

Math Document Template

C ANISH

Abstract—This is a document explaining for a question about the concept of congruence.

Download all python codes from

```
svn co https://github.com/chakki1234/summer
-2020/trunk/traingle/codes
```

and latex-tikz codes from

```
svn co https://github.com/chakki1234/summer
-2020/trunk/traingle/figs
```

Design Parameters	
Parameters	Value
AC	5
$\angle A$	80
λ	6

TABLE 2.2: Quadilateral $ACBD$

1 PROBLEM

In quadilateral $ACBD$, $AC = AD$ and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD ?

2 CONSTRUCTION

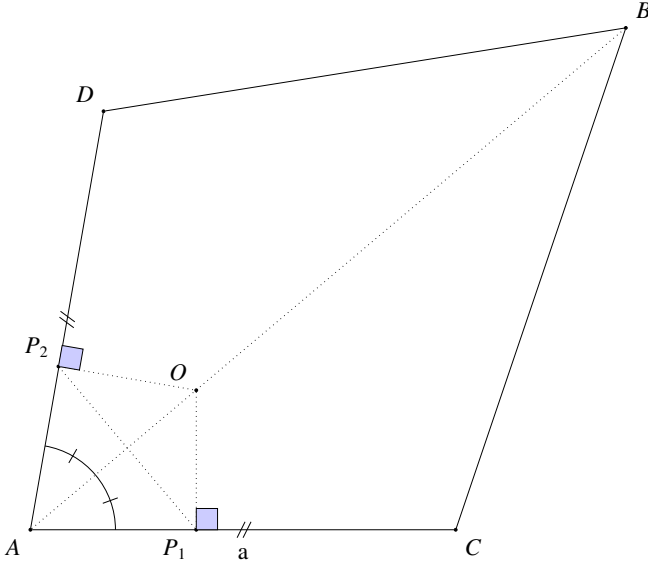


Fig. 2.0: Quadilateral by Latex-Tikz

2.1. The figure obtained looks like Fig. 2.0.

$AC = AD = a$, $\angle DAB = \angle BAC$ and $\angle A = \theta$.

2.2. The design parameters used for construction

Solution: See Table. 2.2.

2.3. Find the coordinates of the various points in Fig

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.3.1)$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad (2.3.2)$$

$$\mathbf{D} = a \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2.3.3)$$

2.4. **Proof:** To find the angle bisector if three points are know.

Solution: Consider $\triangle DAC$ from the quadilateral, let O be the incenter and P_1 be the point where the perpendicular from O intersects AC . Similarly let P_2 be the point where the perpendicular from O intersects AD . Let A be at the origin.

$$\mathbf{P}_1 = \frac{\mathbf{C}}{\|\mathbf{C}\|} \quad (2.4.1)$$

$$\mathbf{P}_2 = \frac{\mathbf{D}}{\|\mathbf{D}\|} \quad (2.4.2)$$

$$\mathbf{P}_1 \mathbf{P}_2 \perp \mathbf{AB} \quad (2.4.3)$$

$$\mathbf{B} \in (\mathbf{P}_1 - \mathbf{P}_2) \quad (2.4.4)$$

$$\|\mathbf{P}_1\| = \|\mathbf{P}_2\| \quad (2.4.5)$$

$$\|\mathbf{P}_1\|^2 = \|\mathbf{P}_2\|^2 \quad (2.4.6)$$

$$(\mathbf{P}_1 - \mathbf{P}_2)^T (\mathbf{P}_1 + \mathbf{P}_2) = 0 \quad (2.4.7)$$

$$\mathbf{P}_1 + \mathbf{P}_2 \perp \mathbf{P}_1 - \mathbf{P}_2 \quad (2.4.8)$$

From 2.4.4 and 2.4.8, The equation of AB is:

$$\mathbf{AB} = \lambda (\mathbf{P}_1 + \mathbf{P}_2) \quad (2.4.9)$$

$$\Rightarrow \mathbf{B} = \lambda \left(\frac{\mathbf{C}}{\|\mathbf{C}\|} + \frac{\mathbf{D}}{\|\mathbf{D}\|} \right) \quad (2.4.10)$$

If A is not at the origin :

$$\mathbf{B} = \mathbf{A} + \lambda \left(\frac{\mathbf{C}}{\|\mathbf{C}\|} + \frac{\mathbf{D}}{\|\mathbf{D}\|} \right) \quad (2.4.11)$$

λ is the length of the angle bisector

Solution: From the given information, The values are listed in 2.4

Ouput values	
Parameter	value
B	$\begin{pmatrix} 7 \\ 5.9 \end{pmatrix}$

TABLE 2.4: **B**

The above latex code can be compiled as a standalone document as

figs/quad_final.tex

3 SOLUTION

$\triangle ABC \cong \triangle ABD$ by SAS congruence:

$$AD = AC$$

$$\angle DAB = \angle BAC$$

DB is common

$$\Rightarrow BC = BD$$

2.5. Draw Fig. 2.5.

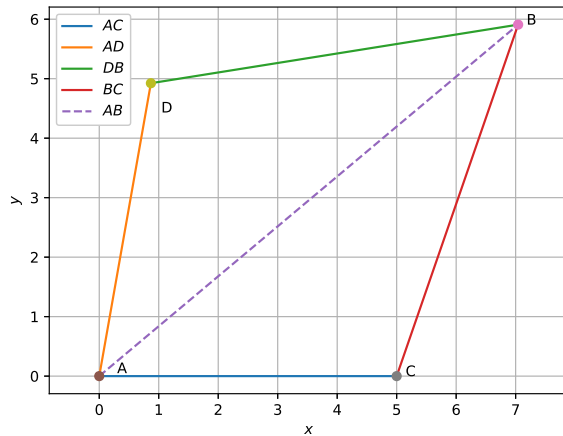


Fig. 2.5: quadilateral generated using python

Solution: The following Python code generates Fig. 2.5

codes/quad.py

and the equivalent latex-tikz code generating Fig. 2.5 is

figs/quad_fig.tex