

# Assignment 11

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Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment11/Codes>

and latex-tikz codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment11>

## 1 QUESTION No. 2.57

Solve  $4x+3y \leq 60, y \geq 2x, x \geq 3, x, y \geq 0$

## 2 SOLUTION

The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -4 & -3 \\ -2 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ 0 \\ 3 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.1)$$

which can be further simplified into

$$\begin{pmatrix} -4 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ 3 \\ 6 \end{pmatrix} \quad (2.0.2)$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \quad (2.0.3)$$

1)

$$\begin{pmatrix} -4 & -3 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ 3 \end{pmatrix} \quad (2.0.4)$$

$$\Rightarrow \begin{pmatrix} -4 & -3 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -60 \\ 3 \end{pmatrix} + \mathbf{u} \quad (2.0.5)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -4 & -3 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -60 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 & -3 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.6)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 3 \\ 16 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -\frac{1}{3} & \frac{4}{3} \end{pmatrix} \mathbf{u} \quad (2.0.7)$$

2)

$$\begin{pmatrix} -4 & -3 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ 6 \end{pmatrix} \quad (2.0.8)$$

$$\Rightarrow \begin{pmatrix} -4 & -3 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -60 \\ 6 \end{pmatrix} + \mathbf{u} \quad (2.0.9)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -4 & -3 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -60 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 & -3 \\ 0 & 1 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.10)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} \frac{21}{6} \\ \frac{2}{6} \end{pmatrix} + \begin{pmatrix} -\frac{1}{4} & -\frac{3}{4} \\ 0 & 1 \end{pmatrix} \mathbf{u} \quad (2.0.11)$$

Now, solution region which is common to regions of eq. (2.0.7) and eq. (2.0.11), is given by

$$\mathbf{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{1}{12} & -\frac{13}{12} \end{pmatrix} \mathbf{u} \quad (2.0.12)$$

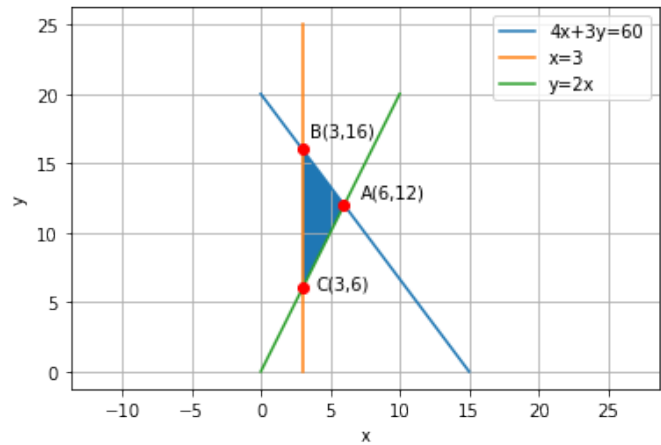


Fig. 2.1: Solution Region

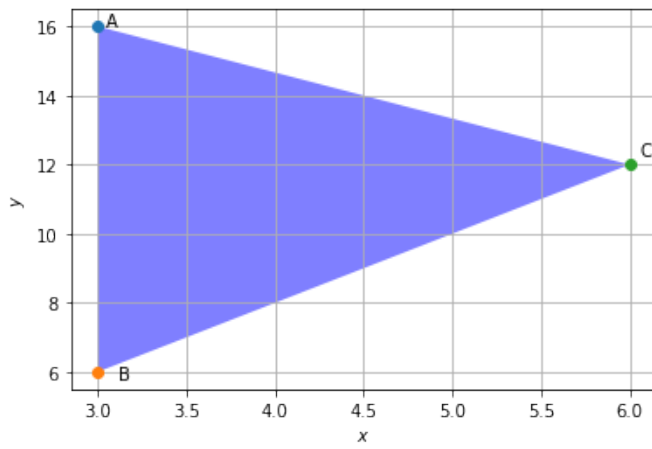


Fig. 2.2: Magnified Solution Region