Vector Properties

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CONTENTS

***Abstract*—This book provides a computational approach to school geometry based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

1 Direction Vector

1.1. Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points $\mathbf{P} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$. **Solution:**

1.2. The slope of a line is double of the slope of another line. If the tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines. **Solution:**

1.3. Find a unit vector that makes an angle of $90°, 60°$ and $30°$ with the positive x, y and z axis respectively.

Solution: The direction vector is

$$\mathbf{x} = \begin{pmatrix} \cos 90° \\ \cos 60° \\ \cos 30° \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{1.3.1}$$

$\because \|\mathbf{x}\| = 1$, it is the desired unit vector.

1.4. Find the direction vectors and slopes of the lines passing through the points

a) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

b) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ -2 \end{pmatrix}$.

c) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

d) Making an inclination of $60°$ with the positive direction of the x-axis.

Solution:

a) If the direction vector is

$$\begin{pmatrix} 1 \\ m \end{pmatrix}, \tag{1.4.1}$$

the slope is m. Thus, the direction vector is

$$\begin{pmatrix} -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{1.4.2}$$

$$= \begin{pmatrix} 1 \\ -\frac{3}{2} \end{pmatrix} \implies m = -\frac{3}{2} \tag{1.4.3}$$

b) The direction vector is

$$\begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{1.4.4}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \implies m = 0 \tag{1.4.5}$$

c) The direction vector is

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{1.4.6}$$

$$= \begin{pmatrix} 1 \\ \infty \end{pmatrix} \implies m = \infty \tag{1.4.7}$$

d) The slope is $m = \tan 60° = \sqrt{3}$ and the direction vector is

$$\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \tag{1.4.8}$$

1.5. If the angle between two lines is $\frac{\pi}{4}$ and the slope of one of the lines is $\frac{1}{4}$ find the slope of the other line.

Solution: The angle θ between two lines is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} \tag{1.5.1}$$

$$\implies 1 = \frac{m_1 - \frac{1}{4}}{1 + \frac{m_1}{4}} \tag{1.5.2}$$

$$\text{or } m_1 = \frac{5}{3} \tag{1.5.3}$$

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1.6. If the points $\mathbf{A} = \begin{pmatrix}6\\1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}8\\2\end{pmatrix}, \mathbf{C} = \begin{pmatrix}9\\4\end{pmatrix}, \mathbf{D} = \begin{pmatrix}p\\3\end{pmatrix}$ are the vertices of a parallelogram, taken in order, find the value of p.
Solution: In the parallelogram $ABCD$, AC and BD bisect each other. This can be used to find p.

1.7. Without using distance formula, show that points $\begin{pmatrix}-2\\-1\end{pmatrix}$, $\begin{pmatrix}4\\0\end{pmatrix}$, $\begin{pmatrix}3\\3\end{pmatrix}$ and $\begin{pmatrix}-3\\2\end{pmatrix}$ are the vertices of a parallelogram.
Solution:

1.8. The two opposite vertices of a square are $\begin{pmatrix}-1\\2\end{pmatrix}$, $\begin{pmatrix}3\\2\end{pmatrix}$. Find the coordinates of the other two vertices.
Solution:

1.9. Find the direction vectors of the sides of a triangle with vertices $\mathbf{A} = \begin{pmatrix}3\\5\\-4\end{pmatrix}, \mathbf{B} = \begin{pmatrix}-1\\1\\2\end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix}-5\\-5\\-2\end{pmatrix}$
Solution:

1.10. Find a unit vector in the direction of

$$\begin{pmatrix}1\\1\\-2\end{pmatrix}. \tag{1.10.1}$$

Solution:

1.11. Find a unit vector in the direction of $\begin{pmatrix}2\\-1\\-2\end{pmatrix}$.
Solution:

1.12. Find a unit vector in the direction of the line passing through $\begin{pmatrix}-2\\4\\-5\end{pmatrix}$ and $\begin{pmatrix}1\\2\\3\end{pmatrix}$.
Solution:

2 Norm

2.1. Find the unit normal vector of the plane

$$\begin{pmatrix}6 & -3 & -2\end{pmatrix}\mathbf{x} = 1. \tag{2.1.1}$$

Solution: The normal vector is

$$\mathbf{n} = \begin{pmatrix}6 & -3 & -2\end{pmatrix} \tag{2.1.2}$$

$$\because \|\mathbf{n}\| = 7, \tag{2.1.3}$$

the unit normal vector is

$$\frac{\mathbf{n}}{\|\mathbf{n}\|} = \frac{1}{7}\begin{pmatrix}6 & -3 & -2\end{pmatrix} \tag{2.1.4}$$

2.2. Find the condition for $\mathbf{x} = \begin{pmatrix}x_1\\x_2\end{pmatrix}$ to be equidistant from the points $\begin{pmatrix}7\\1\end{pmatrix}, \begin{pmatrix}3\\5\end{pmatrix}$.
Solution: From the given information,

$$\left\|\mathbf{x} - \begin{pmatrix}7\\1\end{pmatrix}\right\|^2 = \left\|\mathbf{x} - \begin{pmatrix}3\\5\end{pmatrix}\right\|^2 \tag{2.2.1}$$

$$\implies \|\mathbf{x}\|^2 + \left\|\begin{pmatrix}7\\1\end{pmatrix}\right\|^2 - 2\begin{pmatrix}7 & 1\end{pmatrix}\mathbf{x}$$
$$= \|\mathbf{x}\|^2 + \left\|\begin{pmatrix}3\\5\end{pmatrix}\right\|^2 - 2\begin{pmatrix}3 & 5\end{pmatrix}\mathbf{x} \tag{2.2.2}$$

which can be simplified to obtain

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 2 \tag{2.2.3}$$

which is the desired condition. The following code plots Fig. **??**clearly showing that the above equation is the perpendicular bisector of AB.

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codes/line/line_perp_bisect.py
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2.3. Find a point on the y-axis which is equidistant from the points $\mathbf{A} = \begin{pmatrix}6\\5\end{pmatrix}, \mathbf{B} = \begin{pmatrix}-4\\3\end{pmatrix}$.
Solution:

2.4. Find the equation of set of points $\mathbf{P}$ such that

$$PA^2 + PB^2 = 2k^2, \tag{2.4.1}$$

$$\mathbf{A} = \begin{pmatrix}3\\4\\5\end{pmatrix}, \mathbf{B} = \begin{pmatrix}-1\\3\\-7\end{pmatrix}, \tag{2.4.2}$$

respectively. **Solution:**

2.5. Find the equation of the set of points $\mathbf{P}$ such that its distances from the points $\mathbf{A} = \begin{pmatrix}3\\4\\-5\end{pmatrix}, \mathbf{B} = \begin{pmatrix}-2\\1\\4\end{pmatrix}$ are equal.
Solution: