Linear Inequalities

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***Abstract*—This book provides a computational approach to school geometry based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    ncert/computation/codes
```

1 Examples

1.1. Solve $30x < 200$ when
   a) x is a natural number,
   b) x is an integer.

   Solution: From the given information,

$$30x < 200 \implies x < \frac{20}{3} \tag{1.1.1}$$

   If x is a natural number, $x \in \{1, 2, 3, 4, 5, 6\}$. If x is an integer, then the solution set includes 0 as well as all negative integers.

1.2. Solve $5x - 3 < 3x + 1$ when
   a) x is an integer,
   b) x is a real number.

   Solution:

$$5x - 3 < 3x + 1 \implies x < 2 \tag{1.2.1}$$

   If x is real, then $x \in (-\infty, 2)$.

1.3. Solve the following system of linear inequalities graphically.

$$\begin{aligned} x + y &\geq 5 \\ x - y &\leq 3 \end{aligned} \tag{1.3.1}$$

   Solution: Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \tag{1.3.2}$$

   (1.3.1) can then be expressed as

$$\begin{aligned} x + y &\geq 5 \\ -x + y &\geq -3 \end{aligned} \tag{1.3.3}$$

$$\implies \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} 5 \\ -3 \end{pmatrix} \tag{1.3.4}$$

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} \tag{1.3.5}$$

$$\text{or, } \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + \mathbf{u} \tag{1.3.6}$$

   resulting in

$$\mathbf{x} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1} \mathbf{u} \tag{1.3.7}$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{u} \tag{1.3.8}$$

   after obtaining the inverse. Fig. 1.3 generated using the following python code shows the region satisfying (1.3.1)

```
codes/line/line_ineq.py
```

1.4. Solve

$$\begin{aligned} 2x + y &\geq 4 \\ x + y &\leq 3 \\ 2x - 3y &\leq 6 \end{aligned} \tag{1.4.1}$$

   Solution: Fig. 1.4 generated using the following python code shows the region satisfying (1.4.1)

```
codes/line/line_ineq_mult.py
```

1.5. Solve $x + y < 5$ graphically.

   Solution: The following python code generates Fig. 1.5.

```
./solutions/5/codes/lines/q6.py
```

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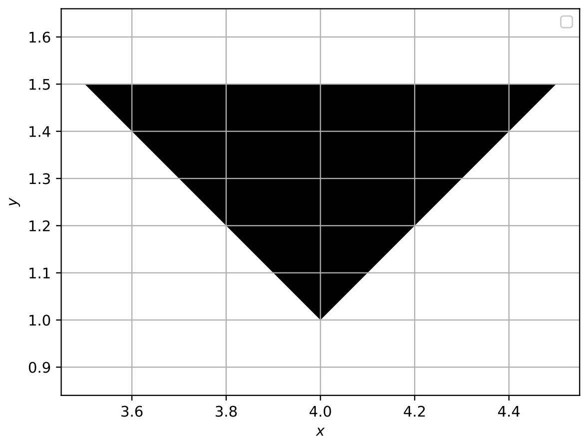

Fig. 1.3

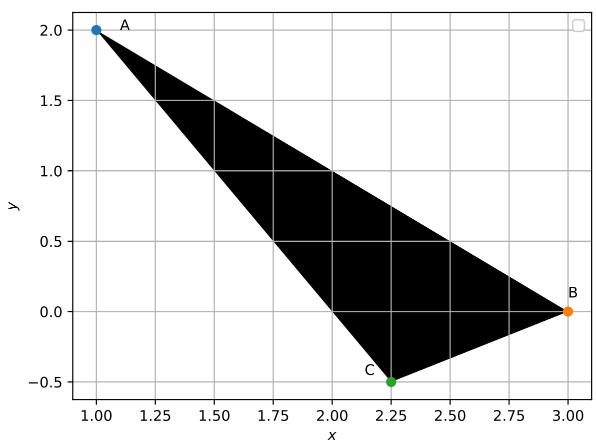

Fig. 1.4

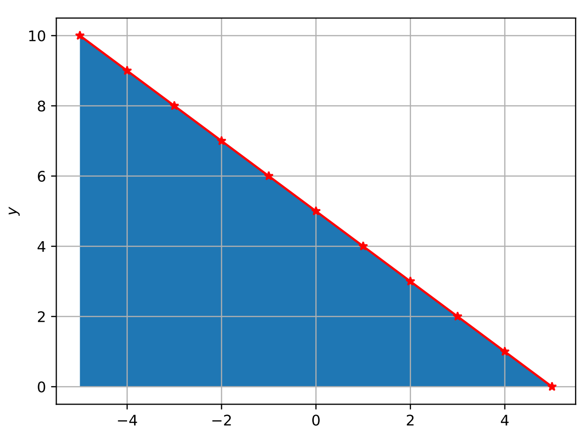

Fig. 1.5: x+y<5

1.6. Solve

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \\ 1 & 0 \\ 0 & -1 \\ -1 & 0 \end{pmatrix} \mathbf{x} \preceq \begin{pmatrix} 150 \\ 80 \\ 15 \\ 0 \\ 0 \end{pmatrix} \tag{1.6.1}$$

1.7. Solve x ≥ 3, y ≥ 2 graphically.
Solution: From the given information, for

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \succeq \mathbf{0}, \tag{1.7.1}$$

the given conditions can be expressed as

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \succeq \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{1.7.2}$$

$$\implies \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{1.7.3}$$

$$\text{or, } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \mathbf{u} \tag{1.7.4}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \mathbf{u} \tag{1.7.5}$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \mathbf{u} \tag{1.7.6}$$

after obtaining the inverse. Fig. 1.7 generated using the following python code shows the desired region

```
solutions/1/codes/line/line_eq.py
```

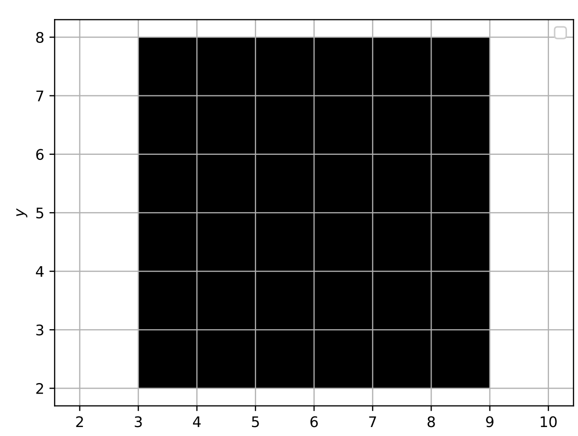

Fig. 1.7

1.8. Solve 7x+3 < 5x+9. Show the graph of the solutions on number line.
Solution:

$$7x+3 < 5x+9 \tag{1.8.1}$$
$$2x-6 < 0 \tag{1.8.2}$$
$$x < 3 \tag{1.8.3}$$
$$\therefore \quad x \in \{3, -\infty\} \tag{1.8.4}$$

The following Python code to generate Fig 1.8

```
solutions/2/codes/line_ex/lin_ineq/
    dist_btw_pts.py
```

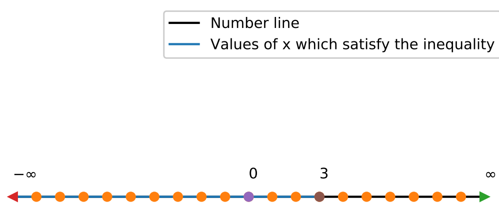

Fig. 1.8

1.9. Solve $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$. Show the graph of the solutions on number line.
Solution: Let

$$\frac{3x-4}{2} = \frac{x+1}{4} - 1 + s, \quad s \geq 0 \tag{1.9.1}$$

Then,

$$5x - 5 - 4s = 0 \tag{1.9.2}$$
$$\implies x = 1 + \frac{4s}{5} \tag{1.9.3}$$
$$\implies x \geq 1 \tag{1.9.4}$$

The following code marks the solution of inequality on numberline as shown in figure 1.9

```
codes/line/ineq/ineq.py
```

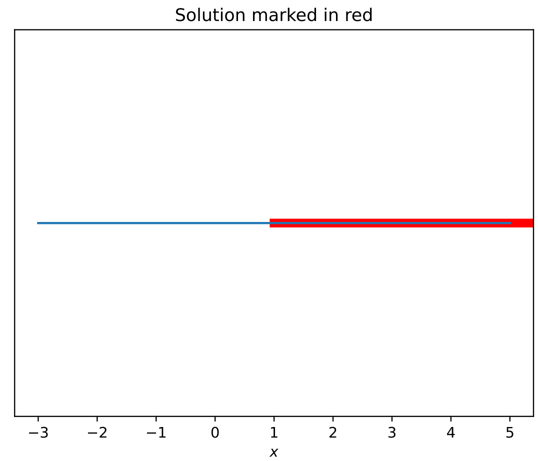

Fig. 1.9: Solution of the inequality

1.10. The marks obtained by a student of Class XI in first and second terminal examination are 62 and 48, respectively. Find the minimum marks he should get in the annual examination to have an average of at least 60 marks.
Solution: If x be the student marks,

$$\frac{62+48+x}{3} \geq 60 \tag{1.10.1}$$
$$\implies x \geq 70 \tag{1.10.2}$$

1.11. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.
Solution:
Let x be an odd natural number and y be the odd natural number consecutive to x.

$$\therefore y = x + 2 \tag{1.11.1}$$

We need to find x and y such that

$$x, y > 10 \text{ and } x + y < 40$$
$$\therefore x + x + 2 < 40$$
$$2x + 2 < 40$$
$$x + 1 < 20$$
$$x < 19 \tag{1.11.2}$$

Hence the condition is satisfied when $x > 10$ and $x < 19$
The following python code computes the required pairs of consecutive odd natural numbers which satisfy the required condition, shown in Fig.1.11.

```
./solutions/5/codes/lines/q15.py
```

1.12. Solve 3x+2y > 6 graphically.

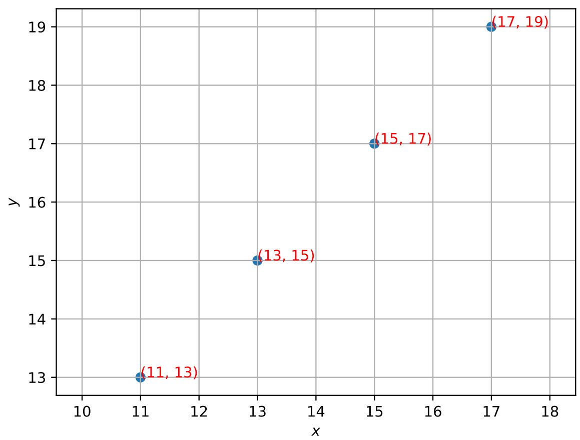

Fig. 1.11

Solution: Let $3x+2y=6$ intersects the x-axis and y-axis at $\mathbf{A}$ and $\mathbf{B}$ respectively.

a) Let $\mathbf{A} = \begin{pmatrix} x \\ 0 \end{pmatrix}$

$$3x = 6 \tag{1.12.1}$$
$$\implies x = 2 \tag{1.12.2}$$
$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{1.12.3}$$

b) Let $\mathbf{B} = \begin{pmatrix} 0 \\ y \end{pmatrix}$

$$2y = 6 \tag{1.12.4}$$
$$\implies y = 3 \tag{1.12.5}$$
$$\mathbf{B} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{1.12.6}$$

c) Origin $= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ does not satisfy the equation $3x+2y<6$.
$\implies$ The solution is the right side of the line $3x+2y=6$

d) The following python code is the diagrammatic representation of the solution in Fig.1.12

```
solutions/6/codes/linear_inequalities/
    linear_inequalities.py
```

1.13. Solve 3x-6 $\geq$ 0 graphically in a two dimensional plane.
Solution:

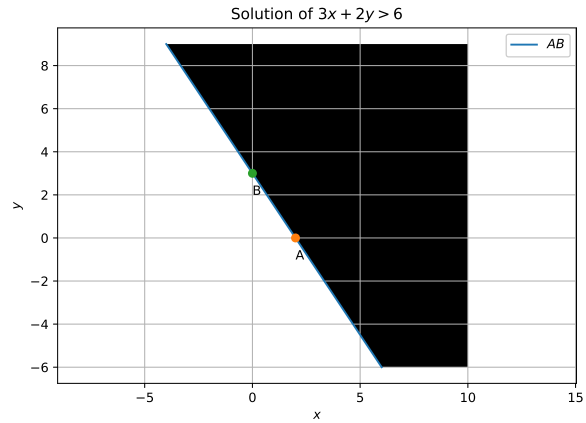

Fig. 1.12

The given inequality can be expressed as

$$\begin{pmatrix} 3 & 0 \end{pmatrix} \mathbf{x} - 6 \geq 0 \implies \mathbf{x} \geq \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{1.13.1}$$

The python code for Fig. 1.13 is

```
solutions/7/codes/line/lin_ineq/lin_ineq1.py
```

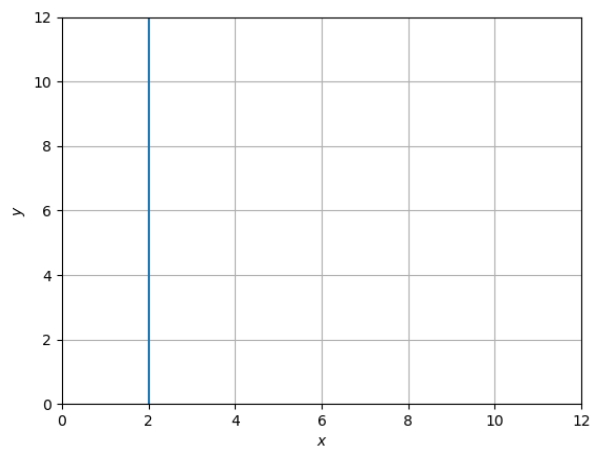

Fig. 1.13

1.14. 2x+y $\geq$ 6, 3x+4y $\leq$ 12.
Solution:
The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -1 & -2 \\ -1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -10 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \tag{1.14.1}$$

which can be further simplified into

$$\begin{pmatrix}-1 & -2\\ 1 & 0\\ 0 & 1\end{pmatrix}\mathbf{x} \geq \begin{pmatrix}-10\\ \frac{-1}{2}\\ \frac{-1}{2}\end{pmatrix} \tag{1.14.2}$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix}u_1\\ u_2\end{pmatrix} \geq 0 \tag{1.14.3}$$

a)

$$\begin{pmatrix}-1 & -2\\ 1 & 0\end{pmatrix}\mathbf{x} \geq \begin{pmatrix}-10\\ \frac{-1}{2}\end{pmatrix} \tag{1.14.4}$$

$$\implies \begin{pmatrix}-1 & -2\\ 1 & 0\end{pmatrix}\mathbf{x} = \begin{pmatrix}-10\\ \frac{-1}{2}\end{pmatrix} + \mathbf{u} \tag{1.14.5}$$

resulting in

$$\mathbf{x} = \begin{pmatrix}-1 & -2\\ 1 & 0\end{pmatrix}^{-1}\begin{pmatrix}-10\\ \frac{-1}{2}\end{pmatrix} + \begin{pmatrix}-1 & -2\\ 1 & 0\end{pmatrix}^{-1}\mathbf{u} \tag{1.14.6}$$

$$\implies \mathbf{x} = \begin{pmatrix}\frac{1}{2}\\ \frac{19}{4}\end{pmatrix} + \begin{pmatrix}0 & 1\\ \frac{-1}{2} & \frac{-1}{2}\end{pmatrix}\mathbf{u} \tag{1.14.7}$$

b)

$$\begin{pmatrix}-1 & -2\\ 0 & 1\end{pmatrix}\mathbf{x} \geq \begin{pmatrix}-10\\ \frac{-1}{2}\end{pmatrix} \tag{1.14.8}$$

$$\implies \begin{pmatrix}-1 & -2\\ 0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}-10\\ \frac{-1}{2}\end{pmatrix} + \mathbf{u} \tag{1.14.9}$$

resulting in

$$\mathbf{x} = \begin{pmatrix}-1 & -2\\ 0 & 1\end{pmatrix}^{-1}\begin{pmatrix}-10\\ \frac{-1}{2}\end{pmatrix} + \begin{pmatrix}-1 & -2\\ 0 & 1\end{pmatrix}^{-1}\mathbf{u} \tag{1.14.10}$$

$$\implies \mathbf{x} = \begin{pmatrix}9\\ \frac{1}{2}\end{pmatrix} + \begin{pmatrix}-1 & -2\\ 0 & 1\end{pmatrix}\mathbf{u} \tag{1.14.11}$$

Now,solution region which is common to regions of eq. (1.14.7) and eq. (1.14.11),is given by

$$\boxed{\mathbf{x} = \begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\end{pmatrix} + \begin{pmatrix}0 & 1\\ \frac{-1}{2} & 1\end{pmatrix}\mathbf{u}} \tag{1.14.12}$$

1.15. 2x-y > 1, x-2y < -1.

Solution:

Let

$$\begin{aligned}2x - y &> 1,\\ -x + 2y &> 1.\end{aligned} \tag{1.15.1}$$

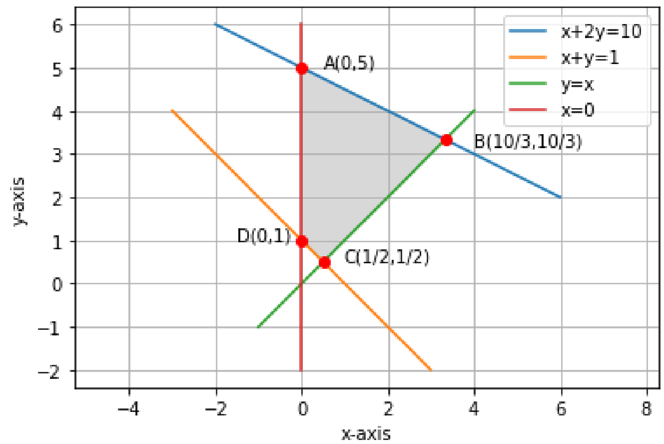

Fig. 1.14: Graphical Solution

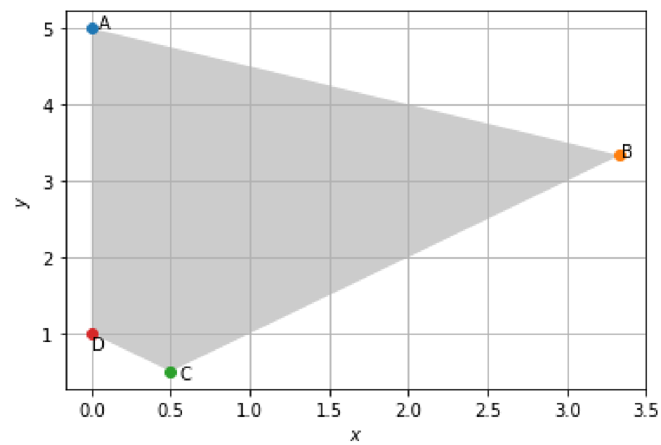

Fig. 1.14: Magnified Solution region

Let $u_1 > 0, u_2 > 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix}u_1\\ u_2\end{pmatrix} > \mathbf{0} \tag{1.15.2}$$

Now we have,

$$\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}\mathbf{x} > \begin{pmatrix}1\\ 1\end{pmatrix} \tag{1.15.3}$$

$$\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}\mathbf{x} - \mathbf{u} = \begin{pmatrix}1\\ 1\end{pmatrix} \tag{1.15.4}$$

$$\text{or, } \begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}\mathbf{x} = \begin{pmatrix}1\\ 1\end{pmatrix} + \mathbf{u} \tag{1.15.5}$$

Resulting in

$$\mathbf{x} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}^{-1} \mathbf{u} \quad (1.15.6)$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{u} \quad (1.15.7)$$

Thus , the solution of the system of inequalities can be determined graphically and the desired region is the shaded triangle which is represented in Fig. 1.15

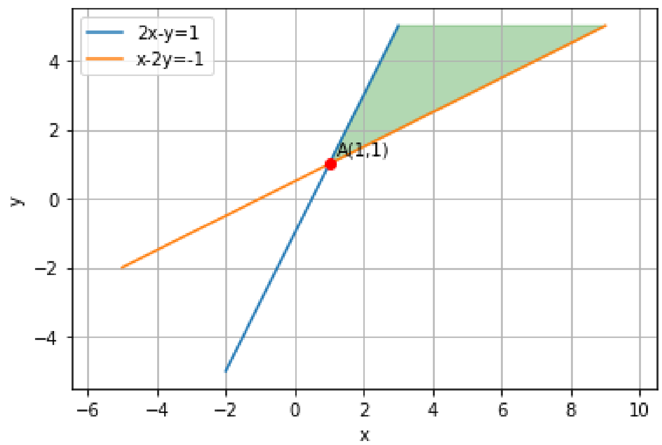

Fig. 1.15: Graphical Solution

1.16. 2x+y $\geq$ 8, x+2y $\geq$ 10.
Solution: Let $u_1 \geq 0$ and $u_2 \geq 0$.This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \quad (1.16.1)$$

From the given inequalities we have,

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} 8 \\ 10 \end{pmatrix} \quad (1.16.2)$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 8 \\ 10 \end{pmatrix} \quad (1.16.3)$$

Now we have,

$$\mathbf{x} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ 10 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (1.16.4)$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (1.16.5)$$

Thus the solution of the system of inequalities can be determined graphically and is represented in Fig. 1.16,

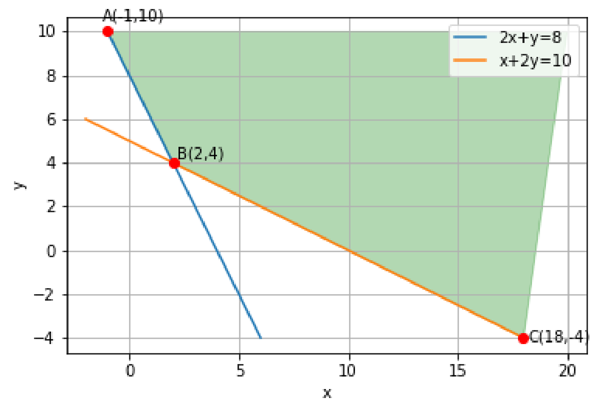

Fig. 1.16: Graphical solution

1.17. 3x+4y $\leq$ 60, x+3y $\leq$ 30, x $\geq$ 0, y $\geq$ 0.
Solution:

From the given inequalities we have,

$$\begin{pmatrix} -3 & -4 \\ -1 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ -30 \\ 0 \\ 0 \end{pmatrix} \quad (1.17.1)$$

Which can be further written as

$$\begin{pmatrix} -3 & -4 \\ -1 & -3 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ -30 \end{pmatrix} \quad (1.17.2)$$

Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \quad (1.17.3)$$

Now we have,

$$\begin{pmatrix} -3 & -4 \\ -1 & -3 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ -30 \end{pmatrix} + \mathbf{u} \quad (1.17.4)$$

$$\mathbf{x} = \begin{pmatrix} -3 & -4 \\ -1 & -3 \end{pmatrix}^{-1} \begin{pmatrix} -60 \\ -30 \end{pmatrix} + \begin{pmatrix} -3 & -4 \\ -1 & -3 \end{pmatrix}^{-1} \mathbf{u} \quad (1.17.5)$$

$$\implies \mathbf{x} = \frac{1}{5} \begin{pmatrix} 60 \\ 30 \end{pmatrix} + \frac{1}{5} \begin{pmatrix} -3 & 4 \\ 1 & -3 \end{pmatrix} \mathbf{u} \quad (1.17.6)$$

$$\mathbf{x} = \begin{pmatrix} 12 \\ 6 \end{pmatrix} + \frac{1}{5} \begin{pmatrix} -3 & 4 \\ 1 & -3 \end{pmatrix} \mathbf{u} \quad (1.17.7)$$

Thus the solution of the system of inequalities can be determined graphically, which is represented in Fig. 1.17.

1.18. x-2y $\leq$ 3, 3x+4y $\geq$ 12, x $\geq$ 0, y $\geq$ 1.
Solution:

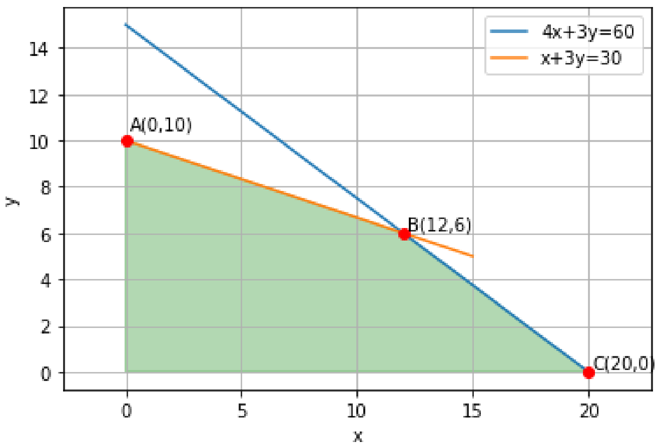

Fig. 1.17: Graphical solution

a) Solving first pair of inequality:

$$\begin{aligned} -x + 2y &\geq -3 \\ 3x + 4y &\geq 12 \end{aligned} \tag{1.18.1}$$

Solution: Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \tag{1.18.2}$$

(1.18.1) can then be expressed as

$$\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -3 \\ 12 \end{pmatrix} \tag{1.18.3}$$

$$\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} -3 \\ 12 \end{pmatrix} \tag{1.18.4}$$

$$\text{or, } \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -3 \\ 12 \end{pmatrix} + \mathbf{u} \tag{1.18.5}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \begin{pmatrix} -3 \\ 12 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \mathbf{u} \tag{1.18.6}$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 3.6 \\ 0.3 \end{pmatrix} + \frac{-1}{10} \begin{pmatrix} 4 & -2 \\ -3 & -1 \end{pmatrix} \mathbf{u} \tag{1.18.7}$$

b) Similarly,Solving second pair of inequality:

$$\begin{aligned} x &\geq 0 \\ y &\geq 1 \end{aligned} \tag{1.18.8}$$

Solution: Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \tag{1.18.9}$$

(1.18.8) can then be expressed as

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.18.10}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.18.11}$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mathbf{u} \tag{1.18.12}$$

From (1.18.7) and (1.18.12),solution of the given system of inequalities can be found out graphically by intersection as shown by the below figures generated by Python: As seen from Fig. 1.18 the solution region is bounded by line segments AB and BC and the line $\begin{pmatrix} 1 & -2 \end{pmatrix} \mathbf{x} = 3$.Beyond A the region expands infinitely along the Y axis,Beyond C the region includes all the portion above the line $\begin{pmatrix} 1 & -2 \end{pmatrix} \mathbf{x} = 3$.

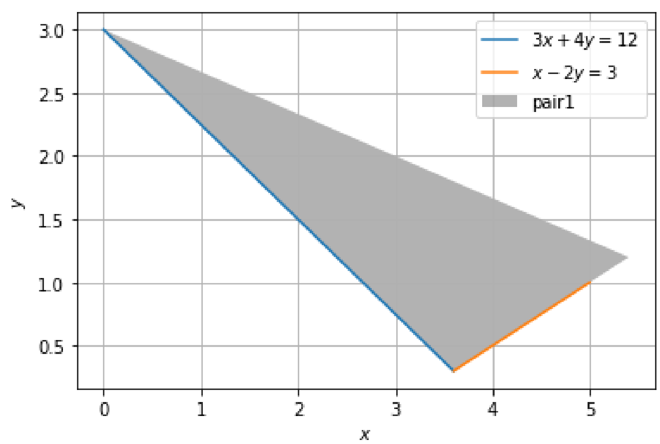

Fig. 1.18: Inequality pair 1

The common region shown by 1.18 is the solution of set of inequalities.

1.19. 4x+3y ≤ 60, y ≥ 2x, x ≥ 3, x,y ≥ 0.
Solution:
The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -4 & -3 \\ -2 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ 0 \\ 3 \\ 0 \\ 0 \end{pmatrix} \tag{1.19.1}$$

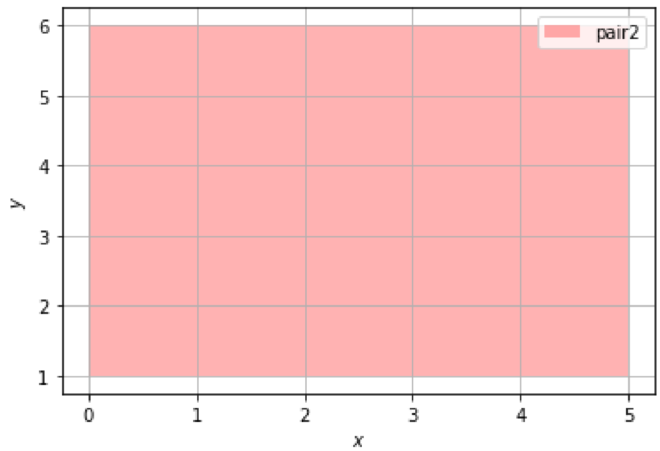

Fig. 1.18: Inequality pair 2

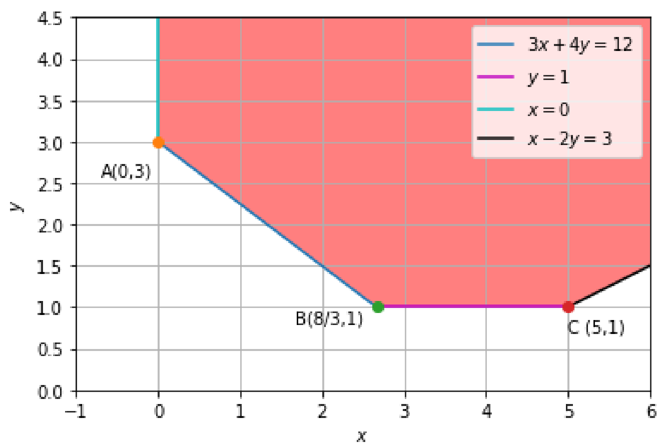

Fig. 1.18: Intersection of 1.18 and 1.18

which can be further simplified into

$$\begin{pmatrix} -4 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ 3 \\ 6 \end{pmatrix} \tag{1.19.2}$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \tag{1.19.3}$$

a)

$$\begin{pmatrix} -4 & -3 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ 3 \end{pmatrix} \tag{1.19.4}$$

$$\implies \begin{pmatrix} -4 & -3 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -60 \\ 3 \end{pmatrix} + \mathbf{u} \tag{1.19.5}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -4 & -3 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -60 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 & -3 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u} \tag{1.19.6}$$

$$\implies \mathbf{x} = \begin{pmatrix} 3 \\ 16 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{-1}{3} & \frac{-4}{3} \end{pmatrix} \mathbf{u} \tag{1.19.7}$$

b)

$$\begin{pmatrix} -4 & -3 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -60 \\ 6 \end{pmatrix} \tag{1.19.8}$$

$$\implies \begin{pmatrix} -4 & -3 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -60 \\ 6 \end{pmatrix} + \mathbf{u} \tag{1.19.9}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -4 & -3 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -60 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 & -3 \\ 0 & 1 \end{pmatrix}^{-1} \mathbf{u} \tag{1.19.10}$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{21}{2} \\ 6 \end{pmatrix} + \begin{pmatrix} \frac{-1}{4} & \frac{-3}{4} \\ 0 & 1 \end{pmatrix} \mathbf{u} \tag{1.19.11}$$

Now,solution region which is common to regions of eq. (1.19.7) and eq. (1.19.11),is given by

$$\boxed{\mathbf{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{1}{12} & \frac{-13}{12} \end{pmatrix} \mathbf{u}} \tag{1.19.12}$$

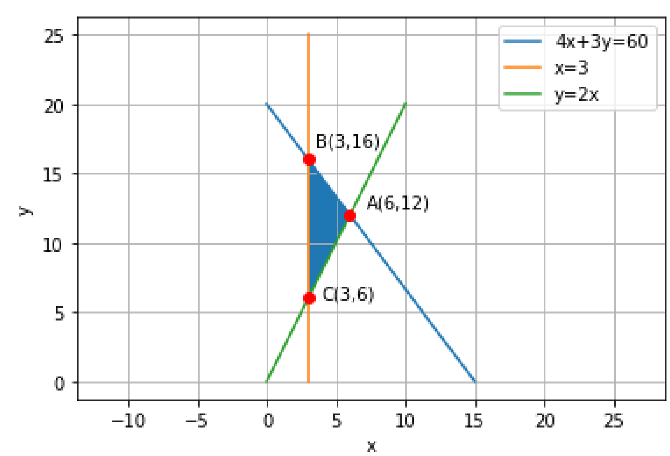

Fig. 1.19: Solution Region

2 Exercises

2.1. Solve y < 2 graphically.

2.2. Solve the following system of inequalities graphically. 5x+4y ≤ 40 x ≥ 2 y ≥ 3

Solution: The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -5 & -4 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -40 \\ 2 \\ 3 \end{pmatrix} \tag{2.2.1}$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \tag{2.2.2}$$

The first pair of inequality can be solved as,

a)

$$\begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -40 \\ 2 \end{pmatrix} \tag{2.2.3}$$

$$\implies \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -40 \\ 2 \end{pmatrix} + \mathbf{u} \tag{2.2.4}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -40 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u} \tag{2.2.5}$$

$$\implies \mathbf{x} = \begin{pmatrix} 2 \\ \frac{15}{2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{-1}{4} & \frac{-5}{4} \end{pmatrix} \mathbf{u} \tag{2.2.6}$$

Similarly, solving 2nd pair of inequality

b)

$$\begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -40 \\ 3 \end{pmatrix} \tag{2.2.7}$$

$$\implies \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -40 \\ 3 \end{pmatrix} + \mathbf{u} \tag{2.2.8}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -40 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix}^{-1} \mathbf{u} \tag{2.2.9}$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{28}{5} \\ 3 \end{pmatrix} + \begin{pmatrix} \frac{-1}{5} & \frac{-4}{5} \\ 0 & 1 \end{pmatrix} \mathbf{u} \tag{2.2.10}$$

Now,solution region which is common to regions of eq. (2.2.6) and eq. (2.2.10),is given by

$$\boxed{\mathbf{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{1}{20} & \frac{-21}{20} \end{pmatrix} \mathbf{u}} \tag{2.2.11}$$

See Fig. 2.2.

2.3. Solve the following system of inequalities graphically. 8x+3y ≤ 100 x ≥ 0 y ≥ 0

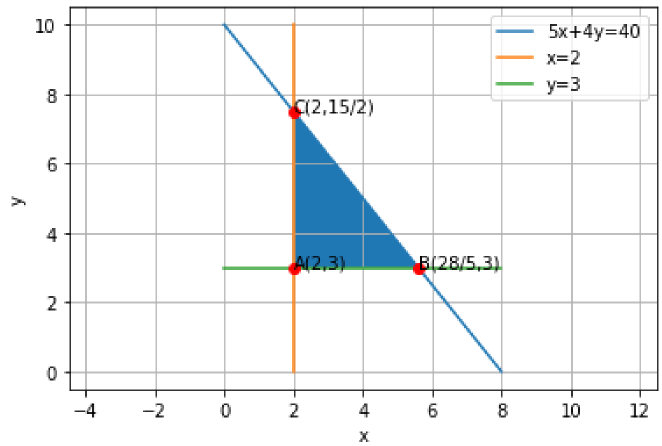

Fig. 2.2: Solution Region

2.4. Solve the following system of inequalities graphically. x+2y ≤ 8 2x+y ≤ 8 x ≥ 0 y ≥ 0
2.5. Solve -8 ≤ 5x-3 < 7.
2.6. Solve -5 ≤ $\frac{5-3x}{2}$ ≤ 8.
2.7. Solve the system inequalities: 3x-7 < 5+x 11-5x ≤ 1 and represent the solutions on the number line.
2.8. Solve 4x+3 < 6x+7.
2.9. Solve $\frac{5-2x}{3} \leq \frac{x}{6} - 5$.
2.10. Solve 24x < 100, when (i) x is a natural number. (ii) x is an integer.
2.11. Solve -12x > 30, when (i) x is a natural number. (ii) x is an integer.
2.12. Solve 5x-3 < 7, when (i) x is an integer. (ii) x is a real number.
2.13. Solve 3x+8 > 2, when (i) x is an integer. (ii) x is a real number
2.14. 4x+3 < 5x+7.
2.15. 3x-7 > 5x-1.
2.16. 3(x-1) ≥ 2(x-3).
2.17. 3(2-x) ≤ 2(1-x).
2.18. x+$\frac{x}{2}$ + $\frac{x}{3}$ < 11.
2.19. $\frac{x}{3}$ $\frac{x}{2}$+1.
2.20. $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$.
2.21. $\frac{1}{2}$($\frac{3x}{5}$+4)≥ $\frac{1}{3}(x-6)$.
2.22. 2(2x+3)-10 < 6(x-2).
2.23. 37-(3x+5) ≥ 9x-8(x-3).
2.24. $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$.
2.25. $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$.
2.26. 3x-2 < 2x+1.
2.27. 5x-3 ≥ 3x-5.
2.28. 3(1-x) < 2(x+4).
2.29. $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$.

2.30. $x+y < 5$.
2.31. $2x+y \geq 6$.
2.32. $3x+4y \leq 12$.
2.33. $y+8 \geq 2x$.
2.34. $x-y \leq 2$.
2.35. $2x-3y > 6$.
2.36. $-3x+2y \geq -6$.
2.37. $3y-5x < 30$.
2.38. $y < -2$.
2.39. $x > -3$.
2.40. $3x+2y \leq 12$, $x \geq 1$, $y \geq 2$.
2.41. $x+y \geq 4$, $2x-y < 0$.
2.42. $x+y \leq 9$, $y > x$, $x \geq 0$.
2.43. $5x+4y \leq 20$, $x \geq 1$, $y \geq 2$.
2.44. $x+2y \leq 10$, $x+y \geq 1$, $x-y \leq 0$, $x \geq 0$, $y \geq 0$.
2.45. $2 \leq 3x-4 \leq 5$.
2.46. $6 \leq -3(2x-40) < 12$.
2.47. $-3 \leq 4-\frac{7x}{2} \leq 18$.
2.48. $-15 < \frac{3(x-2)}{5} \leq 0$.
2.49. $-12 < 4-\frac{3x}{-5} \leq 2$.
2.50. $7 \leq \frac{(3x+11)}{2} \leq 11$.
2.51. $5x+1 > -24$, $5x-1 < 24$.
2.52. $2(x-1) < x+5$, $3(x+2) > 2-x$.
2.53. $3x-7 > 2(x-6)$, $6-x > 11-2x$.
2.54. $5(2x-7)-3(2x+3) \leq 0$, $2x+19 \leq 6x+47$.
2.55. $x+y \leq 6$, $x+y \geq 4$.