Geometric Constructions through Python

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Contents

***Abstract*—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.**

Download all codes for this manual from

```
svn co https://github.com/gadepall/school/trunk/
    geometry/constructions/codes
```

1 Triangle

1.1 Draw a line segement of length 7.6 cm and divide it in the ratio 5 : 8.
Solution: Let the end points of the line be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.6 \\ 0 \end{pmatrix} \tag{1}$$

Then the point $\mathbf{C}$

$$\mathbf{C} = \frac{k\mathbf{A} + \mathbf{B}}{k+1} \tag{2}$$

divides AB in the ration $k : 1$. For the given problem, $k = \frac{5}{8}$. The following code plots Fig. 1.1

```
codes/draw_section.py
```

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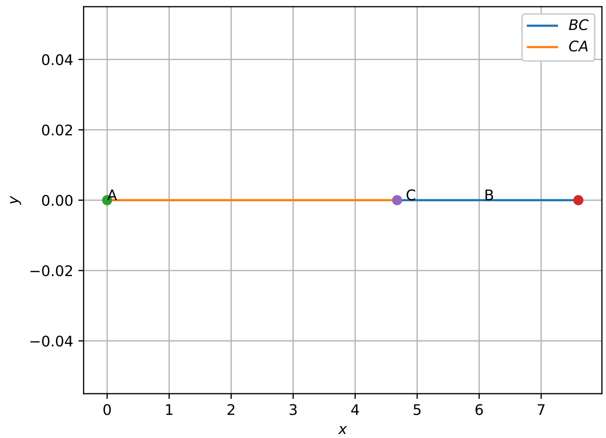

Fig. 1.1

1.2 Draw $\triangle ABC$ where $\angle B = 90°, a = 4$ and $b = 3$.
Solution: The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{3}$$

The following code plots Fig. 1.2

```
codes/rt_triangle.py
```

1.3 Construct a triangle of sides $a = 4$, $b = 5$ and $c = 6$.
Solution: Let the vertices of $\triangle ABC$ be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{4}$$

Then

$$\|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \tag{5}$$

$$\|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \tag{6}$$

$$\|\mathbf{A} - \mathbf{C}\|^2 = b^2 \tag{7}$$

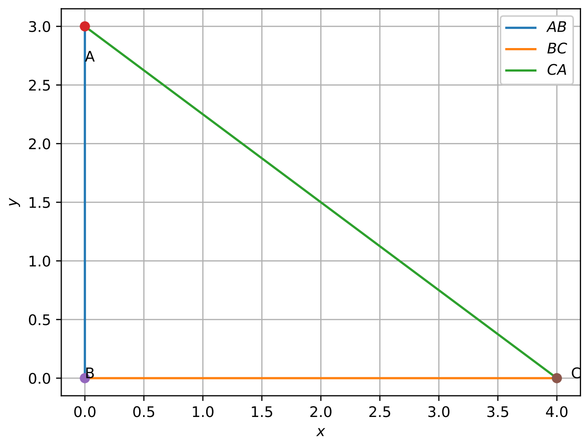

Fig. 1.2

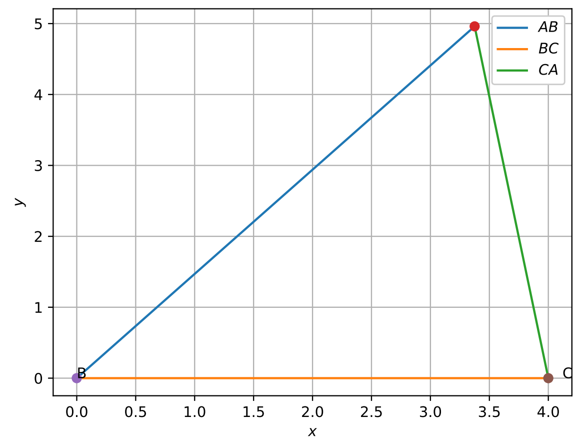

Fig. 1.3

From (7), yielding

$$b^2 = \|\mathbf{A}-\mathbf{C}\|^2 = \|\mathbf{A}-\mathbf{C}\|^T \|\mathbf{A}-\mathbf{C}\| \tag{8}$$
$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{9}$$
$$= a^2 + c^2 - 2ap \tag{10}$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \tag{11}$$

From (5),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \tag{12}$$
$$\implies q = \sqrt{c^2 - p^2} \tag{13}$$

The following code plots Fig. 1.3

```
codes/draw_triangle.py
```

1.4 Construct a triangle of sides $a = 5$, $b = 6$ and $c = 7$. Construct a similar triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the first triangle.
Solution: The sides of the similar triangle are $\frac{7}{5}a, \frac{7}{5}b$ and $\frac{7}{5}c$.

1.5 Construct an isosceles triangle whose base is $a = 8$cm and altitude $AD = p = 4$cm
Solution: Using Baudhayana's theorem,

$$b = c = \sqrt{p^2 + \left(\frac{a}{2}\right)^2} \tag{14}$$

1.6 Draw $\triangle ABC$ with $a = 6, c = 5$ and $\angle B = 60°$.
Solution: In Fig. (1.6), $AD \perp BC$.

$$\cos C = \frac{y}{b}, \tag{15}$$
$$\cos B = \frac{x}{b}, \tag{16}$$

Thus,

$$a = x + y = b\cos C + c\cos B, \tag{17}$$
$$b = c\cos A + a\cos C \tag{18}$$
$$c = b\cos A + a\cos B \tag{19}$$

The above equations can be expressed in matrix form as

$$\begin{pmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{pmatrix} \begin{pmatrix} \cos A \\ \cos B \\ \cos C \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{20}$$

Using the properties of determinants,

$$\cos A = \frac{\begin{vmatrix} a & c & b \\ b & 0 & a \\ c & a & 0 \end{vmatrix}}{\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}} = \frac{ab^2 + ac^2 - a^3}{abc + abc} \tag{21}$$
$$= \frac{b^2 + c^2 - a^2}{2bc} \tag{22}$$

From (22)

$$b^2 = c^2 + a^2 - 2ca\cos B \tag{23}$$

which is computed by the following code

```
codes/cos_form.py
```

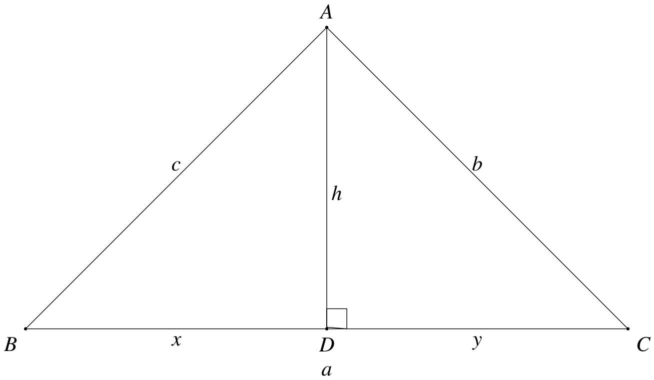

Fig. 1.6: The cosine formula

1.7 Draw $\triangle ABC$ with $a = 7, \angle B = 45°$ and $\angle A = 105°$.
Solution: In Fig. (1.6),

$$\sin B = \frac{h}{c} \quad (24)$$

$$\sin C = \frac{h}{b} \quad (25)$$

which can be used to show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (26)$$

Thus,

$$c = \frac{a \sin C}{\sin A} \quad (27)$$

where

$$C = 180 - A - B \quad (28)$$

1.8 $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find b, c and draw the triangle.
Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \quad (29)$$

$$\implies (18 - c)^2 = 12^2 + c^2 \quad (30)$$

which can be simplified to obtain

$$36c - 180 = 0 \quad (31)$$

$$\implies c = 5 \quad (32)$$

and $b = 13$

1.9 In $\triangle ABC$, $a = 7, \angle B = 75^{\circ}$ and $b + c = 13$. Find b and c and sketch $\triangle ABC$.
Solution: Use cosine formula.

1.10 In $\triangle ABC$, $a = 8, \angle B = 45^{\circ}$ and $c - b = 3.5$. Sketch $\triangle ABC$.

1.11 In $\triangle ABC$, $a = 6, \angle B = 60^{\circ}$ and $b - c = 2$. Sketch $\triangle ABC$.

1.12 In $\triangle ABC$, given that $a + b + c = 11, \angle B = 45^{\circ}$ and $\angle C = 45^{\circ}$, find a, b, c.
Solution: We have

$$a = b \cos C + c \cos B \quad (33)$$

$$b \sin C = c \sin B \quad (34)$$

$$a + b + c = 11 \quad (35)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (36)$$

Solving the equivalent matrix equation gives the desired answer.

1.13 Draw $\triangle ABC$, given that $a+b+c = 11, \angle B = 30^{\circ}$ and $\angle C = 90^{\circ}$.

1.14 Construct $\triangle xyz$ where $xy = 4.5, yz = 5$ and $zx = 6$.

1.15 Draw an equilateral triangle of side 5.5.

1.16 Draw $\triangle PQR$ with $PQ = 4, QR = 3.5$ and $PR = 4$. What type of triangle is this?

1.17 Construct $\triangle ABC$ such that $AB = 2.5, BC = 6$ and $AC = 6.5$. Find $\angle B$.

1.18 Construct $\triangle PQR$, given that $PQ = 3, QR = 5.5$ and $\angle PQR = 60°$.

1.19 Draw $\triangle ABC$ if $AB = 3, AC = 5$ and $\angle C = 30°$.

1.20 Construct $\triangle DEF$ such that $DE = 5, DF = 3$ and $\angle D = 90°$.

1.21 Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110°.

1.22 Construct $\triangle ABC$ with $BC = 7.5, AC = 5$ and $\angle C = 60°$.

1.23 Construct $\triangle XYZ$ if $XY = 6, \angle X = 30°$ and $\angle Y = 100°$.

1.24 If $AC = 7, \angle A = 60°$ and $\angle B = 50°$, can you draw the triangle?

1.25 Construct $\triangle ABC$ given that $\angle A = 60°, \angle B = 30°$ and $AB = 5.8$.

1.26 Construct $\triangle PQR$ if $PQ = 5, \angle Q = 105°$ and $\angle R = 40°$.

1.27 Can you construct $\triangle DEF$ such that $EF = 7.2, \angle E = 110°$ and $\angle F = 180°$?

1.28 Construct $\triangle LMN$ right angled at M such that $LN = 5$ and $MN = 3$.

1.29 Construct $\triangle PQR$ right angled at Q such that $QR = 8$ and $PR = 10$.

1.30 Construct right angled $\triangle$ whose hypotenuse is 6 and one of the legs is 4.

1.31 Construct an isosceles right angled $\triangle ABC$ right angled at C such $AC = 6$.
1.32 Construct the triangles in Table 1.32.

S.No	Triangle	Given Measurements		
1	△ABC	$\angle A = 85°$	$\angle B = 115°$	AB = 5
2	△PQR	$\angle Q = 30°$	$\angle R = 60°$	QR = 4.7
3	△ABC	$\angle A = 70°$	$\angle B = 50°$	AC = 3
4	△LMN	$\angle L = 60°$	$\angle N = 120°$	LM = 5
5	△ABC	BC = 2	AB = 4	AC = 2
6	△PQR	PQ = 2.5	QR = 4	PR = 3.5
7	△XYZ	XY = 3	YZ = 4	XZ = 5
8	△DEF	DE = 4.5	EF = 5.5	DF = 4

TABLE 1.32

2 Circle

2.1 Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents AC and CD to the circle.
Solution: The tangent is perpendicular to the radius. From the given information, in $\triangle ABC, AC \perp AB, a = 10$ and $c = 6$.

$$b = \sqrt{a^2 - c^2} \tag{37}$$

The following code plots Fig. 2.1

```
codes/draw_circle_eg.py
```

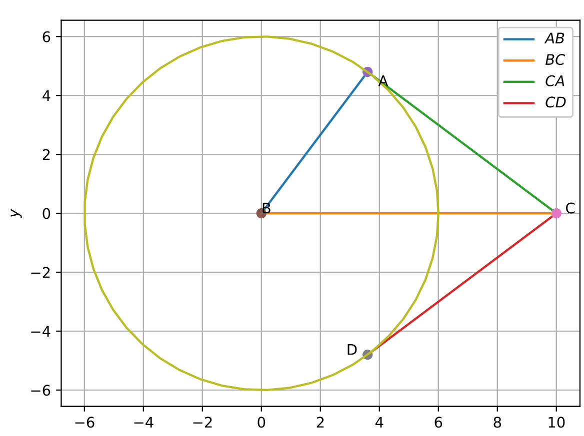

Fig. 2.1

2.2 Draw a circle of diameter 6.1
2.3 Draw a circle of radius 3. Mark any point **A** on the circle, point **B** inside the circle and point **C** outside the circle.
Solution: For any angle θ, a point on the circle with radius 3 has coordinates

$$3\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{38}$$

2.4 With the same centre **O**, draw two circles of radii 4 and 2.5
2.5 Draw a circle of radius 3 and any two of its diameters. draw the ends of these diameters. What figure do you get?
2.6 Let **A** and **B** be two circles of equal radii 3 such that each one of them passes through the centre of the other. Let them intersect at **C** and **D**. Is $AB \perp CD$?
2.7 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.
Solution: Take the centre of both circles to be at the origin.
2.8 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.
Solution: Take the diameter to be on the x-axis.
2.9 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60°.
Solution: The tangent is perpendicular to the radius.
2.10 Draw a line segment AB of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.
Solution: Let

$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{B} = \begin{pmatrix}8\\0\end{pmatrix}. \tag{39}$$

2.11 Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^{\circ}$. BD is the perpendicular from **B** on AC (altitude). The circle through **B**, **C**, **D** (circumcircle of $\triangle BCD$) is drawn. Construct the tangents from **A** to this circle.
2.12 Draw a circle with centre **C** and radius 3.4. Draw any chord. Construct the perpendicular

bisector of the chord and examine if it passes through **C**

3 Quadrilaterals

3.1 Draw $ABCD$ with $AB = a = 4.5, BC = b = 5.5, CD = c = 4, AD = d = 6$ and $AC = e = 7$.
Solution: Fig. 3.1 shows a rough sketch of $ABCD$. Letting

$$\mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{40}$$

it is trivial to sketch $\triangle ABC$ from Problem 1.3. $\triangle ACD$ is can be obtained by rotating an equivalent triangle with AC on the x-axis by an angle θ with

$$\mathbf{D} = \begin{pmatrix} h \\ k \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} e \\ 0 \end{pmatrix} \tag{41}$$

and

$$\cos\theta = \frac{a^2 + e^2 - b^2}{2ae} \tag{42}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} \tag{43}$$

The coordinates of the rotated triangle ACD are

$$\mathbf{D} = \mathbf{P}\begin{pmatrix} h \\ k \end{pmatrix} \tag{44}$$

$$\mathbf{A} = \mathbf{P}\begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{45}$$

$$\mathbf{C} = \mathbf{P}\begin{pmatrix} e \\ 0 \end{pmatrix} \tag{46}$$

where

$$\mathbf{P} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{47}$$

The following code plots quadrilateral $ABCD$ in Fig. 3.1

```
codes/draw_quad.py
```

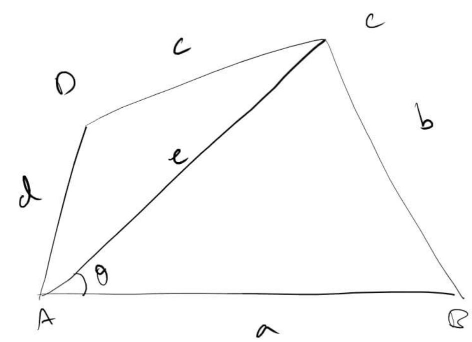

Fig. 3.1

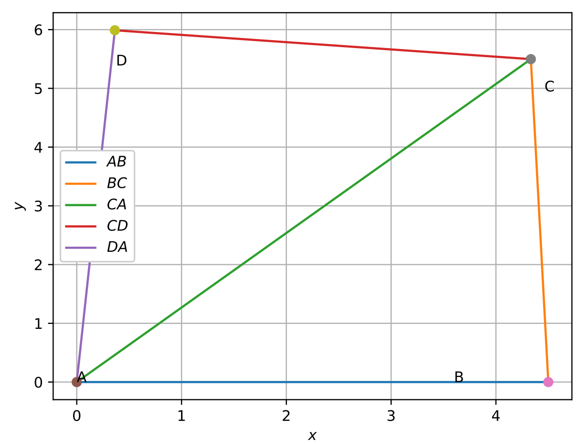

Fig. 3.1

3.2 Construct a quadrilateral $ABCD$ such that $AB = 5, \angle A = 50°, AC = 4, BD = 5$ and $AD = 6$.

3.3 Construct $PQRS$ where $PQ = 4, QR = 6, RS = 5, PS = 5.5$ and $PR = 7$.

3.4 Draw $JUMP$ with $JU = 3.5, UM = 4, MP = 5, PJ = 4.5$ and $PU = 6.5$

3.5 Draw the parallelogram $MORE$ with $OR = 6, RE = 4.5$ and $EO = 7.5$.
Solution: Diagonals of a parallelogram bisect each other. Opposite sides of a parallelogram are equal and parallel .

3.6 Draw the rhombus $BEST$ with $BE = 4.5$ and $ET = 6$.
Solution: Diagonals of a rhombus bisect each other at right angles.

3.7 Construct a quadrilateral $ABCD$ such that $BC = 4.5, AC = 5.5, CD = 5, BD = 7$ and $AD = 5.5$.

3.8 Can you construct a quadrilateral $PQRS$ with $PQ = 3, RS = 3, PS = 7.5, PR = 8$ and $SQ = 4$?

3.9 Construct $LIFT$ such that $LI = 4, IF = 3, TL = 2.5, LF = 4.5, IT = 4$.

3.10 Draw $GOLD$ such that $OL = 7.5, GL = 6, GD = 6, LD = 5, OD = 10$.

3.11 DRAW rhombus $BEND$ such that $BN = 5.6$, $DE = 6.5$.

3.12 construct a quadrilateral MIST where $MI =$

$3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

3.13 Can you construct the above quadrilateral MIST if $\angle M = 100°$ instead of $75°$.

3.14 Can you constrcut the quadrilateral PLAN if $PL = 6, LA = 9.5, \angle P = 75°, \angle L = 150°$ and $\angle A = 140°$?

3.15 Construct $MORE$ where $MO = 6, OR = 4.5, \angle M = 60°, \angle O = 105°, \angle R = 105°$.

3.16 Construct $PLAN$ where $PL = 4, LA = 6.5, \angle P = 90°, \angle A = 110°$ and $\angle N = 85°$.

3.17 Constrcut parallelogram $HEAR$ where $HE = 5, EA = 6, \angle R = 85°$.

3.18 Draw rectangle $OKAY$ with $OK = 7$ and $KA = 5$.

3.19 Construct $ABCd$, where $AB = 4, BC = 5, Cd = 6.5, \angle B = 105°$ and $\angle C = 80°$.

3.20 Construct $DEAR$ with $DE = 4, EA = 5, AR = 4.5, \angle E = 60°$ and $\angle A = 90°$.

3.21 Construct $TRUE$ with $TR = 3.5, RU = 3, UE = 4\angle R = 75°$ and $\angle U = 120°$.

3.22 Draw a square of side 4.5.

3.23 Can you construct a rhombus $ABCD$ with $AC = 6$ and $BD = 7$?

3.24 Construct a kite $EASY$ if $AY = 8, EY = 4$ and $SY = 6$.

Solution: The diagonals of a kite are perpendicular to each other.

3.25 Draw a square $READ$ with $RE = 5.1$.

3.26 Draw a rhombus who diagonals are 5.2 and 6.4.

3.27 Draw a rectangle with adjacent sides 5 and 4.

3.28 Draw a parallelogram $OKAY$ with $OK = 5.5$ and $KA = 4.2$.

4 Properties of a Triangle

4.1 *Cosine Formula:*

$$c^2 = a^2 + b^2 - 2ab\cos C \tag{48}$$

4.2 *Sine Formula:*

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = 2R, \tag{49}$$

where R is the radius of the circumcircle of $\triangle ABC$.

4.3 Sum of two sides is always greater than the third side.

$$a + b > c \tag{50}$$

$$b + c > a \tag{51}$$

$$c + a > b \tag{52}$$