Geometric Programming Assignment

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***Abstract*—This document contains the solution to Question 25 of Exercise 5 in Chapter 6 of the class 12 NCERT textbook.**

1) Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

Solution: We use geometric programming. Taking the radius to be r, height to be h, and slant height ; = 1 without loss of generality, we need to find

$$\max_{r,h} \frac{1}{3}\pi r^2 h \tag{1}$$

$$\text{s.t. } r^2 + h^2 = 1 \tag{2}$$

$$r, h \geq 0 \tag{3}$$

The Python code `codes/gp.py` solves this Disciplined Geometric Programming (DGP) problem using *cvxpy*. The solutions are

$$r_M = \sqrt{\frac{2}{3}},\ h_M = \frac{1}{\sqrt{3}} \tag{4}$$

Hence, from (4), the required semi-verical angle is

$$\alpha = \tan^{-1}\frac{r}{h} = \tan^{-1}\sqrt{2} \tag{5}$$

as required.