Problems On Geometry Of Circle

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***Abstract*—This document prooves a circle theoram with the help of different figures and tables written in python and latex .**

Download all python codes from

```
svn co svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/circle/
    codes
```

and latex-tikz codes from

```
svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/circle/
    figures
```

1 Problem

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.

2 Construction

2.1. We have two equal angles made by a line segment which makes two triangles ABC and BCD.Both the triangles has one angle and a side eqaual.We will draw a triangle with sides a ,b and c and calculate the angle $\angle\theta$ and then after that with this angle and base a we will draw one more triangle.

2.2. Values of all three sides of the triangle are as given in the table .

Parameter	Value
a	5
b	6
c	4
d	4

TABLE 2.2: To construct circumecircle

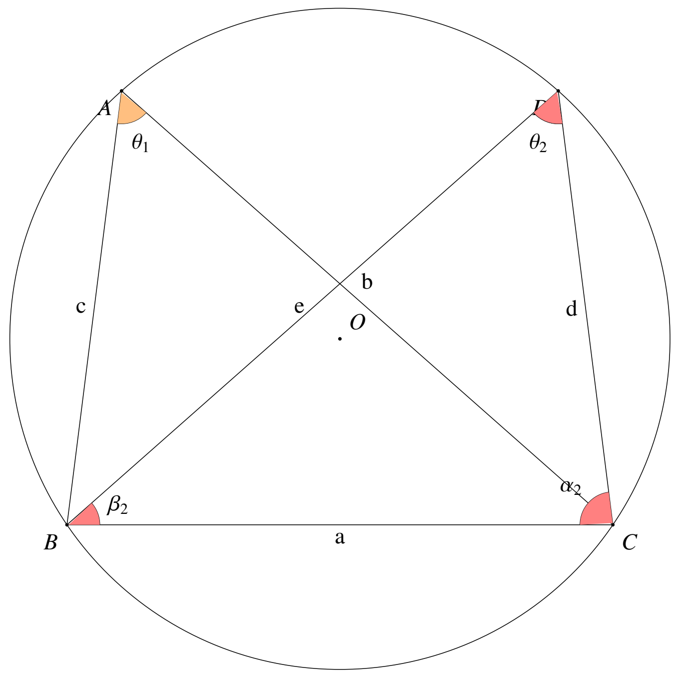

Fig. 2.2: circumecircle generated by latex

2.3. Finding out the coordinates of the various points in Fig. 2.2

$$x_1 = \frac{\left(a^2 + c^2 - b^2\right)}{2 * a} \tag{2.0.1}$$

$$y_1 = \sqrt{c^2 - x_1^2} \tag{2.0.2}$$

$$\left(\mathbf{A}\right) = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 3.9686 \end{pmatrix} \tag{2.0.3}$$

$$\left(\mathbf{B}\right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$

$$\left(\mathbf{C}\right) = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{2.0.5}$$

2.4. Finding out the angle BAC

$$\cos\theta_{\mathbf{1}} = \frac{b^2 + c^2 - a^2}{2dc} \tag{2.0.6}$$

$$\angle A = 55.94 \tag{2.0.7}$$

2.5. Drawing triangle DBC having angle BDC and side a and d using sine rule of the triangle

$$\frac{\sin\theta_2}{a} = \frac{\sin\beta_2}{d} \tag{2.0.8}$$
$$\angle\beta_2 = 41.40 \tag{2.0.9}$$
$$\angle\alpha = 180 - 55.77 - 41.40 = 82.83 \tag{2.0.10}$$
$$\frac{\sin\theta_2}{a} = \frac{\sin\alpha_2}{e} \tag{2.0.11}$$
$$e = 6 \tag{2.0.12}$$
$$x_2 = \frac{\left(a^2 + d^2 - e^2\right)}{2 * a} \tag{2.0.13}$$
$$y_2 = \sqrt{d^2 - x_2^2} \tag{2.0.14}$$
$$\left(\mathbf{A}\right) = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4.5 \\ 3.9686 \end{pmatrix} \tag{2.0.15}$$

The values are listed in Table. 2.5

Derived Values.	
D	$\begin{pmatrix} 4.5 \\ 3.9686 \end{pmatrix}$

TABLE 2.5: circumecentre of the triangle

2.6. Drawing Fig. 2.6.
The following Python code generates Fig. 2.6

```
codes/c_circle.py
```

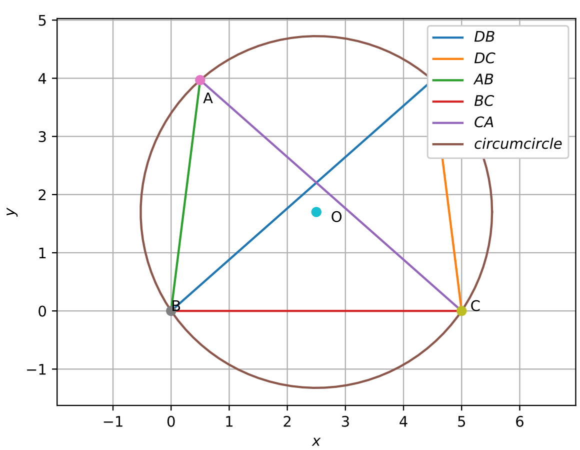

Fig. 2.6: circumecircle generated using python

and the equivalent latex-tikz code generating Fig.2.1 is

```
figs/C_circle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/C_circle_slone.tex
```

3 Solution

3.1. To prove that all four points lie on the circumefarence of the circle first of all we will draw a circume circle for a triangle ABC.

3.2. Finding the circumecentre $\rightarrow$
let assume that circumecentre of the triangle ABC is $\mathbf{O}$

$$\|\mathbf{A} - \mathbf{O}\| = \|\mathbf{B} - \mathbf{O}\| = \|\mathbf{C} - \mathbf{O}\| \tag{3.0.1}$$
$$\|\mathbf{A} - \mathbf{O}\|^2 - \|\mathbf{B} - \mathbf{O}\|^2 = 0 \tag{3.0.2}$$

Which can be simplified as

$$\left(\mathbf{A} - \mathbf{B}\right)^T \mathbf{O} = \frac{(\|A\|^2 - \|B\|^2)}{2} \tag{3.0.3}$$

Similarly,

$$\left(\mathbf{B} - \mathbf{C}\right)^T \mathbf{O} = \frac{(\|B\|^2 - \|C\|^2)}{2} \tag{3.0.4}$$

can be combined to form the matrix equation

$$\mathbf{N}^T = \mathbf{c} \tag{3.0.5}$$
$$\mathbf{O} = \mathbf{N}^{-T}\mathbf{c} \tag{3.0.6}$$
$$\mathbf{O} = \begin{pmatrix} 2.5 \\ 1.7 \end{pmatrix} \tag{3.0.7}$$

Where

$$\mathbf{N} = \begin{pmatrix} \mathbf{A} - \mathbf{B} & \mathbf{B} - \mathbf{C} \end{pmatrix} \tag{3.0.8}$$
$$\mathbf{c} = \frac{1}{2}\left(\|A\|^2 - \|B\|^2 \; \|B\|^2 - \|C\|^2\right) \tag{3.0.9}$$

3.3. Finding $\mathbf{R}$ of circumecircle
area of triangle of ABC $\rightarrow$

$$\frac{1}{2}ab\sin C = \frac{abc}{4R} \tag{3.0.10}$$
$$\implies \mathbf{R} = \frac{abc}{4s(\sqrt{(s-a)(s-b)(s-c)})} \tag{3.0.11}$$
$$\mathbf{R} = 3.023 \tag{3.0.12}$$

3.4. For point D to be on the circumeference of the circume circle it should satisfie the circle equation

$$\|\mathbf{D} - \mathbf{O}\| = \|\mathbf{R}\| \tag{3.0.13}$$

$$\left\|\begin{pmatrix} 4.5 - 2.5 \\ 3.9686 - 1.7 \end{pmatrix}\right\| = 3.023 \tag{3.0.14}$$

$$\left\|\begin{pmatrix} 2 \\ 2.26 \end{pmatrix}\right\| = 3.023 \tag{3.0.15}$$

3.5. thus the point D setisfies the circle equation of the circumecircle of triangle ABC and we can say that all four points lies on the circle.