Quadratic Programming Assignment

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***Abstract*—This document contains the solution to a modification of Question 27 of Exercise 5 in Chapter 6 of the class 12 NCERT textbook.**

1) Find the point on the curve

$$x^2 = 2y \tag{1}$$

which is nearest to the point $\mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Solution: We need to find

$$\min_{\mathbf{x}} g(\mathbf{x}) = \|\mathbf{x} - \mathbf{P}\|^2 \tag{2}$$

$$\text{s.t. } h(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} = 0 \tag{3}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{4}$$

We use the method of Lagrange multipliers to find the optima. Here, we need to find $\lambda \in \mathbb{R}$ such that there exists a $\mathbf{x}$ satisfying

$$\nabla g(\mathbf{x}) = \lambda \nabla h(\mathbf{x}) \tag{5}$$

$$\implies 2(\mathbf{x} - \mathbf{P}) = 2\lambda(\mathbf{V}\mathbf{x} + \mathbf{u}) \tag{6}$$

$$\implies (\mathbf{I} - \lambda\mathbf{V})\mathbf{x} = \lambda\mathbf{u} + \mathbf{P} \tag{7}$$

$$\implies \begin{pmatrix} 1-\lambda & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 1-\lambda \end{pmatrix} \tag{8}$$

From (8), we have two cases:

a) $\lambda \neq 1$. In this case, we form the augmented matrix

$$\begin{pmatrix} 1-\lambda & 0 & 2 \\ 0 & 1 & 1-\lambda \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{1-\lambda}} \begin{pmatrix} 1 & 0 & \frac{2}{1-\lambda} \\ 0 & 1 & 1-\lambda \end{pmatrix} \tag{9}$$

and get that

$$\mathbf{x_m} = \begin{pmatrix} \frac{2}{1-\lambda} \\ 1-\lambda \end{pmatrix} \tag{10}$$

Substituting in (3) with equality gives $\lambda = 1 - 2^{\frac{1}{3}}$. Thus,

$$\mathbf{x_m} = \begin{pmatrix} 2^{\frac{2}{3}} \\ 2^{\frac{1}{3}} \end{pmatrix} \tag{11}$$

b) $\lambda = 1$. In this case, (8) becomes

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{12}$$

which clearly has no solution.

Thus, the required point is

$$\mathbf{x_m} = \begin{pmatrix} 2^{\frac{2}{3}} \\ 2^{\frac{1}{3}} \end{pmatrix} \tag{13}$$