भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology
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Linear Inequalities

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CONTENTS

***Abstract*—This book provides a computational approach to school geometry based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    ncert/computation/codes
```

1 EXAMPLES

1.1. Solve $30x < 200$ when

a) x is a natural number,
b) x is an integer.

Solution: From the given information,

$$30x < 200 \implies x < \frac{20}{3} \tag{1.1.1}$$

If x is a natural number, $x \in \{1, 2, 3, 4, 5, 6\}$. If x is an integer, then the solution set includes 0 as well as all negative integers.

1.2. Solve $5x - 3 < 3x + 1$ when

a) x is an integer,
b) x is a real number.

Solution:

$$5x - 3 < 3x + 1 \implies x < 2 \tag{1.2.1}$$

If x is real, then $x \in (-\infty, 2)$.

1.3. Solve the following system of linear inequalities graphically.

$$\begin{aligned} x + y &\geq 5 \\ x - y &\leq 3 \end{aligned} \tag{1.3.1}$$

Solution: Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \tag{1.3.2}$$

(1.3.1) can then be expressed as

$$\begin{aligned} x + y &\geq 5 \\ -x + y &\geq -3 \end{aligned} \tag{1.3.3}$$

$$\implies \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} 5 \\ -3 \end{pmatrix} \tag{1.3.4}$$

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} \tag{1.3.5}$$

$$\text{or, } \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + \mathbf{u} \tag{1.3.6}$$

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resulting in

$$\mathbf{x} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1} \mathbf{u} \quad (1.3.7)$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{u} \quad (1.3.8)$$

after obtaining the inverse. Fig. 1.3 generated using the following python code shows the region satisfying (1.3.1)

codes/line/line_ineq.py

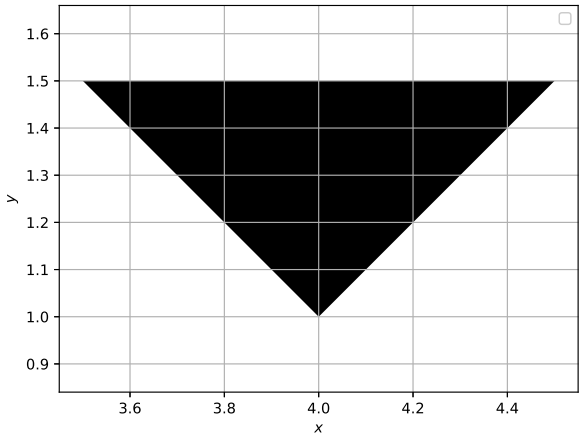

Fig. 1.3

1.4. Solve

$$\begin{aligned} 2x + y &\geq 4 \\ x + y &\leq 3 \\ 2x - 3y &\leq 6 \end{aligned} \quad (1.4.1)$$

Solution: Fig. 1.4 generated using the following python code shows the region satisfying (1.4.1)

codes/line/line_ineq_mult.py

1.5. Solve $x + y < 5$ graphically.
Solution: The following python code generates Fig. 1.5.

./solutions/5/codes/lines/q6.py

1.6. Solve

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \\ 1 & 0 \\ 0 & -1 \\ -1 & 0 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 150 \\ 80 \\ 15 \\ 0 \\ 0 \end{pmatrix} \quad (1.6.1)$$

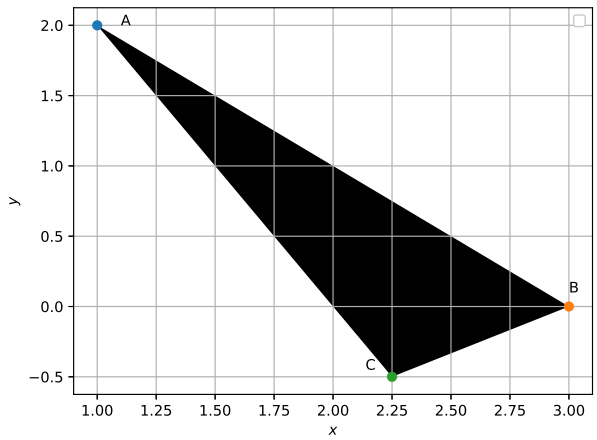

Fig. 1.4

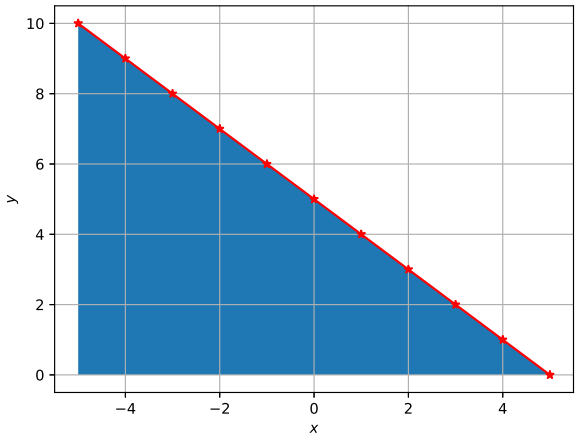

Fig. 1.5: x+y<5

2 Exercises

2.1. Solve x ≥ 3, y ≥ 2 graphically.
Solution: From the given information, for

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0}, \quad (2.1.1)$$

the given conditions can be expressed as

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (2.1.2)$$

$$\implies \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (2.1.3)$$

$$\text{or, } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \mathbf{u} \quad (2.1.4)$$

resulting in

$$\mathbf{x} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}^{-1}\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}^{-1}\mathbf{u} \quad (2.1.5)$$

$$\text{or, } \mathbf{x} = \begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1 & 0\\ 1 & 0\end{pmatrix}\mathbf{u} \quad (2.1.6)$$

after obtaining the inverse. Fig. 2.1 generated using the following python code shows the desired region

solutions/1/codes/line/line_eq.py

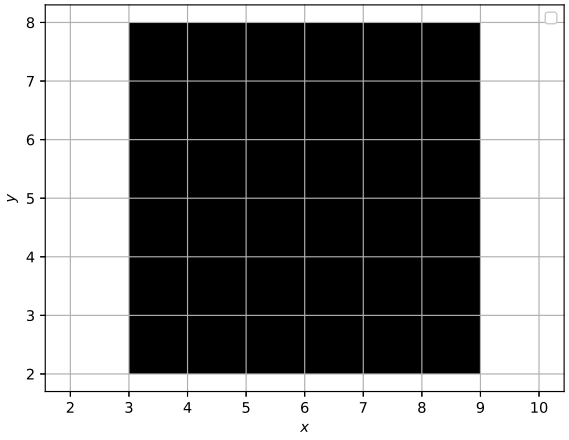

Fig. 2.1

2.2. Solve 7x+3 < 5x+9. Show the graph of the solutions on number line.
Solution:

$$7x + 3 < 5x + 9 \quad (2.2.1)$$

$$2x - 6 < 0 \quad (2.2.2)$$

$$x < 3 \quad (2.2.3)$$

$$\therefore\ x \in \{3, -\infty\} \quad (2.2.4)$$

The following Python code to generate Fig 2.2

solutions/2/codes/line_ex/lin_ineq/dist_btw_pts.py

2.3. Solve $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$. Show the graph of the solutions on number line.
Solution: Let

$$\frac{3x-4}{2} = \frac{x+1}{4} - 1 + s, \quad s \geq 0 \quad (2.3.1)$$

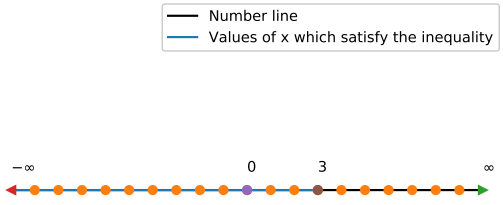

Fig. 2.2

Then,

$$5x - 5 - 4s = 0 \quad (2.3.2)$$

$$\implies x = 1 + \frac{4s}{5} \quad (2.3.3)$$

$$\implies x \geq 1 \quad (2.3.4)$$

The following code marks the solution of inequality on numberline as shown in figure 2.3

codes/line/ineq/ineq.py

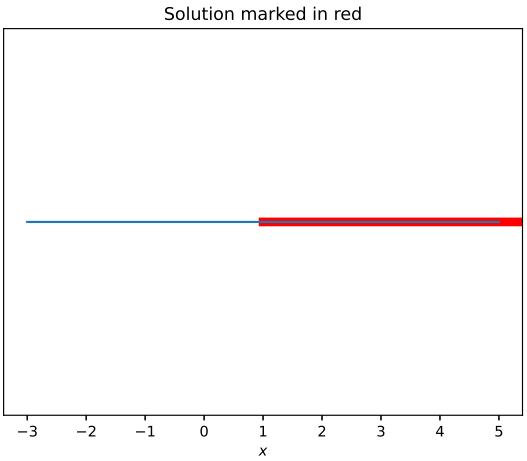

Fig. 2.3: Solution of the inequality

2.4. The marks obtained by a student of Class XI in first and second terminal examination are 62 and 48, respectively. Find the minimum marks he should get in the annual examination to have

an average of at least 60 marks.

Solution: If x be the student marks,

$$\frac{62+48+x}{3} \geq 60 \tag{2.4.1}$$

$$\implies x \geq 70 \tag{2.4.2}$$

2.5. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

Solution:

Let x be an odd natural number and y be the odd natural number consecutive to x.

$$\therefore y = x + 2 \tag{2.5.1}$$

We need to find x and y such that

$$x, y > 10 \text{ and } x + y < 40$$

$$\therefore x + x + 2 < 40$$

$$2x + 2 < 40$$

$$x + 1 < 20$$

$$x < 19 \tag{2.5.2}$$

Hence the condition is satisfied when $x > 10$ and $x < 19$

The following python code computes the required pairs of consecutive odd natural numbers which satisfy the required condition, shown in Fig.2.5.

./solutions/5/codes/lines/q15.py

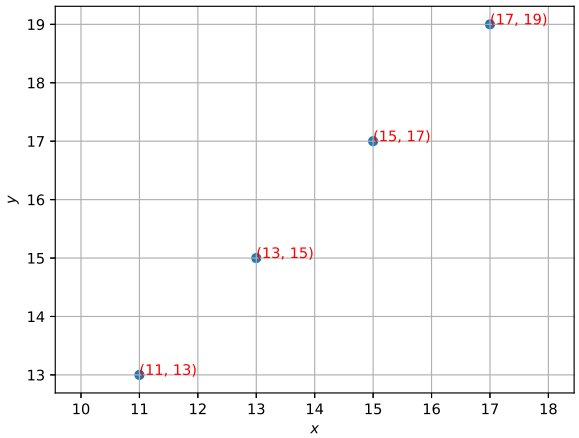

Fig. 2.5

2.6. Solve 3x+2y > 6 graphically.

Solution: Let $3x + 2y = 6$ intersects the x-axis and y-axis at **A** and **B** respectively.

a) Let $\mathbf{A} = \begin{pmatrix} x \\ 0 \end{pmatrix}$

$$3x = 6 \tag{2.6.1}$$

$$\implies x = 2 \tag{2.6.2}$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.6.3}$$

b) Let $\mathbf{B} = \begin{pmatrix} 0 \\ y \end{pmatrix}$

$$2y = 6 \tag{2.6.4}$$

$$\implies y = 3 \tag{2.6.5}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{2.6.6}$$

c) Origin $= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ does not satisfy the equation $3x + 2y < 6$.

$\implies$ The solution is the right side of the line $3x + 2y = 6$

d) The following python code is the diagrammatic representation of the solution in Fig.2.6

solutions/6/codes/linear_inequalities/linear_inequalities.py

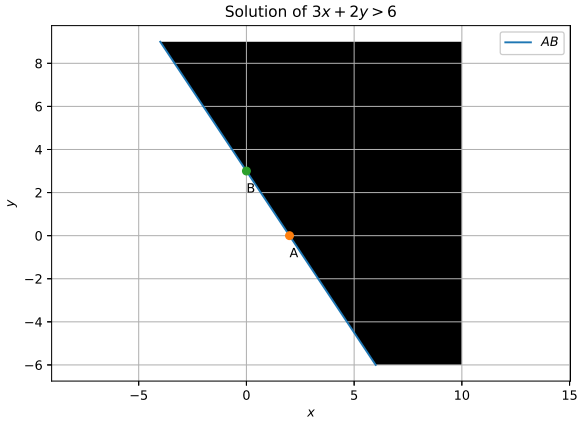

Fig. 2.6

2.7. Solve 3x-6 ≥ 0 graphically in a two dimensional plane.

Solution:

The given inequality can be expressed as

$$\begin{pmatrix}3 & 0\end{pmatrix}\mathbf{x} - 6 \geq 0 \implies \mathbf{x} \geq \begin{pmatrix}2\\0\end{pmatrix} \tag{2.7.1}$$

The python code for Fig. 2.7 is

solutions/7/codes/line/lin_ineq/lin_ineq1.py

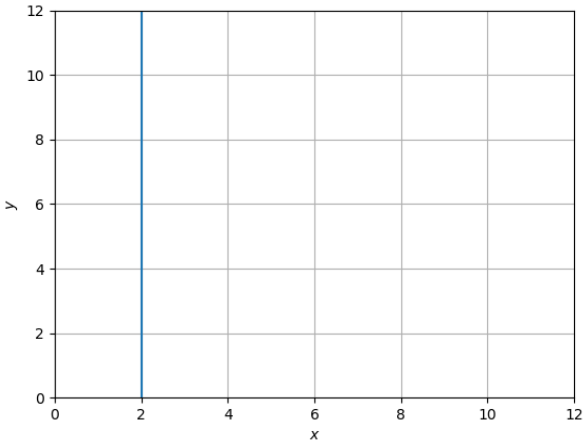

Fig. 2.7

2.8. Solve y < 2 graphically.
2.9. Solve the following system of inequalities graphically. 5x+4y ≤ 40 x ≥ 2 y ≥ 3
2.10. Solve the following system of inequalities graphically. 8x+3y ≤ 100 x ≥ 0 y ≥ 0
2.11. Solve the following system of inequalities graphically. x+2y ≤ 8 2x+y ≤ 8 x ≥ 0 y ≥ 0
2.12. Solve -8 ≤ 5x-3 < 7.
2.13. Solve $-5 \leq \frac{5-3x}{2} \leq 8$.
2.14. Solve the system inequalities: 3x-7 < 5+x 11-5x ≤ 1 and represent the solutions on the number line.
2.15. Solve 4x+3 < 6x+7.
2.16. Solve $\frac{5-2x}{3} \leq \frac{x}{6} - 5$.
2.17. Solve 24x < 100, when (i) x is a natural number. (ii) x is an integer.
2.18. Solve -12x > 30, when (i) x is a natural number. (ii) x is an integer.
2.19. Solve 5x-3 < 7, when (i) x is an integer. (ii) x is a real number.
2.20. Solve 3x+8 > 2, when (i) x is an integer. (ii) x is a real number
2.21. 4x+3 < 5x+7.
2.22. 3x-7 > 5x-1.
2.23. 3(x-1) ≥ 2(x-3).
2.24. 3(2-x) ≤ 2(1-x).
2.25. $x+\frac{x}{2}+\frac{x}{3} < 11$.
2.26. $\frac{x}{3}\ \frac{x}{2}+1$.
2.27. $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$.
2.28. $\frac{1}{2}(\frac{3x}{5}+4) \geq \frac{1}{3}(x-6)$.
2.29. 2(2x+3)-10 < 6(x-2).
2.30. 37-(3x+5) ≥ 9x-8(x-3).
2.31. $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$.
2.32. $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$.
2.33. 3x-2 < 2x+1.
2.34. 5x-3 ≥ 3x-5.
2.35. 3(1-x) < 2(x+4).
2.36. $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$.
2.37. x+y < 5.
2.38. 2x+y ≥ 6.
2.39. 3x+4y ≤ 12.
2.40. y+8 ≥ 2x.
2.41. x-y ≤ 2.
2.42. 2x-3y > 6.
2.43. -3x+2y ≥ -6.
2.44. 3y-5x < 30.
2.45. y < -2.
2.46. x > -3.
2.47. 3x+2y ≤ 12, x ≥ 1, y ≥ 2.
2.48. 2x+y ≥ 6, 3x+4y ≤ 12.
2.49. x+y ≥ 4, 2x-y < 0.
2.50. 2x-y > 1, x-2y < -1.
2.51. x+y ≤ 6, x+y ≥ 4.
2.52. 2x+y ≥ 8, x+2y ≥ 10.
2.53. x+y ≤ 9, y > x, x ≥ 0.
2.54. 5x+4y ≤ 20, x ≥ 1, y ≥ 2.
2.55. 3x+4y ≤ 60, x+3y ≤ 30, x ≥ 0, y ≥ 0.
2.56. x-2y ≤ 3, 3x+4y ≥ 12, x ≥ 0, y ≥ 1.
2.57. 4x+3y ≤ 60, y ≥ 2x, x ≥ 3, x,y ≥ 0.
2.58. x+2y ≤ 10, x+y ≥ 1,x-y ≤ 0, x ≥ 0, y ≥ 0.
2.59. 2 ≤ 3x-4 ≤ 5.
2.60. 6 ≤ -3(2x-40) < 12.
2.61. $-3 \leq 4-\frac{7x}{2} \leq 18$.
2.62. $-15 < \frac{3(x-2)}{5} \leq 0$.
2.63. $-12 < 4-\frac{3x}{5} \leq 2$.
2.64. $7 \leq \frac{(3x+11)}{2} \leq 11$.
2.65. 5x+1 > -24, 5x-1 < 24.
2.66. 2(x-1) < x+5, 3(x+2) > 2-x.
2.67. 3x-7 > 2(x-6), 6-x > 11-2x.
2.68. 5(2x-7)-3(2x+3) ≤ 0, 2x+19 ≤ 6x+47.