Assignment 7

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Download all python codes from

https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment7/Codes

and latex-tikz codes from

https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment7

1 Question No. 2.99

AOBA is the part of the ellipse $\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 36$ in the first quadrant such that $OA = 2$ and $OB = 6$ .Find the area between the arc AB and the chord AB.

2 Solution

Given ellipse is

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 36 \tag{2.0.1}$$

On comparing it with standard form

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{D} = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 36 \tag{2.0.4}$$

$$\lambda_1 = 9 \tag{2.0.5}$$

$$\lambda_2 = 1 \tag{2.0.6}$$

$\therefore$ Semi major and minor axes of ellipse are

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = \sqrt{\frac{36}{1}} = 6 \tag{2.0.7}$$

$$b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} = \sqrt{\frac{36}{9}} = 2 \tag{2.0.8}$$

$\therefore$ Equation of ellipse can be written as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2.0.9}$$

$$\implies \frac{x^2}{4} + \frac{y^2}{36} = 1 \tag{2.0.10}$$

Now,area of ellipse is given by

$$A = \pi ab \tag{2.0.11}$$

$$\implies A = 12\pi \tag{2.0.12}$$

$\therefore$ Area of a quadrant of ellipse is given by

$$A_1 = A/4 = 3\pi \tag{2.0.13}$$

Now,from fig. 2.1 , $AOBA$ is a right angled triangle whose area is given by

$$A_2 = \frac{1}{2}ab = 6 \tag{2.0.14}$$

$\therefore$ Area between arc AB and chord AB is given by

$$A_3 = A_1 - A_2 = 3\pi - 6 \tag{2.0.15}$$

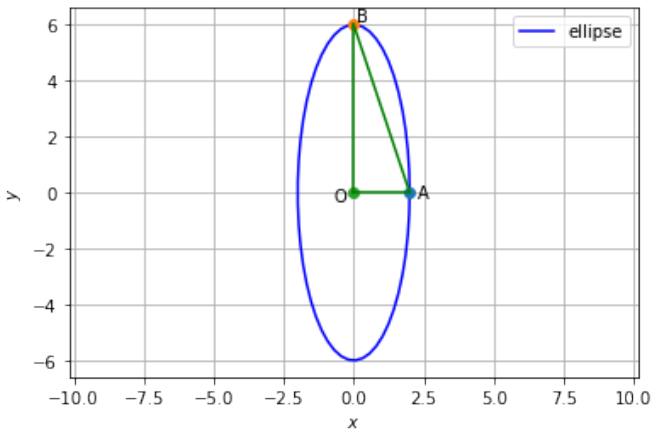

Fig. 2.1: Ellipse $\frac{x^2}{4} + \frac{y^2}{36} = 1$