ASSIGNMENT-11

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_11/blob/master/codes.py

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_11

1 Question No. 2.30

A fruit grower can use two types of fertilizer in his garden, brand P and brand Q.The amounts(in kg) of nitrogen, phosphoric acid,potash and chlorine in a bag of each brand are given in the table.Tests indicate that garden needs atleast 240 kg of phosphoric acid,atleast 270 kg of potash and atmost 310 kg of chlorine. If the grower wants to minimise the amount of nitrogen added to garden, how many bags of each brand should be used?What is the minimum amount of nitrogen added in the ground?

	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric Acid	1	2
Potash	3	1.5
Chlorine	1.5	2

TABLE 1.1: kg per bag

2 Solution

- All the data can be tabularised as:

	Brand P	**Brand Q**	Amounts Required
Nitrogen	3	3.5	?
Phosphoric Acid	1	2	≥ 240 kg
Potash	3	1.5	≥ 270 kg
Chlorine	1.5	2	≤ 310 kg

TABLE 2.1: Requirements of fertilizers

- Let the number of bags of Brand P be x &
- The number of bags of Brand Q be y such that :

$$x \geq 0 \tag{2.0.1}$$
$$y \geq 0 \tag{2.0.2}$$

- From the data given we have:

$$x + 2y \geq 240 \tag{2.0.3}$$
$$\implies -x - 2y \leq -240 \tag{2.0.4}$$

and,

$$3x + 1.5y \geq 270 \tag{2.0.5}$$
$$\implies -x - 0.5y \leq -90 \tag{2.0.6}$$

and,

$$1.5x + 2y \leq 310 \tag{2.0.7}$$
$$\tag{2.0.8}$$

$\therefore$ The minimizing function is:

$$\min_{\mathbf{x}} Z = \begin{pmatrix} 3 & 3.5 \end{pmatrix} \mathbf{x} \tag{2.0.9}$$

$$s.t. \quad \begin{pmatrix} -1 & -2 \\ -1 & -0.5 \\ 1.5 & 2 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} -240 \\ -90 \\ 310 \end{pmatrix} \tag{2.0.10}$$

$$-\mathbf{x} \leq \mathbf{0} \tag{2.0.11}$$

- The Lagrangian function can be given as:

$$\begin{aligned} &L(\mathbf{x}, \boldsymbol{\lambda}) \\ &= \begin{pmatrix} 3 & 3.5 \end{pmatrix} \mathbf{x} + \Big\{ \Big[\begin{pmatrix} -1 & -2 \end{pmatrix} \mathbf{x} + 240 \Big] \\ &+ \Big[\begin{pmatrix} -1 & -0.5 \end{pmatrix} \mathbf{x} + 90 \Big] + \Big[\begin{pmatrix} 1.5 & 2 \end{pmatrix} \mathbf{x} - 310 \Big] \\ &+ \Big[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \Big] + \Big[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \Big] \Big\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.12}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \tag{2.0.13}$$

- Now, we have

$$\nabla L(\mathbf{x},\lambda) = \begin{pmatrix} 3+\begin{pmatrix}-1 & -1 & 1.5 & -1 & 0\end{pmatrix}\lambda \\ 3.5+\begin{pmatrix}-2 & -0.5 & 2 & 0 & -1\end{pmatrix}\lambda \\ \begin{pmatrix}-1 & -2\end{pmatrix}\mathbf{x}+240 \\ \begin{pmatrix}-1 & -0.5\end{pmatrix}\mathbf{x}+90 \\ \begin{pmatrix}1.5 & 2\end{pmatrix}\mathbf{x}-310 \\ \begin{pmatrix}-1 & 0\end{pmatrix}\mathbf{x} \\ \begin{pmatrix}0 & -1\end{pmatrix}\mathbf{x} \end{pmatrix} \tag{2.0.14}$$

$\therefore$ The Lagrangian matrix is given by:-

$$\begin{pmatrix} 0 & 0 & -1 & -1 & 1.5 & -1 & 0 \\ 0 & 0 & -2 & -0.5 & 2 & 0 & -1 \\ -1 & -2 & 0 & 0 & 0 & 0 & 0 \\ -1 & -0.5 & 0 & 0 & 0 & 0 & 0 \\ 1.5 & 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix}\mathbf{x}\\ \lambda\end{pmatrix} = \begin{pmatrix}-3\\ -3.5\\ -240\\ -90\\ 310\\ 0\\ 0\end{pmatrix} \tag{2.0.15}$$

- Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & -2 & -0.5 \\ -1 & -2 & 0 & 0 \\ -1 & -0.5 & 0 & 0 \end{pmatrix}\begin{pmatrix}\mathbf{x}\\ \lambda\end{pmatrix} = \begin{pmatrix}-3\\ -3.5\\ -240\\ -90\end{pmatrix} \tag{2.0.16}$$

$$\implies \begin{pmatrix}\mathbf{x}\\ \lambda\end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & -2 & -0.5 \\ -1 & -2 & 0 & 0 \\ -1 & -0.5 & 0 & 0 \end{pmatrix}^{-1}\begin{pmatrix}-3\\ -3.5\\ -240\\ -90\end{pmatrix} \tag{2.0.17}$$

$$\implies \begin{pmatrix}\mathbf{x}\\ \lambda\end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{4}{3} \\ 0 & 0 & \frac{-2}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{-2}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix}\begin{pmatrix}-3\\ -3.5\\ -240\\ -90\end{pmatrix} \tag{2.0.18}$$

$$\implies \begin{pmatrix}\mathbf{x}\\ \lambda\end{pmatrix} = \begin{pmatrix}40\\ 100\\ \frac{4}{3}\\ \frac{5}{3}\end{pmatrix} \tag{2.0.19}$$

$\because \lambda = \begin{pmatrix}\frac{4}{3}\\ \frac{5}{3}\end{pmatrix} \succ \mathbf{0}$

- The Optimal solution is given by:

$$\mathbf{x} = \begin{pmatrix}40\\ 100\end{pmatrix} \tag{2.0.20}$$

$$Z = \begin{pmatrix}3 & 3.5\end{pmatrix}\mathbf{x} \tag{2.0.21}$$

$$Z = \begin{pmatrix}3 & 3.5\end{pmatrix}\begin{pmatrix}40\\ 100\end{pmatrix} \tag{2.0.22}$$

$$Z = 470 \text{ units} \tag{2.0.23}$$

- So, we get
Bags of brand **P** as $\boxed{x = 40}$ &
Bags of brand **Q** as $\boxed{y = 100}$ so as to minimise the amount of nitrogen added.
- The minimum amount of nitrogen required is $\boxed{Z = 470 \text{ units}}$.

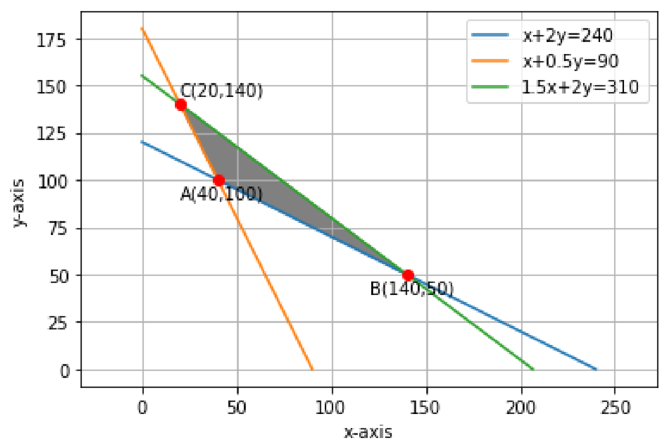

Fig. 2.1: Graphical Solution