

# Constructions using Python

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**Abstract**—This book introduces constructions in high school geometry using Python. The content and exercises are based on NCERT textbooks from Class 6-12. A simple introduction to Python and  $\text{\LaTeX}$  figures is provided in the process.

Download all python codes from

```
svn co https://github.com/gadepall/school/trunk/ncert/constructions/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/ncert/constructions/figs
```

## 1 EXAMPLES

1.1. Draw Fig. 1.1.1 for  $a = 4, c = 3$ .

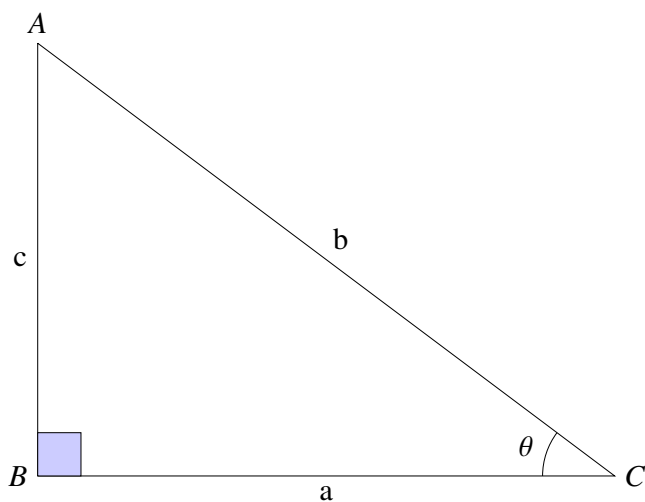


Fig. 1.1.1: Right Angled Triangle

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**Solution:** The vertices of  $\triangle ABC$  are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (1.1.1)$$

The python code for Fig. 1.1.1 is

```
codes/triangle/tri_right_angle.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_right_angle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle/tri_right_angle_alone.tex
```

1.2. Draw Fig. 1.2.1 for  $a = 4, c = 3$ .

**Solution:** The vertex  $\mathbf{A}$  can be expressed in polar coordinate form as

$$\mathbf{A} = b \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (1.2.1)$$

where

$$b = \sqrt{a^2 + c^2} = 5, \tan \theta = \frac{3}{4} \quad (1.2.2)$$

The python code for Fig. 1.2.1 is

```
codes/triangle/tri_polar.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_polar.tex
```

1.3. Draw Fig. 1.3.1 with  $a = 6, b = 5$  and  $c = 4$ .

**Solution:** Let the vertices of  $\triangle ABC$  and  $\mathbf{D}$  be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} p \\ 0 \end{pmatrix} \quad (1.3.1)$$

Then

$$AB = \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0} \quad (1.3.2)$$

$$BC = \|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \quad (1.3.3)$$

$$AC = \|\mathbf{A} - \mathbf{C}\|^2 = b^2 \quad (1.3.4)$$

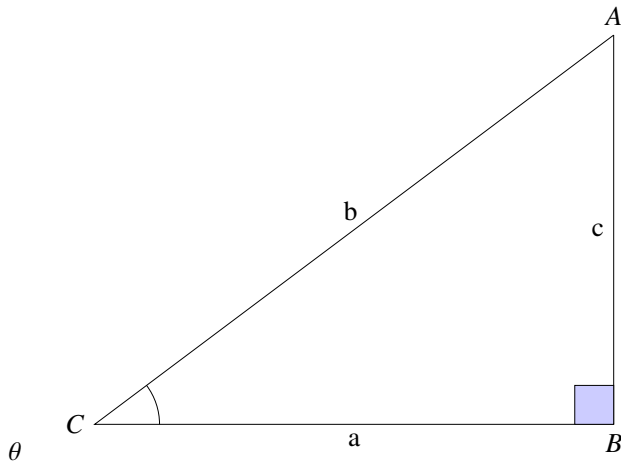


Fig. 1.2.1: Right Angled Triangle

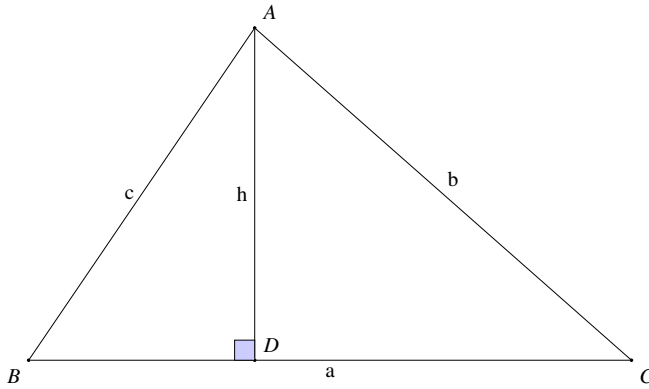


Fig. 1.3.1

From (1.3.4),

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \quad (1.3.5)$$

$$= \mathbf{A}^T \mathbf{A} + \mathbf{C}^T \mathbf{C} - \mathbf{A}^T \mathbf{C} - \mathbf{C}^T \mathbf{A} \quad (1.3.6)$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (\because \mathbf{A}^T \mathbf{C} = \mathbf{C}^T \mathbf{A}) \quad (1.3.7)$$

$$= a^2 + c^2 - 2ap \quad (1.3.8)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \quad (1.3.9)$$

From (1.3.2),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \quad (1.3.10)$$

$$\Rightarrow q = \pm \sqrt{c^2 - p^2} \quad (1.3.11)$$

The python code for Fig. 1.3.1 is

```
codes/triangle/tri_sss.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_sss.tex
```

- 1.4. Construct a triangle of sides  $a = 4$ ,  $b = 5$  and  $c = 6$ .

**Solution:**

The vertex  $\mathbf{A}$  can be expressed in *polar coordinate form* as

$$\mathbf{A} = c \begin{pmatrix} \cos B \\ \sin B \end{pmatrix} \quad (1.4.1)$$

From  $\triangle ABC$ , we use the law of cosines:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \quad (1.4.2)$$

$$= 0.5625 \quad (1.4.3)$$

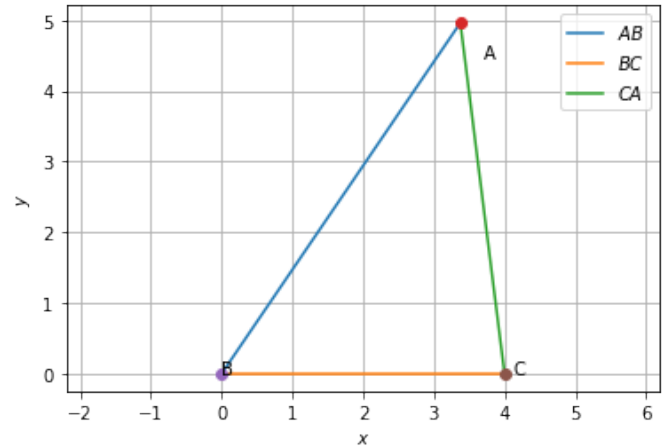
$$\Rightarrow B = 55.771^\circ \quad (1.4.4)$$

Thus,

$$\mathbf{A} = 6 \begin{pmatrix} \cos 55.771 \\ \sin 55.771 \end{pmatrix} \quad (1.4.5)$$

$$\mathbf{A} = \begin{pmatrix} 3.375 \\ 4.960 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}. \quad (1.4.6)$$

which are plotted in Fig. 1.4.1

Fig. 1.4.1:  $\triangle ABC$ 

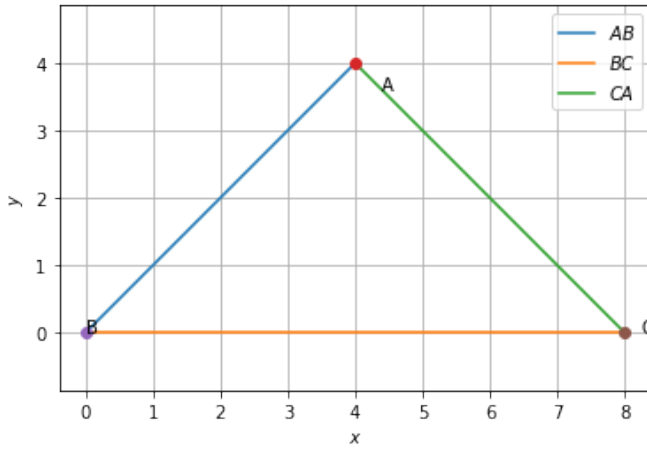
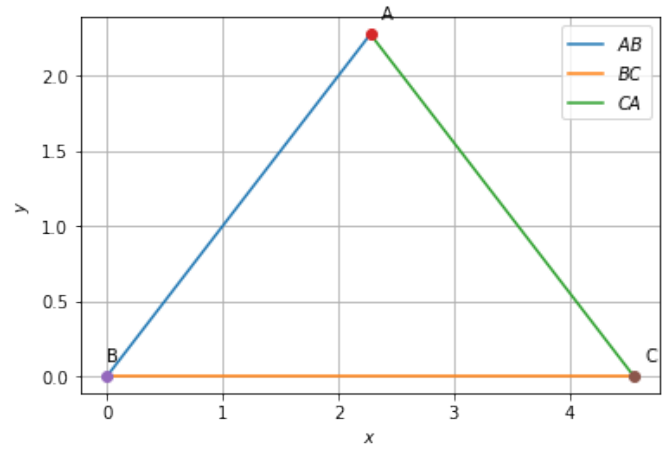
- 1.5. Construct an isosceles triangle whose base is  $a = 8\text{cm}$  and altitude  $AD = h = 4\text{cm}$

**Solution:** From the given information,

$$\mathbf{A} = \begin{pmatrix} a/2 \\ h \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \quad (1.5.1)$$

which are used to plot the triangle in Fig. 1.5.1

- 1.6. In  $\triangle ABC$ , given that  $a+b+c = 11$ ,  $\angle B = 45^\circ$  and  $\angle C = 45^\circ$ , find  $a, b, c$  and sketch the triangle.

Fig. 1.5.1: isosceles triangle  $\triangle ABC$ Fig. 1.6.1:  $\triangle ABC$ 

**Solution:** Use sine formula,

$$b \sin 45 = c \sin 45 \quad (1.6.1)$$

$$\Rightarrow b = c \quad (1.6.2)$$

$$a \sin 45 = b \sin 90 \quad (1.6.3)$$

$$\Rightarrow a = \sqrt{2}b \quad (1.6.4)$$

which can be expressed as the matrix equation

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & -\sqrt{2} & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (1.6.5)$$

solving which yields

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 3.22 \end{pmatrix} \quad (1.6.6)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1.6.7)$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4.55 \\ 0 \end{pmatrix} \quad (1.6.8)$$

resulting in  $\triangle ABC$  plotted in Fig. 1.6.1.

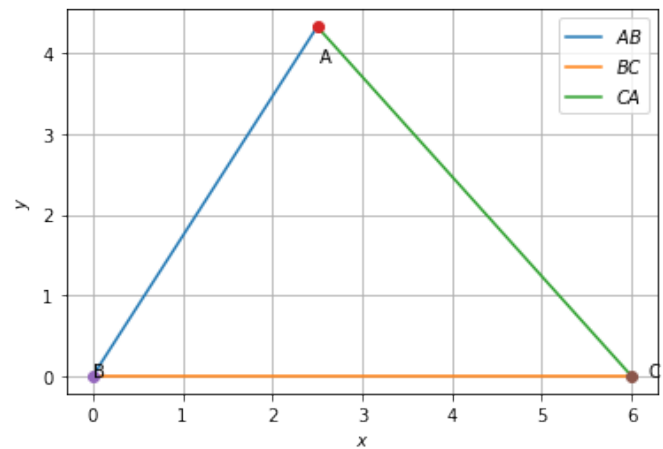
1.7. Draw  $\triangle ABC$  with  $a = 6$ ,  $c = 5$  and  $\angle B = 60^\circ$ .

**Solution:** The vertex  $\mathbf{A}$  can be expressed in polar coordinate form as

$$\mathbf{A} = c \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad (1.7.1)$$

$$\Rightarrow \mathbf{A} = 5 \begin{pmatrix} \cos 60 \\ \sin 60 \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2.5\sqrt{3} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (1.7.2)$$

upon substituting the given values. The triangle is plotted in Fig. 1.7.1.

Fig. 1.7.1:  $\triangle ABC$ 

1.8. Draw  $\triangle ABC$  with  $a = 7$ ,  $\angle B = 45^\circ$  and  $\angle A = 105^\circ$ .

**Solution:** Let

$$\mathbf{A} = c \begin{pmatrix} \cos B \\ \sin B \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (1.8.1)$$

$$\therefore \angle C = 30^\circ, \quad (1.8.2)$$

By law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (1.8.3)$$

$$\Rightarrow c = \frac{7 \sin 30^\circ}{\sin 105^\circ} \quad (1.8.4)$$

$$c = 3.62 \quad (1.8.5)$$

and

$$\mathbf{A} = c \begin{pmatrix} \cos B \\ \sin B \end{pmatrix} \quad (1.8.6)$$

$$= \begin{pmatrix} 2.55 \\ 2.55 \end{pmatrix} \quad (1.8.7)$$

Thus, the vertices of given  $\triangle ABC$  are

$$\mathbf{A} = \begin{pmatrix} 2.55 \\ 2.55 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (1.8.8)$$

and  $\triangle ABC$  is plotted in Fig. 1.8.1.

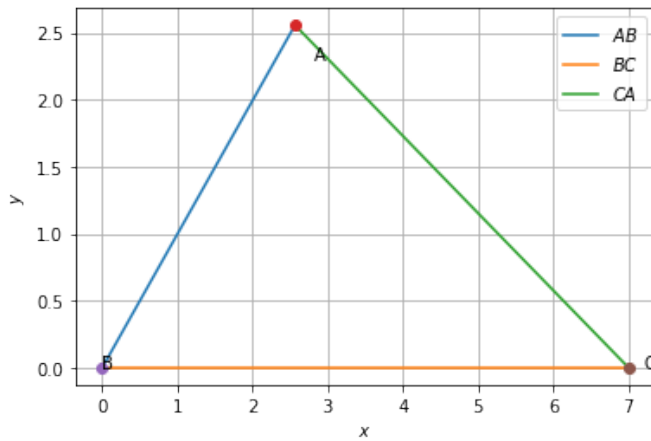


Fig. 1.8.1:  $\triangle ABC$

1.9.  $\triangle ABC$  is right angled at  $\mathbf{B}$ . If  $a = 12$  and  $b+c = 18$ , find  $b, c$  and draw the triangle.

**Solution:** Let,

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (1.9.1)$$

Given,

$$a = 12, b + c = 18 \quad (1.9.2)$$

From  $\triangle ABC$ , using the Baudhayana sutra,

$$b^2 = c^2 + a^2 \quad (1.9.3)$$

$$\Rightarrow b - c = 8 \quad (\because b + c = 18) \quad (1.9.4)$$

Now we have,

$$b + c = 18 \quad (1.9.5)$$

$$b - c = 8 \quad (1.9.6)$$

which can be expressed as the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 18 \\ 8 \end{pmatrix} \quad (1.9.7)$$

Applying row reduction,

$$\begin{pmatrix} 1 & 1 & 18 \\ 1 & -1 & 8 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 18 \\ 0 & -2 & -10 \end{pmatrix} \quad (1.9.8)$$

$$\xrightarrow{R_1 \rightarrow 2R_1 + R_2} \begin{pmatrix} 2 & 0 & 26 \\ 0 & -2 & -10 \end{pmatrix} \xrightarrow{\begin{matrix} R_1 \rightarrow \frac{R_1}{2} \\ R_2 \rightarrow -\frac{R_2}{2} \end{matrix}} \begin{pmatrix} 1 & 0 & 13 \\ 0 & 1 & 5 \end{pmatrix} \quad (1.9.9)$$

Therefore,

$$\begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 13 \\ 5 \end{pmatrix} \quad (1.9.10)$$

Thus,

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \end{pmatrix} \quad (1.9.11)$$

and  $\triangle ABC$  is plotted in Fig. 1.9.1

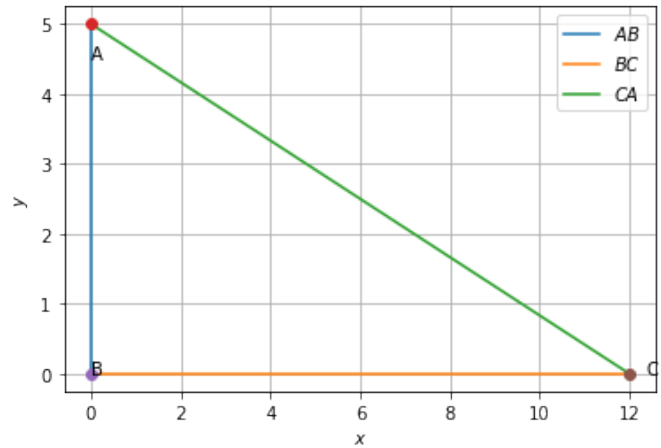


Fig. 1.9.1: Right Angle  $\triangle ABC$

1.10. In  $\triangle ABC$ ,  $a = 8$ ,  $\angle B = 45^\circ$  and  $c - b = 3.5$ . Sketch  $\triangle ABC$ .

**Solution:** Let

$$\mathbf{A} = c \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad (1.10.1)$$

Using the cosine formula in  $\triangle ABC$ ,

$$b^2 = a^2 + c^2 - 2ac \cos B \quad (1.10.2)$$

$$\Rightarrow (c+b)(c-b) + 8^2 - 2 \times 8 \times \left(\frac{1}{\sqrt{2}}\right)c = 0 \quad (1.10.3)$$

$$\Rightarrow (7 - 16\sqrt{2})c + 7b = -128 \quad (1.10.4)$$

upon simplification. From the given information,

$$c - b = \frac{7}{2}, \quad (1.10.5)$$

and the above equations can be expressed as the matrix equation

$$\begin{pmatrix} 7 - 16\sqrt{2} & 7 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} -128 \\ \frac{7}{2} \end{pmatrix} \quad (1.10.6)$$

yielding

$$\begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 11.99 \\ 8.49 \end{pmatrix} \quad (1.10.7)$$

Thus, the vertices of  $\triangle ABC$  are

$$\mathbf{A} = 11.99 \begin{pmatrix} \cos 45^\circ \\ \sin 45^\circ \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}. \quad (1.10.8)$$

which are used to plot Fig. 1.10.1.

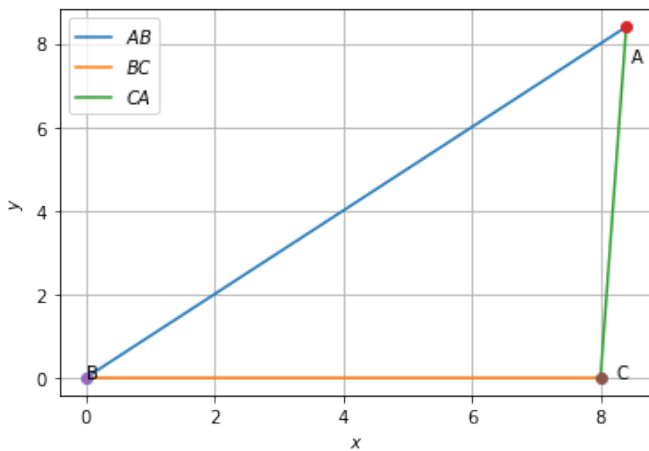


Fig. 1.10.1:  $\triangle ABC$

1.11. In  $\triangle ABC$ ,  $a = 6$ ,  $\angle B = 60^\circ$  and  $b - c = 2$ . Sketch  $\triangle ABC$ .

Let

$$\mathbf{A} = b \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad (1.11.1)$$

Using the cosine formula,

$$b^2 = a^2 + c^2 - 2ac \cos B \quad (1.11.2)$$

$$\Rightarrow (b+c)(b-c) = 6^2 - 2(6)\frac{1}{2}c \quad (\because \angle B = 60^\circ) \quad (1.11.3)$$

$$\Rightarrow (b+c)(2) = 36 - 6c \quad (\because b - c = 2) \quad (1.11.4)$$

$$\text{or, } b + 4c = 18 \quad (1.11.5)$$

From the above, we obtain the matrix equation

$$\begin{pmatrix} 1 & 4 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 18 \\ 2 \end{pmatrix} \quad (1.11.6)$$

By applying row reduction:

$$\begin{pmatrix} 1 & 4 & 18 \\ 1 & -1 & 2 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 1 & 4 & 18 \\ 0 & -5 & -16 \end{pmatrix} \quad (1.11.7)$$

$$\xrightarrow{R_1 \rightarrow 5R_1 + 4R_2} \begin{pmatrix} 5 & 0 & 26 \\ 0 & -5 & -16 \end{pmatrix} \quad (1.11.8)$$

$$\xrightarrow{\substack{R_1 \rightarrow \frac{R_1}{5} \\ R_2 \rightarrow -\frac{R_2}{5}}} \begin{pmatrix} 1 & 0 & \frac{26}{5} \\ 0 & 1 & \frac{16}{5} \end{pmatrix} \quad (1.11.9)$$

$$\therefore \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} \frac{26}{5} \\ \frac{16}{5} \end{pmatrix} \quad (1.11.10)$$

Thus, the vertices of  $\triangle ABC$  are

$$\mathbf{A} = \frac{26}{5} \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} = \begin{pmatrix} 2.6 \\ 4.5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (1.11.11)$$

and the plot of  $\triangle ABC$  is obtained in Fig. 1.11.1

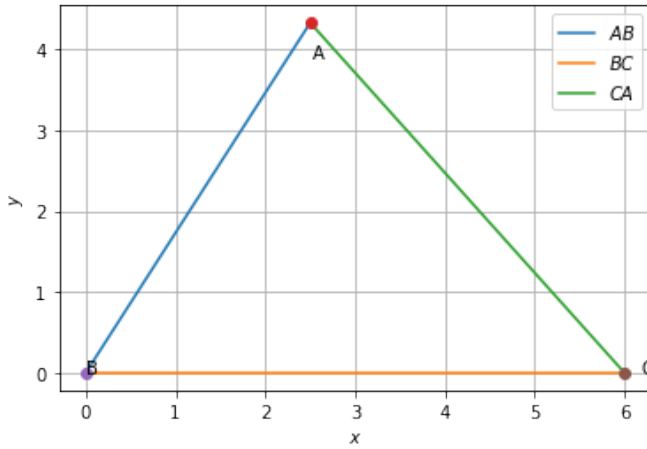
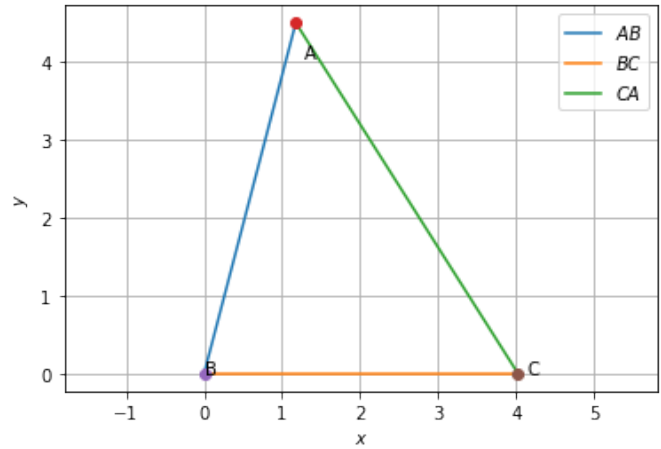
1.12. Draw  $\triangle ABC$ , given that  $a+b+c = 11$ ,  $\angle B = 30^\circ$  and  $\angle C = 90^\circ$ .

**Solution:** Using the sine formula,

$$b \sin C = c \sin B \quad (1.12.1)$$

$$\Rightarrow b \sin 90^\circ = c \sin 30^\circ \quad (1.12.2)$$

$$\text{or, } c = 2b \quad (1.12.3)$$

Fig. 1.11.1:  $\triangle ABC$ Fig. 1.12.1:  $\triangle ABC$ 

Similarly,

$$a \sin B = b \sin A \quad (1.12.4)$$

$$\Rightarrow a = \sqrt{3}b \quad (1.12.5)$$

Formulating the above as a matrix equation

$$\begin{pmatrix} 0 & -2 & 1 \\ 1 & -\sqrt{3} & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (1.12.6)$$

Solving the above,

$$a = 4.026, b = 2.32, c = 4.64 \quad (1.12.7)$$

which are used to obtain the vertices of  $\triangle ABC$  using Problem 1.3.

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 4.64 \end{pmatrix} \quad (1.12.8)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1.12.9)$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4.02 \\ 0 \end{pmatrix} \quad (1.12.10)$$

The desired triangle is plotted in Fig. 1.12.1.

1.13. Construct  $\triangle xyz$  where  $xy = 4.5$ ,  $yz = 5$  and  $zx = 6$ .

**Solution:** Let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} c \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix} \quad (1.13.1)$$

The vertex C can be expressed in polar coordinate form as

$$\mathbf{C} = b \begin{pmatrix} \cos A \\ \sin A \end{pmatrix} \quad (1.13.2)$$

Using the cosine formula,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad (1.13.3)$$

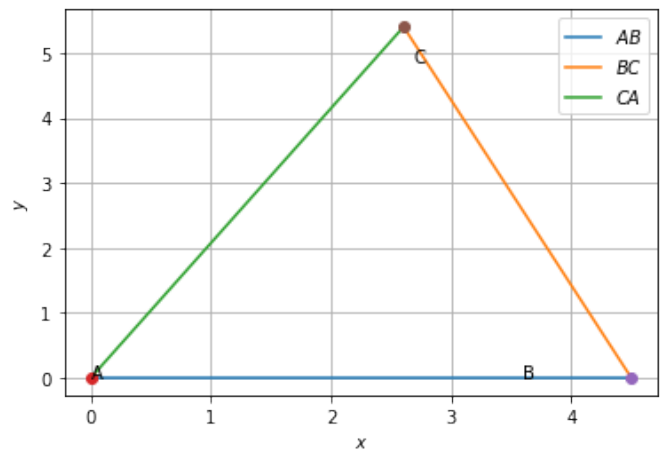
$$\Rightarrow A = 54.640^\circ \quad (1.13.4)$$

Hence,

$$\mathbf{C} = 6 \begin{pmatrix} \cos 54.640^\circ \\ \sin 54.640^\circ \end{pmatrix} = \mathbf{C} = \begin{pmatrix} 3.472 \\ 3.990 \end{pmatrix}, \quad (1.13.5)$$

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} c \\ 0 \end{pmatrix} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix} \quad (1.13.6)$$

which are plotted in Fig. 1.13.1

Fig. 1.13.1:  $\triangle ABC$ 

1.14. Draw an equilateral triangle of side 5.5.

**Solution:**

Let,

$$\mathbf{A} = a \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (1.14.1)$$

$$= 5.5 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix} \quad (1.14.2)$$

after substituting  $\theta = 60^\circ$  and  $a = 5.5$ . The triangle is then plotted in Fig. 1.14.1

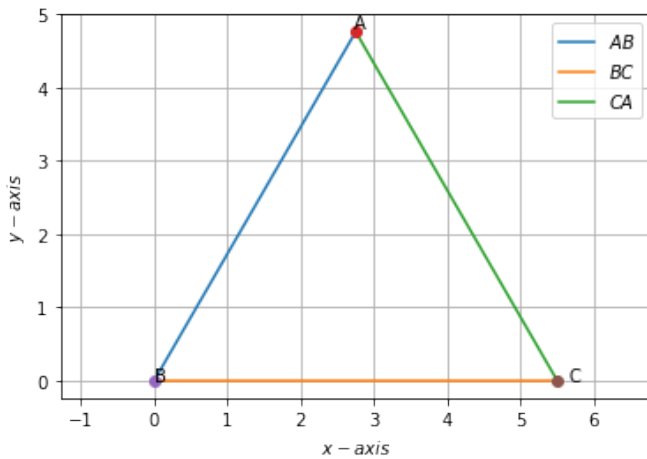


Fig. 1.14.1:  $\triangle ABC$

- 1.15. Draw  $\triangle PQR$  with  $PQ = 4$ ,  $QR = 3.5$  and  $PR = 4$ . What type of triangle is this?

**Solution:** Let

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{R} = PR \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (1.15.1)$$

where,

$$PR \left( \frac{\sin \theta}{2} \right) = \frac{QR}{2} \quad (1.15.2)$$

$$\Rightarrow \theta = 2 \sin^{-1} \left( \frac{QR}{2PR} \right) \quad (1.15.3)$$

$$= 51.88 \quad (1.15.4)$$

Thus, the vertices of  $\triangle PQR$  are

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 2.47 \\ 3.15 \end{pmatrix} \quad (1.15.5)$$

which are used to plot  $\triangle PQR$  in Fig. 1.15.1.

- 1.16. Construct  $\triangle ABC$  such that  $AB = 2.5$ ,  $BC = 6$  and  $AC = 6.5$ . Find  $\angle B$ .

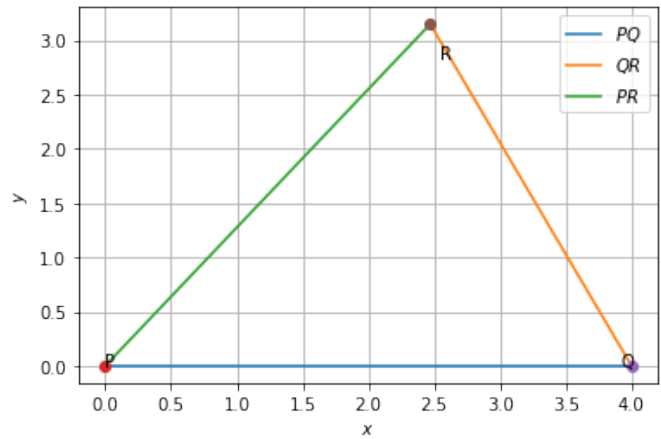


Fig. 1.15.1: isosceles  $\triangle PQR$

**Solution:** From the given information,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \quad (1.16.1)$$

$$\Rightarrow \cos B = 0 \quad (1.16.2)$$

$$\text{or, } \angle B = 90^\circ \quad (1.16.3)$$

Thus, the vertices of  $\triangle ABC$  are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 2.5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (1.16.4)$$

and plotted in Fig. 1.16.1.

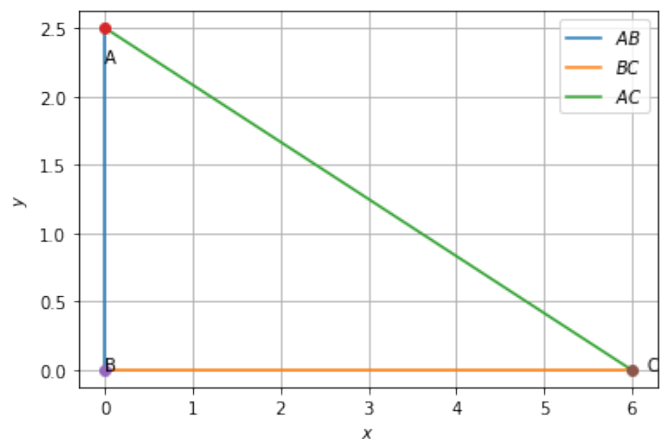


Fig. 1.16.1:  $\triangle ABC$

- 1.17. Construct  $\triangle DEF$  such that  $DE = 5$ ,  $DF = 3$  and  $\angle D = 90^\circ$ .

**Solution:** From the given information, the vertices of  $\triangle DEF$  are

$$\mathbf{E} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (1.17.1)$$

which are used to plot Fig. 1.17.1.

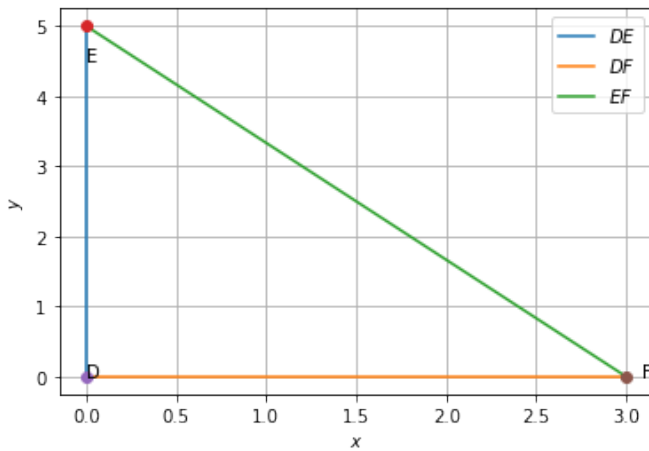


Fig. 1.17.1

- 1.18. Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is  $110^\circ$ .

**Solution:** Let the vertices be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \mathbf{B} = c \begin{pmatrix} \cos A \\ \sin A \end{pmatrix} \quad (1.18.1)$$

Then, the vertices of isosceles  $\triangle ABC$  are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6.5 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2.22313 \\ 6.10798 \end{pmatrix} \quad (1.18.2)$$

which are plotted in Fig. 1.18.1.

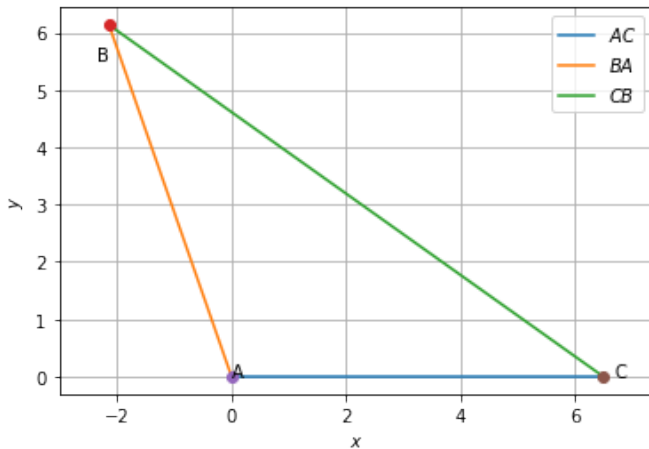


Fig. 1.18.1: Isosceles  $\triangle ABC$

- 1.19. Construct  $\triangle ABC$  given that  $\angle A = 60^\circ$ ,  $\angle B = 30^\circ$  and  $AB = 5.8$ .

**Solution:** From the given information,

$$\angle C = 90^\circ \quad (1.19.1)$$

Hence,

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \sin B \end{pmatrix} \quad (1.19.2)$$

$$= \begin{pmatrix} 0 \\ 2.9 \end{pmatrix} \quad (1.19.3)$$

$$\mathbf{B} = \begin{pmatrix} c \cos B \\ 0 \end{pmatrix} \quad (1.19.4)$$

$$= \begin{pmatrix} 5.02294 \\ 0 \end{pmatrix} \quad (1.19.5)$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1.19.6)$$

which are used to draw  $\triangle ABC$  in Fig. 1.19.1.

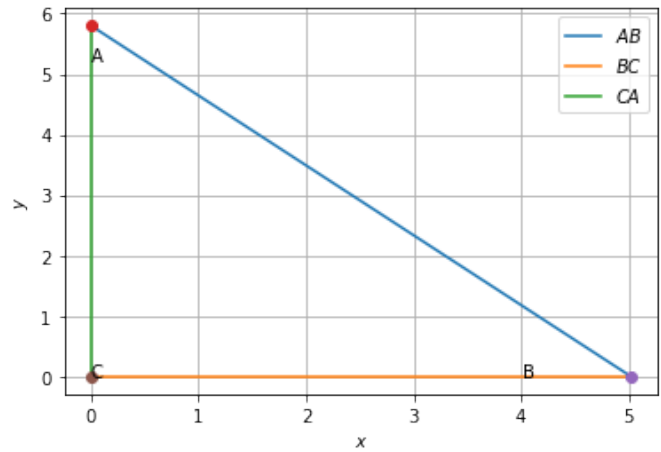


Fig. 1.19.1:  $\triangle ABC$

- 1.20. Construct  $\triangle LMN$  right angled at  $M$  such that  $LN = 5$  and  $MN = 3$ .

**Solution:**

Let

$$\mathbf{L} = \begin{pmatrix} 0 \\ l \end{pmatrix}, \mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{N} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (1.20.1)$$

From the given information,

$$\|\mathbf{N} - \mathbf{M}\|^2 = \|\mathbf{N}\|^2 = 3^2 = 9 \quad (1.20.2)$$

$$\|\mathbf{L} - \mathbf{M}\|^2 = \|\mathbf{L}\|^2 = l^2 \quad (1.20.3)$$

$$\|\mathbf{L} - \mathbf{N}\|^2 = 5^2 = 25 \quad (1.20.4)$$

which can be expressed as

$$\|\mathbf{L} - \mathbf{N}\|^2 = (\mathbf{L} - \mathbf{N})^T (\mathbf{L} - \mathbf{N}) \quad (1.20.5)$$

$$= \|\mathbf{L}\|^2 + \|\mathbf{N}\|^2 - 2\mathbf{L}^T \mathbf{N} \quad (1.20.6)$$

$$\Rightarrow l^2 + 9 = 25 \quad (1.20.7)$$

$$\text{or, } l = \pm 4 \quad (1.20.8)$$



For  $l=4$ ,  $\triangle LMN$  is plotted in the first quadrant in Fig. 1.20.1.

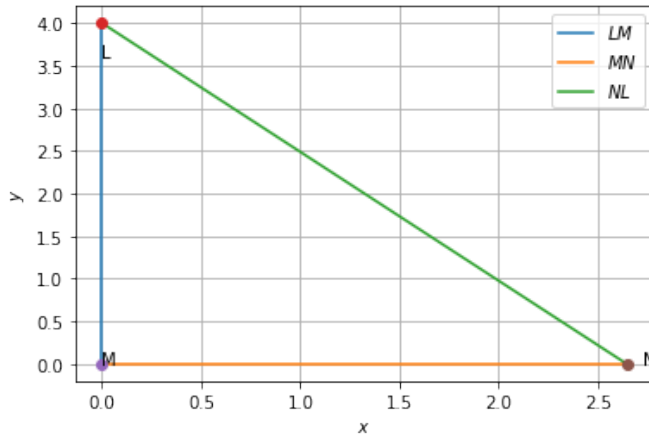


Fig. 1.20.1

- 1.21. Construct  $\triangle PQR$  right angled at  $Q$  such that  $QR = 8$  and  $PR = 10$ .

**Solution:** Let

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ p \end{pmatrix} \quad (1.21.1)$$

Then,

$$\|\mathbf{P} - \mathbf{R}\|^2 = (\mathbf{P} - \mathbf{R})^T (\mathbf{P} - \mathbf{R}) \quad (1.21.2)$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{R}\|^2 \quad (1.21.3)$$

$$\therefore \mathbf{P}^T \mathbf{R} = \mathbf{R}^T \mathbf{P}, \mathbf{R}^T \mathbf{P} = 0 \quad (1.21.4)$$

$$= p^2 + 64 = 10^2 \quad (1.21.5)$$

$$\Rightarrow p = \pm 6 \quad (1.21.6)$$

Since positive area is considered here, only  $p = 6$  is taken into consideration. Thus,

$$\mathbf{P} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (1.21.7)$$

and the desired triangle is plotted in Fig. 1.21.1

- 1.22. Construct right angled  $\triangle$  whose hypotenuse is 6 and one of the legs is 4.

**Solution:** Let us consider  $\triangle PQR$  right angled at  $Q$  and assume that we are restricted to first quadrant such that

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ p \end{pmatrix} \quad (1.22.1)$$

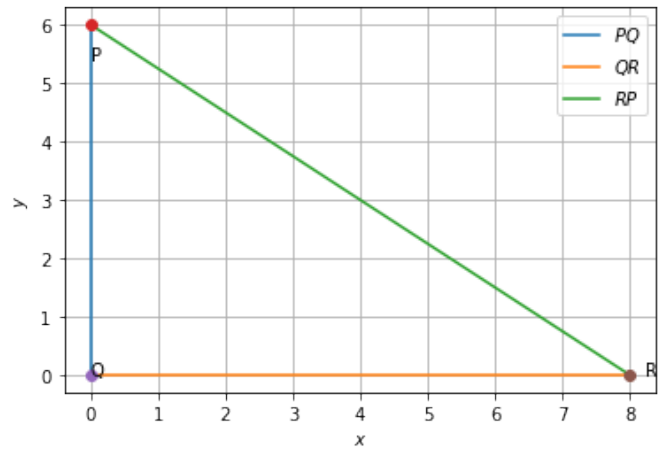


Fig. 1.21.1: Right Angle  $\triangle PQR$

Then,

$$\|\mathbf{P} - \mathbf{R}\|^2 = 36 \quad (1.22.2)$$

$$\Rightarrow p^2 + 16 = 36 \quad (1.22.3)$$

$$\Rightarrow p = \pm 2\sqrt{5} \quad (1.22.4)$$

Since first quadrant was assumed here, only  $p = +2\sqrt{5}$  is taken into consideration. So, the vertices of  $\triangle PQR$  in Fig. 1.22.1 are

$$\mathbf{P} = \begin{pmatrix} 0 \\ 2\sqrt{5} \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (1.22.5)$$

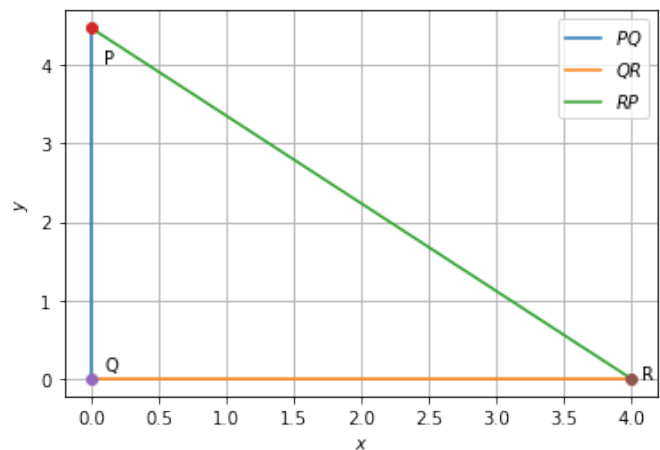


Fig. 1.22.1: Right Angled  $\triangle PQR$

- 1.23. Construct an isosceles right angled  $\triangle ABC$  right angled at  $C$  such that  $AC = 6$ .

**Solution:**

$\therefore \triangle ABC$  is isosceles, its vertices are

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (1.23.1)$$

which are used to plot the desired triangle in Fig. 1.23.1.

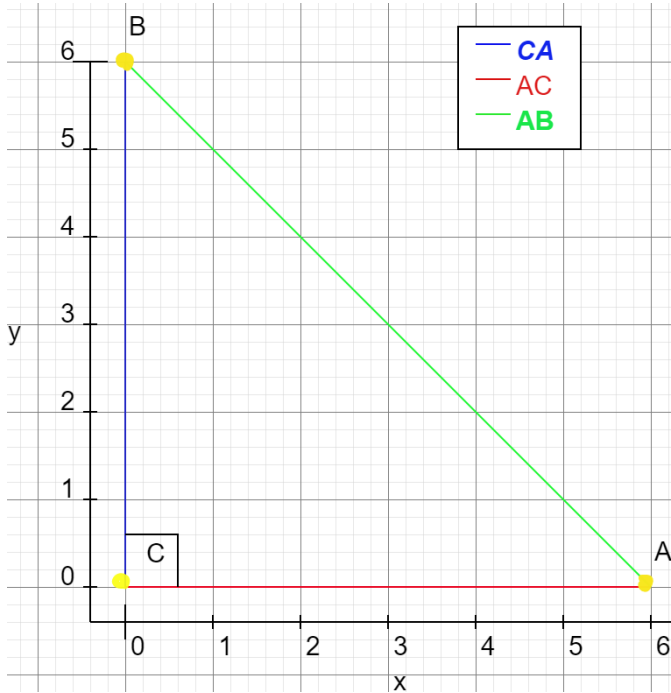


Fig. 1.23.1: Isosceles Right Angle  $\triangle ABC$

1.24. Construct parallelogram  $ABCD$  in Fig. 1.24.1 given that  $BC = 5, AB = 6, \angle C = 85^\circ$ .

**Solution:**  $BD$  is found using the cosine formula and  $\triangle BDC$  is drawn using the approach in Construction 1.3 with

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \quad (1.24.1)$$

Since the diagonals bisect each other,

$$\mathbf{O} = \frac{\mathbf{B} + \mathbf{D}}{2} \quad (1.24.2)$$

$$\mathbf{A} = 2\mathbf{O} - \mathbf{C}. \quad (1.24.3)$$

$AB$  and  $AD$  are then joined to complete the ||gm. The python code for Fig. 1.24.1 is

```
codes/quad/pgm_sas.py
```

and The equivalent latex-tikz code is

```
figs/quad/pgm_sas.tex
```

1.25. Draw the ||gm  $ABCD$  in Fig. 1.25.1 with  $BC = 6, CD = 4.5$  and  $BD = 7.5$ . Show that it is a

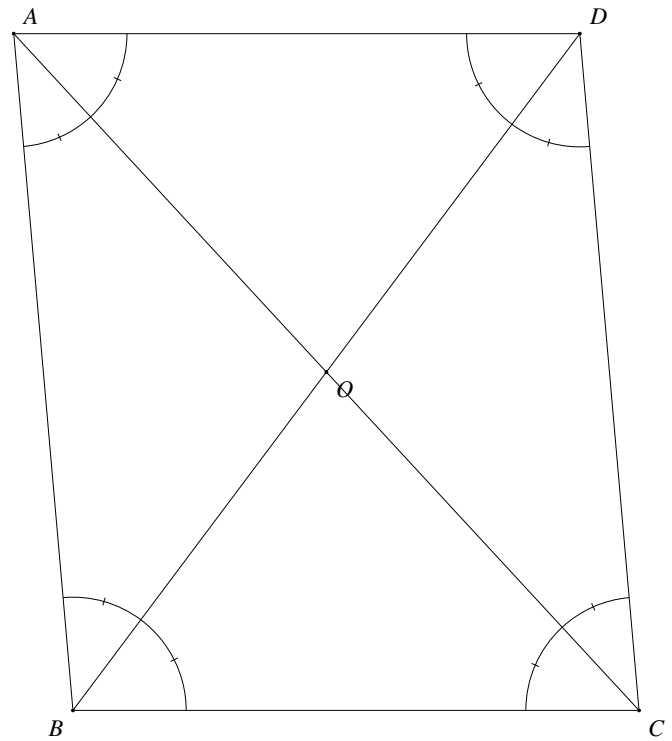


Fig. 1.24.1: Parallelogram Properties

rectangle.

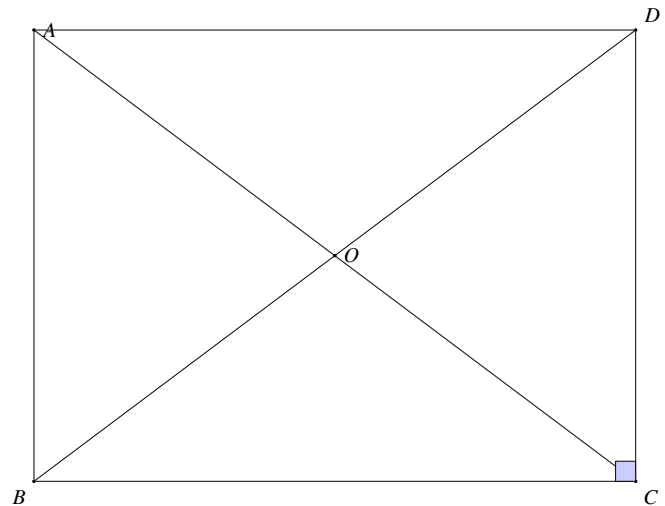


Fig. 1.25.1: Rectangle

**Solution:** It is easy to verify that

$$BD^2 = BC^2 + CD^2 \quad (1.25.1)$$

Hence, using Baudhayana theorem,

$$\angle BCD = 90^\circ \quad (1.25.2)$$

and  $ABCD$  is a rectangle.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \quad (1.25.3)$$

The python code for Fig. 1.25.1 is

```
codes/quad/pgm_sss.py
```

and the equivalent latex-tikz code is

```
figs/quad/pgm_sss.tex
```

- 1.26. Draw the rhombus  $BEST$  with  $BE = 4.5$  and  $ET = 6$ .

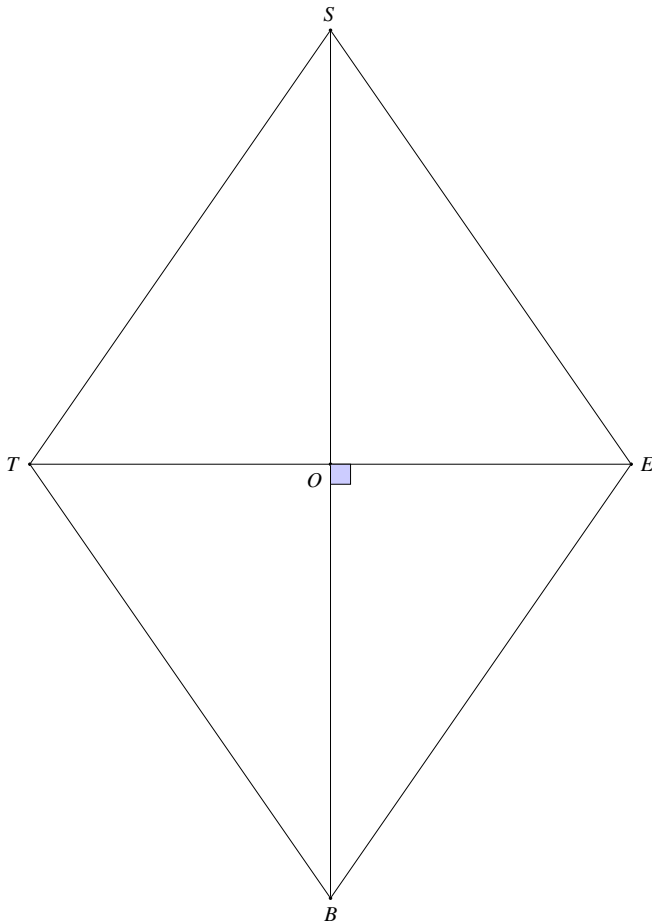


Fig. 1.26.1: Rhombus

**Solution:** The coordinates of the various points in Fig. 1.26.1 are obtained as

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -4.5 \end{pmatrix} \quad (1.26.1)$$

$$\mathbf{E} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \quad (1.26.2)$$

- 1.27. A square is a rectangle whose sides are equal. 1.29. Let  $\mathbf{A}$  and  $\mathbf{B}$  be the centres of two circles of equal radii 3 such that each one of them

**Solution:** The coordinates of the various points in Fig. 1.27.1 are obtained as

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \quad (1.27.1)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 4.5 \\ 4.5 \end{pmatrix} \mathbf{O} = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (1.27.2)$$

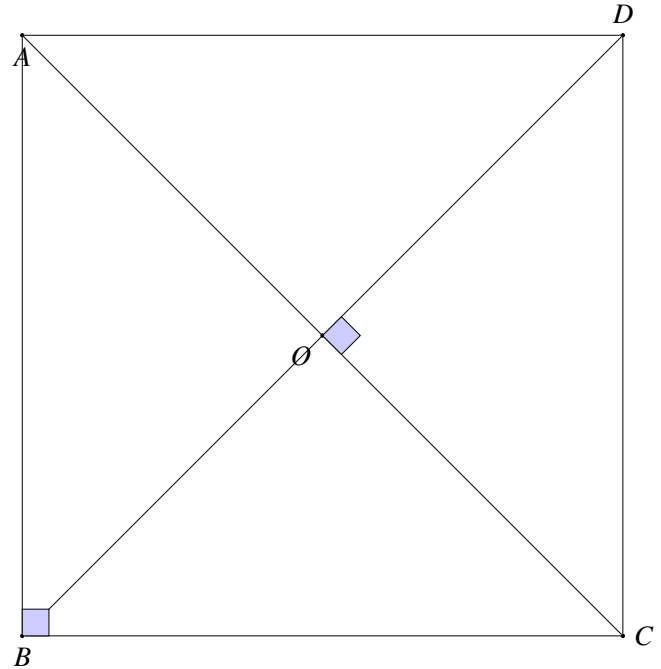


Fig. 1.27.1: Square

- 1.28. With the same centre  $\mathbf{O}$ , draw two circles of radii 4 and 2.5

**Solution:**

All input values required to plot Fig. 1.28.1 are given in Table 1.28.1 as shown below

	Symbols	Circle1	Circle2
Centre	$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	$r_1, r_2$	2.5	4
Polar coordinate	$\mathbf{C}_1, \mathbf{C}_2$	$2.5 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$	$4 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$
Angle	$\theta$	$0-2\pi$	$0-2\pi$

TABLE 1.28.1: Input values

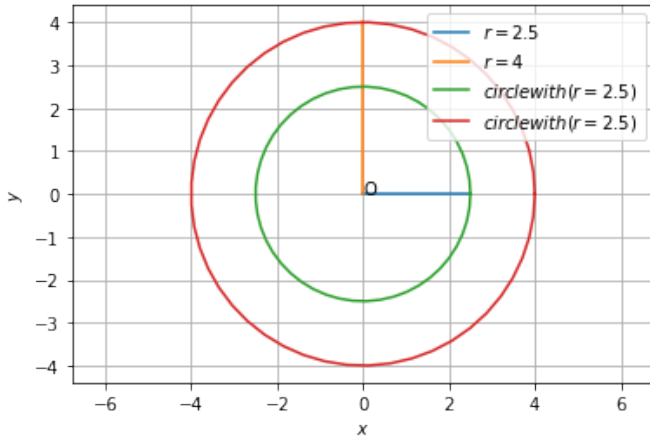


Fig. 1.28.1: Concentric circles with centre as origin and radii 2.5 and 4 respectively

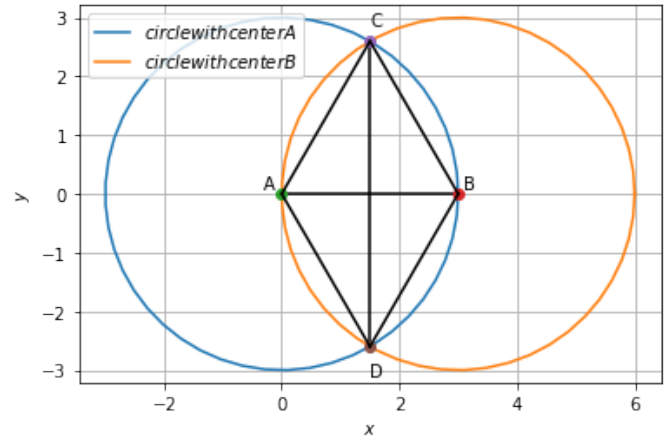


Fig. 1.29.1: Circles with their points of intersection

passes through the centre of the other. Let them intersect at **C** and **D**. Is  $AB \perp CD$ ?

**Solution:** The centers and radii of the two circles without any loss of generality are given in Table 1.29.1

	Circle 1	Circle 2
Centre	$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\mathbf{B} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$
Radius	$r_1 = r_2 = 3$	

TABLE 1.29.1: Input values

Let

$$\mathbf{u} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \theta \in [0, 2\pi]. \quad (1.29.1)$$

Then on Circle 1 and Circle 2 are given by

$$\mathbf{x} = \mathbf{A} + r\mathbf{u} \quad (1.29.2)$$

$$\mathbf{x} = \mathbf{B} + r\mathbf{u} \quad (1.29.3)$$

Fig. 1.29.1 is plotted using the above equations. Fig. 1.29.1

The general equation of Circle 1 is given by

$$\|\mathbf{x} - \mathbf{A}\|^2 = r^2 \quad (1.29.4)$$

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{A}^T \mathbf{x} + \|\mathbf{A}\|^2 - r^2 = 0 \quad (1.29.5)$$

Similarly, for Circle 2,

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{B}^T \mathbf{x} + \|\mathbf{B}\|^2 - r^2 = 0 \quad (1.29.6)$$

Subtracting (1.29.6) from (1.29.5),

$$2\mathbf{B}^T \mathbf{x} = \|\mathbf{B}\|^2 \quad (1.29.7)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = \frac{3}{2} \quad (1.29.8)$$

which can be expressed as

$$\mathbf{x} = \frac{1}{2} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (1.29.9)$$

$$= \mathbf{q} + \lambda \mathbf{m} \text{ where} \quad (1.29.10)$$

$$\mathbf{q} = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (1.29.11)$$

Substituting (1.29.10) in (1.29.5)

$$\|\mathbf{x}\|^2 = r^2 \quad (\because \mathbf{A} = 0) \quad (1.29.12)$$

$$\|\mathbf{q} + \lambda \mathbf{m}\|^2 = r^2 \quad (1.29.13)$$

$$(\mathbf{q} + \lambda \mathbf{m})^T (\mathbf{q} + \lambda \mathbf{m}) = r^2 \quad (1.29.14)$$

$$\Rightarrow \mathbf{q}^T (\mathbf{q} + \lambda \mathbf{m}) + \lambda \mathbf{m}^T (\mathbf{q} + \lambda \mathbf{m}) = r^2 \quad (1.29.15)$$

$$\Rightarrow \|\mathbf{q}\|^2 + \lambda \mathbf{q}^T \mathbf{m} + \lambda \mathbf{m}^T \mathbf{q} + \lambda^2 \|\mathbf{m}\|^2 = r^2 \quad (1.29.16)$$

$$\Rightarrow \|\mathbf{q}\|^2 + 2\lambda \mathbf{q}^T \mathbf{m} + \lambda^2 \|\mathbf{m}\|^2 = r^2 \quad (1.29.17)$$

$$\Rightarrow \lambda = \pm \sqrt{\frac{9 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2}} \quad \because \mathbf{q}^T \mathbf{m} = 0 \quad (1.29.18)$$

Substituting the value of  $\lambda$  in (1.29.10),

$$\mathbf{C} = \mathbf{q} + \lambda \mathbf{m} \quad (1.29.19)$$

$$\mathbf{D} = \mathbf{q} - \lambda \mathbf{m} \quad (1.29.20)$$

$$\Rightarrow (\mathbf{A} - \mathbf{B})^T (\mathbf{C} - \mathbf{D}) = 2 \begin{pmatrix} -3 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \sqrt{6.75} \end{pmatrix} \quad (1.29.21)$$

$$= 0 \quad (1.29.22)$$

$$\Rightarrow AB \perp CD \quad (1.29.23)$$

- 1.30. Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

**Solution:** The given information is summarised in Table 1.30.1. See Fig. 1.30.1. Let P be a

	Symbols	Circle1	Circle2
Centre	<b>O</b>	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	$r_1, r_2$	4	6

TABLE 1.30.1

point on Circle 2 with radius 6. Then

$$\mathbf{P} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (1.30.1)$$

Let  $PQ$  and  $PR$  be tangents from point  $P$  on circle with radius 6 to the points  $Q$  and  $R$  on circle with radius 4. Now,

$$(\mathbf{O} - \mathbf{Q})^T (\mathbf{Q} - \mathbf{P}) = 0 \quad (\because OQ \perp QP) \quad (1.30.2)$$

$$\Rightarrow \mathbf{P}^T \mathbf{Q} = 16 \quad (\because \|\mathbf{Q}\|^2 = 16) \quad (1.30.3)$$

$$\text{or, } \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{Q} = \frac{8}{3} \quad (1.30.4)$$

$$\Rightarrow \mathbf{Q} = \begin{pmatrix} \frac{8}{3} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (1.30.5)$$

$$= \mathbf{q} + \lambda \mathbf{m} \quad (1.30.6)$$

$$\text{where } \mathbf{q} = \begin{pmatrix} \frac{8}{3} \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (1.30.7)$$

We know,

$$\|\mathbf{q} + \lambda \mathbf{m}\|^2 = r_1^2 \quad (1.30.8)$$

$$(\mathbf{q} + \lambda \mathbf{m})^T (\mathbf{q} + \lambda \mathbf{m}) = r_1^2 \quad (1.30.9)$$

$$\lambda^2 = \frac{r_1^2 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \quad (1.30.10)$$

$$\lambda = \pm 2.98 \quad (1.30.11)$$

Substituting the above in (1.30.5),

$$\mathbf{Q} = \begin{pmatrix} \frac{8}{3} \\ 2.98 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \frac{8}{3} \\ -2.98 \end{pmatrix} \quad (1.30.12)$$

The circles as well as the tangents are plotted in Fig. 1.30.1

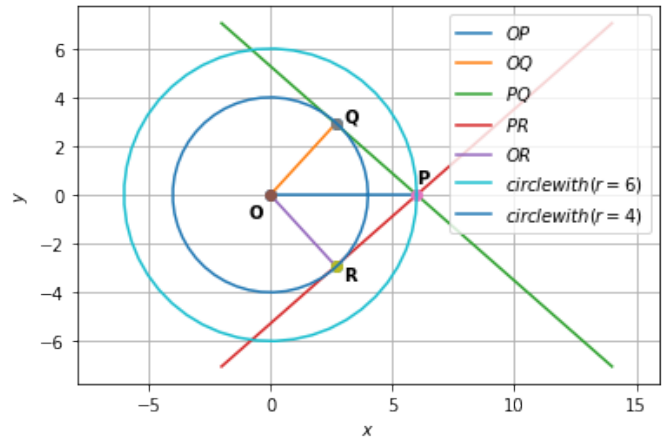


Fig. 1.30.1: Tangent lines to circle of radius 4 units.

- 1.31. Draw a circle with centre  $\mathbf{C}$  and radius 3.4. Draw any chord. Construct the perpendicular bisector of the chord and examine if it passes through  $\mathbf{C}$ .

**Solution:** Data from the given question is available in Table 1.31.1: Let

	Symbols	Circle1
Centre	<b>C</b>	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	$r$	3.4

TABLE 1.31.1: Input values

$$\mathbf{P} = r \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix} = \begin{pmatrix} 1.7 \\ 2.9 \end{pmatrix} \quad (1.31.1)$$

$$\mathbf{Q} = r \begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix} = \begin{pmatrix} -2.4 \\ 2.4 \end{pmatrix} \quad (1.31.2)$$

Then the perpendicular bisector of  $PQ$  passes through

$$\mathbf{M} = \frac{\mathbf{P} + \mathbf{Q}}{2} \quad (1.31.3)$$

and has normal vector

$$\mathbf{n} = \mathbf{P} - \mathbf{Q} \quad (1.31.4)$$

resulting in the equation

$$(\mathbf{P} - \mathbf{Q})^\top \left( \mathbf{x} - \frac{\mathbf{P} + \mathbf{Q}}{2} \right) = 0 \implies (\mathbf{P} - \mathbf{Q})^\top \mathbf{x} = 0 \quad (1.31.5)$$

after simplification. It is obvious that  $\mathbf{O}$  satisfies the above equation as can be verified in Fig. 1.31.1.

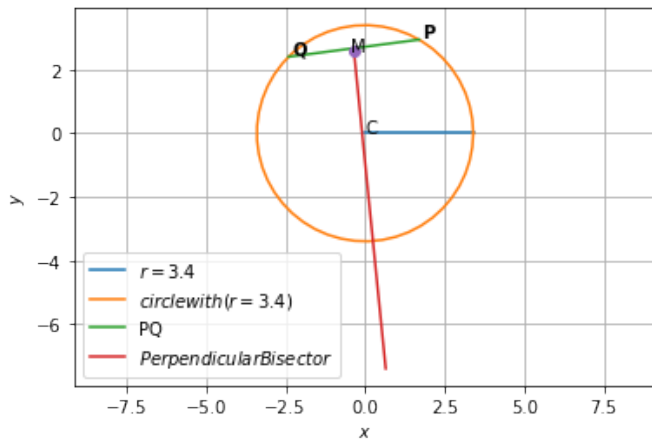


Fig. 1.31.1: perpendicular bisector of the chord passes through the center

- 1.32. Construct a quadrilateral  $ABCD$  such that  $AB = 5$ ,  $\angle A = 50^\circ$ ,  $AC = 4$ ,  $BD = 5$  and  $AD = 6$ .

**Solution:**

The rough figure of the expected quadrilateral  $ABCD$  is given in Fig. 1.32.1

From the given information, in  $\triangle ABD$ ,

$$\cos A = \frac{\|\mathbf{B} - \mathbf{A}\|^2 + \|\mathbf{D} - \mathbf{A}\|^2 - \|\mathbf{D} - \mathbf{B}\|^2}{2 \|\mathbf{B} - \mathbf{A}\| \|\mathbf{D} - \mathbf{A}\|} \quad (1.32.1)$$

$$\implies \angle A = \cos^{-1}(0.6) \approx 53.13^\circ \quad (1.32.2)$$

$$\neq 50^\circ \quad (1.32.3)$$

resulting in a contradiction. Therefore construction of quadrilateral with given measurements is not possible.

- 1.33. Can you construct a quadrilateral  $PQRS$  with  $PQ = 3$ ,  $RS = 3$ ,  $PS = 7.5$ ,  $PR = 8$  and  $SQ =$

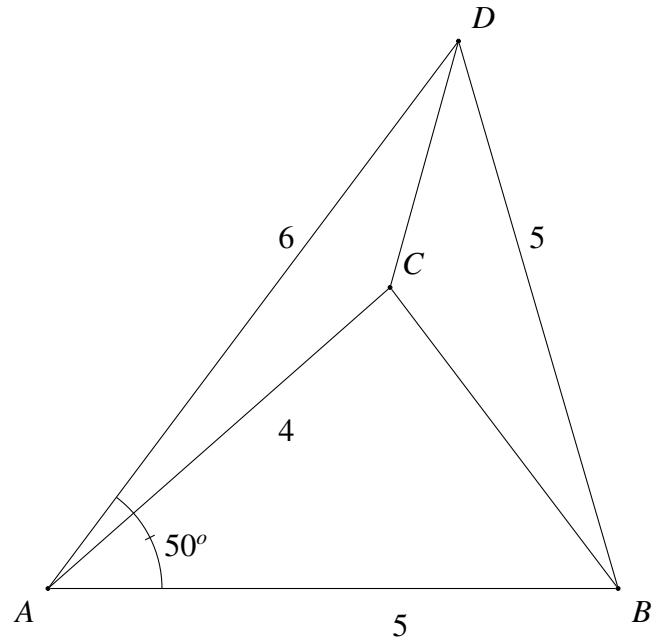


Fig. 1.32.1: Rough Figure

4?

**Solution:** From the given information,

$$\|\mathbf{P} - \mathbf{Q}\| = 3 \quad (1.33.1)$$

$$\|\mathbf{R} - \mathbf{S}\| = 3 \quad (1.33.2)$$

$$\|\mathbf{P} - \mathbf{S}\| = 7.5 \quad (1.33.3)$$

$$\|\mathbf{P} - \mathbf{R}\| = 8 \quad (1.33.4)$$

$$\|\mathbf{S} - \mathbf{Q}\| = 4 \quad (1.33.5)$$

Let quadrilateral  $PQRS$  be made up of two triangles  $\triangle PSQ$  and  $\triangle PSR$  on base  $PS$ .

a) In  $\triangle PSR$ ,

$$\begin{aligned} \|\mathbf{P} - \mathbf{S}\| + \|\mathbf{R} - \mathbf{S}\| &= 7.5 + 3 = 10.5 \\ &> \|\mathbf{P} - \mathbf{R}\| \end{aligned} \quad (1.33.6)$$

$$\|\mathbf{P} - \mathbf{R}\| + \|\mathbf{R} - \mathbf{S}\| = 8 + 3 = 11 > \|\mathbf{P} - \mathbf{S}\| \quad (1.33.7)$$

$$\begin{aligned} \|\mathbf{P} - \mathbf{S}\| + \|\mathbf{P} - \mathbf{R}\| &= 7.5 + 8 = 15.5 \\ &> \|\mathbf{R} - \mathbf{S}\| \end{aligned} \quad (1.33.8)$$

$\therefore$  using triangle inequality, construction of  $\triangle PSR$  is possible.

b) In  $\triangle PSQ$ ,

$$\begin{aligned}\|P - S\| + \|S - Q\| &= 7.5 + 4 = 11.5 \\ &> \|P - Q\| \quad (1.33.9)\end{aligned}$$

$$\begin{aligned}\|P - S\| + \|P - Q\| &= 7.5 + 3 = 10.5 \\ &> \|S - Q\| \quad (1.33.10)\end{aligned}$$

$$\|P - Q\| + \|S - Q\| = 3 + 4 = 7 < \|P - S\| \quad (1.33.11)$$

which violates triangle inequality.  $\therefore$  construction of  $\triangle PSQ$  is not possible.

Fig. 1.33.1 highlights this.

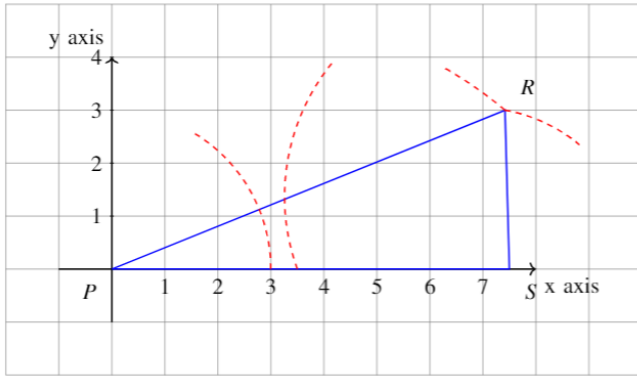


Fig. 1.33.1: Construction of quadrilateral  $PQRS$

1.34. Draw  $GOLD$  such that  $OL = 7.5, GL = 6, GD = 6, LD = 5, OD = 10$ .

**Solution:** In  $\triangle LDO$

$$\|O - L\| + \|O - D\| = 17.5 > \|L - D\| \quad (1.34.1)$$

$$\|O - D\| + \|L - D\| = 15 > \|O - L\| \quad (1.34.2)$$

$$\|O - L\| + \|L - D\| = 12.5 > \|O - D\| \quad (1.34.3)$$

and triangle inequality is satisfied. Similarly, in  $\triangle LDG$

$$\|L - D\| + \|G - L\| = 11 > \|G - D\| \quad (1.34.4)$$

$$\|G - L\| + \|G - D\| = 12 > \|L - D\| \quad (1.34.5)$$

$$\|L - D\| + \|G - D\| = 11 > \|G - L\| \quad (1.34.6)$$

and triangle inequality is satisfied.  $\therefore$  the given sides form a quadrilateral which can be constructed by using the approach in Problem 1.3 to obtain the vertices of  $\triangle LDO$  and  $\triangle LDG$  as

$$L = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, D = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, O = \begin{pmatrix} -1.875 \\ 7.26 \end{pmatrix}, G = \begin{pmatrix} 2.5 \\ 5.5 \end{pmatrix} \quad (1.34.7)$$

and plotting the quadrilateral  $GOLD$  in Fig. 1.34.1

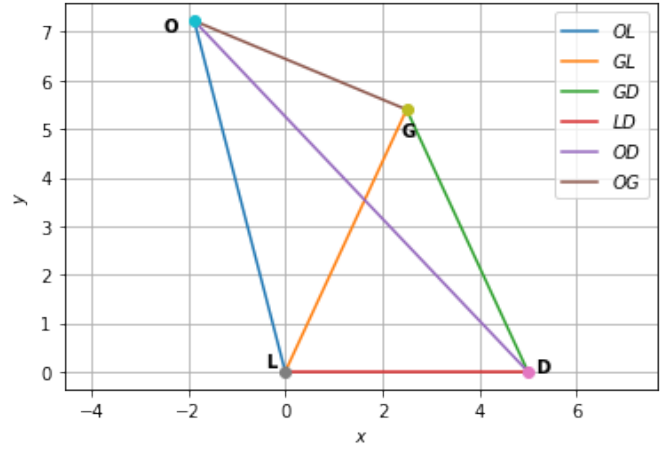


Fig. 1.34.1: Quadrilateral  $GOLD$

1.35. Construct  $ABCD$ , where  $AB = 4, BC = 5, CD = 6.5, \angle B = 105^\circ$  and  $\angle C = 80^\circ$ .

**Solution:**

Let

$$\angle B = 105^\circ = \theta \quad (1.35.1)$$

$$\angle C = 80^\circ = \alpha \quad (1.35.2)$$

$$\|A - B\| = 4 = p \quad (1.35.3)$$

$$\|C - B\| = 5 = q \quad (1.35.4)$$

$$\|D - C\| = 6.5 = r \quad (1.35.5)$$

and

$$B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (1.35.6)$$

**Lemma 1.1.**

$$A = p\mathbf{b} \quad \left( \because B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \quad (1.35.7)$$

$$D = C + r\mathbf{c} \quad (1.35.8)$$

where

$$\mathbf{b} = \begin{pmatrix} \cos B \\ \sin B \end{pmatrix}, \mathbf{c} = \begin{pmatrix} \cos C \\ \sin C \end{pmatrix} \quad (1.35.9)$$

Thus,

$$A = 4 \begin{pmatrix} \cos 105^\circ \\ \sin 105^\circ \end{pmatrix} \quad (1.35.10)$$

$$= \begin{pmatrix} -1.03 \\ 3.86 \end{pmatrix} \quad (1.35.11)$$

and

$$\mathbf{D} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 80 \\ \sin 80 \end{pmatrix} \quad (1.35.12)$$

$$= \begin{pmatrix} 6.12 \\ 6.39 \end{pmatrix} \quad (1.35.13)$$

which are then used to plot Fig. 1.35.1

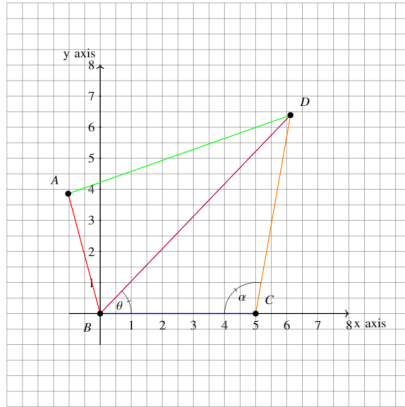


Fig. 1.35.1: Quadrilateral ABCD

- 1.36. Construct  $DEAR$  with  $DE = 4$ ,  $EA = 5$ ,  $AR = 4.5$ ,  $\angle E = 60^\circ$  and  $\angle A = 90^\circ$ .

**Solution:** The given information can be expressed as

$$\angle E = 60^\circ = \theta \quad (1.36.1)$$

$$\angle A = 90^\circ = \alpha \quad (1.36.2)$$

$$\|\mathbf{D} - \mathbf{E}\| = 4 = a \quad (1.36.3)$$

$$\|\mathbf{E} - \mathbf{A}\| = 5 = b \quad (1.36.4)$$

$$\|\mathbf{A} - \mathbf{R}\| = 4.5 = c \quad (1.36.5)$$

Let,

$$\mathbf{E} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (1.36.6)$$

**Lemma 1.2.**

$$\mathbf{D} = a\mathbf{e} \quad \left( \because \mathbf{E} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \quad (1.36.7)$$

$$\mathbf{R} = \mathbf{A} + c\mathbf{a} \quad (1.36.8)$$

where

$$\mathbf{e} = \begin{pmatrix} \cos E \\ \sin E \end{pmatrix}, \mathbf{a} = \begin{pmatrix} \cos A \\ \sin A \end{pmatrix} \quad (1.36.9)$$

Thus, from (1.36.1) and (1.36.3) in (1.36.7),

$$\mathbf{D} = 4 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} \quad (1.36.10)$$

$$= \begin{pmatrix} 2 \\ 3.46 \end{pmatrix} \quad (1.36.11)$$

and from (1.36.2) and (1.36.5) in (1.36.8),

$$\mathbf{R} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + 4.5 \begin{pmatrix} \cos 90^\circ \\ \sin 90^\circ \end{pmatrix} \quad (1.36.12)$$

$$= \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad (1.36.13)$$

Thus

$$\mathbf{D} = \begin{pmatrix} 2 \\ 3.46 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad (1.36.14)$$

and the quadrilateral  $DEAR$  is plotted in Fig. 1.36.1.

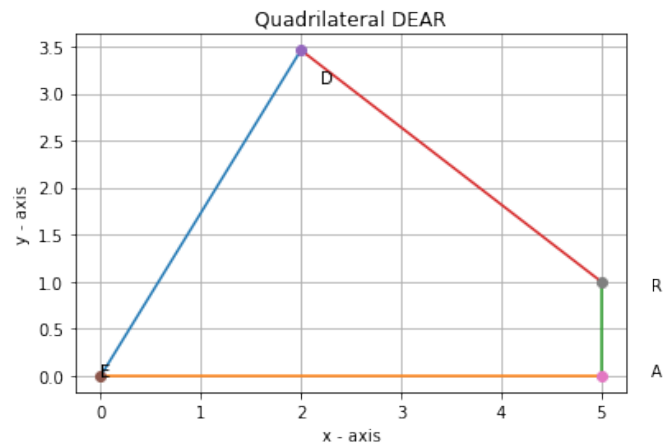


Fig. 1.36.1: Quadrilateral DEAR

- 1.37. Construct  $TRUE$  with  $TR = 3.5$ ,  $RU = 3$ ,  $UE = 4$ ,  $\angle R = 75^\circ$  and  $\angle U = 120^\circ$ .



**Solution:** From the given information,

$$\angle R = 75^\circ = \theta \quad (1.37.1)$$

$$\angle U = 120^\circ = \alpha \quad (1.37.2)$$

$$\|\mathbf{T} - \mathbf{R}\| = 3.5 = a \quad (1.37.3)$$

$$\|\mathbf{U} - \mathbf{R}\| = 3 = b \quad (1.37.4)$$

$$\|\mathbf{E} - \mathbf{U}\| = 4 = c \quad (1.37.5)$$

Let,

$$\mathbf{R} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{U} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (1.37.6)$$

**Lemma 1.3.**

$$\mathbf{T} = C\mathbf{u} \quad \left( \because \mathbf{R} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \quad (1.37.7)$$

$$\mathbf{E} = \mathbf{U} + a\mathbf{r} \quad (1.37.8)$$

where

$$\mathbf{r} = \begin{pmatrix} \cos R \\ \sin R \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \cos U \\ \sin U \end{pmatrix} \quad (1.37.9)$$

Thus,

$$\mathbf{T} = 4 \begin{pmatrix} \cos 120^\circ \\ \sin 120^\circ \end{pmatrix} \quad (1.37.10)$$

$$= \begin{pmatrix} -2 \\ 3.46 \end{pmatrix} \quad (1.37.11)$$

and

$$\mathbf{E} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 3.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (1.37.12)$$

$$= \begin{pmatrix} 3.39 \\ 3.38 \end{pmatrix} \quad (1.37.13)$$

The vertices of given quadrilateral TRUE can be written as,

$$\mathbf{T} = \begin{pmatrix} -2 \\ 3.46 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{U} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 3.39 \\ 3.38 \end{pmatrix} \quad (1.37.14)$$

which are plotted in Fig. 1.37.1.

- 1.38. Can you construct a rhombus  $ABCD$  with  $AC = 6$  and  $BD = 7$ ?

**Solution:** We obtain the vertices of the rhombus as follows

$$\mathbf{A} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -3.5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 3.5 \end{pmatrix} \quad (1.38.1)$$

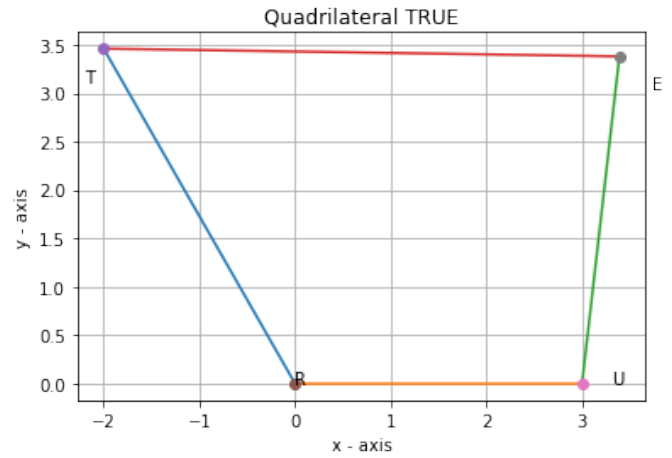


Fig. 1.37.1: Quadrilateral TRUE

which are plotted in Fig. 1.38.1.

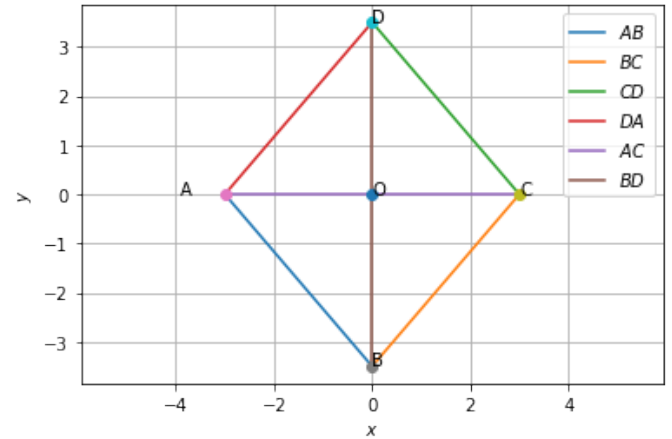


Fig. 1.38.1: Rhombus ABCD

- 1.39. Draw a square  $READ$  with  $RE = 5.1$ .

**Solution:** The vertices are given by

$$\mathbf{R} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 5.1 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 5.1 \\ 5.1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 5.1 \end{pmatrix} \quad (1.39.1)$$

The desired square is plotted in Fig. 1.39.1

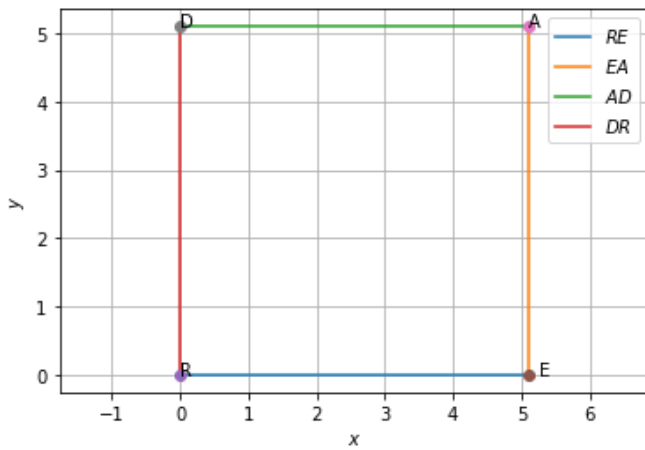
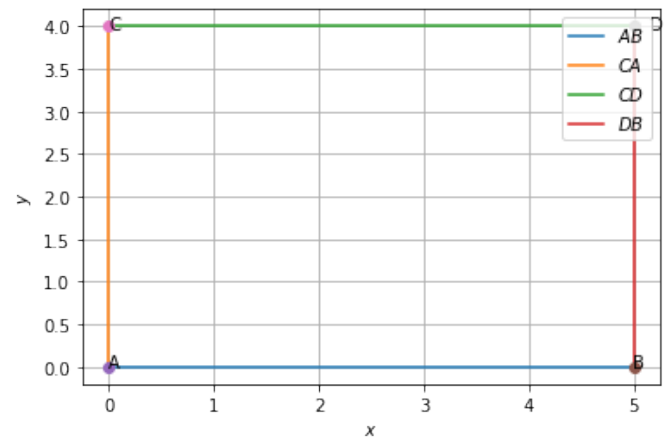
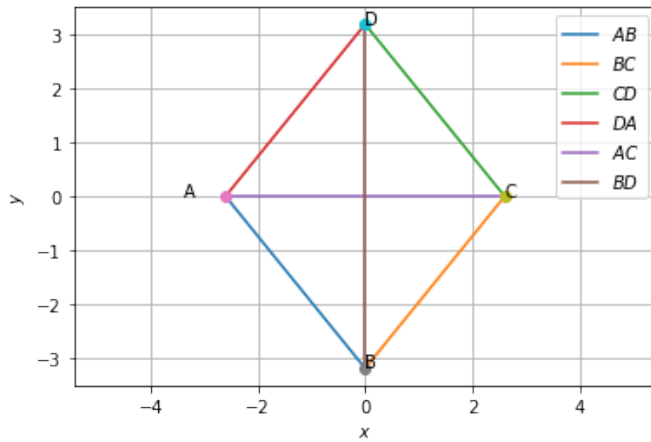
- 1.40. Draw a rhombus whose diagonals are 5.2 and 6.4.

**Solution:** We obtain the vertices of the rhombus as

$$\mathbf{A} = \begin{pmatrix} -2.6 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -3.2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2.6 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 3.2 \end{pmatrix} \quad (1.40.1)$$

which are plotted in Fig. 1.40.1

- 1.41. Draw a rectangle with adjacent sides 5 and 4.

Fig. 1.39.1: Square  $READ$ Fig. 1.41.1: Rectangle  $ABCD$ Fig. 1.40.1: Rhombus  $ABCD$ 

**Solution:** The vertices of rectangle  $ABCD$  are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{D} = \begin{pmatrix} a \\ c \end{pmatrix} \quad (1.41.1)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad (1.41.2)$$

where  $a = 5$  and  $c = 4$ . The rectangle  $ABCD$  is plotted in Fig. 1.41.1

1.42. Draw a parallelogram  $OKAY$  with  $OK = 5.5$  and  $KA = 4.2$ .

**Solution:** There are infinite number of parallelograms that can be drawn. For a unique parallelogram, one angle needs to be specified.

## 2 EXERCISES

2.1. Construct  $\triangle PQR$ , given that  $PQ = 3$ ,  $QR = 5.5$  and  $\angle PQR = 60^\circ$ .

**Solution:**

$$\therefore \mathbf{P} = r \begin{pmatrix} \cos Q \\ \sin Q \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} p \\ 0 \end{pmatrix} \quad (2.1.1)$$

from the given information,

$$\mathbf{P} = 3 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} = \frac{3}{2} \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix} \quad (2.1.2)$$

and plotted in Fig. 2.1.1.

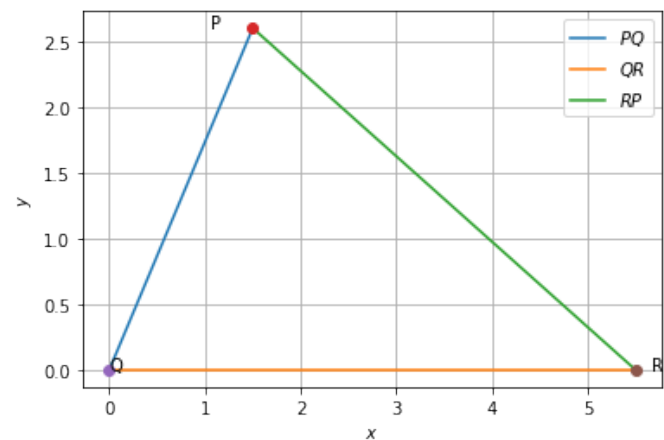


Fig. 2.1.1: The Constructed triangle

2.2. Construct  $\triangle ABC$  with  $BC = 7.5$ ,  $AC = 5$  and  $\angle C = 60^\circ$ .

**Solution:**

$$\therefore \mathbf{A} = b \begin{pmatrix} \cos C \\ \sin C \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (2.2.1)$$

substituting the given values,

$$\mathbf{A} = 5 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2.5\sqrt{3} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.5 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.2.2)$$

which are plotted in Fig. 2.2.1.

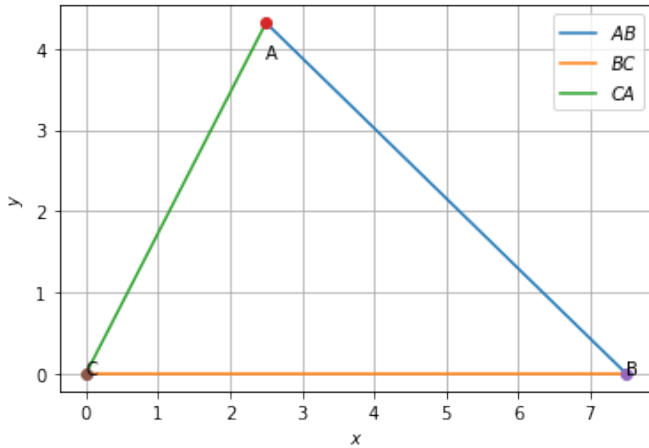


Fig. 2.2.1: The Constructed triangle

- 2.3. Construct  $\triangle XYZ$  if  $XY = 6$ ,  $\angle X = 30^\circ$  and  $\angle Y = 100^\circ$ .

**Solution:** Let

$$XY = z, YZ = x, XZ = y \quad (2.3.1)$$

From the given information,

$$y = z \left( \frac{\sin Y}{\sin Z} \right) = 6 \left( \frac{\sin 100^\circ}{\sin 50^\circ} \right) = 7.7134 \quad (2.3.2)$$

and the vertices are

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{Y} = z \begin{pmatrix} \cos X^\circ \\ \sin X^\circ \end{pmatrix}, \mathbf{Z} = \begin{pmatrix} y \\ 0 \end{pmatrix} \quad (2.3.3)$$

$$(2.3.4)$$

$$\Rightarrow \mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{Y} = 6 \begin{pmatrix} \cos 30^\circ \\ \sin 30^\circ \end{pmatrix} = \begin{pmatrix} 3\sqrt{3} \\ 3 \end{pmatrix}, \mathbf{Z} = \begin{pmatrix} 7.7134 \\ 0 \end{pmatrix} \quad (2.3.5)$$

and plotted in Fig. 2.3.1.

- 2.4. If  $AC = 7$ ,  $\angle A = 60^\circ$  and  $\angle B = 50^\circ$ , can you draw the triangle?

**Solution:** From the given information,

$$\angle C = 70^\circ \quad (2.4.1)$$

and

$$a = b \left( \frac{\sin A}{\sin B} \right) \quad (2.4.2)$$

$$= 7.913611 \quad (2.4.3)$$

Thus, the coordinates are

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = a \begin{pmatrix} \cos 70^\circ \\ \sin 70^\circ \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (2.4.4)$$

and plotted in Fig. 2.4.1.

- 2.5. Construct  $\triangle PQR$  if  $PQ = 5$ ,  $\angle Q = 105^\circ$  and  $\angle R = 40^\circ$ .

- 2.6. Can you construct  $\triangle DEF$  such that  $EF = 7.2$ ,  $\angle E = 110^\circ$  and  $\angle F = 180^\circ$ ?

- 2.7. Construct the triangles in Table 2.7.1. **Solu-**

S.No	Triangle	Given Measurements		
1	$\triangle ABC$	$\angle A = 85^\circ$	$\angle B = 115^\circ$	$AB = 5$
2	$\triangle PQR$	$\angle Q = 30^\circ$	$\angle R = 60^\circ$	$QR = 4.7$
3	$\triangle ABC$	$\angle A = 70^\circ$	$\angle B = 50^\circ$	$AC = 3$
4	$\triangle LMN$	$\angle L = 60^\circ$	$\angle N = 120^\circ$	$LM = 5$
5	$\triangle ABC$	$BC = 2$	$AB = 4$	$AC = 2$
6	$\triangle PQR$	$PQ = 2.5$	$QR = 4$	$PR = 3.5$
7	$\triangle XYZ$	$XY = 3$	$YZ = 4$	$XZ = 5$
8	$\triangle DEF$	$DE = 4.5$	$EF = 5.5$	$DF = 4$

TABLE 2.7.1

**tion:**

a)

- b) **Solution:** From the given information,  $\triangle PQR$  is a right angled triangle. Let  $QR = p$  and  $\theta = 30^\circ$ . Then the vertices of the triangle

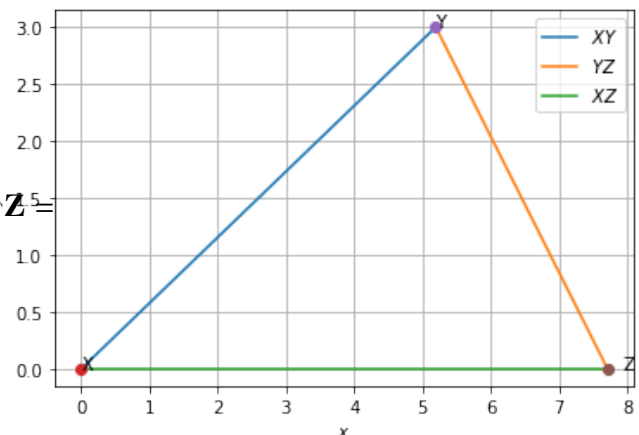
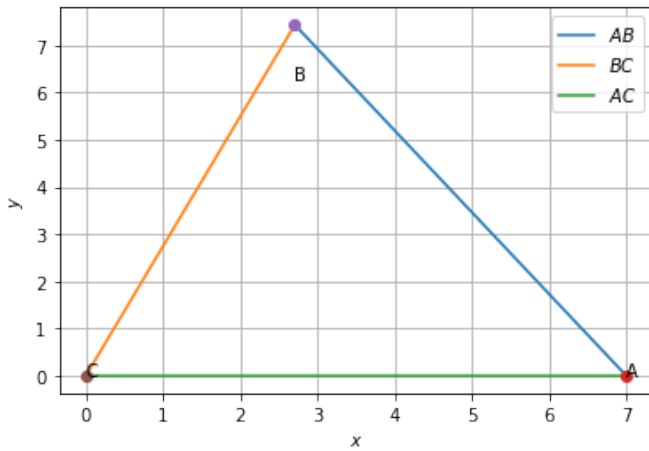


Fig. 2.3.1: Constructed Triangle

Fig. 2.4.1:  $\triangle ABC$ 

are

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.7.1)$$

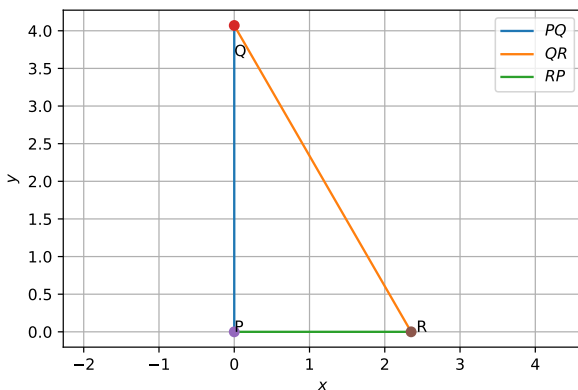
$$\mathbf{Q} = \begin{pmatrix} 0 \\ p \cos \theta \end{pmatrix} \quad (2.7.2)$$

$$= \begin{pmatrix} 0 \\ 4.07 \end{pmatrix} \quad (2.7.3)$$

$$\mathbf{R} = \begin{pmatrix} p \sin \theta \\ 0 \end{pmatrix} \quad (2.7.4)$$

$$= \begin{pmatrix} 2.35 \\ 0 \end{pmatrix} \quad (2.7.5)$$

The triangle is plotted in Fig. 2.7.1

Fig. 2.7.1:  $\triangle PQR$  constructed using python

c) **Solution:** From the given information,

$$\angle C = 60^\circ \quad (2.7.6)$$

Using the sine formula,

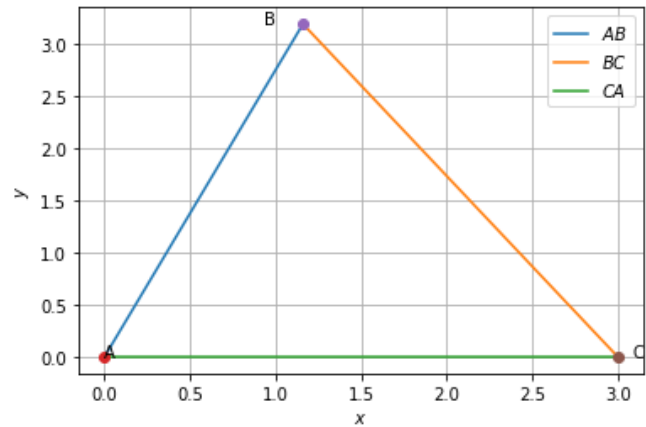
$$c = b \left( \frac{\sin C}{\sin B} \right) \quad (2.7.7)$$

$$= 3.3915 \quad (2.7.8)$$

the vertices of  $\triangle ABC$  are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = c \begin{pmatrix} \cos 70^\circ \\ \sin 70^\circ \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (2.7.9)$$

and plotted in Fig. 2.7.2.

Fig. 2.7.2: Plot of  $\triangle ABC$ 

- 2.8. Construct  $PQRS$  where  $PQ = 4$ ,  $QR = 6$ ,  $RS = 5$ ,  $PS = 5.5$  and  $PR = 7$ .
- 2.9. Draw  $JUMP$  with  $JU = 3.5$ ,  $UM = 4$ ,  $MP = 5$ ,  $PJ = 4.5$  and  $PU = 6.5$
- 2.10. Construct a quadrilateral  $ABCD$  such that  $BC = 4.5$ ,  $AC = 5.5$ ,  $CD = 5$ ,  $BD = 7$  and  $AD = 5.5$ .
- 2.11. Construct  $LIFT$  such that  $LI = 4$ ,  $IF = 3$ ,  $TL = 2.5$ ,  $LF = 4.5$ ,  $IT = 4$ .
- 2.12. DRAW rhombus  $BEND$  such that  $BN = 5.6$ ,  $DE = 6.5$ .
- 2.13. construct a quadrilateral MIST where  $MI = 3.5$ ,  $IS = 6.5$ ,  $\angle M = 75^\circ$ ,  $\angle I = 105^\circ$  and  $\angle S = 120^\circ$ .
- 2.14. Can you construct the above quadrilateral MIST if  $\angle M = 100^\circ$  instead of  $75^\circ$ .
- 2.15. Can you construct the quadrilateral PLAN if  $PL = 6$ ,  $LA = 9.5$ ,  $\angle P = 75^\circ$ ,  $\angle L = 150^\circ$  and  $\angle A = 140^\circ$ ?
- 2.16. Construct  $MORE$  where  $MO = 6$ ,  $OR = 4.5$ ,  $\angle M = 60^\circ$ ,  $\angle O = 105^\circ$ ,  $\angle R = 105^\circ$ .
- 2.17. Construct  $PLAN$  where  $PL = 4$ ,  $LA = 6.5$ ,  $\angle P = 90^\circ$ ,  $\angle A = 110^\circ$  and  $\angle N = 85^\circ$ .

- 2.18. Draw rectangle  $OKAY$  with  $OK = 7$  and  $KA = 5$ .
- 2.19. Construct a kite  $EASY$  if  $AY = 8$ ,  $EY = 4$  and  $SY = 6$ .
- 2.20. Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.
- 2.21. Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of  $60^\circ$ .
- 2.22. Let  $ABC$  be a right triangle in which  $a = 8$ ,  $c = 6$  and  $\angle B = 90^\circ$ .  $BD$  is the perpendicular from **B** on  $AC$  (altitude). The circle through **B**, **C**, **D** (circumcircle of  $\triangle BCD$ ) is drawn. Construct the tangents from **A** to this circle.
- 2.23. Draw a circle of diameter 6.1
- 2.24. Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents  $AC$  and  $CD$  to the circle.
- 2.25. Draw a circle of radius 3 and any two of its diameters. Draw the ends of these diameters. What figure do you get?
- 2.26. Draw a line segment  $AB$  of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.