

# Constructions using Python

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**Abstract**—This book introduces constructions in high school geometry using Python. The content and exercises are based on NCERT textbooks from Class 6-12. A simple introduction to Python and  $\text{\LaTeX}$  figures is provided in the process.

Download all python codes from

```
svn co https://github.com/gadepall/school/trunk/ncert/constructions/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/ncert/constructions/figs
```

## 1 EXAMPLES

1.1. Draw Fig. 1.1.1 for  $a = 4, c = 3$ .

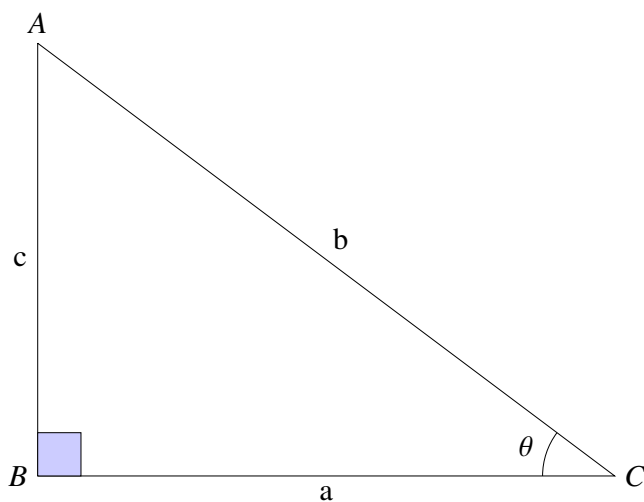


Fig. 1.1.1: Right Angled Triangle

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**Solution:** The vertices of  $\triangle ABC$  are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (1.1.1)$$

The python code for Fig. 1.1.1 is

```
codes/triangle/tri_right_angle.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_right_angle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle/tri_right_angle_alone.tex
```

1.2. Draw Fig. 1.2.1 for  $a = 4, c = 3$ .

**Solution:** The vertex  $\mathbf{A}$  can be expressed in polar coordinate form as

$$\mathbf{A} = b \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (1.2.1)$$

where

$$b = \sqrt{a^2 + c^2} = 5, \tan \theta = \frac{3}{4} \quad (1.2.2)$$

The python code for Fig. 1.2.1 is

```
codes/triangle/tri_polar.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_polar.tex
```

1.3. Draw Fig. 1.3.1 with  $a = 6, b = 5$  and  $c = 4$ .

**Solution:** Let the vertices of  $\triangle ABC$  and  $\mathbf{D}$  be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} p \\ 0 \end{pmatrix} \quad (1.3.1)$$

Then

$$AB = \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0} \quad (1.3.2)$$

$$BC = \|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \quad (1.3.3)$$

$$AC = \|\mathbf{A} - \mathbf{C}\|^2 = b^2 \quad (1.3.4)$$

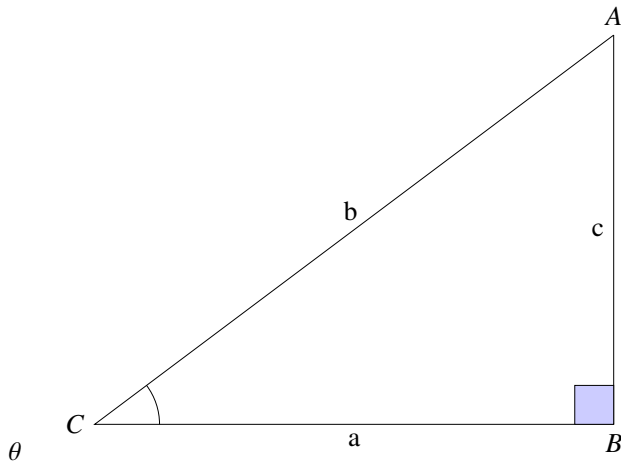


Fig. 1.2.1: Right Angled Triangle

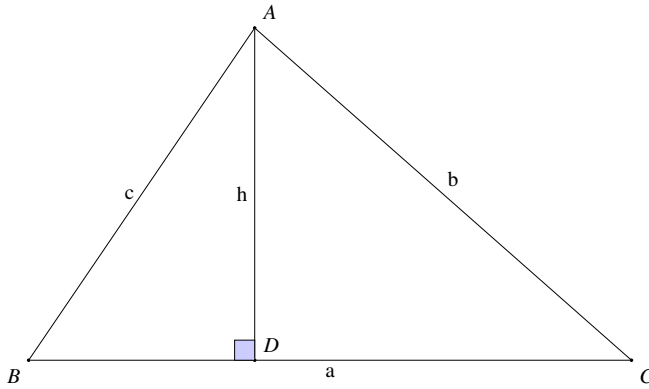


Fig. 1.3.1

From (1.3.4),

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \quad (1.3.5)$$

$$= \mathbf{A}^T \mathbf{A} + \mathbf{C}^T \mathbf{C} - \mathbf{A}^T \mathbf{C} - \mathbf{C}^T \mathbf{A} \quad (1.3.6)$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (\because \mathbf{A}^T \mathbf{C} = \mathbf{C}^T \mathbf{A}) \quad (1.3.7)$$

$$= a^2 + c^2 - 2ap \quad (1.3.8)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \quad (1.3.9)$$

From (1.3.2),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \quad (1.3.10)$$

$$\Rightarrow q = \pm \sqrt{c^2 - p^2} \quad (1.3.11)$$

The python code for Fig. 1.3.1 is

```
codes/triangle/tri_sss.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_sss.tex
```

1.4. Construct parallelogram  $ABCD$  in Fig. 1.4.1 given that  $BC = 5, AB = 6, \angle C = 85^\circ$ .

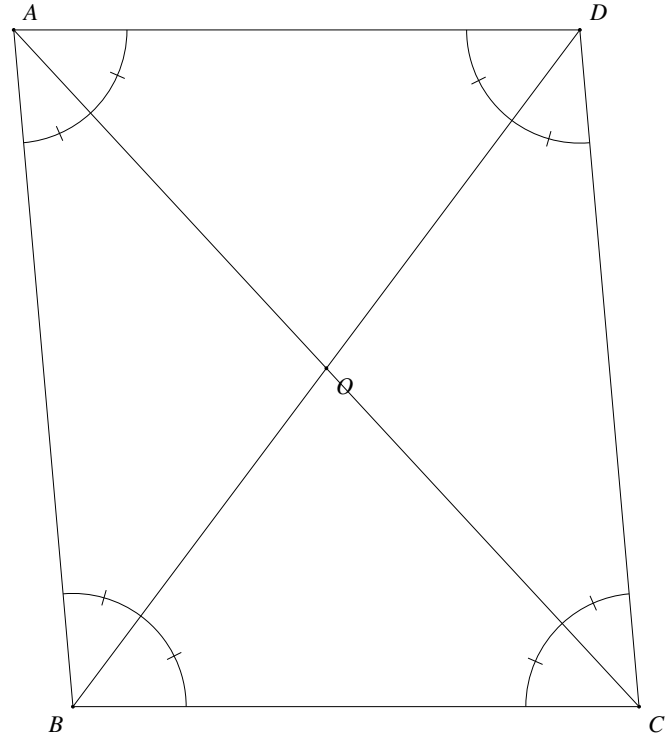


Fig. 1.4.1: Parallelogram Properties

**Solution:**  $BD$  is found using the cosine formula and  $\triangle BDC$  is drawn using the approach in Construction 1.3 with

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \quad (1.4.1)$$

Since the diagonals bisect each other,

$$\mathbf{O} = \frac{\mathbf{B} + \mathbf{D}}{2} \quad (1.4.2)$$

$$\mathbf{A} = 2\mathbf{O} - \mathbf{C}. \quad (1.4.3)$$

$AB$  and  $AD$  are then joined to complete the ||gm. The python code for Fig. 1.4.1 is

```
codes/quad/pgm_sas.py
```

and The equivalent latex-tikz code is

```
figs/quad/pgm_sas.tex
```

1.5. Draw the ||gm  $ABCD$  in Fig. 1.5.1 with  $BC = 6, CD = 4.5$  and  $BD = 7.5$ . Show that it is a rectangle.

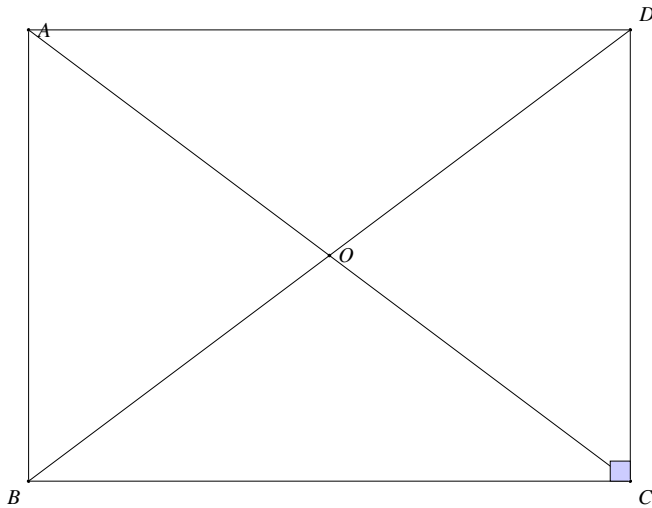


Fig. 1.5.1: Rectangle

**Solution:** It is easy to verify that

$$BD^2 = BC^2 + C^2 \quad (1.5.1)$$

Hence, using Baudhayana theorem,

$$\angle BCD = 90^\circ \quad (1.5.2)$$

and  $ABCD$  is a rectangle.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \quad (1.5.3)$$

The python code for Fig. 1.5.1 is

```
codes/quad/pgm_sss.py
```

and the equivalent latex-tikz code is

```
figs/quad/pgm_sss.tex
```

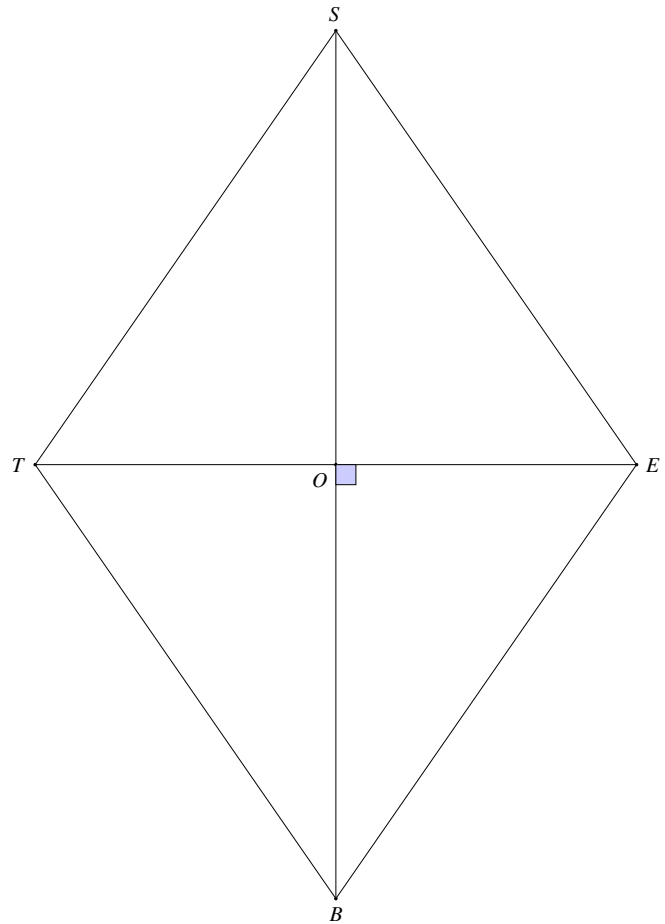


Fig. 1.6.1: Rhombus

in Fig. 1.7.1 are obtained as

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \quad (1.7.1)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 4.5 \\ 4.5 \end{pmatrix} \mathbf{O} = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (1.7.2)$$

- 1.6. Draw the rhombus  $BEST$  with  $BE = 4.5$  and  $ET = 6$ .

**Solution:** The coordinates of the various points in Fig. 1.6.1 are obtained as

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -4.5 \end{pmatrix} \quad (1.6.1)$$

$$\mathbf{E} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \quad (1.6.2)$$

- 1.7. A square is a rectangle whose sides are equal. Draw a square of side 4.5.

**Solution:** The coordinates of the various points

## 2 EXERCISES

- 2.1. Construct a triangle of sides  $a = 4$ ,  $b = 5$  and  $c = 6$ .
- 2.2. Construct an isosceles triangle whose base is  $a = 8\text{cm}$  and altitude  $AD = h = 4\text{cm}$
- 2.3. In  $\triangle ABC$ , given that  $a+b+c = 11$ ,  $\angle B = 45^\circ$  and  $\angle C = 45^\circ$ , find  $a, b, c$  and sketch the triangle.
- 2.4. Draw  $\triangle ABC$  with  $a = 6$ ,  $c = 5$  and  $\angle B = 60^\circ$ .
- 2.5. Draw  $\triangle ABC$  with  $a = 7$ ,  $\angle B = 45^\circ$  and  $\angle A = 105^\circ$ .

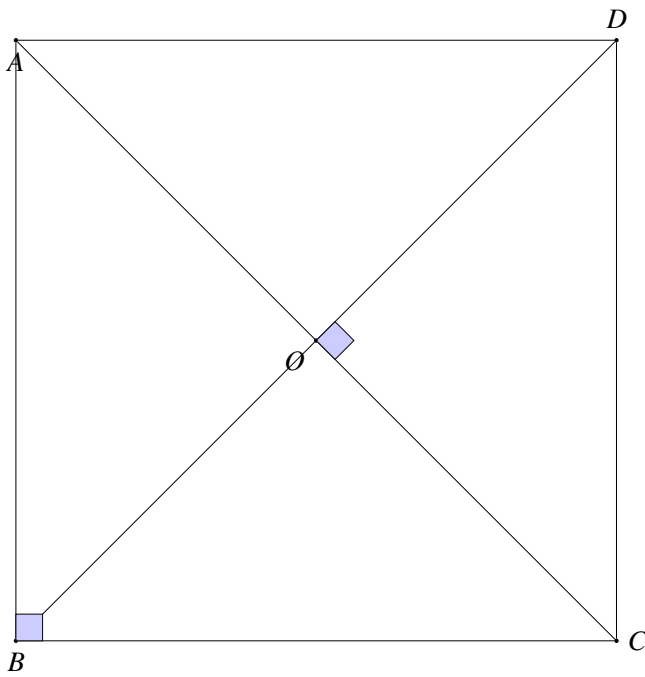


Fig. 1.7.1: Square

- 2.6.  $\triangle ABC$  is right angled at **B**. If  $a = 12$  and  $b+c = 18$ , find  $b, c$  and draw the triangle.
- 2.7. In  $\triangle ABC$ ,  $a = 8, \angle B = 45^\circ$  and  $c - b = 3.5$ . Sketch  $\triangle ABC$ .
- 2.8. In  $\triangle ABC$ ,  $a = 6, \angle B = 60^\circ$  and  $b - c = 2$ . Sketch  $\triangle ABC$ .
- 2.9. Draw  $\triangle ABC$ , given that  $a+b+c = 11, \angle B = 30^\circ$  and  $\angle C = 90^\circ$ .
- 2.10. Construct  $\triangle xyz$  where  $xy = 4.5, yz = 5$  and  $zx = 6$ .
- 2.11. Draw an equilateral triangle of side 5.5.
- 2.12. Draw  $\triangle PQR$  with  $PQ = 4, QR = 3.5$  and  $PR = 4$ . What type of triangle is this?
- 2.13. Construct  $\triangle ABC$  such that  $AB = 2.5, BC = 6$  and  $AC = 6.5$ . Find  $\angle B$ .
- 2.14. Construct  $\triangle PQR$ , given that  $PQ = 3, QR = 5.5$  and  $\angle PQR = 60^\circ$ .
- 2.15. Construct  $\triangle DEF$  such that  $DE = 5, DF = 3$  and  $\angle D = 90^\circ$ .
- 2.16. Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is  $110^\circ$ .
- 2.17. Construct  $\triangle ABC$  with  $BC = 7.5, AC = 5$  and  $\angle C = 60^\circ$ .
- 2.18. Construct  $\triangle XYZ$  if  $XY = 6, \angle X = 30^\circ$  and  $\angle Y = 100^\circ$ .
- 2.19. If  $AC = 7, \angle A = 60^\circ$  and  $\angle B = 50^\circ$ , can you draw the triangle?
- 2.20. Construct  $\triangle ABC$  given that  $\angle A = 60^\circ, \angle B = 30^\circ$  and  $AB = 5.8$ .
- 2.21. Construct  $\triangle PQR$  if  $PQ = 5, \angle Q = 105^\circ$  and  $\angle R = 40^\circ$ .
- 2.22. Can you construct  $\triangle DEF$  such that  $EF = 7.2, \angle E = 110^\circ$  and  $\angle F = 180^\circ$ ?
- 2.23. Construct  $\triangle LMN$  right angled at  $M$  such that  $LN = 5$  and  $MN = 3$ .

**Solution:**

Let

$$\mathbf{L} = \begin{pmatrix} 0 \\ l \end{pmatrix}, \mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{N} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (2.23.1)$$

From the given information,

$$\|\mathbf{N} - \mathbf{M}\|^2 = \|\mathbf{N}\|^2 = 3^2 = 9 \quad (2.23.2)$$

$$\|\mathbf{L} - \mathbf{M}\|^2 = \|\mathbf{L}\|^2 = l^2 \quad (2.23.3)$$

$$\|\mathbf{L} - \mathbf{N}\|^2 = 5^2 = 25 \quad (2.23.4)$$

which can be expressed as

$$\|\mathbf{L} - \mathbf{N}\|^2 = (\mathbf{L} - \mathbf{N})^T (\mathbf{L} - \mathbf{N}) \quad (2.23.5)$$

$$= \|\mathbf{L}\|^2 + \|\mathbf{N}\|^2 - 2\mathbf{L}^T \mathbf{N} \quad (2.23.6)$$

$$\Rightarrow l^2 + 9 = 25 \quad (2.23.7)$$

$$\text{or, } l = \pm 4 \quad (2.23.8)$$

For  $l=4$ ,  $\triangle LMN$  is plotted in the first quadrant in Fig. 2.23.1.

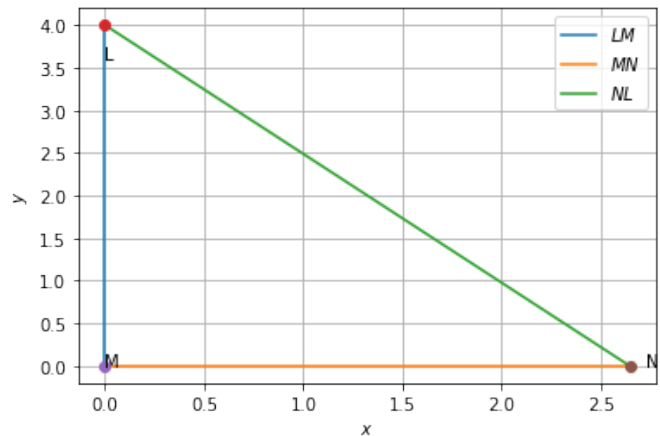


Fig. 2.23.1

- 2.24. Construct  $\triangle PQR$  right angled at  $Q$  such that  $QR = 8$  and  $PR = 10$ .

**Solution:** Let

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ p \end{pmatrix} \quad (2.24.1)$$

Then,

$$\|\mathbf{P} - \mathbf{R}\|^2 = (\mathbf{P} - \mathbf{R})^T (\mathbf{P} - \mathbf{R}) \quad (2.24.2)$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{R}\|^2 \quad (2.24.3)$$

$$\therefore \mathbf{P}^T \mathbf{R} = \mathbf{R}^T \mathbf{P}, \mathbf{R}^T \mathbf{P} = 0 \quad (2.24.4)$$

$$= p^2 + 64 = 10^2 \quad (2.24.5)$$

$$\Rightarrow p = \pm 6 \quad (2.24.6)$$

Since positive area is considered here, only  $p = 6$  is taken into consideration. Thus,

$$\mathbf{P} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (2.24.7)$$

and the desired triangle is plotted in Fig. 2.24.1

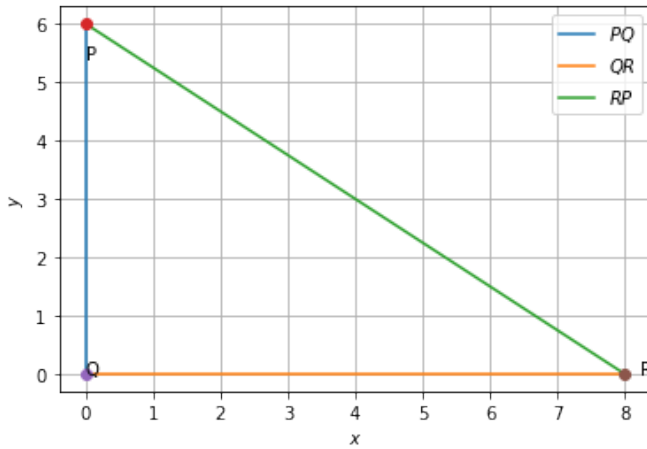


Fig. 2.24.1: Right Angle  $\triangle PQR$

- 2.25. Construct right angled  $\triangle$  whose hypotenuse is 6 and one of the legs is 4.

**Solution:** Let us consider  $\triangle PQR$  right angled at  $Q$  and assume that we are restricted to first quadrant such that

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ p \end{pmatrix} \quad (2.25.1)$$

Then,

$$\|\mathbf{P} - \mathbf{R}\|^2 = 36 \quad (2.25.2)$$

$$\Rightarrow p^2 + 16 = 36 \quad (2.25.3)$$

$$\Rightarrow p = \pm 2\sqrt{5} \quad (2.25.4)$$

Since first quadrant was assumed here, only  $p = +2\sqrt{5}$  is taken into consideration. So, the

vertices of  $\triangle PQR$  in Fig. 2.25.1 are

$$\mathbf{P} = \begin{pmatrix} 0 \\ 2\sqrt{5} \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (2.25.5)$$

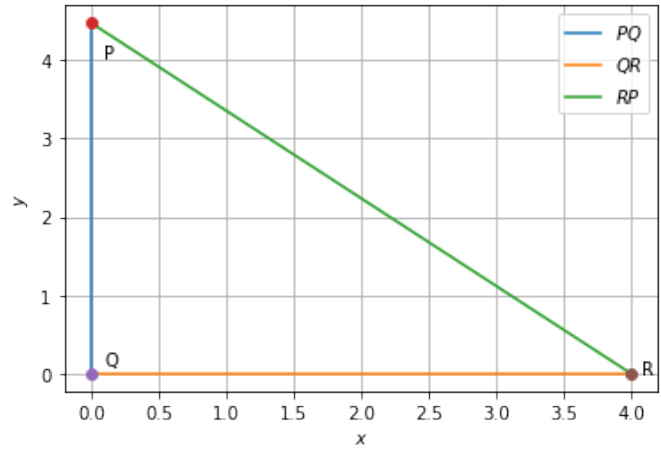


Fig. 2.25.1: Right Angled  $\triangle PQR$

- 2.26. Construct an isosceles right angled  $\triangle ABC$  right angled at  $C$  such that  $AC = 6$ .

**Solution:**

$\therefore \triangle ABC$  is isosceles, its vertices are

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (2.26.1)$$

which are used to plot the desired triangle in Fig. 2.26.1.

- 2.27. Construct the triangles in Table 2.27.1.

S.No	Triangle	Given Measurements		
1	$\triangle ABC$	$\angle A = 85^\circ$	$\angle B = 115^\circ$	$AB = 5$
2	$\triangle PQR$	$\angle Q = 30^\circ$	$\angle R = 60^\circ$	$QR = 4.7$
3	$\triangle ABC$	$\angle A = 70^\circ$	$\angle B = 50^\circ$	$AC = 3$
4	$\triangle LMN$	$\angle L = 60^\circ$	$\angle N = 120^\circ$	$LM = 5$
5	$\triangle ABC$	$BC = 2$	$AB = 4$	$AC = 2$
6	$\triangle PQR$	$PQ = 2.5$	$QR = 4$	$PR = 3.5$
7	$\triangle XYZ$	$XY = 3$	$YZ = 4$	$XZ = 5$
8	$\triangle DEF$	$DE = 4.5$	$EF = 5.5$	$DF = 4$

TABLE 2.27.1

- 2.28. Construct a quadrilateral  $ABCD$  such that  $AB = 5$ ,  $\angle A = 50^\circ$ ,  $AC = 4$ ,  $BD = 5$  and  $AD = 6$ .
- 2.29. Construct  $PQRS$  where  $PQ = 4$ ,  $QR = 6$ ,  $RS = 5$ ,  $PS = 5.5$  and  $PR = 7$ .
- 2.30. Draw  $JUMP$  with  $JU = 3.5$ ,  $UM = 4$ ,  $MP = 5$ ,  $PJ = 4.5$  and  $PU = 6.5$

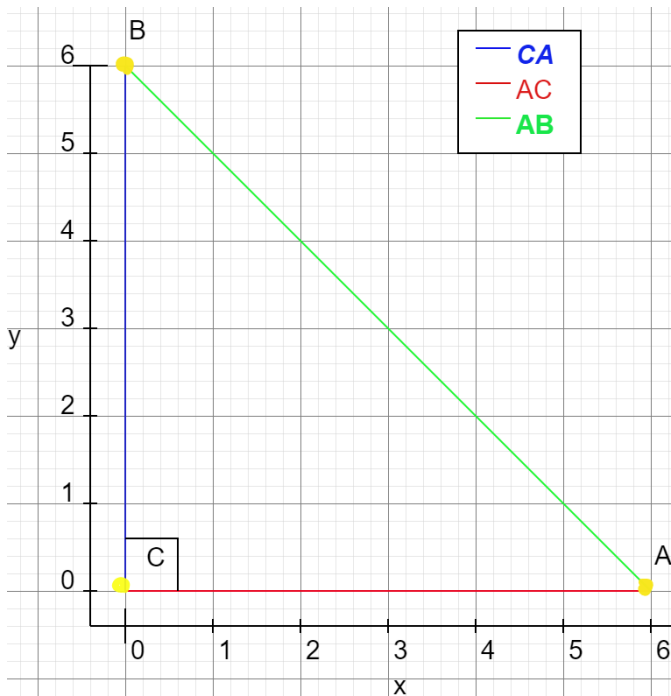


Fig. 2.26.1: Isosceles Right Angle  $\triangle ABC$

2.31. Construct a quadrilateral  $ABCD$  such that  $BC = 4.5, AC = 5.5, CD = 5, BD = 7$  and  $AD = 5.5$ .

2.32. Can you construct a quadrilateral  $PQRS$  with  $PQ = 3, RS = 3, PS = 7.5, PR = 8$  and  $SQ = 4$ ?

**Solution:** From the given information,

$$\|P - Q\| = 3 \quad (2.32.1)$$

$$\|R - S\| = 3 \quad (2.32.2)$$

$$\|P - S\| = 7.5 \quad (2.32.3)$$

$$\|P - R\| = 8 \quad (2.32.4)$$

$$\|S - Q\| = 4 \quad (2.32.5)$$

Let quadrilateral  $PQRS$  be made up of two triangles  $\triangle PSQ$  and  $\triangle PSR$  on base  $PS$ .

a) In  $\triangle PSR$ ,

$$\begin{aligned} \|P - S\| + \|R - S\| &= 7.5 + 3 = 10.5 \\ &> \|P - R\| \end{aligned} \quad (2.32.6)$$

$$\begin{aligned} \|P - R\| + \|R - S\| &= 8 + 3 = 11 > \|P - S\| \\ &\quad (2.32.7) \end{aligned}$$

$$\begin{aligned} \|P - S\| + \|P - R\| &= 7.5 + 8 = 15.5 \\ &> \|R - S\| \end{aligned} \quad (2.32.8)$$

$\therefore$  using triangle inequality, construction of  $\triangle PSR$  is possible.

b) In  $\triangle PSQ$ ,

$$\begin{aligned} \|P - S\| + \|S - Q\| &= 7.5 + 4 = 11.5 \\ &> \|P - Q\| \end{aligned} \quad (2.32.9)$$

$$\begin{aligned} \|P - S\| + \|P - Q\| &= 7.5 + 3 = 10.5 \\ &> \|S - Q\| \end{aligned} \quad (2.32.10)$$

$$\|P - Q\| + \|S - Q\| = 3 + 4 = 7 < \|P - S\| \quad (2.32.11)$$

which violates triangle inequality.  $\therefore$  construction of  $\triangle PSQ$  is not possible.

Fig. 2.32.1 highlights this.

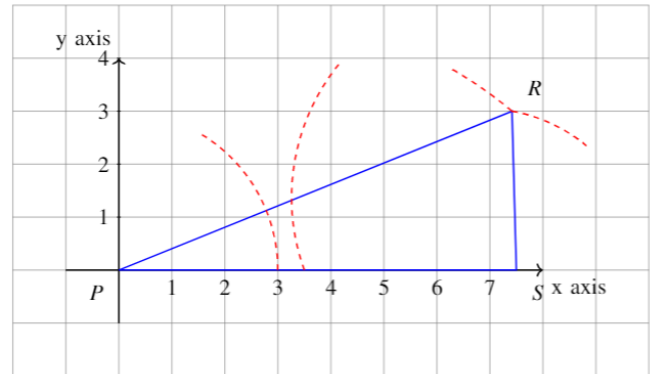


Fig. 2.32.1: Construction of quadrilateral  $PQRS$

2.33. Construct  $LIFT$  such that  $LI = 4, IF = 3, TL = 2.5, LF = 4.5, IT = 4$ .

2.34. Draw  $GOLD$  such that  $OL = 7.5, GL = 6, GD = 6, LD = 5, OD = 10$ .

2.35. DRAW rhombus  $BEND$  such that  $BN = 5.6, DE = 6.5$ .

2.36. construct a quadrilateral MIST where  $MI = 3.5, IS = 6.5, \angle M = 75^\circ, \angle I = 105^\circ$  and  $\angle S = 120^\circ$ .

2.37. Can you construct the above quadrilateral MIST if  $\angle M = 100^\circ$  instead of  $75^\circ$ .

2.38. Can you construct the quadrilateral PLAN if  $PL = 6, LA = 9.5, \angle P = 75^\circ, \angle L = 150^\circ$  and  $\angle A = 140^\circ$ ?

2.39. Construct  $MORE$  where  $MO = 6, OR = 4.5, \angle M = 60^\circ, \angle O = 105^\circ, \angle R = 105^\circ$ .

2.40. Construct  $PLAN$  where  $PL = 4, LA = 6.5, \angle P = 90^\circ, \angle A = 110^\circ$  and  $\angle N = 85^\circ$ .

2.41. Draw rectangle  $OKAY$  with  $OK = 7$  and  $KA = 5$ .

2.42. Construct  $ABCD$ , where  $AB = 4, BC = 5, CD = 6.5, \angle B = 105^\circ$  and  $\angle C = 80^\circ$ .

2.43. Construct  $DEAR$  with  $DE = 4, EA = 5, AR =$

4.5,  $\angle E = 60^\circ$  and  $\angle A = 90^\circ$ .

2.44. Construct *TRUE* with  $TR = 3.5$ ,  $RU = 3$ ,  $UE = 4$ ,  $\angle R = 75^\circ$  and  $\angle U = 120^\circ$ .

2.45. Can you construct a rhombus *ABCD* with  $AC = 6$  and  $BD = 7$ ?

2.46. Draw a square *READ* with  $RE = 5.1$ .

2.47. Draw a rhombus whose diagonals are 5.2 and 6.4.

2.48. Draw a rectangle with adjacent sides 5 and 4.

2.49. Draw a parallelogram *OKAY* with  $OK = 5.5$  and  $KA = 4.2$ .

2.50. Construct a kite *EASY* if  $AY = 8$ ,  $EY = 4$  and  $SY = 6$ .

2.51. Draw a circle of diameter 6.1

2.52. With the same centre **O**, draw two circles of radii 4 and 2.5

**Solution:**

All input values required to plot Fig. 2.52.1 are given in Table 2.52.1 as shown below

	Symbols	Circle1	Circle2
Centre	<b>O</b>	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	$r_1, r_2$	2.5	4
Polar coordinate	$C_1, C_2$	$2.5 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$	$4 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$
Angle	$\theta$	$0-2\pi$	$0-2\pi$

TABLE 2.52.1: Input values

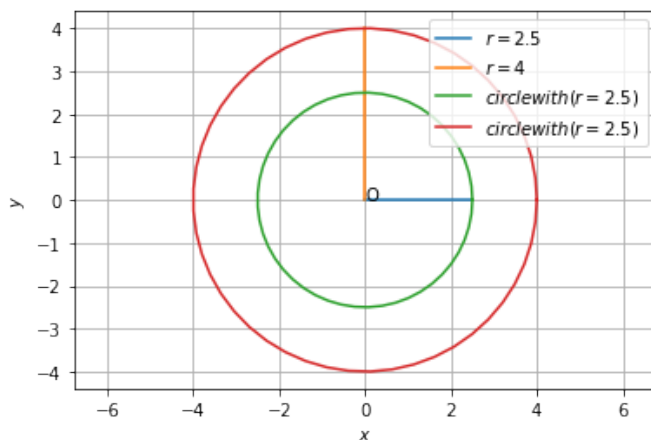


Fig. 2.52.1: Concentric circles with centre as origin and radii 2.5 and 4 respectively

2.53. Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents *AC* and *CD* to the circle.

2.54. Draw a circle of radius 3 and any two of its diameters. Draw the ends of these diameters. What figure do you get?

2.55. Let **A** and **B** be the centres of two circles of equal radii 3 such that each one of them passes through the centre of the other. Let them intersect at **C** and **D**. Is  $AB \perp CD$ ?

2.56. Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

**Solution:** Take the centre of both circles to be at the origin.

2.57. Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.

**Solution:** Take the diameter to be on the *x*-axis.

2.58. Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of  $60^\circ$ .

**Solution:** The tangent is perpendicular to the radius.

2.59. Draw a line segment *AB* of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

**Solution:** Let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}. \quad (2.59.1)$$

2.60. Let *ABC* be a right triangle in which  $a = 8$ ,  $c = 6$  and  $\angle B = 90^\circ$ . *BD* is the perpendicular from **B** on *AC* (altitude). The circle through **B**, **C**, **D** (circumcircle of  $\triangle BCD$ ) is drawn. Construct the tangents from **A** to this circle.

2.61. Draw a circle with centre **C** and radius 3.4. Draw any chord. Construct the perpendicular bisector of the chord and examine if it passes through **C**