ASSIGNMENT-1

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Download all python codes from

```
https://github.com/nikhithakaspa/assignment-1/
    blob/main/ASSIGNMENT%201.py
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and latex-tikz codes from

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https://github.com/nikhithakaspa/assignment-1/
    blob/main/main.tex
```

1 Question No.2.8

In $\triangle ABC$, $a = 6, \angle B = 60^{\circ}$ and $b - c = 2$. Sketch $\triangle ABC$.

2 Solution

The vertex $\mathbf{A}$ can be expressed in *polar coordinate form* as

$$\mathbf{A} = b\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}a\\0\end{pmatrix}, \tag{2.0.1}$$

From $\triangle ABC$,we use the law of cosines:

$$b^2 = a^2 + c^2 - 2ac\cos B \tag{2.0.2}$$

$$b^2 - c^2 = a^2 - 2ac\cos B \tag{2.0.3}$$

$$(b+c)(b-c) = 6^2 - 2(6)\frac{1}{2}c \quad \left(\because \angle B = 60^{\circ}\right) \tag{2.0.4}$$

$$(b+c)(2) = 36 - 6c \quad (\because b - c = 2) \tag{2.0.5}$$

$$\implies b + 4c = 18 \tag{2.0.6}$$

And we have,

$$b - c = 2 \tag{2.0.7}$$

which can be expressed as the matrix equation

$$\begin{pmatrix}1 & 4\\ 1 & -1\end{pmatrix}\begin{pmatrix}b\\c\end{pmatrix} = \begin{pmatrix}18\\2\end{pmatrix} \tag{2.0.8}$$

By applying row reduction:

$$\begin{pmatrix}1 & 4 & 18\\ 1 & -1 & 2\end{pmatrix} \tag{2.0.9}$$

$$\xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix}1 & 4 & 18\\ 0 & -5 & -16\end{pmatrix} \tag{2.0.10}$$

$$\xrightarrow{R_1 \to 5R_1 + 4R_2} \begin{pmatrix}5 & 0 & 26\\ 0 & -5 & -16\end{pmatrix} \tag{2.0.11}$$

$$\xrightarrow[R_2 \to -\frac{R_2}{5}]{R_1 \to \frac{R_1}{5}} \begin{pmatrix}1 & 0 & \frac{26}{5}\\ 0 & 1 & \frac{16}{5}\end{pmatrix} \tag{2.0.12}$$

Therefore,

$$\begin{pmatrix}b\\c\end{pmatrix} = \begin{pmatrix}\frac{26}{5}\\ \frac{16}{5}\end{pmatrix} \tag{2.0.13}$$

Now vertices of $\triangle ABC$ can be written as

$$\mathbf{A} = \frac{26}{5}\begin{pmatrix}\cos 60\\ \sin 60\end{pmatrix} = \begin{pmatrix}2.6\\4.5\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}6\\0\end{pmatrix} \tag{2.0.14}$$

Plot of the $\triangle ABC$:

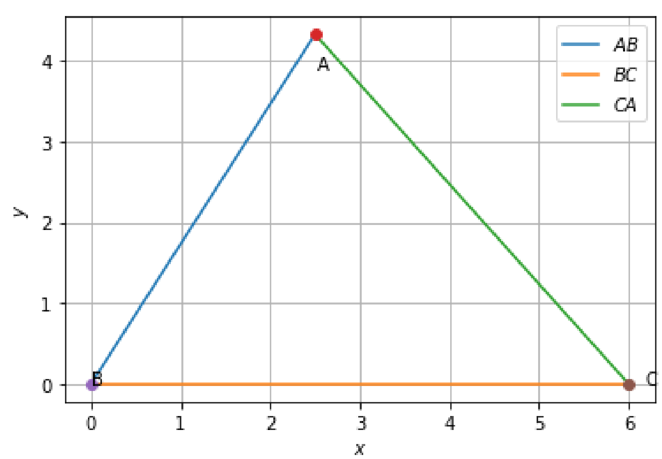

Fig. 2.1: $\triangle ABC$