Optimization Assignment

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***Abstract*—This document contains the solution to Question 4 of Exercise 2 in Chapter 10 of the class 11 NCERT textbook.**

1) Find the coordinates of the foot of perpendicular from the point

$$\mathbf{P} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{1}$$

to the line

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = 16 \tag{2}$$

Solution: Any point on (2) is clearly of the form

$$\mathbf{Q} = \mathbf{A} + \lambda \mathbf{m} \tag{3}$$

where $\lambda \in \mathbb{R}$ and

$$\mathbf{A} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{4}$$

Thus,

$$f(\lambda) = \|\mathbf{Q} - \mathbf{P}\|^2 \tag{5}$$
$$= \|\mathbf{A} - \mathbf{P} + \lambda \mathbf{m}\|^2 \tag{6}$$
$$= \|\mathbf{m}\|^2 \lambda^2 + 2\mathbf{m}^\top (\mathbf{A} - \mathbf{P}) \lambda + \|\mathbf{A} - \mathbf{P}\|^2 \tag{7}$$

Since the coefficient of λ^2 in $f(\lambda)$ is positive, it follows that $f(\lambda)$ is convex. Hence, the minima is achieved at

$$f'(\lambda_m) = 2\left(\|\mathbf{m}\|^2 \lambda_m + \mathbf{m}^\top (\mathbf{A} - \mathbf{P})\right) = 0 \tag{8}$$
$$\implies \lambda_m = -\frac{\mathbf{m}^\top (\mathbf{A} - \mathbf{P})}{\|\mathbf{m}\|^2} \tag{9}$$

Thus,

$$\mathbf{Q_m} = \mathbf{A} + \lambda_m \mathbf{m} \tag{10}$$
$$= \mathbf{A} - \frac{\mathbf{m}^\top (\mathbf{A} - \mathbf{P})}{\|\mathbf{m}\|^2} \mathbf{m} \tag{11}$$

Thus, substituting (4) into (11), we get

$$\mathbf{Q_m} = \frac{1}{25} \begin{pmatrix} 68 \\ -49 \end{pmatrix} \tag{12}$$

The value of λ_m is verified in Fig. 1, plotted by the Python code `codes/convex.py`.

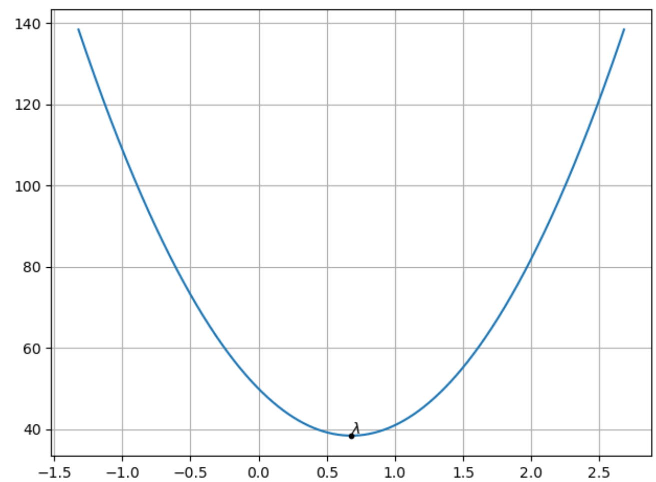

Fig. 1: This convex function achieves its minimum at λ_m.