Quadratic Programming Assignment

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***Abstract*—This document contains the solution to Question 27 of Exercise 5 in Chapter 6 of the class 12 NCERT textbook.**

1) Show that the point on the curve

$$x^2 = 2y \tag{1}$$

which is nearest to the point $\mathbf{P} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ is a nonconvex optimization problem.
Solution: We need to find

$$\min_{\mathbf{x}} g(\mathbf{x}) = \|\mathbf{x} - \mathbf{P}\|^2 \tag{2}$$

$$\text{s.t. } h(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \ f = 0 \tag{4}$$

Suppose $\mathbf{x_1}$ and $\mathbf{x_2}$ satisfy $h(\mathbf{x}) = 0$. Then,

$$\mathbf{x_1}^\top \mathbf{V} \mathbf{x_1} + 2\mathbf{u}^\top \mathbf{x_1} + f = 0 \tag{5}$$

$$\mathbf{x_2}^\top \mathbf{V} \mathbf{x_2} + 2\mathbf{u}^\top \mathbf{x_2} + f = 0 \tag{6}$$

Then, for any $0 \le \lambda \le 1$, substituting

$$\mathbf{x}_\lambda \leftarrow \lambda \mathbf{x_1} + (1 - \lambda) \mathbf{x_2} \tag{7}$$

into (3), we get

$$h(\mathbf{x}_\lambda) = \lambda (\lambda - 1) (\mathbf{x_1} - \mathbf{x_2})^\top \mathbf{V} (\mathbf{x_1} - \mathbf{x_2}) \neq 0 \tag{8}$$

since $\mathbf{x_1} - \mathbf{x_2}$ can be arbitrary. Hence, the optimization problem is nonconvex as the set of points on the parabola do not form a convex set. The constraints throw an error when *cvxpy* is used, as shown in the erroneous Python code `codes/parab_cvx.py`.