

Constructions using Python

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Abstract—This book introduces constructions in high school geometry using Python. The content and exercises are based on NCERT textbooks from Class 6-12. A simple introduction to Python and \LaTeX figures is provided in the process.

Download all python codes from

```
svn co https://github.com/gadepall/school/trunk/ncert/constructions/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/ncert/constructions/figs
```

1 EXAMPLES

1.1. Draw Fig. 1.1.1 for $a = 4, c = 3$.

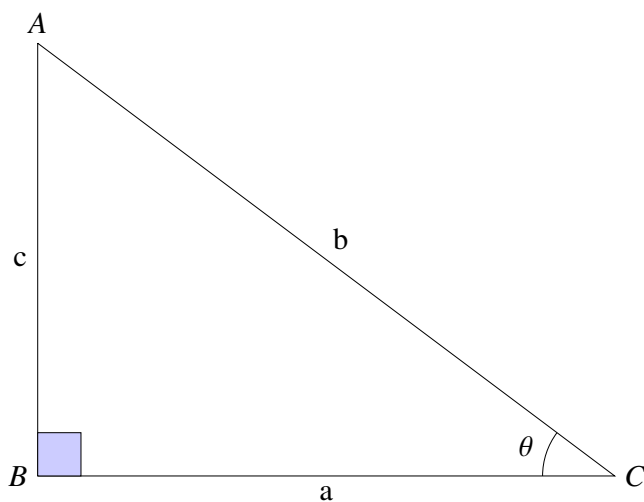


Fig. 1.1.1: Right Angled Triangle

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Solution: The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (1.1.1)$$

The python code for Fig. 1.1.1 is

```
codes/triangle/tri_right_angle.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_right_angle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle/tri_right_angle_alone.tex
```

1.2. Draw Fig. 1.2.1 for $a = 4, c = 3$.

Solution: The vertex \mathbf{A} can be expressed in polar coordinate form as

$$\mathbf{A} = b \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (1.2.1)$$

where

$$b = \sqrt{a^2 + c^2} = 5, \tan \theta = \frac{3}{4} \quad (1.2.2)$$

The python code for Fig. 1.2.1 is

```
codes/triangle/tri_polar.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_polar.tex
```

1.3. Draw Fig. 1.3.1 with $a = 6, b = 5$ and $c = 4$.

Solution: Let the vertices of $\triangle ABC$ and \mathbf{D} be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} p \\ 0 \end{pmatrix} \quad (1.3.1)$$

Then

$$AB = \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0} \quad (1.3.2)$$

$$BC = \|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \quad (1.3.3)$$

$$AC = \|\mathbf{A} - \mathbf{C}\|^2 = b^2 \quad (1.3.4)$$

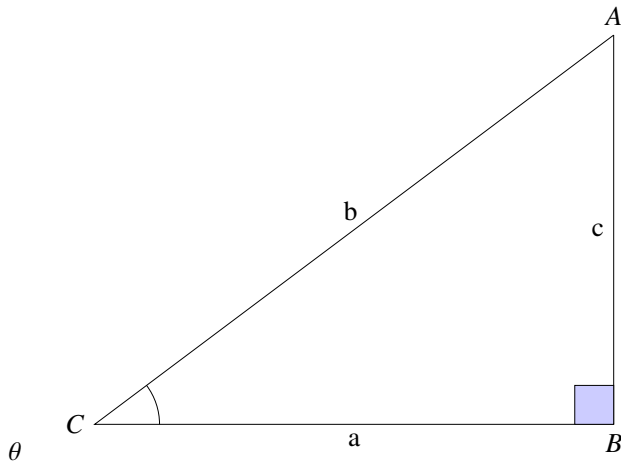


Fig. 1.2.1: Right Angled Triangle

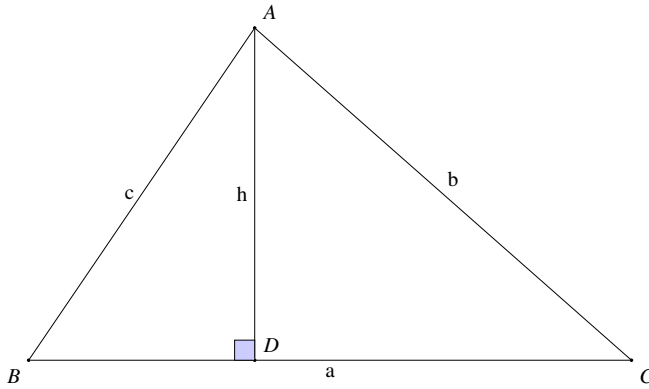


Fig. 1.3.1

From (1.3.4),

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \quad (1.3.5)$$

$$= \mathbf{A}^T \mathbf{A} + \mathbf{C}^T \mathbf{C} - \mathbf{A}^T \mathbf{C} - \mathbf{C}^T \mathbf{A} \quad (1.3.6)$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (\because \mathbf{A}^T \mathbf{C} = \mathbf{C}^T \mathbf{A}) \quad (1.3.7)$$

$$= a^2 + c^2 - 2ap \quad (1.3.8)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \quad (1.3.9)$$

From (1.3.2),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \quad (1.3.10)$$

$$\Rightarrow q = \pm \sqrt{c^2 - p^2} \quad (1.3.11)$$

The python code for Fig. 1.3.1 is

```
codes/triangle/tri_sss.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_sss.tex
```

1.4. Construct parallelogram $ABCD$ in Fig. 1.4.1 given that $BC = 5$, $AB = 6$, $\angle C = 85^\circ$.

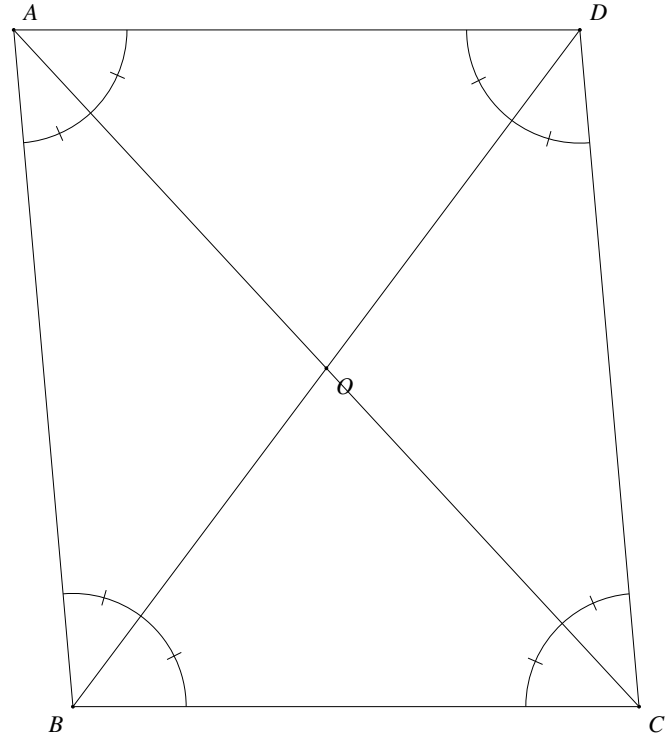


Fig. 1.4.1: Parallelogram Properties

Solution: BD is found using the cosine formula and $\triangle BDC$ is drawn using the approach in Construction 1.3 with

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \quad (1.4.1)$$

Since the diagonals bisect each other,

$$\mathbf{O} = \frac{\mathbf{B} + \mathbf{D}}{2} \quad (1.4.2)$$

$$\mathbf{A} = 2\mathbf{O} - \mathbf{C}. \quad (1.4.3)$$

AB and AD are then joined to complete the ||gm. The python code for Fig. 1.4.1 is

```
codes/quad/pgm_sas.py
```

and The equivalent latex-tikz code is

```
figs/quad/pgm_sas.tex
```

1.5. Draw the ||gm $ABCD$ in Fig. 1.5.1 with $BC = 6$, $CD = 4.5$ and $BD = 7.5$. Show that it is a rectangle.

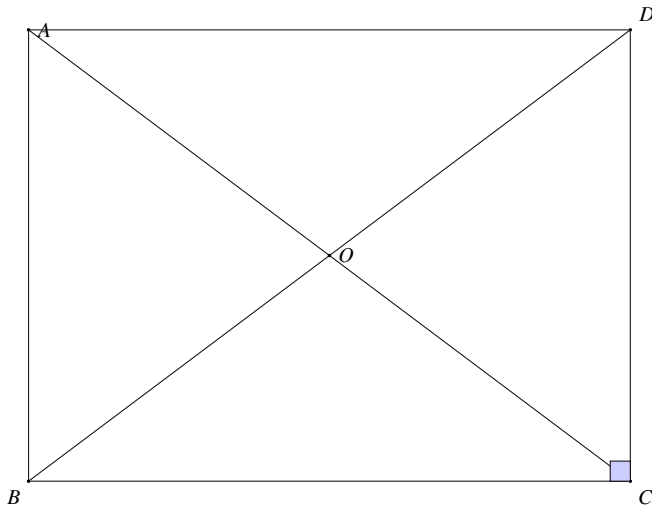


Fig. 1.5.1: Rectangle

Solution: It is easy to verify that

$$BD^2 = BC^2 + C^2 \quad (1.5.1)$$

Hence, using Baudhayana theorem,

$$\angle BCD = 90^\circ \quad (1.5.2)$$

and $ABCD$ is a rectangle.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \quad (1.5.3)$$

The python code for Fig. 1.5.1 is

```
codes/quad/pgm_sss.py
```

and the equivalent latex-tikz code is

```
figs/quad/pgm_sss.tex
```

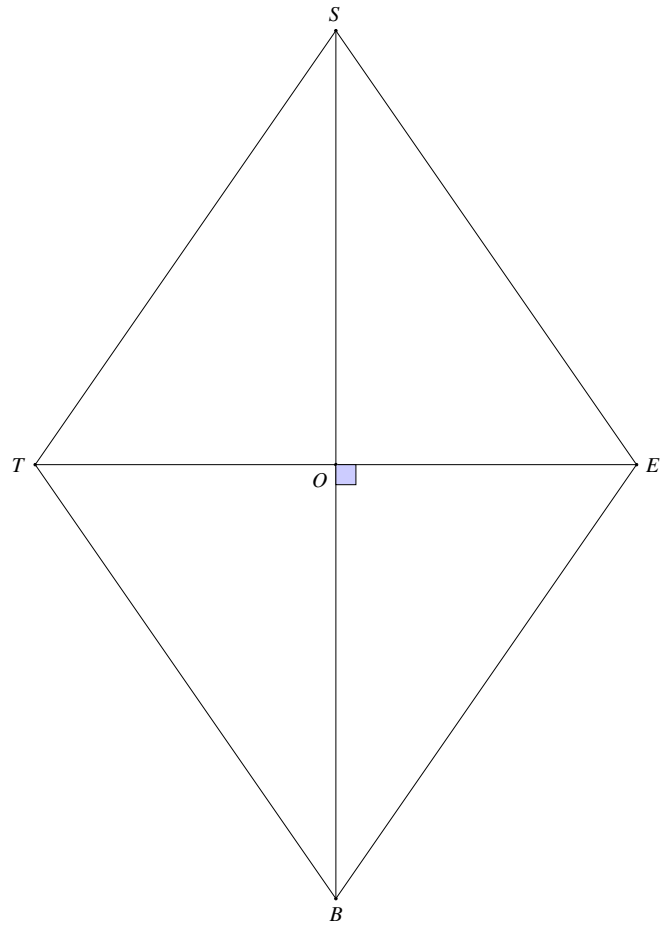


Fig. 1.6.1: Rhombus

in Fig. 1.7.1 are obtained as

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \quad (1.7.1)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 4.5 \\ 4.5 \end{pmatrix} \mathbf{O} = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (1.7.2)$$

- 1.6. Draw the rhombus $BEST$ with $BE = 4.5$ and $ET = 6$.

Solution: The coordinates of the various points in Fig. 1.6.1 are obtained as

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -4.5 \end{pmatrix} \quad (1.6.1)$$

$$\mathbf{E} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \quad (1.6.2)$$

- 1.7. A square is a rectangle whose sides are equal. Draw a square of side 4.5.

Solution: The coordinates of the various points

2 EXERCISES

- 2.1. Construct a triangle of sides $a = 4$, $b = 5$ and $c = 6$.
- 2.2. Construct an isosceles triangle whose base is $a = 8\text{cm}$ and altitude $AD = h = 4\text{cm}$
- 2.3. In $\triangle ABC$, given that $a+b+c = 11$, $\angle B = 45^\circ$ and $\angle C = 45^\circ$, find a, b, c and sketch the triangle.
- 2.4. Draw $\triangle ABC$ with $a = 6$, $c = 5$ and $\angle B = 60^\circ$.
- 2.5. Draw $\triangle ABC$ with $a = 7$, $\angle B = 45^\circ$ and $\angle A = 105^\circ$.

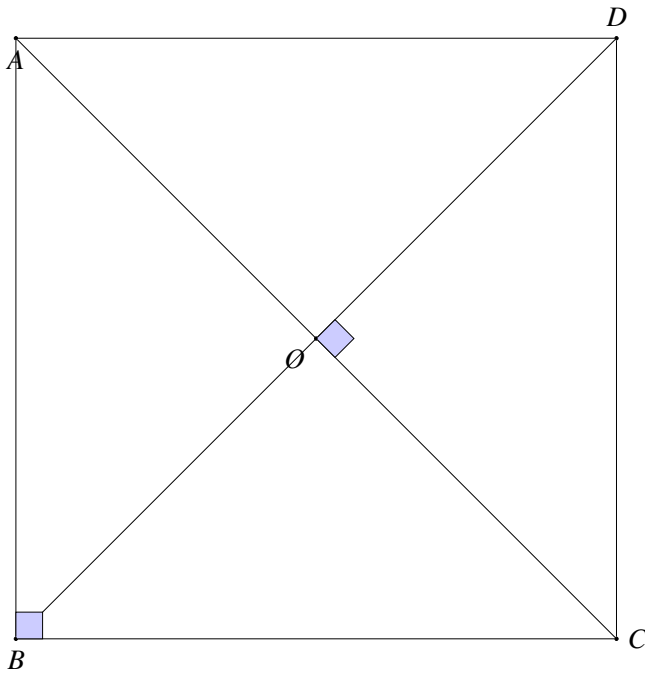


Fig. 1.7.1: Square

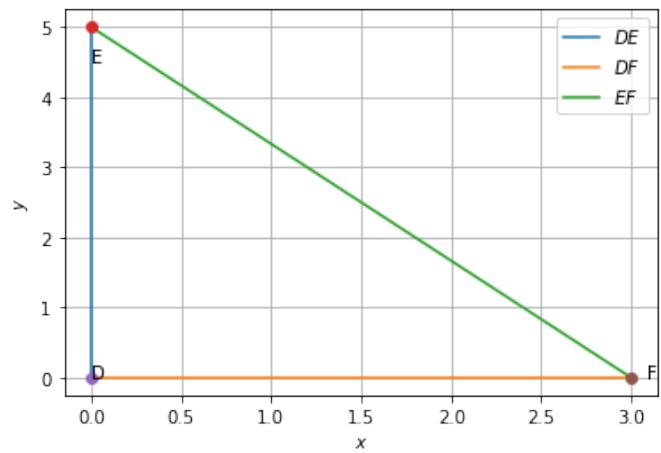


Fig. 2.15.1

- 2.6. $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find b, c and draw the triangle.
- 2.7. In $\triangle ABC$, $a = 8, \angle B = 45^\circ$ and $c - b = 3.5$. Sketch $\triangle ABC$.
- Solution:**
- 2.8. In $\triangle ABC$, $a = 6, \angle B = 60^\circ$ and $b - c = 2$. Sketch $\triangle ABC$.
- 2.9. Draw $\triangle ABC$, given that $a+b+c = 11, \angle B = 30^\circ$ and $\angle C = 90^\circ$.
- 2.10. Construct $\triangle xyz$ where $xy = 4.5, yz = 5$ and $zx = 6$.
- 2.11. Draw an equilateral triangle of side 5.5.
- 2.12. Draw $\triangle PQR$ with $PQ = 4, QR = 3.5$ and $PR = 4$. What type of triangle is this?
- 2.13. Construct $\triangle ABC$ such that $AB = 2.5, BC = 6$ and $AC = 6.5$. Find $\angle B$.
- 2.14. Construct $\triangle PQR$, given that $PQ = 3, QR = 5.5$ and $\angle PQR = 60^\circ$.
- 2.15. Construct $\triangle DEF$ such that $DE = 5, DF = 3$ and $\angle D = 90^\circ$.

Solution: From the given information, the vertices of $\triangle DEF$ are

$$E = \begin{pmatrix} 0 \\ 5 \end{pmatrix}, D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, F = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (2.15.1)$$

which are used to plot Fig. 2.15.1.

- 2.16. Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle

between them is 110° .

- 2.17. Construct $\triangle ABC$ with $BC = 7.5, AC = 5$ and $\angle C = 60^\circ$.
- 2.18. Construct $\triangle XYZ$ if $XY = 6, \angle X = 30^\circ$ and $\angle Y = 100^\circ$.
- 2.19. If $AC = 7, \angle A = 60^\circ$ and $\angle B = 50^\circ$, can you draw the triangle?
- 2.20. Construct $\triangle ABC$ given that $\angle A = 60^\circ, \angle B = 30^\circ$ and $AB = 5.8$.
- 2.21. Construct $\triangle PQR$ if $PQ = 5, \angle Q = 105^\circ$ and $\angle R = 40^\circ$.
- 2.22. Can you construct $\triangle DEF$ such that $EF = 7.2, \angle E = 110^\circ$ and $\angle F = 180^\circ$?
- 2.23. Construct $\triangle LMN$ right angled at M such that $LN = 5$ and $MN = 3$.

Solution:

Let

$$\mathbf{L} = \begin{pmatrix} 0 \\ l \end{pmatrix}, \mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{N} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (2.23.1)$$

From the given information,

$$\|\mathbf{N} - \mathbf{M}\|^2 = \|\mathbf{N}\|^2 = 3^2 = 9 \quad (2.23.2)$$

$$\|\mathbf{L} - \mathbf{M}\|^2 = \|\mathbf{L}\|^2 = l^2 \quad (2.23.3)$$

$$\|\mathbf{L} - \mathbf{N}\|^2 = 5^2 = 25 \quad (2.23.4)$$

which can be expressed as

$$\|\mathbf{L} - \mathbf{N}\|^2 = (\mathbf{L} - \mathbf{N})^T (\mathbf{L} - \mathbf{N}) \quad (2.23.5)$$

$$= \|\mathbf{L}\|^2 + \|\mathbf{N}\|^2 - 2\mathbf{L}^T \mathbf{N} \quad (2.23.6)$$

$$\Rightarrow l^2 + 9 = 25 \quad (2.23.7)$$

$$\text{or, } l = \pm 4 \quad (2.23.8)$$

For $l=4$, $\triangle LMN$ is plotted in the first quadrant

in Fig. 2.23.1.

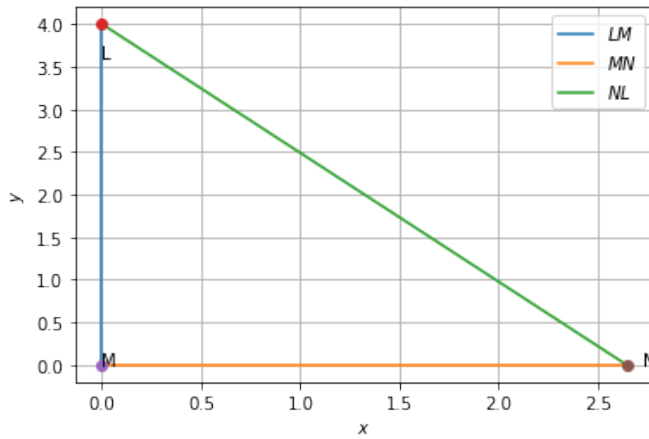


Fig. 2.23.1

2.24. Construct $\triangle PQR$ right angled at Q such that $QR = 8$ and $PR = 10$.

Solution: Let

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ p \end{pmatrix} \quad (2.24.1)$$

Then,

$$\|\mathbf{P} - \mathbf{R}\|^2 = (\mathbf{P} - \mathbf{R})^T (\mathbf{P} - \mathbf{R}) \quad (2.24.2)$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{R}\|^2 \quad (2.24.3)$$

$$\therefore \mathbf{P}^T \mathbf{R} = \mathbf{R}^T \mathbf{P}, \mathbf{R}^T \mathbf{P} = 0 \quad (2.24.4)$$

$$= p^2 + 64 = 10^2 \quad (2.24.5)$$

$$\Rightarrow p = \pm 6 \quad (2.24.6)$$

Since positive area is considered here, only $p = 6$ is taken into consideration. Thus,

$$\mathbf{P} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (2.24.7)$$

and the desired triangle is plotted in Fig. 2.24.1

2.25. Construct right angled \triangle whose hypotenuse is 6 and one of the legs is 4.

Solution: Let us consider $\triangle PQR$ right angled at Q and assume that we are restricted to first quadrant such that

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ p \end{pmatrix} \quad (2.25.1)$$

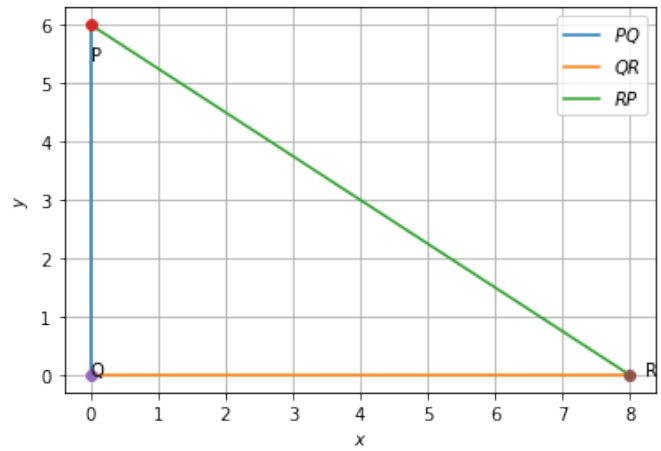


Fig. 2.24.1: Right Angle $\triangle PQR$

Then,

$$\|\mathbf{P} - \mathbf{R}\|^2 = 36 \quad (2.25.2)$$

$$\Rightarrow p^2 + 16 = 36 \quad (2.25.3)$$

$$\Rightarrow p = \pm 2\sqrt{5} \quad (2.25.4)$$

Since first quadrant was assumed here, only $p = +2\sqrt{5}$ is taken into consideration. So, the vertices of $\triangle PQR$ in Fig. 2.25.1 are

$$\mathbf{P} = \begin{pmatrix} 0 \\ 2\sqrt{5} \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (2.25.5)$$

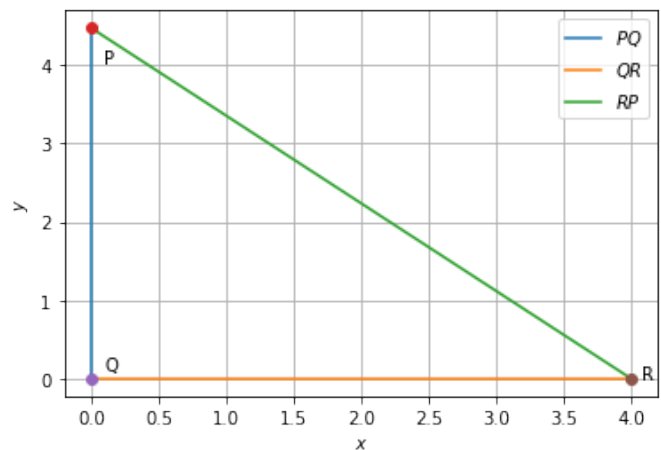


Fig. 2.25.1: Right Angled $\triangle PQR$

2.26. Construct an isosceles right angled $\triangle ABC$ right angled at C such that $AC = 6$.

Solution:

$\therefore \triangle ABC$ is isosceles, its vertices are

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (2.26.1)$$

which are used to plot the desired triangle in Fig. 2.26.1.

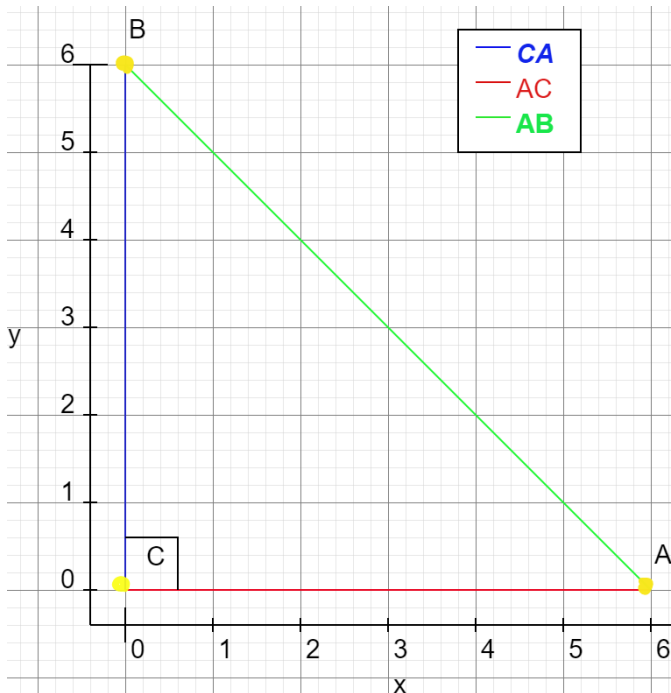


Fig. 2.26.1: Isosceles Right Angle $\triangle ABC$

2.27. Construct the triangles in Table 2.27.1.

S.No	Triangle	Given Measurements		
1	$\triangle ABC$	$\angle A = 85^\circ$	$\angle B = 115^\circ$	$AB = 5$
2	$\triangle PQR$	$\angle Q = 30^\circ$	$\angle R = 60^\circ$	$QR = 4.7$
3	$\triangle ABC$	$\angle A = 70^\circ$	$\angle B = 50^\circ$	$AC = 3$
4	$\triangle LMN$	$\angle L = 60^\circ$	$\angle N = 120^\circ$	$LM = 5$
5	$\triangle ABC$	$BC = 2$	$AB = 4$	$AC = 2$
6	$\triangle PQR$	$PQ = 2.5$	$QR = 4$	$PR = 3.5$
7	$\triangle XYZ$	$XY = 3$	$YZ = 4$	$XZ = 5$
8	$\triangle DEF$	$DE = 4.5$	$EF = 5.5$	$DF = 4$

TABLE 2.27.1

2.28. Construct a quadrilateral $ABCD$ such that $AB = 5$, $\angle A = 50^\circ$, $AC = 4$, $BD = 5$ and $AD = 6$.

Solution:

The rough figure of the expected quadrilateral $ABCD$ is given in Fig. 2.28.1

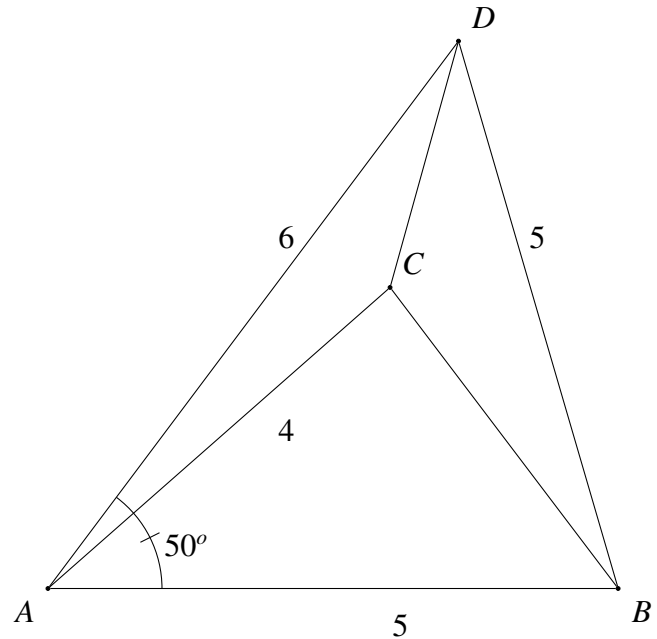


Fig. 2.28.1: Rough Figure

From the given information, in $\triangle ABD$,

$$\cos A = \frac{\|\mathbf{B} - \mathbf{A}\|^2 + \|\mathbf{D} - \mathbf{A}\|^2 - \|\mathbf{D} - \mathbf{B}\|^2}{2 \|\mathbf{B} - \mathbf{A}\| \|\mathbf{D} - \mathbf{A}\|} \quad (2.28.1)$$

$$\Rightarrow \angle A = \cos^{-1}(0.6) \approx 53.13^\circ \quad (2.28.2)$$

$$\neq 50^\circ \quad (2.28.3)$$

resulting in a contradiction. Therefore construction of quadrilateral with given measurements is not possible.

2.29. Construct $PQRS$ where $PQ = 4$, $QR = 6$, $RS = 5$, $PS = 5.5$ and $PR = 7$.

2.30. Draw $JUMP$ with $JU = 3.5$, $UM = 4$, $MP = 5$, $PJ = 4.5$ and $PU = 6.5$

2.31. Construct a quadrilateral $ABCD$ such that $BC = 4.5$, $AC = 5.5$, $CD = 5$, $BD = 7$ and $AD = 5.5$.

2.32. Can you construct a quadrilateral $PQRS$ with $PQ = 3$, $RS = 3$, $PS = 7.5$, $PR = 8$ and $SQ = 4$?

Solution: From the given information,

$$\|\mathbf{P} - \mathbf{Q}\| = 3 \quad (2.32.1)$$

$$\|\mathbf{R} - \mathbf{S}\| = 3 \quad (2.32.2)$$

$$\|\mathbf{P} - \mathbf{S}\| = 7.5 \quad (2.32.3)$$

$$\|\mathbf{P} - \mathbf{R}\| = 8 \quad (2.32.4)$$

$$\|\mathbf{S} - \mathbf{Q}\| = 4 \quad (2.32.5)$$

Let quadrilateral $PQRS$ be made up of two triangles $\triangle PSQ$ and $\triangle PSR$ on base PS .

a) In $\triangle PSR$,

$$\begin{aligned}\|P - S\| + \|R - S\| &= 7.5 + 3 = 10.5 \\ &> \|P - R\| \quad (2.32.6)\end{aligned}$$

$$\|P - R\| + \|R - S\| = 8 + 3 = 11 > \|P - S\| \quad (2.32.7)$$

$$\begin{aligned}\|P - S\| + \|P - R\| &= 7.5 + 8 = 15.5 \\ &> \|R - S\| \quad (2.32.8)\end{aligned}$$

\therefore using triangle inequality, construction of $\triangle PSR$ is possible.

b) In $\triangle PSQ$,

$$\begin{aligned}\|P - S\| + \|S - Q\| &= 7.5 + 4 = 11.5 \\ &> \|P - Q\| \quad (2.32.9)\end{aligned}$$

$$\begin{aligned}\|P - S\| + \|P - Q\| &= 7.5 + 3 = 10.5 \\ &> \|S - Q\| \quad (2.32.10)\end{aligned}$$

$$\|P - Q\| + \|S - Q\| = 3 + 4 = 7 < \|P - S\| \quad (2.32.11)$$

which violates triangle inequality. \therefore construction of $\triangle PSQ$ is not possible.

Fig. 2.32.1 highlights this.

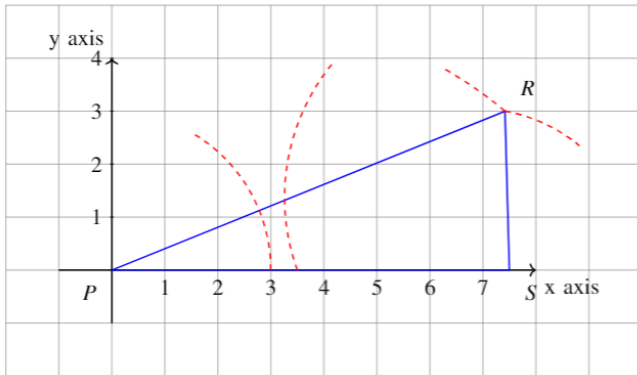


Fig. 2.32.1: Construction of quadrilateral $PQRS$

- 2.33. Construct $LIFT$ such that $LI = 4, IF = 3, TL = 2.5, LF = 4.5, IT = 4$.
- 2.34. Draw $GOLD$ such that $OL = 7.5, GL = 6, GD = 6, LD = 5, OD = 10$.

Solution: In $\triangle LDO$

$$\|O - L\| + \|O - D\| = 17.5 > \|L - D\| \quad (2.34.1)$$

$$\|O - D\| + \|L - D\| = 15 > \|O - L\| \quad (2.34.2)$$

$$\|O - L\| + \|L - D\| = 12.5 > \|O - D\| \quad (2.34.3)$$

and triangle inequality is satisfied. Similarly, in $\triangle LDG$

$$\|L - D\| + \|G - L\| = 11 > \|G - D\| \quad (2.34.4)$$

$$\|G - L\| + \|G - D\| = 12 > \|L - D\| \quad (2.34.5)$$

$$\|L - D\| + \|G - D\| = 11 > \|G - L\| \quad (2.34.6)$$

and triangle inequality is satisfied. \therefore the given sides form a quadrilateral which can be constructed by using the approach in Problem 1.3 to obtain the vertices of $\triangle LDO$ and $\triangle LDG$ as

$$L = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, D = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, O = \begin{pmatrix} -1.875 \\ 7.26 \end{pmatrix}, G = \begin{pmatrix} 2.5 \\ 5.5 \end{pmatrix} \quad (2.34.7)$$

and plotting the quadrilateral $GOLD$ in Fig. 2.34.1

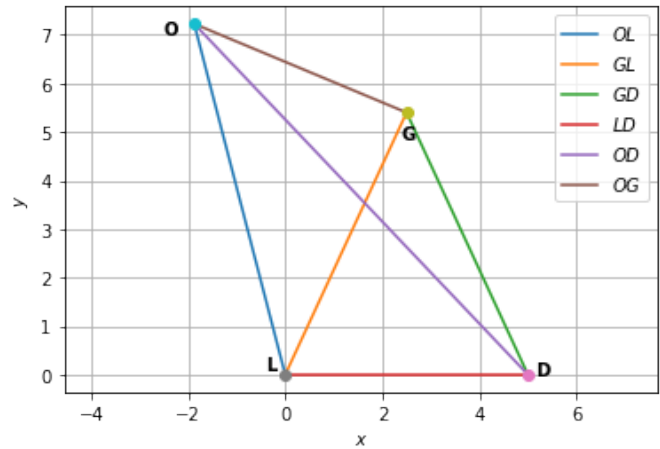


Fig. 2.34.1: Quadrilateral $GOLD$

- 2.35. DRAW rhombus $BEND$ such that $BN = 5.6, DE = 6.5$.
- 2.36. construct a quadrilateral $MIST$ where $MI = 3.5, IS = 6.5, \angle M = 75^\circ, \angle I = 105^\circ$ and $\angle S = 120^\circ$.
- 2.37. Can you construct the above quadrilateral $MIST$ if $\angle M = 100^\circ$ instead of 75° .
- 2.38. Can you construct the quadrilateral $PLAN$ if $PL = 6, LA = 9.5, \angle P = 75^\circ, \angle L = 150^\circ$ and $\angle A = 140^\circ$?

- 2.39. Construct *MORE* where $MO = 6, OR = 4.5, \angle M = 60^\circ, \angle O = 105^\circ, \angle R = 105^\circ$.
- 2.40. Construct *PLAN* where $PL = 4, LA = 6.5, \angle P = 90^\circ, \angle A = 110^\circ$ and $\angle N = 85^\circ$.
- 2.41. Draw rectangle *OKAY* with $OK = 7$ and $KA = 5$.
- 2.42. Construct *ABCD*, where $AB = 4, BC = 5, Cd = 6.5, \angle B = 105^\circ$ and $\angle C = 80^\circ$.
- 2.43. Construct *DEAR* with $DE = 4, EA = 5, AR = 4.5, \angle E = 60^\circ$ and $\angle A = 90^\circ$.
- 2.44. Construct *TRUE* with $TR = 3.5, RU = 3, UE = 4, \angle R = 75^\circ$ and $\angle U = 120^\circ$.
- 2.45. Can you construct a rhombus *ABCD* with $AC = 6$ and $BD = 7$?
- 2.46. Draw a square *READ* with $RE = 5.1$.
- 2.47. Draw a rhombus who diagonals are 5.2 and 6.4.
- 2.48. Draw a rectangle with adjacent sides 5 and 4.
- 2.49. Draw a parallelogram *OKAY* with $OK = 5.5$ and $KA = 4.2$.
- 2.50. Construct a kite *EASY* if $AY = 8, EY = 4$ and $SY = 6$.
- 2.51. Draw a circle of diameter 6.1
- 2.52. With the same centre **O**, draw two circles of radii 4 and 2.5

Solution:

All input values required to plot Fig. 2.52.1 are given in Table 2.52.1 as shown below

	Symbols	Circle1	Circle2
Centre	O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	r_1, r_2	2.5	4
Polar coordinate	C_1, C_2	$2.5 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$	$4 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$
Angle	θ	$0-2\pi$	$0-2\pi$

TABLE 2.52.1: Input values

- 2.53. Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents *AC* and *CD* to the circle.
- 2.54. Draw a circle of radius 3 and any two of its diameters. Draw the ends of these diameters. What figure do you get?
- 2.55. Let **A** and **B** be the centres of two circles of equal radii 3 such that each one of them passes through the centre of the other. Let them intersect at **C** and **D**. Is $AB \perp CD$?

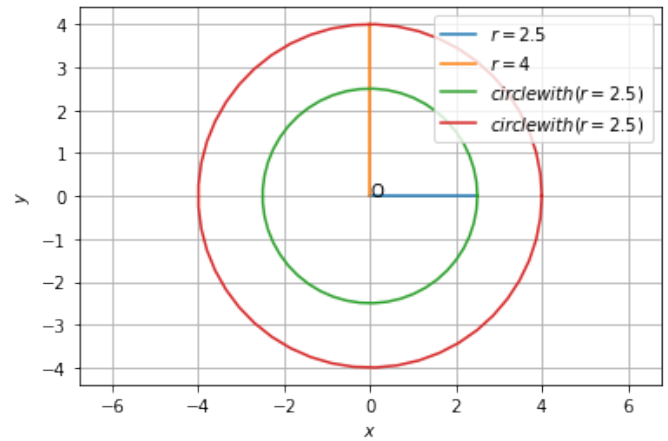


Fig. 2.52.1: Concentric circles with centre as origin and radii 2.5 and 4 respectively

- 2.56. Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.
Solution: Take the centre of both circles to be at the origin.
- 2.57. Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.
Solution: Take the diameter to be on the x -axis.
- 2.58. Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60° .
Solution: The tangent is perpendicular to the radius.
- 2.59. Draw a line segment *AB* of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.
Solution: Let
- $$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}. \quad (2.59.1)$$
- 2.60. Let *ABC* be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^\circ$. *BD* is the perpendicular from **B** on *AC* (altitude). The circle through **B, C, D** (circumcircle of $\triangle BCD$) is drawn. Construct the tangents from **A** to this circle.
- 2.61. Draw a circle with centre **C** and radius 3.4. Draw any chord. Construct the perpendicular

bisector of the chord and examine if it passes through **C**