2\mathbf{e_1^T V e_1}} \tag{2.0.12}$$

7) Putting the values we get:

$$\mathbf{x} = \begin{pmatrix} 2.6180 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.38196 \\ 0 \end{pmatrix} \tag{2.0.13}$$

8) The nature of the roots of equation $x^2-3x+1 = 0$:

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} \frac{-3}{2} \\ \frac{-1}{2} \end{pmatrix} \mathbf{x} + 1 = 0 \tag{2.0.14}$$

Here,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{-3}{2} \\ \frac{-1}{2} \end{pmatrix}, f = 1 \tag{2.0.15}$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.16}$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p_1}^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.17}$$

∴Vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} \frac{-3}{2} & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -1 \\ \frac{-3}{2} \\ 0 \end{pmatrix} \tag{2.0.18}$$

$$\implies \begin{pmatrix} -\frac{-3}{2} & -1 \\ 1 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -1 \\ \frac{3}{2} \end{pmatrix} \tag{2.0.19}$$

$$\implies \mathbf{c} = \begin{pmatrix} \frac{3}{2} \\ \frac{-5}{4} \end{pmatrix} \tag{2.0.20}$$

Now,

$$\mathbf{p_1}^T\mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{3}{2} \\ \frac{-5}{4} \end{pmatrix} \tag{2.0.21}$$

$$= \frac{-5}{4} \tag{2.0.22}$$

and,

$$\mathbf{p_2}^T\mathbf{V}\mathbf{p_2} = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.23}$$

$$= 1 \tag{2.0.24}$$

$\because$

$$(\mathbf{p_1}^T\mathbf{c})(\mathbf{p_2}^T\mathbf{V}\mathbf{p_2}) = \frac{-5}{4} < 0 \tag{2.0.25}$$

Hence,the given equation has real and distinct roots.

9) The values of $\mathbf{x}$ is calculated using python:

$$\mathbf{x} = \begin{pmatrix} 2.6180 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.38196 \\ 0 \end{pmatrix} \tag{2.0.26}$$

10) The plot of the quadratic equation is:

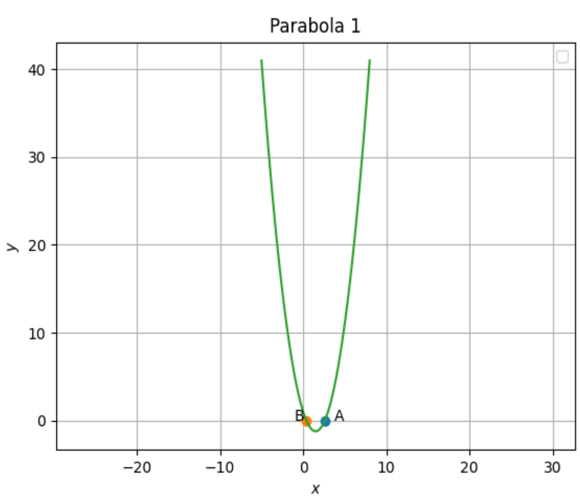

Fig. 2.1: curve