Cross Product

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Contents

***Abstract*—This manual provides an introduction to the cross product, based on the NCERT textbooks from Class 6-12.**

1 Cross

1.1. Find the area of a triangle having the points $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ as its vertices.
Solution:

1.2. Find the area of a triangle with vertices $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}$
Solution:

1.3. Find the area of the triangle whose vertices are
a) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \end{pmatrix}$
b) $\begin{pmatrix} -5 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \end{pmatrix}$
Solution:

1.4. Find the area of the triangle formed by joining the mid points of the sides of a triangle whose vertices are $\begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}$.
Solution:

1.5. Verify that the median of $\triangle ABC$ with vertices $\mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ divides it into two triangles of equal areas.
Solution:

1.6. Find the area of a triangle whose vertices are $\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$.
Solution: Using Hero's formula, the following code computes the area of the triangle as 24.

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codes/triangle/area_tri.py
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1.7. Find the area of a triangle formed by the vertices $\mathbf{A} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$.
Solution: The area of $\triangle ABC$ is also obtained in terms of the *magnitude* of the determinant of the matrix $\mathbf{M}$ in (**??**) as

$$\frac{1}{2}\left|\mathbf{M}\right| \tag{1.7.1}$$

The computation is done in **area_tri.py**

1.8. Find the area of a triangle formed by the points $\mathbf{P} = \begin{pmatrix} -1.5 \\ 3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$.
Solution: Another formula for the area of $\triangle ABC$ is

$$\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{vmatrix} \tag{1.8.1}$$

1.9. Find the area of a triangle having the points

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \tag{1.9.1}$$

as its vertices.
Solution: The area of a triangle using the *vector product* is obtained as

$$\frac{1}{2}\left\|(\mathbf{B}-\mathbf{A}) \times (\mathbf{C}-\mathbf{A})\right\| \tag{1.9.2}$$

For any two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$,

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{1.9.3}$$

The following code computes the area using the vector product.

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codes/triangle/area_tri_vec.py
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1.10. Find the area of a parallelogram whose adjacent sides are given by the vectors $\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.
Solution: The area is given by

$$\frac{1}{2}\left\|\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right\| \tag{1.10.1}$$

1.11. Draw a quadrilateral in the Cartesian plane, whose vertices are $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$, $\begin{pmatrix} 5 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$. Also, find its area.
Solution:

1.12. Find the area of a rhombus if its vertices are

$$\mathbf{P} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \tag{1.12.1}$$

$$\mathbf{R} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{1.12.2}$$

taken in order.
Solution:

1.13. Find the area of the quadrilateral whose vertices, taken in order, are $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.
Solution:

1.14. Find the area of a parallelogram whose adjacent sides are given by the vectors $\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.
Solution:

1.15. Find the area of a rectangle $ABCD$ with vertices $\mathbf{A} = \begin{pmatrix} -1 \\ \frac{1}{2} \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ \frac{1}{2} \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -\frac{1}{2} \\ 4 \end{pmatrix}$.
Solution:

1.16. The two adjacent sides of a parallelogram are $\begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$. Find the unit vector parallel to its diagonal. Also, find its area.
Solution:

1.17. If $\mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ -6 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, find the area of the quadrilateral $ABCD$.
Solution: The area of $ABCD$ is the sum of the areas of trianges ABD and CBD and is given by

$$\frac{1}{2}\left\|(\mathbf{A}-\mathbf{B}) \times (\mathbf{A}-\mathbf{D})\right\| + \frac{1}{2}\left\|(\mathbf{C}-\mathbf{B}) \times (\mathbf{C}-\mathbf{D})\right\| \tag{1.17.1}$$