

Math Document Template

C ANISH

Abstract—This is a document explaining a question about the concept of cyclic quadrilateral.

Download all python codes from

svn co <https://github.com/chakki1234/summer-2020/trunk/Circle/codes>

and latex-tikz codes from

svn co <https://github.com/chakki1234/summer-2020/trunk/Circle/figs>

Design Parameters	
Parameters	Value
b	5
h	3
$\angle A$	70

TABLE 2.2: Trapezium $ABCD$

1 PROBLEM

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

2 CONSTRUCTION

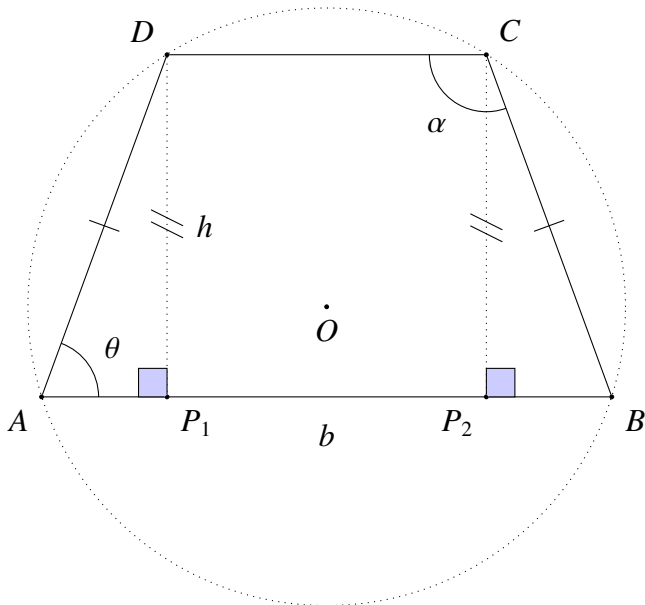


Fig. 2.0: Trapezium by Latex-Tikz

2.1. The figure obtained looks like Fig. 2.0.

$AD = BC$, $AB \parallel DC$.

2.2. The design parameters used for construction See Table. 2.2.

2.3. Find the coordinates of the various points in Fig

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.3.1)$$

$$\mathbf{B} = \mathbf{A} + \begin{pmatrix} b \\ 0 \end{pmatrix}, \quad (2.3.2)$$

$$\mathbf{C} = \begin{pmatrix} b - h \cot \theta \\ h \end{pmatrix} \quad (2.3.3)$$

$$\mathbf{D} = \begin{pmatrix} h \cot \theta \\ h \end{pmatrix} \quad (2.3.4)$$

2.4. **Solution:** From the given information, The values are listed in 2.4

Ouput values	
Parameter	Value
C	$\begin{pmatrix} 3.9 \\ 3 \end{pmatrix}$
D	$\begin{pmatrix} 1.09 \\ 3 \end{pmatrix}$

TABLE 2.4: Values of \mathbf{C} and \mathbf{D}

2.5. Draw Fig. 2.0.

Solution: The following Python code generates Fig. 2.5

codes/quad.py

and the equivalent latex-tikz code generating Fig. 2.5 is

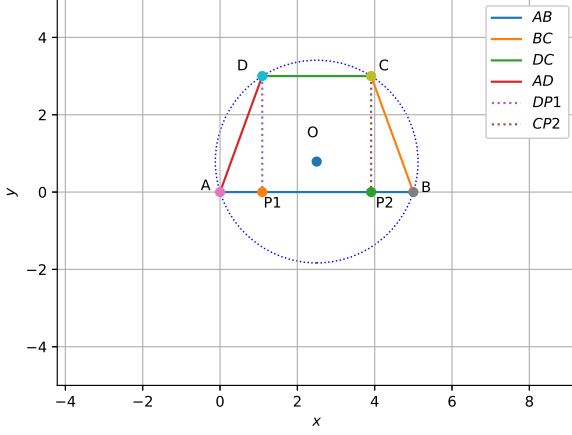


Fig. 2.5: Trapezium generated using python

```
figs/trapezium_altitude_fig.tex
```

The above latex code can be compiled as a standalone document as

```
figs/trapezium_final_altitude.tex
```

3 SOLUTION

Theorem 3.1. *In a cyclic quadrilateral, the sum of each pair of opposite angles is 180° .*

Solution: From theorem 3.1 to prove $ABCD$ is a cyclic quadrilateral, it is sufficient to prove that sum of opposite angles is 180° .

From 2.3.1, 2.3.2, 2.3.3, 2.3.4

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} b \\ 0 \end{pmatrix} \quad (5.1)$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 2h \cot \theta - b \\ 0 \end{pmatrix} \quad (5.2)$$

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix} h \cot \theta \\ h \end{pmatrix} \quad (5.3)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} h \cot \theta \\ -h \end{pmatrix} \quad (5.4)$$

Finding the scalar products:

$$(\mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{A}) = \|\mathbf{D} - \mathbf{A}\| \|\mathbf{B} - \mathbf{A}\| \cos \theta \quad (5.5)$$

$$(\mathbf{D} - \mathbf{C})^T (\mathbf{B} - \mathbf{C}) = \|\mathbf{D} - \mathbf{C}\| \|\mathbf{B} - \mathbf{C}\| \cos \alpha \quad (5.6)$$

Dividing 5.6 with 5.5:

$$\frac{(\mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{A})}{(\mathbf{D} - \mathbf{C})^T (\mathbf{B} - \mathbf{C})} = \frac{\|\mathbf{D} - \mathbf{A}\| \|\mathbf{B} - \mathbf{A}\| \cos \theta}{\|\mathbf{D} - \mathbf{C}\| \|\mathbf{B} - \mathbf{C}\| \cos \alpha} \quad (5.7)$$

Since $\|\mathbf{D} - \mathbf{A}\| = \|\mathbf{B} - \mathbf{C}\|$, 5.7 can be simplified to the form:

$$\frac{(\mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{A})}{(\mathbf{D} - \mathbf{C})^T (\mathbf{B} - \mathbf{C})} = \frac{\|\mathbf{B} - \mathbf{A}\| \cos \theta}{\|\mathbf{D} - \mathbf{C}\| \cos \alpha} \quad (5.8)$$

Substituting values from 5.1, 5.2, 5.3 and 5.4:

$$\frac{bh \cot \theta}{(2h \cot \theta - b) h \cot \theta} = \frac{b \cos \theta}{b - 2h \cot \theta} \quad (5.9)$$

$$\Rightarrow \cos \alpha = -\cos \theta \quad (5.10)$$

$$\Rightarrow \alpha + \theta = 180^\circ \quad (5.11)$$

$\therefore ABCD$ is a cyclic quadrilateral.