Solution For Problem 8.1.26

Yogesh Choudhary

***Abstract*—This document includes different problems and solution on geometry from trigonometry and linear algebra.It also provides the imformation about the python and latex codes of figures.**

Download all python codes from

svn co svn co https://github.com/yogi13995/yogesh_training/tree/master/Geometry/triangle2/codes

and latex-tikz codes from

svn co https://github.com/yogi13995/yogesh_training/tree/master/Geometry/triangle2/figures

1 Problem

Line l is the bisector of $\angle A$ and **B** is any point on l. **BP** and **BQ** are perpendiculars from **B** to the arms of $\angle A$ show that :

(a) $\Delta APB \cong \Delta AQB$
(b) **BP** = **BQ**

2 Construction

2.1. We need to draw an angle having bisector .point A is at the origin .Length of the sides of the angle are 3 .With help of the polar coardinates and linear algebra we will draw the points and lines.Input values are as given in the following table.

Input values	
Parameter	Value
A	$\begin{pmatrix}0\\0\end{pmatrix}$
P	$\begin{pmatrix}3\\0\end{pmatrix}$
$\angle PAQ$	60

TABLE 2.1: To construct $\angle QAB$

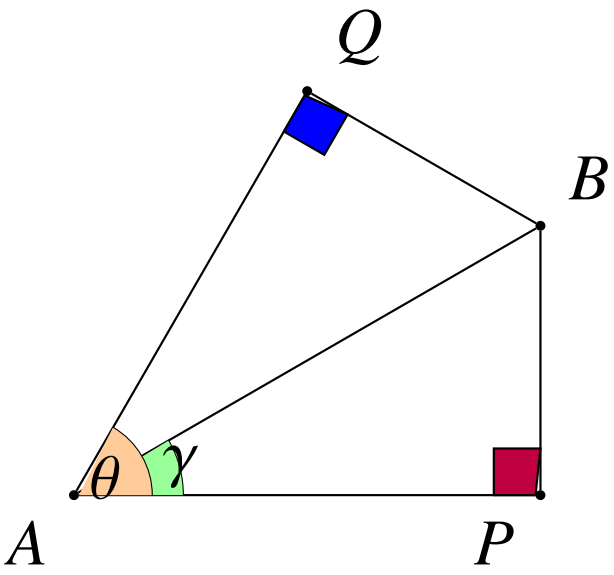

Fig. 2.1: angle by Latex-Tikz

2.2. Finding out the coordinates of the various points in Fig. 2.1

$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.1}$$

here r = 3 and $\theta = 0$ for **P**

(2.0.2)

$$\mathbf{P} = \begin{pmatrix}r * \cos 0\\r * \sin 0\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix} \tag{2.0.3}$$

Similarly we can get **Q** for r = 3 and $\theta = 60$

$$\mathbf{Q} = \begin{pmatrix}r * \cos 60\\r * \sin 60\end{pmatrix} = \begin{pmatrix}1.5\\2.598\end{pmatrix} \tag{2.0.4}$$

equation for the bisector of the $\angle\theta$

$$\mathbf{AB} = b\left(\frac{\mathbf{AP}}{\|\mathbf{AP}\|} + \frac{\mathbf{AQ}}{\|\mathbf{AQ}\|}\right) \tag{2.0.5}$$

2.3. finding out the point **B**

$$\left(\mathbf{B}-\mathbf{P}\right)^{T}\left(\mathbf{A}-\mathbf{P}\right) = 0 \tag{2.0.6}$$

$$\angle\gamma = \frac{\angle\theta}{2} = 30 \tag{2.0.7}$$

$$\begin{pmatrix} b*\cos\gamma - 3 \\ b*\sin\gamma - 0 \end{pmatrix}^{T}\begin{pmatrix} 0-3 \\ 0-0 \end{pmatrix} = 0 \tag{2.0.8}$$

$$3*(b*\cos\gamma - 3) = 0 \tag{2.0.9}$$

$$b*\cos\gamma = 3 \tag{2.0.10}$$

$$b = 3.4641 \tag{2.0.11}$$

$$\mathbf{B} = \begin{pmatrix} 3.4641*\cos 30 \\ 3.4641*\sin 30 \end{pmatrix} = \begin{pmatrix} 3 \\ 1.732 \end{pmatrix} \tag{2.0.12}$$

Derived values	
Parameter	Value
B	$\begin{pmatrix} 3 \\ 1.732 \end{pmatrix}$
Q	$\begin{pmatrix} 1.5 \\ 2.598 \end{pmatrix}$
P	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
$\angle PAB$	30

TABLE 2.3: To construct $\angle QAB$

2.4. Drawing Fig. 2.1.
The following Python code generates Fig. 2.1

./codes/angle.py

and the equivalent latex-tikz code generating Fig.2.1 is

./figs/angle.tex

The above latex code can be compiled as a standalone document as

./figs/angle_fig.tex

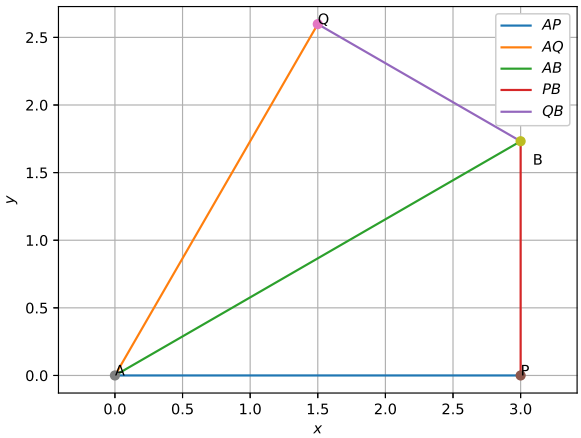

Fig. 2.4: angles generated using python

3 Solution

3.1. given that $\rightarrow$

$$\mathbf{BQ} \perp \mathbf{AQ} \tag{3.0.1}$$

$$\mathbf{BP} \perp \mathbf{AP} \tag{3.0.2}$$

3.1 Solution.a)

3.1. from the ΔAPB and ΔAQB...

$$\|\mathbf{A}-\mathbf{P}\| = \|\mathbf{A}-\mathbf{P}\| \tag{3.1.1}$$

$$\angle AQB = \angle APB \tag{3.1.2}$$

AB is bisector of $\angle QAP$

$$\implies \angle AQB = \angle APB \tag{3.1.3}$$

thus from ASA congurancy

$$\Delta APB \cong \Delta AQB \tag{3.1.4}$$

3.2 Solution.b)

3.1. from equation (3.1.4)...

$$\Delta APB \cong \Delta AQB \tag{3.2.1}$$

$$\implies \|\mathbf{BQ}\| = \|\mathbf{BP}\| \tag{3.2.2}$$

Hence proved