My Presentation

Using Beamer

Valli Devi Bolla

NIPER Hyderabad

September 16, 2021

Question 2.7

Find the area of the region bounded by curve

$$y^2 = x \tag{1}$$

and the lines x = 1, x = 4 and x-axis in the first quadrant.

Solution

Given,

$$y^2 = x \tag{2}$$

can be written as

$$y^2 - x = 0 \tag{3}$$

The matrix parameters are

$$V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} u = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} f = 0 \tag{4}$$

Solution

Given x = 1

$$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} x = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} x = 1 \tag{7}$$

$$Similarlyx = 4, weget \tag{8}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} x = 4 \tag{9}$$

The direction vector and normal vectors are

$$m = \begin{pmatrix} 0 \\ 1 \end{pmatrix} n = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{10}$$

Vertex of Parabola

$$\begin{pmatrix} u^T + np^T \\ V \end{pmatrix} c = \begin{pmatrix} -f \\ np - u \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} \frac{-1}{4} & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} c = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} c = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{12}$$

$$k = \frac{p_1^T u}{p_1^T n} \implies \frac{-1}{2} \tag{13}$$

Point of contact q,

$$\begin{pmatrix} u + kn^T \\ V \end{pmatrix} q = \begin{pmatrix} -f \\ kn - u \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} \frac{-1}{2} & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} q = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \implies q = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{15}$$

Point of intersection

$$x_i = q + \mu_i m \tag{16}$$

$$\mu_i = \frac{1}{m^T V m}(-m^T V q + u) \tag{17}$$

$$\pm\sqrt{[m^T(Vq+u)]^2 - (q^T V q + 2u^T q + f)(m^T V m)} \tag{18}$$

$$\mu = 1, -1 \tag{19}$$

Substituting values for x = 1 we get

$$x_i = \begin{pmatrix}1\\0\end{pmatrix} + \mu \begin{pmatrix}0\\1\end{pmatrix} \implies K_1 = \begin{pmatrix}1\\1\end{pmatrix} L_1 = \begin{pmatrix}1\\-1\end{pmatrix}$$

Substituting values for x = 4 we get

$$x_i = \begin{pmatrix}4\\0\end{pmatrix} + \mu \begin{pmatrix}0\\1\end{pmatrix} \implies K_1 = \begin{pmatrix}4\\2\end{pmatrix} L_1 = \begin{pmatrix}4\\-2\end{pmatrix}$$

Area enclosed by parabola

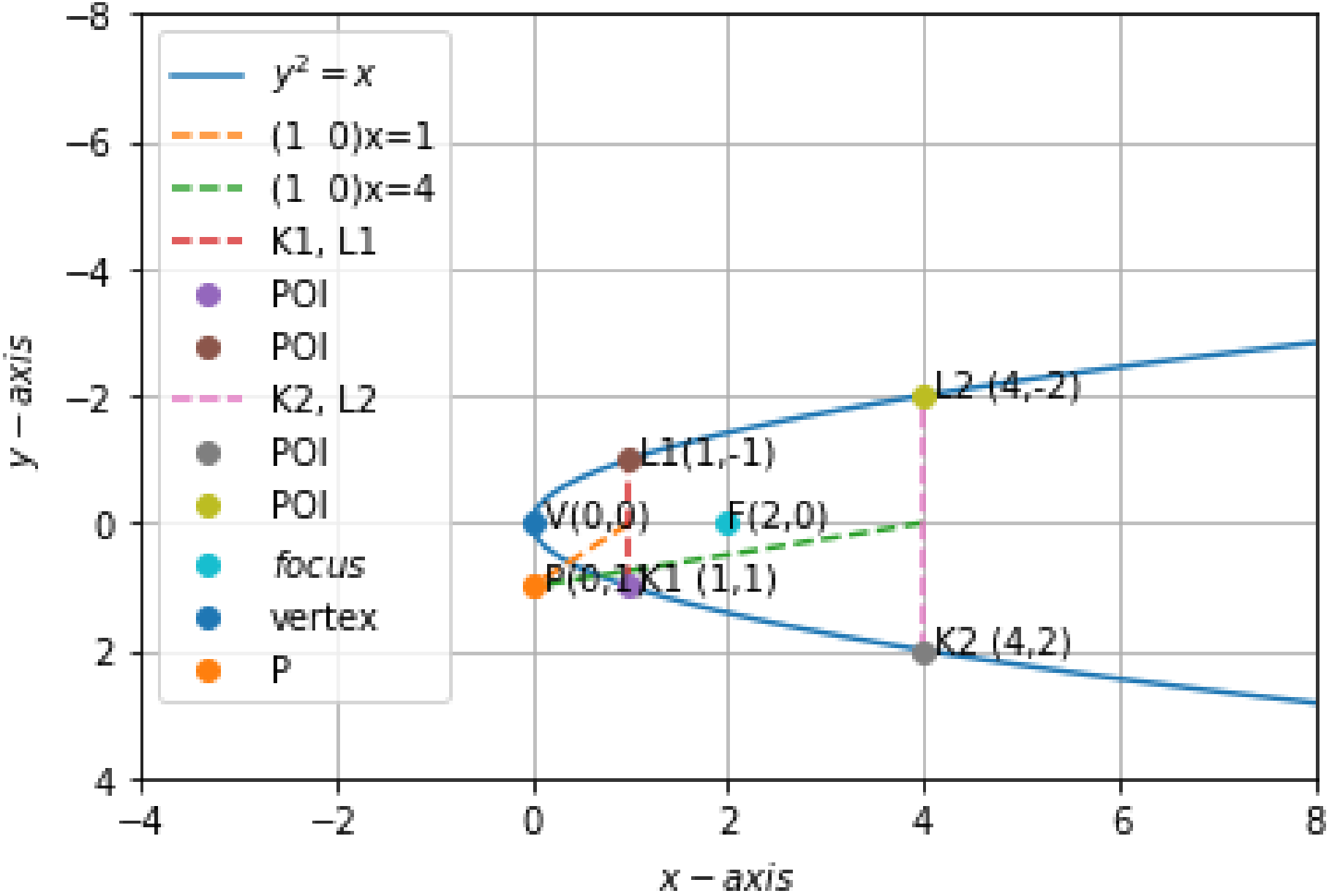
A = Area under line - Area under curve

$A_1 = \frac{16}{3}, A_2 = \frac{2}{3}$

$A = A_1 - A_2$

$\implies A = 4.67 units$

Output


$y^2 = x$
(1 0)x=1
(1 0)x=4
K1, L1
POI
POI
K2, L2
POI
POI
focus
vertex
P
L2 (4,-2)
L1(1,-1)
V(0,0)
F(2,0)
P(0,-1)
K1 (1,1)
K2 (4,2)
$y-axis$
$x-axis$

Codes References

1. Question taken from
   - https://github.com/gadepall/ncert/blob/main/linalg/quadratic$_f$orms/g
     $Q.no.2.7$
2. Download all python codes
   - https://github.com/Vallidevibolla/Assignment-5/blob/main/code.py
3. Download all latex-tikz codes
   - https://github.com/Vallidevibolla/Assignment-5/blob/main/main.tex