ASSIGNMENT 5

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Download all python codes from

```
https://github.com/V−Gopireddy/EE3900/blob/
    main/Assignment5/codes/Assignment−5.py
```

and latex-tikz codes from

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https://github.com/V−gopireddy/EE3900/blob/main
    /Assignment5/Assignment−5.tex
```

1 Quadratic forms 2.27

Find the coordinates of the foci, the vertices, the lengths of major and minor axes and the eccentricity of the ellipse

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 36 \tag{1.0.1}$$

2 SOLUTION

Given ellipse is

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 36 \tag{2.0.1}$$

On comparing it with standard form we have,

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}, \mathbf{u} = 0, f = -36 \tag{2.0.2}$$

$$\implies \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 36 \tag{2.0.3}$$

$$\implies \mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$

$$\tag{2.0.5}$$

The eigen vector decomposition of

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \tag{2.0.6}$$

is given by

$$\mathbf{D} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \implies \lambda_1 = 9, \lambda_2 = 4 \tag{2.0.7}$$

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \implies \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.8}$$

Since

$$\lambda_1 > \lambda_2 \tag{2.0.9}$$

Eccentricity of the ellipse is,

$$e = \sqrt{1 - \frac{\lambda_2}{\lambda_1}} = \frac{\sqrt{5}}{3} \tag{2.0.10}$$

Semi major and minor axes of ellipse are,

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = 3 \tag{2.0.11}$$

$$b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} = 2 \tag{2.0.12}$$

The co-ordinates of vertices are,

$$\pm \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{2.0.13}$$

The co-ordinates of foci are given by,

$$\mathbf{F} = \frac{ce^2\mathbf{n} - \mathbf{u}}{\lambda_1} \tag{2.0.14}$$

Where,

$$\mathbf{n} = \sqrt{\lambda_1}\mathbf{p}_2 \tag{2.0.15}$$

$$c = \frac{e\mathbf{u}^\top\mathbf{n} \pm \sqrt{e^2\left(\mathbf{u}^\top\mathbf{n}\right)^2 - \lambda_2\left(e^2-1\right)\left(\|\mathbf{u}\|^2 - \lambda_2 f\right)}}{\lambda_2 e\left(e^2-1\right)} \tag{2.0.16}$$

Substituting we have,

$$\mathbf{n} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{2.0.17}$$

$$c = \pm\frac{27}{\sqrt{5}} \tag{2.0.18}$$

$$\mathbf{F} = \pm\begin{pmatrix} 0 \\ \sqrt{5} \end{pmatrix}. \tag{2.0.19}$$

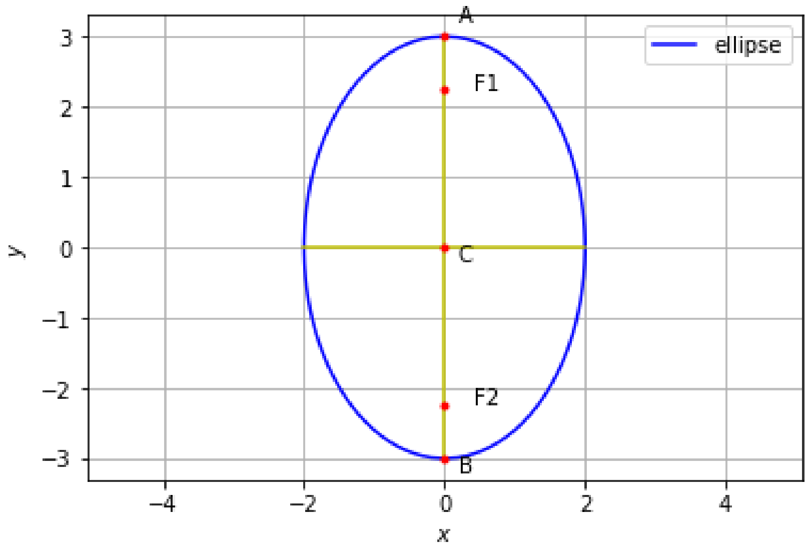

Fig. 0: Plot of the ellipse