ASSIGNMENT 9

B.Ramana

Download all python codes from

```
https://github.com/BatharajuRamana/
    ASSIGNMENT9/tree/main/CODES
```

and latex-tikz codes from

```
https://github.com/BatharajuRamana/
    ASSIGNMENT9/tree/main
```

1 Question No 2.15

One kind of cake requires 200g of flour and 25g of fat and another kind of cake requires 100g flour and 50g of fat. Find the maximum number of cake which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cake.

2 SOLUTION

kind of cake	No.of cakes	Flour	Fat
1st	x	200g	25g
2nd	y	100g	50g
Total	x+y	5 kg=5000g	1kg=1000g

TABLE 2.1: Ingredients used in making the cake is flour and fat

Let the 1st kind be x and the 2nd kind be y such that

$$x \geq 0 \tag{2.0.1}$$
$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$2x + y \leq 50 \tag{2.0.3}$$
$$x + 2y \leq 40 \tag{2.0.4}$$

$\therefore$ Our problem is

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} \tag{2.0.5}$$
$$s.t. \quad \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 50 \\ 40 \end{pmatrix} \tag{2.0.6}$$

Lagrangian function is given by

$$\begin{aligned} &L(\mathbf{x}, \boldsymbol{\lambda}) \\ &= \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 50 \right] \right. \\ &+ \left[\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} - 40 \right] \\ &+ \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.7}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \tag{2.0.8}$$

Now,

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 1 + \begin{pmatrix} 2 & 1 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 1 + \begin{pmatrix} 1 & 2 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 50 \\ \begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} - 40 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.9}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 2 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -1 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 50 \\ 4\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.10}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 5\ 0 \\ 40 \end{pmatrix} \tag{2.0.11}$$

resulting in,

$$\begin{pmatrix}\mathbf{x}\\ \lambda\end{pmatrix} = \begin{pmatrix}0 & 0 & 2 & 1\\ 0 & 0 & 1 & 2\\ 2 & 1 & 0 & 0\\ 1 & 2 & 0 & 0\end{pmatrix}^{-1}\begin{pmatrix}-1\\ -1\\ 50\\ 40\end{pmatrix} \tag{2.0.12}$$

$$\implies \begin{pmatrix}\mathbf{x}\\ \lambda\end{pmatrix} = \begin{pmatrix}0 & 0 & \frac{2}{3} & \frac{-1}{3}\\ 0 & 0 & \frac{-1}{3} & \frac{2}{3}\\ \frac{2}{3} & \frac{-1}{3} & 0 & 0\\ \frac{-1}{3} & \frac{2}{3} & 0 & 0\end{pmatrix}\begin{pmatrix}-1\\ -1\\ 50\\ 40\end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix}\mathbf{x}\\ \lambda\end{pmatrix} = \begin{pmatrix}20\\ 10\\ -0.3\\ -0.3\end{pmatrix} \tag{2.0.14}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix}-0.3\\ -0.3\end{pmatrix} \succ \mathbf{0}$

$\therefore$ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix}20\\ 10\end{pmatrix} \tag{2.0.15}$$

$$Z = \begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} \tag{2.0.16}$$

$$= \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}20\\ 10\end{pmatrix} \tag{2.0.17}$$

$$= 60 \tag{2.0.18}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix}20\\ 10\end{pmatrix} \tag{2.0.19}$$

$$Z = 60 \tag{2.0.20}$$

Hence No.of cakes $\boxed{x = 20}$ 1st kind and .of cakes $\boxed{y = 10}$ 2nd kind should be used to maximum No. of cakes $\boxed{Z = 60}$.

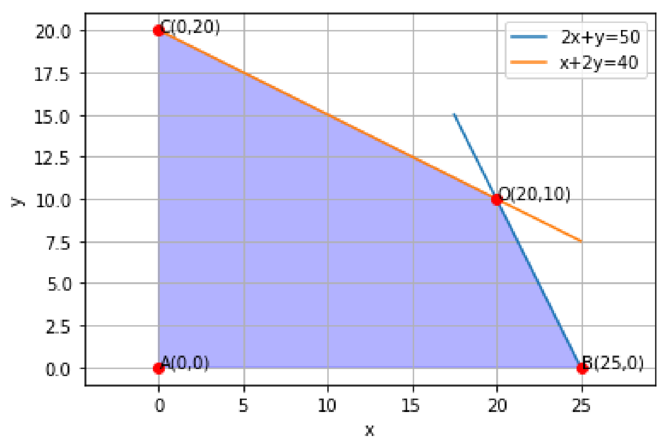

Fig. 2.1: Graphical Solution