ASSIGNMENT 4

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Download all python codes from

```
https://github.com/vishwahurakadli/EE3900/blob/main/Assignment_4/EE3900_Assignment_4.ipynb
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and latex-tikz codes from

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https://github.com/vishwahurakadli/EE3900/blob/main/Assignment_4/EE3900_Assignment_4.tex
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1 Problem

(Linear Forms-2.27)
Find the equation of the plane passing through the intersection of the planes $\begin{pmatrix} 2 & 2 & -3 \end{pmatrix}\mathbf{x} = 7$ and $\begin{pmatrix} 2 & 5 & 3 \end{pmatrix}\mathbf{x} = 9$ and the point $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

2 Solution

General equation of plane is given by

$$\mathbf{n}^T\mathbf{x} = c \tag{2.0.1}$$

The equation of a plane P passing through the intersection of two planes P_1 and P_2 is given by,

$$P : P_1 + \lambda P_2 \tag{2.0.2}$$

Lemma 2.1. *The equation of a plane passing through the intersection of two planes and given point will be*

$$P : P_1 + \lambda P_2 \tag{2.0.3}$$

Let planes P_1 and P_2 respectively given by

$$\mathbf{n_1}^T\mathbf{x} = c_1 \tag{2.0.4}$$
$$\mathbf{n_2}^T\mathbf{x} = c_2 \tag{2.0.5}$$

Plane P is given by

$$\mathbf{n^T}\mathbf{x} = c \tag{2.0.6}$$

Point through which Plane P is passing is A where

$$\mathbf{n}^T = \mathbf{n_1}^T + \left(\frac{c_1 - \mathbf{n_1}^T\mathbf{A}}{\mathbf{n_2}^T\mathbf{A} - c_2}\right)\mathbf{n_2}^T \tag{2.0.7}$$
$$c = c_1 + \left(\frac{c_1 - \mathbf{A}\mathbf{n_1}^T}{\mathbf{A}\mathbf{n_2}^T - c_2}\right)c_2 \tag{2.0.8}$$

Proof. Plane P passing through the intersection of two planes will be

$$\mathbf{n_1}^T\mathbf{x} + \lambda\left(\mathbf{n_2}^T\mathbf{x}\right) = c_1 + \lambda\left(c_2\right) \tag{2.0.9}$$
$$\implies \left(\mathbf{n_1} + \lambda\mathbf{n_2}\right)^T\mathbf{x} = c_1 + \lambda\left(c_2\right) \tag{2.0.10}$$

Then

$$\mathbf{n}^T = \mathbf{n_1}^T + \lambda\mathbf{n_2}^T \tag{2.0.11}$$
$$c = c_1 + \lambda c_2 \tag{2.0.12}$$

Given that plane P passes through point $\mathbf{A}$ then

$$\left(\mathbf{n_1} + \lambda\mathbf{n_2}\right)^T\mathbf{A} = c_1 + \lambda\left(c_2\right) \tag{2.0.13}$$
$$\implies \lambda = \frac{c_1 - \mathbf{n_1}^T\mathbf{A}}{\mathbf{n_2}^T\mathbf{A} - c_2} \tag{2.0.14}$$

Substituting λ in (2.0.11)

$$\mathbf{n}^T = \mathbf{n_1}^T + \left(\frac{c_1 - \mathbf{n_1}^T\mathbf{A}}{\mathbf{n_2}^T\mathbf{A} - c_2}\right)\mathbf{n_2}^T \tag{2.0.15}$$
$$c = c_1 + \left(\frac{c_1 - \mathbf{n_1}^T\mathbf{A}}{\mathbf{n_2}^T\mathbf{A} - c_2}\right)c_2 \tag{2.0.16}$$

□

For the given problem

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \tag{2.0.17}$$
$$\mathbf{n_2} = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} \tag{2.0.18}$$
$$c_1 = 7 \tag{2.0.19}$$
$$c_2 = 9 \tag{2.0.20}$$

By solving the given values

$$\lambda = \frac{10}{9} \tag{2.0.21}$$

$$\mathbf{n} = \begin{pmatrix} \frac{38}{9} \\ \frac{68}{9} \\ \frac{1}{3} \end{pmatrix} \tag{2.0.22}$$

$$c = 17 \tag{2.0.23}$$

So the equation of plane P is given by

$$\begin{pmatrix} \frac{38}{9} & \frac{68}{9} & \frac{1}{3} \end{pmatrix} \mathbf{x} = 17 \tag{2.0.24}$$

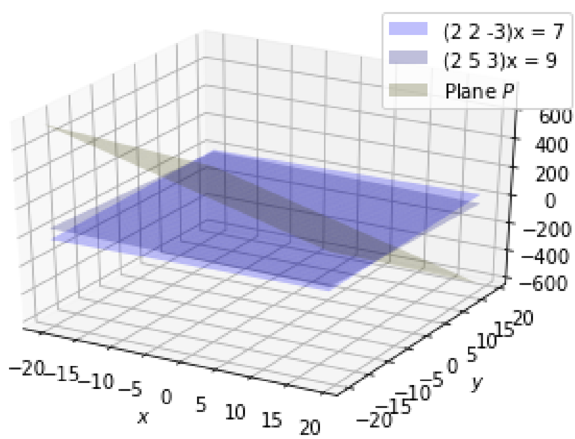

Fig. 0: Plane P passing through intersection of P_1 and P_2 and through a point $\mathbf{A}$