ASSIGNMENT 1

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Download all python codes from

https://github.com/vaibhavchhabra25/EE3900–course/blob/main/Assignment–1/codes

and latex-tikz codes from

https://github.com/vaibhavchhabra25/EE3900–course/blob/main/Assignment–1/main.tex

1 Problem

(Vectors-2.19) Find the ratio in which the line segment joining the points $\begin{pmatrix}4\\8\\10\end{pmatrix}$ and $\begin{pmatrix}6\\10\\-8\end{pmatrix}$ is divided by the YZ plane.

2 Solution

Let $\mathbf{A} = \begin{pmatrix}4\\8\\10\end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix}6\\10\\-8\end{pmatrix}$.
Let $\mathbf{P}$ be the intersecting point of line $\mathbf{AB}$ and the YZ plane. Let the ratio in which $\mathbf{P}$ divides $\mathbf{AB}$ be $k : 1$.
Then,

$$\mathbf{P} - \mathbf{A} = k(\mathbf{B} - \mathbf{P}) \tag{2.0.1}$$

$$\mathbf{P} = \frac{\mathbf{A} + k\mathbf{B}}{k+1} \tag{2.0.2}$$

Vector equation of YZ plane with normal vector $\mathbf{n}$ and perpendicular distance from origin d is

$$\mathbf{n}^\top \mathbf{X} = d \tag{2.0.3}$$

Since $\mathbf{P}$ lies on YZ plane,

$$\mathbf{n}^\top \mathbf{P} = d \tag{2.0.4}$$

$$\implies \mathbf{n}^\top \left(\frac{\mathbf{A} + k\mathbf{B}}{k+1}\right) = d \tag{2.0.5}$$

$$\implies \mathbf{n}^\top \mathbf{A} + k\mathbf{n}^\top \mathbf{B} = d(k+1) \tag{2.0.6}$$

$$\implies k = \frac{d - \mathbf{n}^\top \mathbf{A}}{\mathbf{n}^\top \mathbf{B} - d} \tag{2.0.7}$$

For YZ plane, $\mathbf{n} = \begin{pmatrix}1\\0\\0\end{pmatrix}$ and $d = 0$. So,

$$k = \frac{0 - \begin{pmatrix}1 & 0 & 0\end{pmatrix}\mathbf{A}}{\begin{pmatrix}1 & 0 & 0\end{pmatrix}\mathbf{B} - 0} \tag{2.0.8}$$

$$\implies k = \frac{-\begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}4\\8\\10\end{pmatrix}}{\begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}6\\10\\-8\end{pmatrix}} \tag{2.0.9}$$

$$\implies k = \frac{-4}{6} \tag{2.0.10}$$

$$\implies k = -2/3 \tag{2.0.11}$$

So, YZ plane divides line segment $\mathbf{AB}$ externally in the ratio 2:3.
Also, using (2.0.2)

$$\mathbf{P} = \frac{\mathbf{A} - (2/3)\mathbf{B}}{(-2/3) + 1} = 3\mathbf{A} - 2\mathbf{B} \tag{2.0.12}$$

$$\implies \mathbf{P} = 3\begin{pmatrix}4\\8\\10\end{pmatrix} - 2\begin{pmatrix}6\\10\\-8\end{pmatrix} = \begin{pmatrix}0\\4\\46\end{pmatrix} \tag{2.0.13}$$

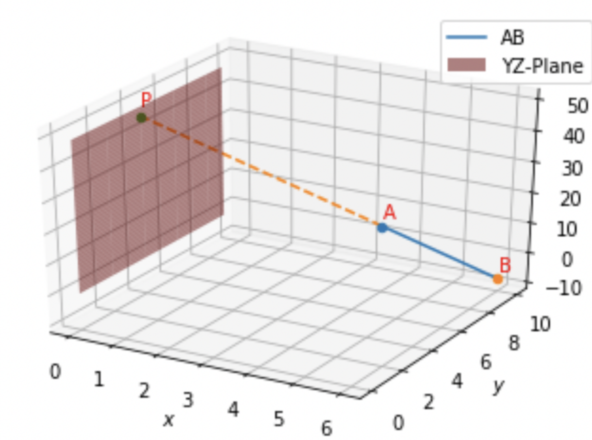

Fig. 0: 3D plot