ASSIGNMENT 2

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Download all python codes from

https://github.com/ooharapolu/ASSIGNMENT2/tree/main/ASSIGNMENT2/CODES

and latex-tikz codes from

https://github.com/ooharapolu/ASSIGNMENT2/tree/main/ASSIGNMENT2

1 Question No 2.8

Which of the following pairs of linear equations are consistent/inconsistent, obtain the solution:

1)

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 6$$
$$\begin{pmatrix}4 & -2\end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

2)

$$\begin{pmatrix}2 & -2\end{pmatrix}\mathbf{x} = 2$$
$$\begin{pmatrix}4 & -4\end{pmatrix}\mathbf{x} = 5 \tag{1.0.2}$$

2 SOLUTION

1)

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 6$$
$$\begin{pmatrix}4 & -2\end{pmatrix}\mathbf{x} = 4 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}2 & 1\\4 & -2\end{pmatrix}\mathbf{x} = \begin{pmatrix}6\\4\end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}2 & 1 & 6\\4 & -2 & 4\end{pmatrix} \xleftrightarrow{R_2 \to \frac{R_1}{2} - \frac{R_2}{4}} \begin{pmatrix}2 & 1 & 6\\0 & 1 & 2\end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix}2 & 1 & 6\\0 & 1 & 2\end{pmatrix} \xleftrightarrow{R_1 \to R_1 - R_2} \begin{pmatrix}2 & 0 & 4\\0 & 1 & 2\end{pmatrix} \tag{2.0.4}$$

$$\begin{pmatrix}2 & 0 & 4\\0 & 1 & 2\end{pmatrix} \xleftrightarrow{R_1 \to \frac{R_1}{2}} \begin{pmatrix}1 & 0 & 2\\0 & 1 & 2\end{pmatrix} \tag{2.0.5}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}2 & 1 & 6\\4 & -2 & 4\end{pmatrix} \tag{2.0.6}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}2 & 1\\4 & -2\end{pmatrix} \tag{2.0.7}$$

is also 2.

$$\because Rank\begin{pmatrix}2 & 1\\4 & -2\end{pmatrix} = Rank\begin{pmatrix}2 & 1 & 6\\4 & -2 & 4\end{pmatrix} = 2$$
$$= dim\begin{pmatrix}2 & 1\\4 & -2\end{pmatrix} = 2 \tag{2.0.8}$$

$\therefore$ Given lines (1.0.1) have unique solution so we can say they intersect. The given lines are consistent.

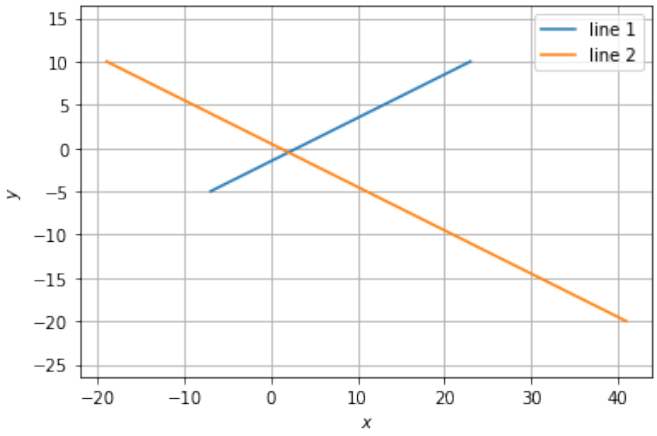

Fig. 2.1: INTERSECTING LINES

2)

$$\begin{pmatrix}2 & -2\end{pmatrix}\mathbf{x} = 2$$
$$\begin{pmatrix}4 & -4\end{pmatrix}\mathbf{x} = 5 \tag{2.0.9}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}2 & -2\\4 & -4\end{pmatrix}\mathbf{x} = \begin{pmatrix}2\\5\end{pmatrix} \tag{2.0.10}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & -2 & 2 \\ 4 & -4 & 5 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 2 & -2 & 2 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.11}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 2 & -2 & 2 \\ 4 & -4 & 5 \end{pmatrix} \tag{2.0.12}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & -2 \\ 4 & -4 \end{pmatrix} \tag{2.0.13}$$

is also 1.

$$\begin{aligned} \because Rank \begin{pmatrix} 2 & -2 \\ 4 & -4 \end{pmatrix} \neq Rank \begin{pmatrix} 2 & -2 & 2 \\ 4 & -4 & 5 \end{pmatrix} &= 1 \\ < dim \begin{pmatrix} 2 & -2 \\ 4 & -4 \end{pmatrix} &= 2 \end{aligned} \tag{2.0.14}$$

$\therefore$ Given lines (1.0.2) have no solution so we say they are parallel. The givens lines are inconsistent. PLOT OF GIVEN LINES -

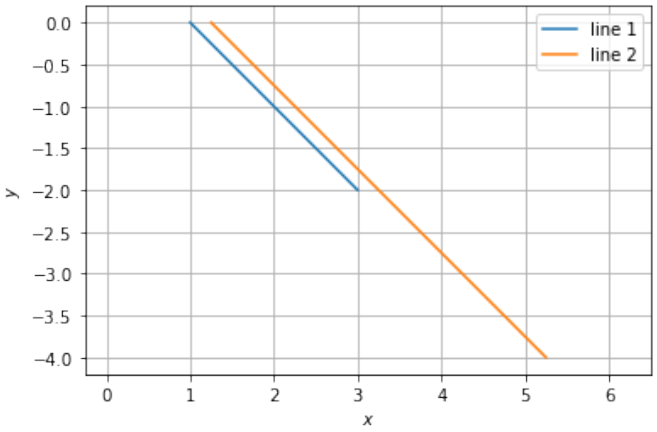

Fig. 2.2: PARALLEL LINES