

# Math Document Template

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**Abstract**—This is a document explaining for a question on the concept of concentric circles.

Download all python codes from

svn co [https://github.com/Ashuwin/Summer\\_20/trunk/circle/codes](https://github.com/Ashuwin/Summer_20/trunk/circle/codes)

and latex-tikz codes from

svn co [https://github.com/Ashuwin/Summer\\_20/trunk/circle/figs](https://github.com/Ashuwin/Summer_20/trunk/circle/figs)

## 1 PROBLEM

If a line intersects two concentric circles(circles with same centre) with centre  $O$  at  $A$ ,  $B$ ,  $C$  and  $D$ . Prove that  $AB = CD$ .

## 2 CONSTRUCTION

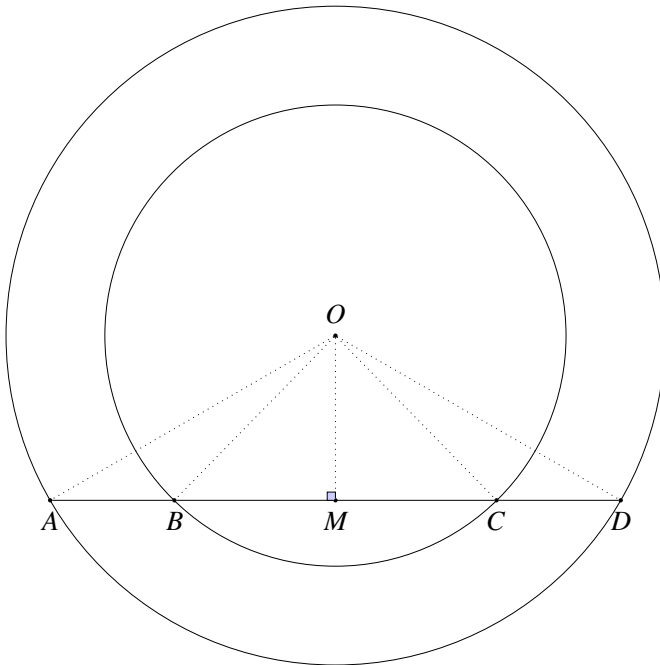


Fig. 2.0: Concentric circles by Latex-Tikz

2.1. The design parameters used for construction

**Solution:** See Table. 2.1.

Design Parameters	
Parameters	Values
Radius 1	7
Radius 2	10
<b>O</b>	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
<b>A</b>	$\begin{pmatrix} -\sqrt{75} \\ -5 \end{pmatrix}$
<b>B</b>	$\begin{pmatrix} -\sqrt{24} \\ -5 \end{pmatrix}$
<b>C</b>	$\begin{pmatrix} \sqrt{24} \\ -5 \end{pmatrix}$
<b>D</b>	$\begin{pmatrix} \sqrt{75} \\ -5 \end{pmatrix}$

TABLE 2.1: Concentric circles and line  $ABCD$

2.2. Finding Coordinates of Point **M** Let **M** be the mid-point of  $AD$ (or  $BC$ )

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$$

2.3. The derived parameters used for construction

**Solution:** From the given information, The values are listed in 2.3

Derived values	
Parameter	values
<b>S</b>	$\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

TABLE 2.3: Concentric circles and line  $ABCD$

2.4. Draw fig. 2.4.

**Solution:** The following Python code generates Fig. 2.4

codes/circle.py

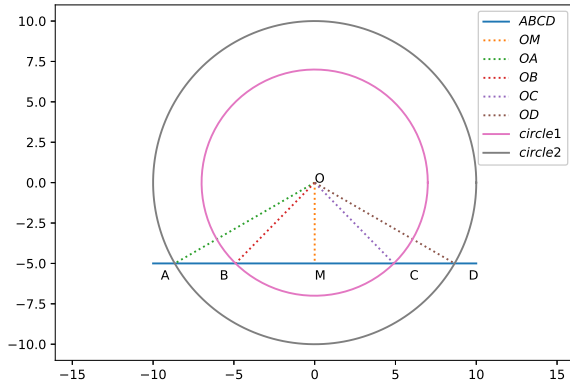


Fig. 2.4: Concentric circles generated using python

and the equivalent latex-tikz code generating Fig. 2.4 is

```
figs/circle.tex
```

### 3 SOLUTION

Finding scalar product between line  $OM$  and  $AD$

$$\begin{aligned}
 &(\mathbf{M} - \mathbf{O})^T(\mathbf{D} - \mathbf{A}) \\
 &\left(\begin{pmatrix} 0 \\ -5 \end{pmatrix}\right)^T \left(\begin{pmatrix} 2\sqrt{74} \\ 0 \end{pmatrix}\right) = 0 \\
 &\implies OM \perp AD
 \end{aligned}$$

$\triangle OMB \cong \triangle OMC$  by RHS congruency because:

- 1)  $\angle OMB = \angle OMC = 90^\circ$
  - 2)  $OB = OC = 7$  (Radius of circle 1)
  - 3)  $OM = OM$  (Common side)
- $$\implies BM = CM$$

$\triangle OMA \cong \triangle OMD$  by RHS congruency because:

- 1)  $\angle OMA = \angle OMD = 90^\circ$
  - 2)  $OA = OD = 10$  (Radius of circle 2)
  - 3)  $OM = OM$  (Common side)
- $$\implies AM = DM$$

$\therefore BM = CM$  and  $AM = DM$ ,

$$\implies AB = CD$$

Hence proved.