Assignment-1

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Download all python codes from

https://github.com/satyasm45/Summer−Internship/tree/main/Assignment−1/Codes

and latex-tikz codes from

https://github.com/satyasm45/Summer−Internship/tree/main/Assignment−1

1 Question No. 2.25

Construct a right angled $\triangle$ whose hypotenuse is 6 and one of the legs is 4

2 Explanation

Let us consider $\triangle PQR$ right angled at Q and assume that we are restricted to first quadrant:

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ p \end{pmatrix} \tag{2.0.1}$$

Then,

$$\|\mathbf{R} - \mathbf{Q}\| = \|\mathbf{R}\| = 4 \quad (\because \mathbf{Q} = 0) \tag{2.0.2}$$

This indicates that length of leg QR is 4.

$$\begin{aligned} \|\mathbf{P} - \mathbf{R}\|^2 &= (\mathbf{P} - \mathbf{R})^T(\mathbf{P} - \mathbf{R}) && (2.0.3)\\ &= \mathbf{P}^T\mathbf{P} + \mathbf{R}^T\mathbf{R} - \mathbf{P}^T\mathbf{R} - \mathbf{R}^T\mathbf{P} && (2.0.4)\\ &= \|\mathbf{P}\|^2 + \|\mathbf{R}\|^2 - 2\mathbf{P}^T\mathbf{R} \quad \left(\because \mathbf{P}^T\mathbf{R} = \mathbf{R}^T\mathbf{P}\right) && (2.0.5)\\ &= \|\mathbf{P}\|^2 + \|\mathbf{R}\|^2 \quad \left(\because \mathbf{R}^T\mathbf{P} = 0\right) && (2.0.6)\\ &= p^2 + 16 && (2.0.7) \end{aligned}$$

Also hypotenuse is 6,

$$\implies \|\mathbf{P} - \mathbf{R}\|^2 = 6^2 = 36 \tag{2.0.8}$$

Therefore,

$$\begin{aligned} p^2 + 16 &= 36 && (2.0.9)\\ \implies p^2 &= 20 && (2.0.10)\\ \implies p &= \pm 2\sqrt{5} && (2.0.11) \end{aligned}$$

Since first quadrant was assumed here,only $p = +2\sqrt{5}$ is taken into consideration. So,the vertices of $\triangle PQR$ in fig. 2.1 are:

$$\mathbf{P} = \begin{pmatrix} 0 \\ 2\sqrt{5} \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{2.0.12}$$

Lines PQ ,QR and RP are then generated and plotted using these coordinates to form $\triangle PQR$. Plot of the right angled $\triangle PQR$ is given below:

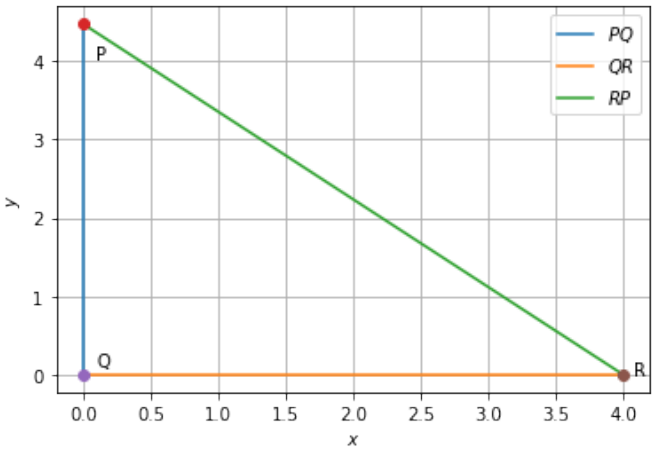

Fig. 2.1: Right Angled $\triangle PQR$