

Gradient Descent

1 11th Maths - Chapter 10

This is Problem-3.1 from Exercise 10.3

1. Reduce $x - \sqrt{3}y + 8 = 0$ into normal form. Find its perpendicular distance from the origin and angle between perpendicular and the positive x-axis.

Solution: Equation for a line can be represented in parametric form as

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \quad (1)$$

We choose

$$\mathbf{A} = \begin{pmatrix} -8 \\ 0 \end{pmatrix} \quad (2)$$

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{3}} \end{pmatrix} \quad (4)$$

$$\text{yielding } f(\lambda) = \frac{4}{3}\lambda^2 - 16\lambda + 64 \quad (5)$$

$$f'(\lambda) = \frac{8}{3}\lambda - 16 \quad (6)$$

Computing λ_{min} using Gradient Descent method:

$$\lambda_{n+1} = \lambda_n - \alpha f'(\lambda_n) \quad (7)$$

$$\lambda_{n+1} = \lambda_n \left(1 - \frac{8}{3}\alpha \right) + 16\alpha \quad (8)$$

Taking the one-sided Z -transform on both sides of (8),

$$z\Lambda(z) = \left(1 - \frac{8}{3}\alpha\right) \Lambda(z) + \frac{16\alpha}{1 - z^{-1}} \quad (9)$$

$$\Lambda(z) = \frac{16\alpha z^{-1}}{(1 - z^{-1}) \left(1 - \left(1 - \frac{8}{3}\alpha\right) z^{-1}\right)} \quad (10)$$

$$= 6 \left(\frac{1}{1 - z^{-1}} - \frac{1}{1 - \left(1 - \frac{8}{3}\alpha\right) z^{-1}} \right) \quad (11)$$

$$= 6 \sum_{k=0}^{\infty} \left(1 - \left(1 - \frac{8}{3}\alpha\right)^k \right) z^{-k} \quad (12)$$

From (12), the ROC is

$$|z| > \max \left\{ 1, \left| 1 - \frac{8}{3}\alpha \right| \right\} \quad (13)$$

$$\implies -1 < \left| 1 - \frac{8}{3}\alpha \right| < 1 \quad (14)$$

$$\implies 0 < \alpha < \frac{3}{4} \quad (15)$$

Thus, if α satisfies (15), then from (12),

$$\lim_{n \rightarrow \infty} \lambda_n = 6 \quad (16)$$

Choosing

- (a) $\alpha = 0.001$
- (b) precision = 0.0000001
- (c) n = 10000000
- (d) $\lambda_0 = -5$

$$\lambda_{min} = 6 \quad (17)$$

Substituting the values of \mathbf{A} , \mathbf{m} and λ_{min} in equation (1)

$$\mathbf{x}_{min} = \mathbf{P} = \begin{pmatrix} -8 \\ 0 \end{pmatrix} + 6 \begin{pmatrix} 1 \\ \frac{1}{\sqrt{3}} \end{pmatrix} \quad (18)$$

$$= \begin{pmatrix} -8 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ \frac{6}{\sqrt{3}} \end{pmatrix} \quad (19)$$

$$= \begin{pmatrix} -2 \\ 2\sqrt{3} \end{pmatrix} \quad (20)$$

$$OP = \|\mathbf{P} - \mathbf{O}\|^2 \quad (21)$$

$$= \left\| \begin{pmatrix} -2 \\ 2\sqrt{3} \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\| \quad (22)$$

$$= \sqrt{2^2 + 12} = \sqrt{16} = 4 \quad (23)$$

The angle θ made by this perpendicular with x-axis is given by

$$\theta = \tan^{-1} \left(\frac{2\sqrt{3}}{-2} \right) \quad (24)$$

$$= \tan^{-1} (-\sqrt{3}) \quad (25)$$

$$= 120^\circ \quad (26)$$

The normal form of equation for straight line is given by

$$\begin{pmatrix} \cos 120^\circ \\ \sin 120^\circ \end{pmatrix}^\top \mathbf{x} = 4 \quad (27)$$

The relevant figure is as shown in 1 and 2

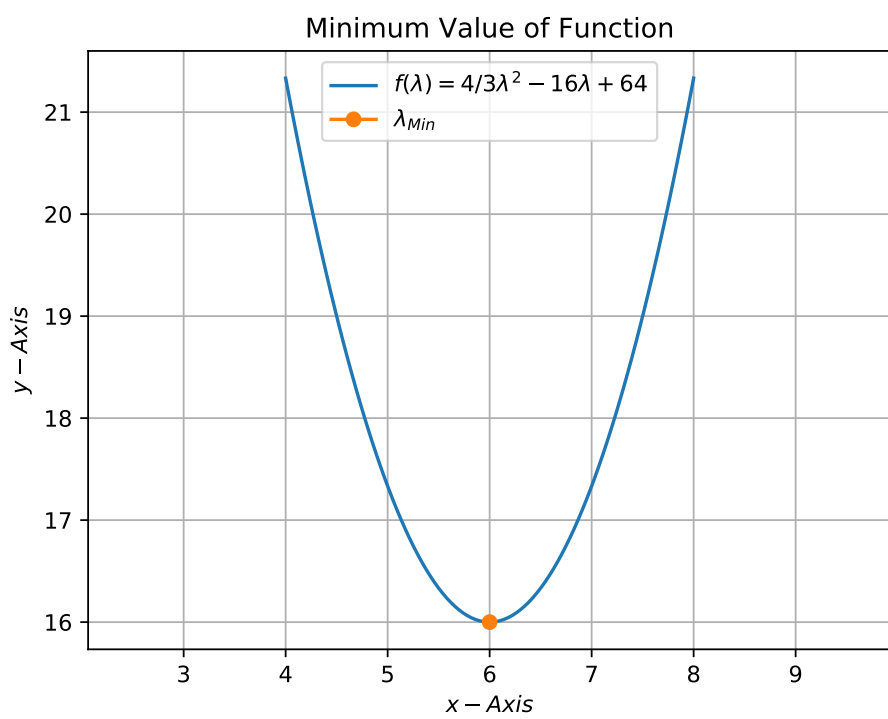


Figure 1

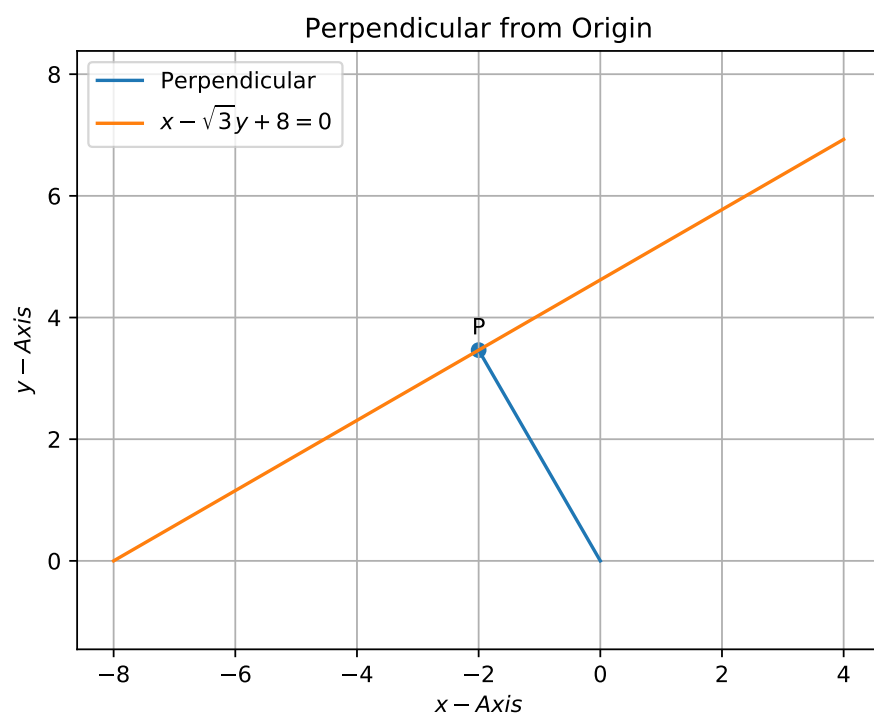


Figure 2