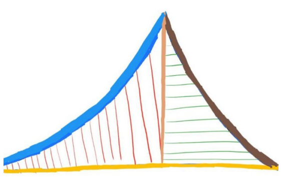
Random Variables in High School Probability



G. V. V. Sharma

About this Book

This book attempts to introduce matrices through high school coordinate geometry. All problems in the book are from NCERT mathematics textbooks from Class 9-12. The content is sufficient for industry jobs. There is no copyright, so readers are free to print and share.

July 9, 2024

Contents

1 Axioms

1.1 Formulae

1.1.1 Boolean Axioms:

$$A + A' = 1 \tag{1.1.1.1}$$
$$AA' = 0 \tag{1.1.1.2}$$

1.1.2 De Morgan's Law

$$A'B' = (A + B)' \tag{1.1.2.1}$$

1.1.3 Axioms of Probability

a)

$$0 \leq \Pr(A) \leq 1 \tag{1.1.3.1}$$

b) If $AB = 0$,

$$\Pr(A + B) = \Pr(A) + \Pr(B). \tag{1.1.3.2}$$

1.1.4 If A, B are independent,

$$\Pr(AB) = \Pr(A)\Pr(B) \tag{1.1.4.1}$$

1.1.5

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{1.1.5.1}$$

1.1.6 Substituting (1.1.13.2) in (1.1.12.2),

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{1.1.6.1}$$

1.1.7

$$A + B = A(B + B') + B \tag{1.1.7.1}$$
$$= B(A + 1) + AB' \tag{1.1.7.2}$$
$$= B + AB' \tag{1.1.7.3}$$

1.1.8

$$A = A(B + B') = AB + AB' \tag{1.1.8.1}$$

and

$$(AB)(AB') = 0, \because BB' = 0 \tag{1.1.8.2}$$

Hence, AB and AB' are mutually exclusive.

1.1.9 Let A, B and C be three events. Let X be the event that exactly one of A, B and C occurs. Let Y be the event that at least one of A, B or C occur. Let Z be the event that at least two of A, B or C occur.

$$Y = A + B + C \tag{1.1.9.1}$$

Similarly,

$$Z = AB + BC + CA \tag{1.1.9.2}$$

And,

$$X = (AB'C' + A'BC' + A'B'C) \tag{1.1.9.3}$$

1.1.10

$$\Pr(A') = 1 - \Pr(A). \tag{1.1.10.1}$$

1.1.11

$$\Pr(A'B') = \Pr((A + B)') \tag{1.1.11.1}$$
$$= 1 - \Pr(A + B) \tag{1.1.11.2}$$

1.1.12

$$\Pr(A + B) = \Pr(B + AB') \tag{1.1.12.1}$$
$$= \Pr(B) + \Pr(AB') \tag{1.1.12.2}$$
$$\because B(AB') = 0 \tag{1.1.12.3}$$

1.1.13

$$\Pr(A) = \Pr(AB) + \Pr(AB') \tag{1.1.13.1}$$
$$\implies \Pr(AB') = \Pr(A) - \Pr(AB) \tag{1.1.13.2}$$

1.2 Examples

1.1 Which of the following cannot be the probability of an event ?

a) $\frac{2}{3}$
b) -1.5
c) 15%
d) 0.7

Solution: From the axioms of probability,

$$0 \leq \Pr(E) \leq 1 \tag{1.1.1}$$

a) $\Pr(E) = \frac{2}{3}$

$$\because 0 \leq \frac{2}{3} \leq 1 \tag{1.1.2}$$

from (1.1.1), it can be probability of an event.

b) $\Pr(E) = -1.5$

$$\because -1.5 < 0 \tag{1.1.3}$$

from (1.1.1), it cannot be a probability of any event.

c)

$$\Pr(E) = \frac{15}{100} \tag{1.1.4}$$
$$\because 0 \leq \frac{15}{100} \leq 1, \tag{1.1.5}$$

from (1.1.1), it can be probability of an event.

d) $\Pr(E) = 0.7$

$$\because 0 \leq 0.7 \leq 1 \tag{1.1.6}$$

from (1.1.1), it can be a probability of an event.

1.2 If P(E) = 0.05, what is the probability of 'not E'?

Solution:

1.3 A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

a) an orange flavoured candy?
b) a lemon flavoured candy?

Solution:

1.4 It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Solution:

1.5 A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is

(i) red ?
(ii) not red?

Solution:

1.6 Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish . What is the probability that the fish taken out is a male fish?
Solution:

1.7 A box contains 12 balls, out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.
Solution:

1.8 A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is

a) a vowel
b) a consonant

. **Solution:**

1.9 In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prizes in the game? [Hint : order of the numbers is not important.]
Solution:

1.10 Check whether the following probabilities $\Pr(A)$ and $\Pr(B)$ are consistently defined

a) $\Pr(A) = 0.5, \Pr(B) = 0.7, \Pr(A \cap B) = 0.6$
b) $\Pr(A) = 0.5, \Pr(B) = 0.7, \Pr(A \cup B) = 0.8$

Solution:

1.11 Given $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{1}{5}$. Find $\Pr(A + B)$ if A and B are mutually exclusive events.
Solution:

1.12 If E and F are events such that $\Pr(E) = \frac{1}{4}$, $\Pr(F) = \frac{1}{2}$ and $\Pr(EF) = \frac{1}{8}$, find

a) $\Pr(E + F)$
b) $\Pr(E'F')$

Solution:

1.13 Events E and F are such that $\Pr(E' + F') = 0.25$, state whether E and F are mutually exclusive.
Solution:

1.14 A and B are events such that $\Pr(A) = 0.42, \Pr(B) = 0.48$ and $\Pr(\text{A and B}) = 0.16$.
Determine

a) $\Pr(\text{not A})$
b) $\Pr(\text{not B})$
c) $\Pr(\text{A or B})$

Solution:

1.15 If A and B are two independent events with $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{4}{9}$ then, $\Pr(A'B')$

a) $\frac{4}{15}$
b) $\frac{8}{45}$
c) $\frac{1}{3}$
d) $\frac{2}{9}$

Solution:

1.16 In class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class,find the probability that he will be studying Mathematics or Biology
Solution:

1.17 In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing atleast one of them is 0.95. What is the probability of passing both?
Solution:

1.18 In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

a) The student opted for NCC or NSS.
b) The student has opted neither NCC nor NSS.
c) The student has opted NSS but not NCC.

Solution:

1.19 A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

a) $\Pr(2)$
b) $\Pr(1 \text{ or } 3)$
c) $\Pr(\text{not } 3)$

Solution:

1.20 A and B are two events such that $\Pr(A) = 0.54$, $\Pr(B) = 0.69$ and $\Pr(AB) = 0.35$. Find

a) $\Pr(A + B)$
b) $\Pr(A'B')$
c) $\Pr(AB')$
d) $\Pr(BA')$

Solution:

1.21 If $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{1}{5}$ find $\Pr(A \cap B)$ if A and B are independent events.
Solution:

1.22 A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event,'the number is even,' and B be the event, 'the number is red'. Are A and B independent?
Solution:

1.23 Let E and F be events with $\Pr(E) = \frac{3}{5}, \Pr(F) = \frac{3}{10}$ and $\Pr(EF) = \frac{1}{5}$. Are E and F independent?
Solution:

1.24 Given that the events A and B are such that $P(A) = \frac{1}{2}, P(A + B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are

a) mutually exclusive
b) independent

Solution:

1.25 If A and B are two events such that $\Pr(A) = \frac{1}{4}, \Pr(B) = \frac{1}{2}$ and $\Pr(AB) = \frac{1}{8}$, find $\Pr(\text{not A and not B})$.
Solution:

1.26 Events A and B are such that

$$\Pr(A) = \frac{1}{2}, \Pr(B) = \frac{7}{12} \text{and} \Pr(A' + B') = \frac{1}{4}. \quad (1.26.1)$$

State whether A and B are independent.

Solution:

1.27 A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

1.28 A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even, ' and B be the event, 'the number is red'. Are A and B independent?

1.29 Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?

1.30 Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are

a) mutually exclusive
b) independent

1.31 If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P(\text{not } A \text{ and not } B)$

1.32 Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$. State whether A and B are independent?

1.33 Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find

a) $P(A \text{ and } B)$
b) $P(A \text{ and not } B)$
c) $P(A \text{ or } B)$
d) $P(\text{neither } A \text{ nor } B)$

1.34 Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

a) the problem is solved
b) exactly one of them solves the problem

Solution:

1.35 One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent ?

a) E: 'the card drawn is spade'
   F: 'the card drawn is an ace'
b) E: 'the card drawn is black'
   F : 'the card drawn is a king'
c) E : 'the card drawn is a king or queen'
   F : 'the card drawn is a queen or jack'

Solution:

Choose the correct answer in the following exercises

1.36 The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

a) 0
b) $\frac{1}{3}$
c) $\frac{1}{12}$
d) $\frac{1}{36}$

1.37 Two events A and B will be independent, if

a) A and B are mutually exclusive
b) P(not $A \cap$ not B) = $[1 - P(A)]\ [1 - P(B)]$
c) $P(A) = P(B)$
d) $P(A) + P(B) = 1$

Solution:

1.38 If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

1.39 State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

TABLE 1.39

a)

TABLE 1.39

b)

TABLE 1.39

c)

TABLE 1.39

d)

Solution:

1.40 If a leap year is selected at random, what is the chance that it will contain 53 tuesdays?

Solution:

1.41 Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Solution:

1.42 An electronic assembley consists of two subsystems,say A and B.From previous testing procedures, the following probabilities are assumed to be known

$$\Pr(\text{A fails}) = 0.20 \quad (1.42.1)$$
$$\Pr(\text{B alone fails}) = 0.15 \quad (1.42.2)$$
$$\Pr(\text{A and B fails}) = 0.15 \quad (1.42.3)$$

Evaluate the following probabilities

a) Pr (A fails given B has failed)
b) Pr (A fails alone)

Solution:

1.43 In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

a) Find the probability that she reads neither Hindi nor English newspapers.
b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Solution:

1.44 If the probabilities for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is

a) > 0.5
b) 0.5
c) ≤ 0.5
d) 0

Solution:

1.45 The probability that at least one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $\Pr(A') + \Pr(B')$.

Solution:

1.46 Prove that

a) $\Pr(A) = \Pr(AB) + \Pr(AB')$
b) $\Pr(A + B) = \Pr(AB) + \Pr(AB') + \Pr(A'B)$

1.47 A and B are events such that $\Pr(A) = 0.4$ and $\Pr(B) = 0.3$ and $\Pr(A + B) = 0.5$ .Then $\Pr(B'A)$ is equal to

Solution:

1.48 State True or False for the statement.
If $\Pr(A) > 0$ and $\Pr(B) > 0$. Then A and B can be mutually exclusive and independent.

Solution:

1.49 If A and B are independent events, then A' and B' are also independent.

Solution:

1.3 Conditional Probability

1.1 A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very-complex, complex, routine, simple or very-simple are respectively, 0.15, 0.20, 0.31, 0.26, .08. Find the probabilities that a particular surgery will be rated

a) complex or very-complex
b) neither very-complex nor very simple
c) routine or complex
d) routine or simple

Solution:

1.2 If A and B are mutually exclusive events, $\Pr(A) = 0.35$ and $\Pr(B) = 0.45$ then find

a) $\Pr(A')$
b) $\Pr(B')$
c) $\Pr(A + B)$
d) $\Pr(AB)$
e) $\Pr(AB')$
f) $\Pr(A'B')$

1.3 The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, $\Pr(AB) = 0.7$. Determine

a) $\Pr(A)$
b) $\Pr(BC')$
c) $\Pr(A + B)$
d) $\Pr(AB')$
e) $\Pr(BC)$
f) Probability of exactly one of the three occurs

Fig. 1.3.1: generated by Latextikz

Solution:

1.4 One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

a) What is the probability that two black balls are chosen?
b) What is the probability that two balls of opposite colour are chosen?

Solution:

1.5 Events E and F are such that P(not E or not F) = 0.25, State whether E and F are mutually exclusive.

1.6 Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9.

1.7 The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $\Pr(A') + \Pr(B')$is

a) 0.4
b) 0.8
c) 1.2
d) 1.6

Solution:

1.8 State whether the statement is True or False.
The probability that a person visiting a zoo will see the giraffe is 0.72, the probability that he will see the bears is 0.84 and the probability that he will see both is 0.52.

Solution:

1.9 The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is _____.

1.10 If A and B are mutually exclusive events,then

a) $\Pr(A) \leq \Pr(B')$
b) $\Pr(A) \geq \Pr(B')$
c) $\Pr(A) < \Pr(B')$
d) none of these

1.11 State whether the statement is True or False. The probabilities that a typist will make 0, 1, 2, 3, 4, 5 or

more mistakes in typing a report are, respectively, 0.12, 0.25, 0.36, 0.14, 0.08, 0.11.
Solution:

1.12 If A and B are two candidates seeking admission in an engineering College. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. Is it possible that the probability of B getting selected is 0.7?

1.13 Prove if the given statement is true or false - The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.

1.14 The probability of an occurrence of event A is .7 and that of the occurrence of event B is .3 and the probability of occurrence of both is .4.Is this statement true or false?
Solution:

1.15 If $\Pr(A+B) = \Pr(AB)$ for any two events A and B , then

A) $\Pr(A)$=$\Pr(B)$
B) $\Pr(A) > \Pr(B)$
C) $\Pr(A) < \Pr(B)$
D) none of these

1.16 Let E_1 and E_2 be two independent events such that $\Pr(E_1) = p_1$ and $\Pr(E_2) = p_2$ Describe in words the events whose probabilities are:

a) $p_1 p_2$
b) $(1-p_1)p_2$
c) $1-(1-p_1)(1-p_2)$
d) $p_1 + p_2 - 2p_1p_2$

1.17 Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

1.18

1.19 If $\Pr(A) = \frac{2}{5}$,$\Pr(B) = \frac{3}{10}$ and $\Pr(AB) = \frac{1}{5}$, then $\Pr(A'|B').\Pr(B'|A')$ is equal to

(A) $\frac{5}{6}$

(B) $\frac{5}{7}$

(C) $\frac{25}{42}$

(D) 1

1.20 A and B are two events such that $\Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{3}$ and $\Pr(AB) = \frac{1}{4}$.
Find:

i $\Pr(A|B)$
ii $\Pr(B|A)$
iii $\Pr(A'|B)$
iv $\Pr(A'|B')$

Solution: :

1.21 Let $\Pr(A) = \frac{7}{13}$, $\Pr(B) = \frac{9}{13}$, $\Pr(AB) = \frac{4}{13}$. Then $\Pr(A'|B)$ is equal to

(a) $\frac{6}{13}$
(b) $\frac{4}{13}$
(c) $\frac{4}{9}$
(d) $\frac{5}{9}$

1.22 If $\Pr(A) = \frac{3}{10}, \Pr(B) = \frac{2}{5}$ and $\Pr(A+B) = \frac{3}{5}$, then $\Pr(B|A) + \Pr(A|B)$ equals

a) $\frac{1}{4}$
b) $\frac{1}{3}$
c) $\frac{5}{12}$
d) $\frac{7}{12}$

1.23 The probability distribution of a discrete random variable X is given below. The value of k is equal to:

(a) 8
(b) 16
(c) 32
(d) 48

1.24 If A and B are such that
$\Pr(A' \cup B') = \frac{2}{3}$ and $\Pr(A \cup B) = \frac{5}{9}$
then $\Pr(A') + \Pr(B') =$
Solution:

1.25 state True or False for the given statement: Two independent events are always mutually exclusive.
Solution:

1.26 If A and B are independent, then
$\Pr(exactlyoneof A, B occurs)$ =$\Pr(B)\Pr(A')$+$\Pr(A)\Pr(B')$

1.27 If A and B are two events and A $\neq \phi$, B $\neq \phi$, then

a) $\Pr(A|B) = \Pr(A).\Pr(B)$
b) $\Pr(A|B) = \frac{\Pr(A\cap B)}{\Pr(B)}$
c) $\Pr(A|B).\Pr(B|A) = 1$
d) $\Pr(A|B) = \frac{\Pr(A)}{\Pr(B)}$

1.28 Let A and B be two events such that $\Pr(A) = \frac{3}{8}$, $\Pr(B) = \frac{5}{8}$ and $\Pr(A+B) = \frac{3}{4}$. Then $\Pr(A|B).\Pr(A'|B)$ is equal to

(a) $\frac{2}{5}$
(b) $\frac{3}{8}$
(c) $\frac{3}{20}$
(d) $\frac{6}{25}$

1.29 If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

a) 0.24
b) 0.3
c) 0.48
d) 0.96

Solution:

1.30 If A and B aret two events such that $\Pr(A) = \frac{1}{2}$ $\Pr(B) = \frac{1}{3}$, $\Pr(A|B)$=$\frac{1}{4}$, Then $\Pr(A'B')$ equals

a) $\frac{1}{12}$
b) $\frac{3}{4}$
c) $\frac{1}{4}$
d) $\frac{3}{16}$

Solution:

1.31 If A and B are such events that $\Pr(A) > 0$ and $\Pr(B) \neq 1$, then $\Pr(A'|B')$ is

a) $1 - \Pr(A|B)$
b) $1 - \Pr(A'|B)$
c) $\frac{1-\Pr(A+B)}{\Pr(B')}$
d) $\frac{\Pr(A')}{\Pr(B')}$

1.32 Two events E and F are independent. If $\Pr(E) = 0.3$, $\Pr(E+F) = 0.5$, then $\Pr(E|F) - \Pr(F|E)$ equals

(a) $\frac{2}{7}$
(b) $\frac{3}{35}$
(c) $\frac{1}{70}$
(d) $\frac{1}{7}$

Solution:

1.33 You are given that A and B are two events such that $\Pr(B)=\frac{3}{5}$ $\Pr(A \mid B)=\frac{1}{2}$ $\Pr(A+B)=\frac{4}{5}$ and $\Pr(A)=\frac{1}{2}$ $\Pr(B \mid A')$ is equal to

1.34 If A and B' are independent events, then $\Pr(A'+B) = 1 - \Pr(A)\Pr(B')$.

1.35 If A and B are two events such that $\Pr(A|B) = p, \Pr(A) = p, \Pr(B) = \frac{1}{3}$ and $\Pr(A+B) = \frac{5}{9}$,then $p =$

Solution:

1.36 If A and B are two events such that $\Pr(A) > 0$ and $\Pr(A)+\Pr(B) > 1$, then

$$\Pr(B|A) \geq 1 - \frac{\Pr(B')}{\Pr(A)}$$

Solution:

1.37 If

$$\Pr(B) = \frac{3}{5}, \Pr(A|B) = \frac{1}{2} \text{and} \Pr(A+B) = \frac{4}{5}, \text{then} \Pr(A+B)' + \Pr(A'+B) \tag{1.37.1}$$

Solution:

1.38 Let A and B be two events. If

$$\Pr(A|B) = \Pr(A), \tag{1.38.1}$$

then A is of B.

Solution:

1.39 If $\Pr(A|B) > \Pr(A)$, then which of the following is correct :

(A) $\Pr(B|A) < \Pr(B)$

(B) $\Pr(AB) < \Pr(A)\Pr(B)$

(C) $\Pr(B|A) > \Pr(B)$

(D) $\Pr(B|A) = \Pr(B)$

1.40 Let and B be independent events with $\Pr(A) = 0.3$ and $\Pr(B) = 0.4$. Find

a) $\Pr(AB)$
b) $\Pr(A+B)$
c) $\Pr(A|B)$
d) $\Pr(B|A)$

Solution:

1.41 Compute $\Pr(A|B)$, if $\Pr(B) = 0.5$ and $\Pr(AB) = 0.32$.

Solution:

1.42 A fair die is rolled. Consider events $E = 1,3,5$, $F = 2,3$ and $G = 2,3,4,5$. Find

a) $\Pr(E \mid F)$ and $\Pr(F \mid E)$
b) $\Pr(E \mid G)$ and $\Pr(G \mid E)$
c) $\Pr(E \cup F \mid G)$ and $\Pr(E \cap F \mid G)$

Solution:

1.43 If A and B are two events such that $P(A) \neq 0$ and $P(B \mid A) = 1$, then

a) $A \subset B$

b) $B \subset A$

c) $B = \phi$

d) $A = \phi$

Solution:

1.44 If $\Pr(A \mid B) > \Pr(A)$, then which of the following is correct

a) $\Pr(B \mid A) < \Pr((B))$

b) $\Pr(AB) < \Pr(A)\Pr(B)$

c) $\Pr(B \mid A) > \Pr(B)$

d) $\Pr(B \mid A) = \Pr(B)$

Solution:

1.45 You are given that A and B are two events such that $\Pr(B) = \frac{3}{5}, \Pr(A|B) = \frac{1}{2}$ and $\Pr(A+B) = \frac{4}{5}$,then $\Pr(A)$ equals

Solution:

1.46 If A and B are any two events such that $\Pr(A)$+$\Pr(B)$-$\Pr(AB)$=$\Pr(A)$, then choose the correct option

a) $\Pr(B|A) = 1$
b) $\Pr(A|B) = 1$
c) $\Pr(B|A) = 0$
d) $\Pr(A|B) = 0$

Solution:

1.47 Three events A, B and C have probabilities $\frac{2}{5}$,$\frac{1}{3}$ and $\frac{1}{2}$ respetively.Given that $\Pr(AC) = \frac{1}{5}$ and $\Pr(BC) = \frac{1}{4}$,find the values of $\Pr(C|B)$ and $\Pr(A'C')$

Solution:

1.48 Two Coins are tossed once, where
(i) E : Tail appears on one coin, F : one coin shows head
(ii) E : no tail appears, F : no head appears
Determine $\Pr(E \mid F)$.

Appendix