Assignment 5

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Download all python codes from

https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment5/Codes

and latex-tikz codes from

https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment5

1 Question No. 2.1

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point $\begin{pmatrix}2\\3\end{pmatrix}$.

2 Solution

Let $\mathbf{O}$ be the centre and r be the radius of the given circle.

$\therefore$

$$\mathbf{O} = \begin{pmatrix}p\\0\end{pmatrix} \tag{2.0.1}$$

$$r = 5 \tag{2.0.2}$$

General equation of a circle is given by

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{O}^T\mathbf{x} + \|\mathbf{O}\|^2 - r^2 = 0 \tag{2.0.3}$$

$\therefore$ The equation of the given circle is

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}p & 0\end{pmatrix}\mathbf{x} + p^2 - 25 = 0 \tag{2.0.4}$$

$\because$ Point $\mathbf{A}=\begin{pmatrix}2\\3\end{pmatrix}$ lies on this equation and satisfies it.

$\therefore$

$$\begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} - 2\begin{pmatrix}p & 0\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} + p^2 - 25 = 0 \tag{2.0.5}$$

$$\implies 13 - 4p + p^2 - 25 = 0 \tag{2.0.6}$$

$$\implies p^2 - 4p - 12 = 0 \tag{2.0.7}$$

$$\implies p = 6 \tag{2.0.8}$$

$$\text{or, } p = -2 \tag{2.0.9}$$

Hence,the equation of the circle can be written as

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}6 & 0\end{pmatrix}\mathbf{x} + 11 = 0 \tag{2.0.10}$$

$$\text{or, } \mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}-2 & 0\end{pmatrix}\mathbf{x} - 21 = 0 \tag{2.0.11}$$

Plot of the equations (2.0.10) and (2.0.11)

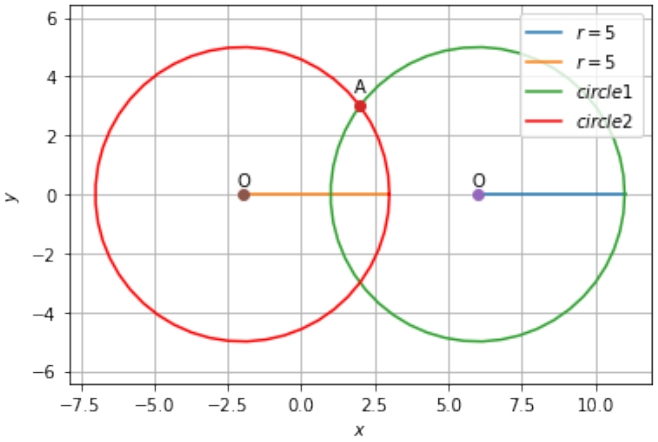

Fig. 2.1: Circles with centres (6,0) and (-2,0) respectively