Matrices

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Contents

***Abstract*—This book provides a computational approach to school geometry based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    ncert/computation/codes
```

1 Definitions

1.1. For a 2×2 matrix

$$\mathbf{A} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}, \tag{1.1.1}$$

the inverse is given by

$$\mathbf{A}^{-1} = \frac{1}{\left|\mathbf{A}\right|} \begin{pmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{pmatrix}, \tag{1.1.2}$$

1.2. For higher order matrices, the inverse should be calculated using row operations.

2 Examples

2.1. Using elementary transforamtions,find the inverse of $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

Solution:
Given that

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \tag{2.1.1}$$

The augmented matrix $[\mathbf{A}|\mathbf{I}]$ is as given below:-

$$\left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right) \tag{2.1.2}$$

We apply the elementary row operations on $[\mathbf{A}|\mathbf{I}]$ as follows :-

$$[\mathbf{A}|\mathbf{I}] = \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right) \tag{2.1.3}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 5 & -2 & 1 \end{array}\right) \tag{2.1.4}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{5}} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{1}{5} \end{array}\right) \tag{2.1.5}$$

$$\xleftrightarrow{R_2 \leftarrow R_1 + R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & \frac{1}{5} \\ 0 & 1 & \frac{-2}{5} & \frac{1}{5} \end{array}\right) \tag{2.1.6}$$

By performing elementary transformations on augmented matrix$[\mathbf{A}|\mathbf{I}]$, we obtained the augmented matrix in the form $[\mathbf{I}|\mathbf{A}]$. Hence we can conclude that the matrix A is invertible and

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix} \tag{2.1.7}$$

2.2. Using elementary transforamtions, find the inverse of $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

Solution:
Given that

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \tag{2.2.1}$$

The augmented matrix $[\mathbf{A}|\mathbf{I}]$ is as given below:-

$$\left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right) \tag{2.2.2}$$

We apply the elementary row operations on $[\mathbf{A}|\mathbf{I}]$ as follows :-

$$[\mathbf{A}|\mathbf{I}] = \left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right) \tag{2.2.3}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{array}\right) \tag{2.2.4}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right) \tag{2.2.5}$$

By performing elementary transformations on augmented matrix$[\mathbf{A}|\mathbf{I}]$, we obtained the augmented matrix in the form $[\mathbf{I}|\mathbf{A}]$. Hence we

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can conclude that the matrix A is invertible and inverse of the matrix is

$$\mathbf{A^{-1}} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \tag{2.2.6}$$

2.3. Obtain the inverse of the following matrix using elementary operations
A=$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$.
Solution: Given that

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}, \tag{2.3.1}$$

The augmented matrix $[\mathbf{A}|\mathbf{I}]$ is

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.3.2}$$

Applying elementary row operations on $[\mathbf{A}|\mathbf{I}]$,

$$[\mathbf{A}|\mathbf{I}] = \left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.3.3}$$

$$\xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.3.4}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right) \tag{2.3.5}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right) \tag{2.3.6}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + 5R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right) \tag{2.3.7}$$

$$\xleftrightarrow{R_3 \leftarrow R_3/2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{array}\right) \tag{2.3.8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{array}\right) \tag{2.3.9}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{array}\right) \tag{2.3.10}$$

By performing elementary transformations on augmented matrix$[\mathbf{A}|\mathbf{I}]$, we obtained the augmented matrix in the form $[\mathbf{I}|\mathbf{A}]$. Hence we can conclude that the matrix A is invertible and inverse of the matrix is

$$\mathbf{A^{-1}} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{pmatrix} \tag{2.3.11}$$

2.4. Find P^{-1}, if it exists, given
P=$\begin{pmatrix} 10 & -2 \\ -5 & 1 \end{pmatrix}$.

Solution: Using row reduction,

$$\begin{pmatrix} 10 & -2 \\ -5 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + \frac{R_1}{2}} \begin{pmatrix} 10 & -2 \\ 0 & 0 \end{pmatrix} \quad (2.4.1)$$

Since we obtain a zero row, $\mathbf{P}^{-1}$ does not exist.