# Quadratic Programming Assignment

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***Abstract*—This document contains the solution to Question 27 of Exercise 5 in Chapter 6 of the class 12 NCERT textbook.**

1) The point on the curve

$$x^2 = 2y \tag{1}$$

which is nearest to the point $\mathbf{P} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ is

a) $\begin{pmatrix} 2\sqrt{2} \\ 4 \end{pmatrix}$
b) $\begin{pmatrix} 2\sqrt{2} \\ 0 \end{pmatrix}$
c) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
d) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

**Solution:** We need to find

$$\min_{\mathbf{x}} g(\mathbf{x}) = \|\mathbf{x} - \mathbf{P}\|^2 \tag{2}$$

$$\text{s.t. } h(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} = 0 \tag{3}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{4}$$

Since the given optimization problem is nonconvex, we use the method of Lagrange multipliers to find the optima. Here, we need to find $\lambda \in \mathbb{R}$ such that there exists a $\mathbf{x}$ satisfying

$$\nabla g(\mathbf{x}) = \lambda \nabla h(\mathbf{x}) \tag{5}$$

$$\implies 2(\mathbf{x} - \mathbf{P}) = 2\lambda(\mathbf{A}\mathbf{x} + \mathbf{u}) \tag{6}$$

$$\implies (\mathbf{I} - \lambda\mathbf{A})\mathbf{x} = \lambda\mathbf{u} + \mathbf{P} \tag{7}$$

$$\implies \begin{pmatrix} 1-\lambda & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 5-\lambda \end{pmatrix} \tag{8}$$

From (8), we have two cases:

a) $\lambda \neq 1$. In this case, we form the augmented matrix

$$\begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 1 & 5-\lambda \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{1-\lambda}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 5-\lambda \end{pmatrix} \tag{9}$$

and get that

$$\mathbf{x_m} = \begin{pmatrix} 0 \\ 5-\lambda \end{pmatrix} \tag{10}$$

Substituting in (3) gives $\lambda = 5$. Thus, $\mathbf{x_m} = \mathbf{0}$.

b) $\lambda = 1$. In this case, (8) becomes

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{11}$$

$$\implies \mathbf{e_2}^\top \mathbf{x} = 4 \tag{12}$$

Substituting (12) into (3) becomes

$$\left(\mathbf{e_1}^\top \mathbf{x}\right)^2 = 8 \tag{13}$$

$$\implies \mathbf{e_1}^\top \mathbf{x} = \pm 2\sqrt{2} \tag{14}$$

Using (14) and (12),

$$\mathbf{x_m} = \begin{pmatrix} \pm 2\sqrt{2} \\ 4 \end{pmatrix} \tag{15}$$

Using these values of $\mathbf{x_m}$, the distances are

$$\left\| \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\| = 5 \tag{16}$$

$$\left\| \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} \pm 2\sqrt{2} \\ 4 \end{pmatrix} \right\| = 3 \tag{17}$$

Thus, the correct answer is **a)**.