Physical Layer Design for a Narrow Band Communication System

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Abstract—**The design and implementation of a simple narrow band communication system is provided in this document. TCM-8PSK is used for modulation, followed by channel estimation and equalization in the presence of Rayleigh fading. Through simulation results, it is shown that the proposed system is robust.**

1 Specifications

The specifications for the communication system to be designed are listed in Table 1.

Parameter	Value
Hardware	FPGA based baseband
MODEM	8PSK-TCM
Modem Rate	555Kbps
SNR	7.6 db at 1e5
Channel (V/UHF)	30Mhz - 512Mhz
Bandwidth	250khz
Bit Duration	2.7us
Throughput	100kbps (Throughput at application Layer)
Ramp up time	116 us (Junk symbols will be sent)
Propagation Delay	100 us (Junk symbols will be sent)
Training sequence	421.2us(provided time for training sequence)
Frame Slot	2 ms
Frame SOM	8 bytes
Payload	32 bytes (692 us)

TABLE 1: Specifications

2 Frame Design

3 8-PSK

3.1 Modulation

3.1.1. See Fig. 3.1.1.1 for the constellation diagram. The transmitted symbol set is given by

$$\mathbf{s}_m = \begin{pmatrix} \cos\frac{2m\pi}{8} \\ \sin\frac{2m\pi}{8} \end{pmatrix}, \quad m \in \{0, 1, \ldots, 7\}. \tag{3.1.1.1}$$

The numerical values for $\mathbf{s}_m$ are listed in Table 3.1.2.1

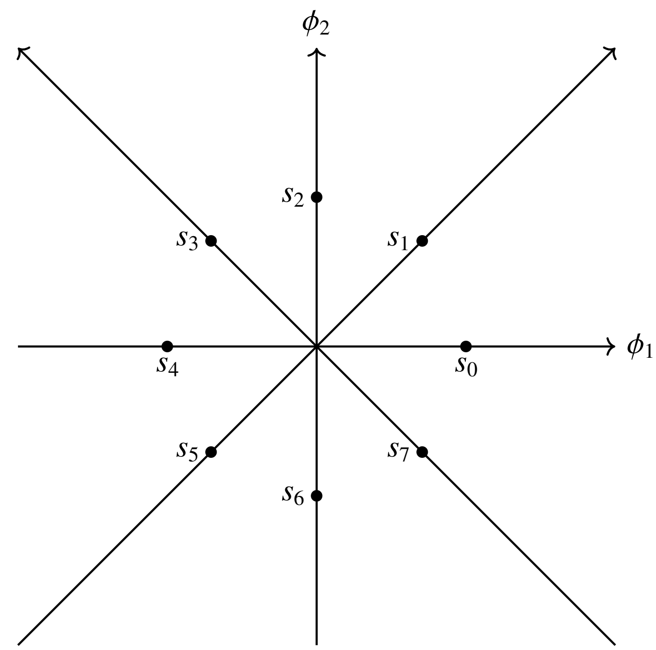

Fig. 3.1.1.1: Constellation diagram

3.1.2. The gray code shown in Table 3.1.2.1 is used for encoding the 8-PSK symbols.

3.1.3. The received symbol is then obtained as

$$\mathbf{y} = \sqrt{E_s}\mathbf{s} + \mathbf{n} \tag{3.1.3.1}$$

where E_s is the symbol energy and

Symbol	Gray Code	Value
s_0	000	$\begin{pmatrix}1\\0\end{pmatrix}$
s_1	001	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_2	011	$\begin{pmatrix}0\\1\end{pmatrix}$
s_3	010	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_4	110	$\begin{pmatrix}-1\\0\end{pmatrix}$
s_5	111	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$
s_6	101	$\begin{pmatrix}0\\-1\end{pmatrix}$
s_7	100	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$

TABLE 3.1.2.1: Gray coding

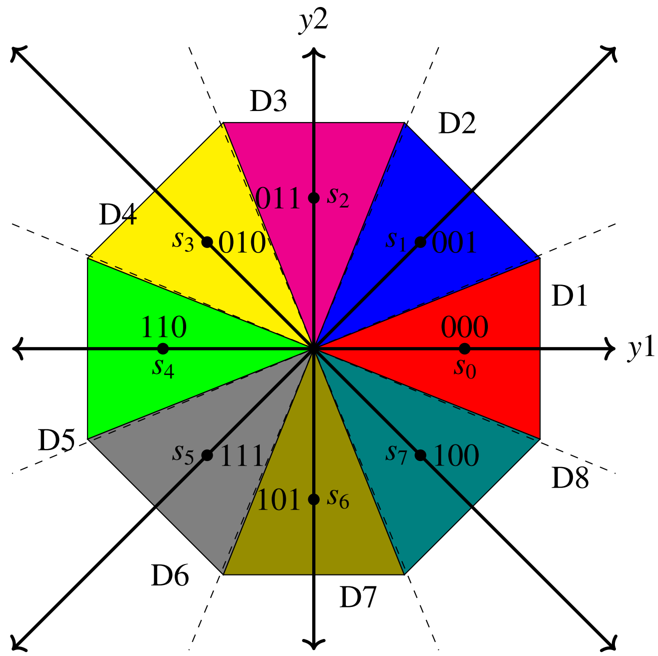

Fig. 3.1.4.1: decision regions

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \frac{N_0}{2}\mathbf{I}\right) \tag{3.1.3.2}$$

$$\mathbf{s} \in \{\mathbf{s}_m\}_{m=0}^{7} \tag{3.1.3.3}$$

3.1.4. Using the ML criterion, the decision rule for each symbol is given by Fig. 3.1.4.1. For $\mathbf{s}_0$, this can be expressed as

$$\|\mathbf{y} - s_0\|^2 \leq \|\mathbf{y} - s_i\|^2, \quad i = 1, \dots, 7 \tag{3.1.4.1}$$

$$\implies (\mathbf{s}_0 - \mathbf{s}_i)^T \mathbf{y} \geq 0 \tag{3.1.4.2}$$

$$\tag{3.1.4.3}$$

which can be simplified to obtain the matrix inequality

$$\begin{pmatrix}(\mathbf{s}_0 - \mathbf{s}_1)^T\\(\mathbf{s}_0 - \mathbf{s}_2)^T\\(\mathbf{s}_0 - \mathbf{s}_3)^T\\(\mathbf{s}_0 - \mathbf{s}_4)^T\\(\mathbf{s}_0 - \mathbf{s}_5)^T\\(\mathbf{s}_0 - \mathbf{s}_6)^T\\(\mathbf{s}_0 - \mathbf{s}_7)^T\end{pmatrix}\mathbf{y} \geq \mathbf{0} \tag{3.1.4.4}$$

resulting in

$$\begin{pmatrix}\sqrt{2}-1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \geq \mathbf{0} \tag{3.1.4.5}$$

Similarly the decisions for all symbols are available in Table 3.1.4.1

Symbol	Decision region	Inequality
s_0	D1	$y_2 + (\sqrt{2}-1)y_1 > 0,\ y_2 - (\sqrt{2}-1)y_1 < 0$
s_1	D2	$y_2 - (\sqrt{2}+1)y_1 < 0,\ y_2 - (\sqrt{2}-1)y_1 > 0$
s_2	D3	$y_2 - (\sqrt{2}+1)y_1 > 0,\ y_2 + (\sqrt{2}+1)y_1 > 0$
s_3	D4	$y_2 + (\sqrt{2}-1)y_1 > 0,\ y_2 + (\sqrt{2}+1)y_1 < 0$
s_4	D5	$y_2 + (\sqrt{2}-1)y_1 < 0,\ y_2 - (\sqrt{2}-1)y_1 > 0$
s_5	D6	$y_2 - (\sqrt{2}+1)y_1 > 0,\ y_2 - (\sqrt{2}-1)y_1 < 0$
s_6	D7	$y_2 - (\sqrt{2}+1)y_1 < 0,\ y_2 + (\sqrt{2}+1)y_1 < 0$
s_7	D8	$y_2 + (\sqrt{2}-1)y_1 < 0,\ y_2 + (\sqrt{2}+1)y_1 > 0$

TABLE 3.1.4.1: Decision regions and their inequalities

4 QPSK

4.1 Modulation

4.1.1. See Fig. 3.1.1.1 for the constellation diagram. The transmitted symbol set is given by

$$\mathbf{s}_m = \begin{pmatrix}\cos\frac{2m\pi}{8}\\\sin\frac{2m\pi}{8}\end{pmatrix}, \quad m \in \{0, 1, \dots, 3\}. \tag{4.1.1.1}$$

The numerical values for $\mathbf{s}_m$ are listed in Table **??**

4.1.2. See Table **??** for the encoding scheme.

4.2 Demodulation

4.2.1. The received symbol is then obtained as

$$\mathbf{y} = \sqrt{E_s}\mathbf{s} + \mathbf{n} \tag{4.2.1.1}$$

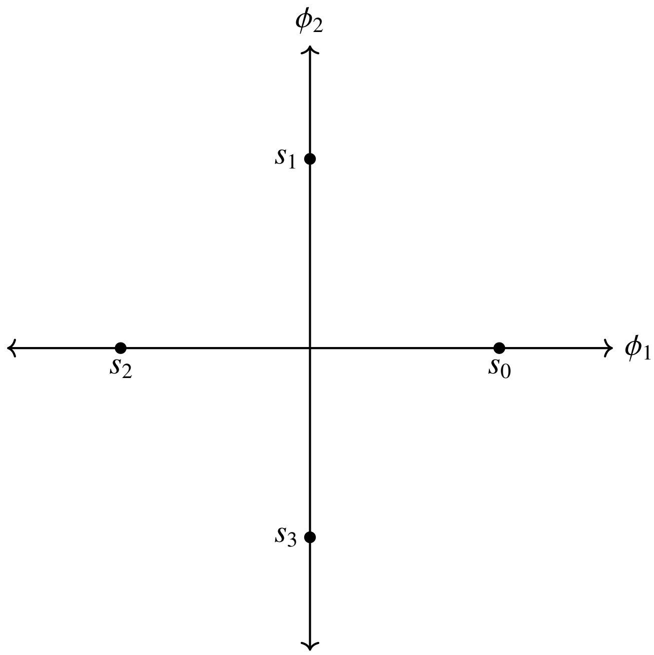

Fig. 4.1.1.1: constellation diagram

where E_s is the symbol energy and

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \frac{N_0}{2}\mathbf{I}\right) \tag{4.2.1.2}$$

$$\mathbf{s} \in \{\mathbf{s}_m\}_{m=0}^{3} \tag{4.2.1.3}$$

4.2.2. The decision rule is given by Fig. 3.1.4.1 and can be expressed as
Let $\mathbf{r}$ be the received bits, $\mathbf{r} = [r_1, r_2]$.

$$r_1 = \begin{cases} 0, & \mathbf{y} \in D1 \cup D2 \iff y_1 + y_2 > 0 \\ 1, & \mathbf{y} \in D3 \cup D4 \iff y_1 + y_2 < 0 \end{cases} \tag{4.2.2.1}$$

$$r_2 = \begin{cases} 0, & \mathbf{y} \in D1 \cup D4 \iff y_2 - y_1 < 0 \\ 1, & \mathbf{y} \in D2 \cup D3 \iff y_2 - y_1 > 0 \end{cases} \tag{4.2.2.2}$$

From eq.4.2.2.1 and eq.4.2.2.2
For detecting s_0, $y_1 > -y_2$ and $y_1 > y_2$.
For detecting s_1, $y_1 > -y_2$ and $y_1 < y_2$.
For detecting s_2, $y_1 < -y_2$ and $y_1 < y_2$.
For detecting s_3, $y_1 < -y_2$ and $y_1 > y_2$.

4.2.3. The following code has simulation of QPSk.

```
codes/qpsk.py
```

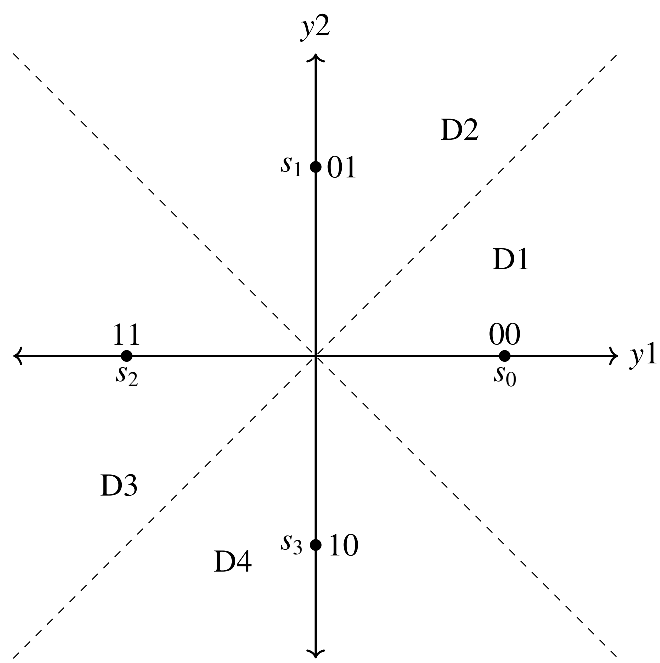

Fig. 4.2.2.1: decision regions