ASSIGNMENT 4

Vojeswitha Gopireddy
AI20BTECH11024

Download all python codes from

```
https://github.com/V−Gopireddy/EE3900/blob/
    main/Assignment4/codes/Assignment−4.py
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and latex-tikz codes from

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https://github.com/V−gopireddy/EE3900/blob/main
    /Assignment4/Assignment−4.tex
```

1 Linear forms 2.28

Find the equation of the plane through the intersection of the planes $\begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} = 1$ and $\begin{pmatrix}2 & 3 & 4\end{pmatrix}\mathbf{x} = 5$ which is perpendicular to the plane $\begin{pmatrix}1 & -1 & 1\end{pmatrix}\mathbf{x} = 0$

2 Solution

The equation of a plane P passing through the line of intersection of the planes P_1 and P_2 has the form,

$$P : P_1 + \lambda P_2 \tag{2.0.1}$$

General equation of plane is given by

$$\mathbf{n}^T\mathbf{x} = c \tag{2.0.2}$$

Where $\mathbf{n}$ is normal vector to the plane

Lemma 2.1. *The equation of a plane passing through intersection of planes*

$$\mathbf{n_1}^T\mathbf{x} = c_1 \tag{2.0.3}$$

$$\mathbf{n_2}^T\mathbf{x} = c_2 \tag{2.0.4}$$

and perpendicular to plane

$$\mathbf{n_3}^T\mathbf{x} = c_3 \tag{2.0.5}$$

is given by

$$\mathbf{n_4}^T\mathbf{x} = c_4 \tag{2.0.6}$$

where

$$\mathbf{n_4} = \mathbf{n_1} - \left(\frac{\mathbf{n_3^T n_1}}{\mathbf{n_3^T n_2}}\right)\mathbf{n_2} \tag{2.0.7}$$

$$c_4 = c_1 - \left(\frac{\mathbf{n_3^T n_1}}{\mathbf{n_3^T n_2}}\right)c_2 \tag{2.0.8}$$

Proof. Let P be the plane that passes through intersection 2 given planes.
From (2.0.1), equation of P has the form,

$$\mathbf{n_1}^T\mathbf{x} + \lambda\left(\mathbf{n_2}^T\mathbf{x}\right) = c_1 + \lambda\left(c_2\right) \tag{2.0.9}$$

$$\implies \left(\mathbf{n_1} + \lambda\mathbf{n_2}\right)^T\mathbf{x} = c_1 + \lambda\left(c_2\right) \tag{2.0.10}$$

Normal vector to plane P is,

$$\mathbf{n_4} = \mathbf{n_1} + \lambda\mathbf{n_2} \tag{2.0.11}$$

As P is perpendicular to the third plane i.e. angle between normal vectors is 90°,

$$\cos\left(90°\right) = 0 = \frac{\mathbf{n_3}^T\mathbf{n_4}}{\|\mathbf{n_3}\|\|\mathbf{n_4}\|} \tag{2.0.12}$$

$$\implies \mathbf{n_3}^T\mathbf{n_4} = 0 \tag{2.0.13}$$

$$\implies \mathbf{n_3}^T\left(\mathbf{n_1} + \lambda\mathbf{n_2}\right) = 0 \tag{2.0.14}$$

$$\implies \lambda = \frac{-\mathbf{n_3^T n_1}}{\mathbf{n_3^T n_2}} \tag{2.0.15}$$

Therefore equation of plane P is,

$$\left(\mathbf{n_1} - \left(\frac{\mathbf{n_3^T n_1}}{\mathbf{n_3^T n_2}}\right)\mathbf{n_2}\right)^T\mathbf{x} = c_1 - \left(\frac{\mathbf{n_3^T n_1}}{\mathbf{n_3^T n_2}}\right)c_2 \tag{2.0.16}$$

□

For the given problem,

$$\mathbf{n_1} = \begin{pmatrix}1\\1\\1\end{pmatrix} \tag{2.0.17}$$

$$c_1 = 1 \tag{2.0.18}$$

$$\mathbf{n_2} = \begin{pmatrix}2\\3\\4\end{pmatrix} \tag{2.0.19}$$

$$c_2 = 5 \tag{2.0.20}$$

$$\mathbf{n_3} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{2.0.21}$$

$$c_3 = 0 \tag{2.0.22}$$

Solving the above we get,

$$\lambda = \frac{-1}{3} \tag{2.0.23}$$

$$\mathbf{n_4} = \begin{pmatrix} \frac{1}{3} \\ 0 \\ \frac{-1}{3} \end{pmatrix} \tag{2.0.24}$$

$$c_4 = \frac{-2}{3} \tag{2.0.25}$$

We have equation of the plane as,

$$\begin{pmatrix} \frac{1}{3} & 0 & \frac{-1}{3} \end{pmatrix} \mathbf{x} = \frac{-2}{3} \tag{2.0.26}$$

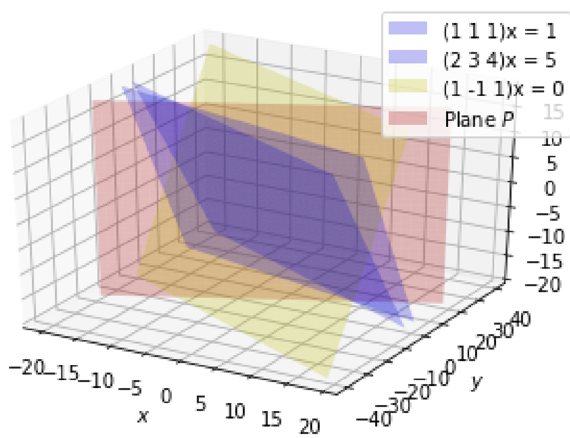

Fig. 0: Plane P passing through intersection of P_1 and P_2 and perpendicular to P_3