

# Assignment No.5

Valli Devi Bolla

Download all python codes from

<https://github.com/Vallidevibolla/Assignment-5/blob/main/code.py>

and latex-tikz codes from

<https://github.com/Vallidevibolla/Assignment-5/blob/main/main.tex>

Question taken from

[https://github.com/gadepall/ncert/blob/main/linalg/quadratic\\_forms/gvv\\_ncert\\_quadratic\\_forms.pdf](https://github.com/gadepall/ncert/blob/main/linalg/quadratic_forms/gvv_ncert_quadratic_forms.pdf)—Q.no.2.7

## 1 QUESTION 2.7

Find the area of the region bounded by curve

$$y^2 = x \quad (1.0.1)$$

and the lines  $x=1, x=4$  and  $x$ -axis in the first quadrant.

## 2 SOLUTION

(1.0.1) can be written as

$$y^2 - x = 0 \quad (2.0.1)$$

The matrix parameters are

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \mathbf{f} = 0. \quad (2.0.2)$$

Thus, the given curve is a parabola.  $\therefore \mathbf{V}$  is diagonal and in standard form, Also,

$$\mathbf{V}\mathbf{p} = \mathbf{0} \quad (2.0.3)$$

$$\Rightarrow \mathbf{p} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.4)$$

with eigen parameters

$$\lambda_1 = 0, \lambda_2 = 1 \quad (2.0.5)$$

Given  $x=1$  Using (1.0.1) we get

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.6)$$

$$(1 \ 0)\mathbf{x} = (1 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{y} (1 \ 0) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.7)$$

$$(1 \ 0)\mathbf{x} = 1 \quad (2.0.8)$$

Similarly,  $x=4$  we get

$$(4 \ 0)\mathbf{x} = 16 \quad (2.0.9)$$

$$\Rightarrow (1 \ 0)\mathbf{x} = 4 \quad (2.0.10)$$

The direction vector and normal vectors are

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (2.0.11)$$

The equation of parabola is

$$\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f = 0 \quad (2.0.12)$$

The vertex of conic section in (2.0.12) is given by  $\mathbf{c}$  using

$$\begin{pmatrix} \mathbf{u}^T + \mathbf{n}\mathbf{p}^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \mathbf{n}\mathbf{p} - \mathbf{u} \end{pmatrix} \quad (2.0.13)$$

$$\begin{pmatrix} -\frac{1}{4} & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \begin{pmatrix} -\frac{1}{4} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.15)$$

$$\text{or, } \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.16)$$

$$\kappa = \frac{\mathbf{p}_1^T \mathbf{u}}{\mathbf{p}_1^T \mathbf{n}}, \quad (2.0.17)$$

From (2.0.17), (2.0.11) and (2.0.4),

$$\kappa = \frac{-1}{2} \quad (2.0.18)$$

which, upon substitution in (2.0.19)

$$\begin{pmatrix} \mathbf{u} + \kappa \mathbf{n}^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa \mathbf{n} - \mathbf{u} \end{pmatrix} \quad (2.0.19)$$

and simplification yields the matrix equation

$$\begin{pmatrix} \frac{-1}{2} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.20)$$

$$\Rightarrow \begin{pmatrix} \frac{-1}{2} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.21)$$

$$\text{or, } \mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.22)$$

*Secant:* The points of intersection of the line

$$L: \mathbf{x} = \mathbf{q} + \mu \mathbf{m} \quad \mu \in \mathbb{R} \quad (2.0.23)$$

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \quad (2.0.24)$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left( -\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u})]^2 - (\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f)(\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \quad (2.0.25)$$

$\therefore \mathbf{q}$  is the point of contact,  $\mathbf{q}$  satisfies parabola equation

Given the point of contact  $\mathbf{q}$ , the equation of a tangent is

$$(\mathbf{V} \mathbf{q} + \mathbf{u})^T \mathbf{x} + \mathbf{u}^T \mathbf{q} + f = 0 \quad (2.0.26)$$

From (2.0.25) we get

$$\mu_1 = 1, \mu_2 = -1$$

The lines (2.0.8), (2.0.9) can be written in parametric form in (2.0.24) we get

$$\mathbf{x}_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mu_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.27)$$

Substituting  $\mu_1, \mu_2$  value in (2.0.27) we get

$$\mathbf{x}_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.28)$$

$$\Rightarrow \mathbf{K}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.29)$$

$$\mathbf{x}_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + -1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.30)$$

$$\Rightarrow \mathbf{L}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.31)$$

For  $x=4$ ,

$$\mathbf{x}_i = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \mu_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.32)$$

$$\mathbf{x}_i = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.33)$$

$$\Rightarrow \mathbf{K}_2 = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad (2.0.34)$$

$$\mathbf{x}_i = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + -2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.35)$$

$$\Rightarrow \mathbf{L}_2 = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \quad (2.0.36)$$

The area enclosed by parabola and line on x-axis can be given as

$A = \text{Area under line} - \text{Area under curve}$

$$\Rightarrow \boxed{A = A_1 - A_3} \quad (2.0.37)$$

Area under the parabola (1.0.1) i.e.,

$$A = \int x^{\frac{1}{2}} dx$$

Thus area under the lines (2.0.8), (2.0.9) is given by

$$A_1 = \frac{2}{3}, A_2 = \frac{16}{3}$$

Putting these values in (2.0.37) we get

$$\Rightarrow \boxed{A = \frac{14}{3}}$$

$A = 4.67 \text{ units}$

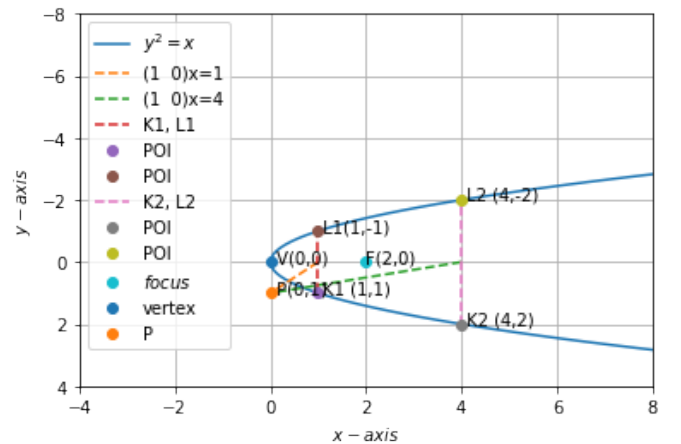


Fig. 2.1: Parabola  $y^2 = x$