

Physical Layer Design for a Narrow Band Communication System

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Abstract—The design and implementation of a simple narrow band communication system is provided in this document. TCM-8PSK is used for modulation, followed by channel estimation and equalization in the presence of Rayleigh fading. Through simulation results, it is shown that the proposed system is robust.

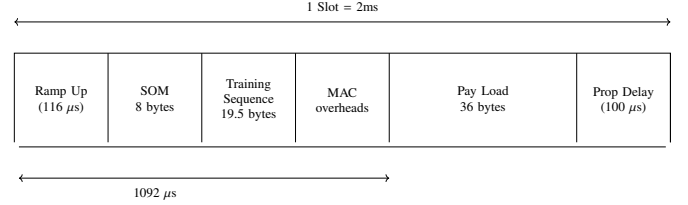


Fig. 2: Physical Layer Frame

1 SPECIFICATIONS

The specifications for the communication system to be designed are listed in Table 1.

Parameter	Value
Hardware	FPGA based baseband
MODEM	8PSK-TCM
Modem Rate	555Kbps
SNR	7.6 db at 1e5
Channel (V/UHF)	30Mhz - 512Mhz
Bandwidth	250khz
Bit Duration	2.7us
Throughput	100kbps (Throughput at application Layer)
Ramp up time	116 us (Junk symbols will be sent)
Propagation Delay	100 us (Junk symbols will be sent)
Training sequence	421.2us(provided time for training sequence)
Frame Slot	2 ms
Frame SOM	8 bytes
Payload	32 bytes (692 us)

TABLE 1: Specifications

2 FRAME DESIGN

The specifications for the communication system to be designed are listed in Table 1.

3 8-PSK

3.1 Modulation

3.1.1. See Fig. 3.1.1.1 for the constellation diagram. The transmitted symbol set is given by

$$s_m = \begin{pmatrix} \cos \frac{2m\pi}{8} \\ \sin \frac{2m\pi}{8} \end{pmatrix}, \quad m \in \{0, 1, \dots, 7\}. \quad (3.1.1.1)$$

The numerical values for s_m are listed in Table 3.1.2.1

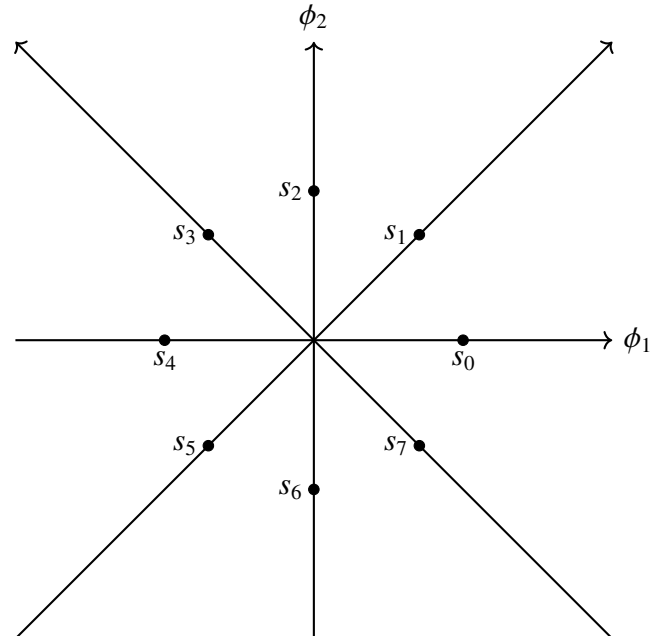


Fig. 3.1.1.1: Constellation diagram

3.1.2. The gray code shown in Table 3.1.2.1 is used for encoding the 8-PSK symbols.

Symbol	Gray Code	Value
s_0	000	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
s_1	001	$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
s_2	011	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
s_3	010	$\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
s_4	110	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
s_5	111	$\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$
s_6	101	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
s_7	100	$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

TABLE 3.1.2.1: Gray coding

3.1.3. The received symbol is then obtained as

$$\mathbf{y} = \sqrt{E_s} \mathbf{s} + \mathbf{n} \quad (3.1.3.1)$$

where E_s is the symbol energy and

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \frac{N_0}{2} \mathbf{I}\right) \quad (3.1.3.2)$$

$$\mathbf{s} \in \{\mathbf{s}_m\}_{m=0}^7 \quad (3.1.3.3)$$

3.1.4. Using the ML criterion, the decision rule for each symbol is given by Fig. 3.1.4.1. For s_0 , this can be expressed as

$$\|\mathbf{y} - s_0\|^2 \leq \|\mathbf{y} - s_i\|^2, \quad i = 1, \dots, 7 \quad (3.1.4.1)$$

$$\Rightarrow (\mathbf{s}_0 - \mathbf{s}_i)^T \mathbf{y} \geq 0 \quad (3.1.4.2)$$

$$(3.1.4.3)$$

which can be simplified to obtain the matrix

inequality

$$\begin{pmatrix} (\mathbf{s}_0 - \mathbf{s}_1)^T \\ (\mathbf{s}_0 - \mathbf{s}_2)^T \\ (\mathbf{s}_0 - \mathbf{s}_3)^T \\ (\mathbf{s}_0 - \mathbf{s}_4)^T \\ (\mathbf{s}_0 - \mathbf{s}_5)^T \\ (\mathbf{s}_0 - \mathbf{s}_6)^T \\ (\mathbf{s}_0 - \mathbf{s}_7)^T \end{pmatrix} \mathbf{y} \geq \mathbf{0} \quad (3.1.4.4)$$

resulting in

$$\begin{pmatrix} \sqrt{2} - 1 & 1 \\ \sqrt{2} - 1 & -1 \end{pmatrix} \mathbf{y} \geq \mathbf{0} \quad (3.1.4.5)$$

after considering the intersection of all the regions and simplifying.

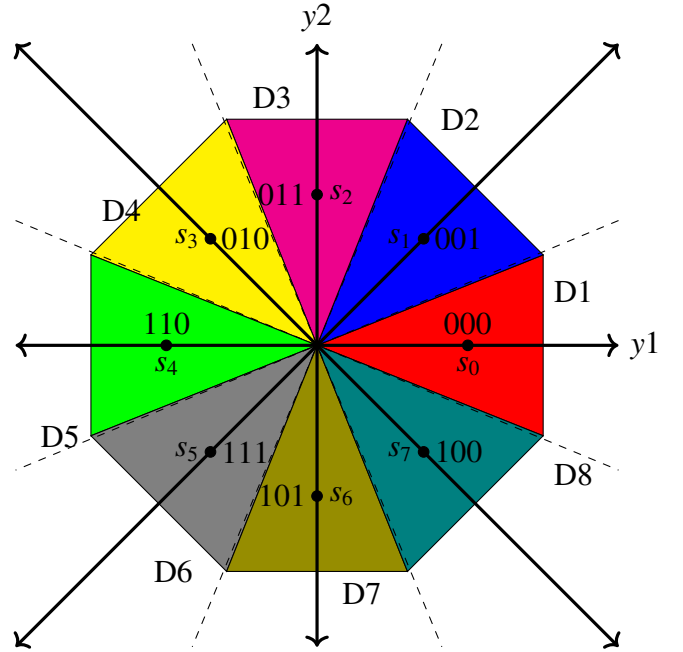


Fig. 3.1.4.1: decision regions

Similarly the decisions for all symbols are available in Table 3.1.4.1

Symbol	Decision region	Inequality	Matrix Inequality
s_0	D1	$y_2 + (\sqrt{2} - 1)y_1 > 0, y_2 - (\sqrt{2} - 1)y_1 < 0$	$\begin{pmatrix} \sqrt{2} - 1 & 1 \\ \sqrt{2} - 1 & -1 \end{pmatrix} \mathbf{y} \geq \mathbf{0}$
s_1	D2	$y_2 - (\sqrt{2} + 1)y_1 < 0, y_2 - (\sqrt{2} - 1)y_1 > 0$	$\begin{pmatrix} \sqrt{2} + 1 & -1 \\ -(\sqrt{2} - 1) & 1 \end{pmatrix} \mathbf{y} \geq \mathbf{0}$
s_2	D3	$y_2 - (\sqrt{2} + 1)y_1 > 0, y_2 + (\sqrt{2} + 1)y_1 > 0$	$\begin{pmatrix} -(\sqrt{2} + 1) & -1 \\ \sqrt{2} + 1 & 1 \end{pmatrix} \mathbf{y} \geq \mathbf{0}$
s_3	D4	$y_2 + (\sqrt{2} - 1)y_1 > 0, y_2 + (\sqrt{2} + 1)y_1 < 0$	$\begin{pmatrix} \sqrt{2} - 1 & 1 \\ \sqrt{2} + 1 & -1 \end{pmatrix} \mathbf{y} \geq \mathbf{0}$
s_4	D5	$y_2 + (\sqrt{2} - 1)y_1 < 0, y_2 - (\sqrt{2} - 1)y_1 > 0$	$\begin{pmatrix} -(\sqrt{2} - 1) & -1 \\ \sqrt{2} - 1 & 1 \end{pmatrix} \mathbf{y} \geq \mathbf{0}$
s_5	D6	$y_2 - (\sqrt{2} + 1)y_1 > 0, y_2 - (\sqrt{2} - 1)y_1 < 0$	$\begin{pmatrix} -(\sqrt{2} + 1) & 1 \\ \sqrt{2} - 1 & -1 \end{pmatrix} \mathbf{y} \geq \mathbf{0}$
s_6	D7	$y_2 - (\sqrt{2} + 1)y_1 < 0, y_2 + (\sqrt{2} + 1)y_1 < 0$	$\begin{pmatrix} \sqrt{2} + 1 & -1 \\ -(\sqrt{2} + 1) & -1 \end{pmatrix} \mathbf{y} \geq \mathbf{0}$
s_7	D8	$y_2 + (\sqrt{2} - 1)y_1 < 0, y_2 + (\sqrt{2} + 1)y_1 > 0$	$\begin{pmatrix} -(\sqrt{2} - 1) & -1 \\ \sqrt{2} + 1 & 1 \end{pmatrix} \mathbf{y} \geq \mathbf{0}$

TABLE 3.1.4.1: Decision rules

3.2 Simulation

3.2.1. Fig. 3.2.1.1 shows the comparison of the SER for 8-PSK for simulation as well as analysis.

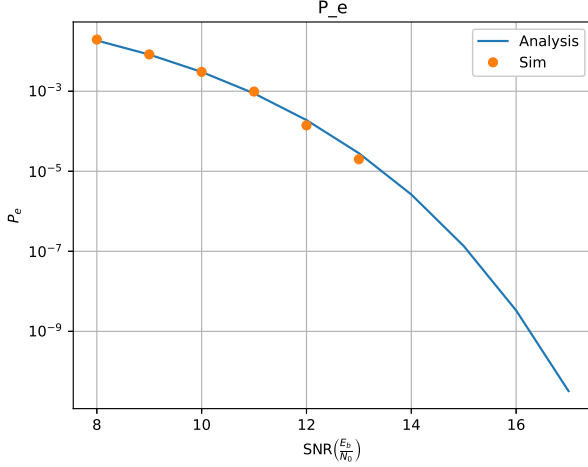


Fig. 3.2.1.1: Constellation diagram

4 CHANNEL

4.1 Estimation

The pilot symbols are used for channel estimation.

4.1.1. $P = 10$ pilot symbols are used at a time for channel estimation. The channel filter has length $L = 5$. See Table 4.1.1.1 for details. The consequent model is

$$\mathbf{y}_p = \mathbf{x}_p * \mathbf{h} + \mathbf{n}_p \quad (4.1.1.1)$$

4.1.2. Let

$$\mathbf{x}_p \xrightarrow{\mathcal{F}} \mathbf{X}_p, \mathbf{y}_p \xrightarrow{\mathcal{F}} \mathbf{Y}_p, \mathbf{h} \xrightarrow{\mathcal{F}} \mathbf{H}_p, \quad (4.1.2.1)$$

be the DFTs of the signals. Then,

$$\mathbf{H}_p = \frac{\mathbf{Y}_p}{\mathbf{X}_p} \quad (4.1.2.2)$$

$$\text{and } \mathbf{H}_p \xrightarrow{\mathcal{F}} \mathbf{h} \quad (4.1.2.3)$$

This is how channel estimation is done \because both \mathbf{x}_p and \mathbf{y}_p are known at the receiver.

4.1.3. While (4.1.2.3) cannot be applied directly, since the lengths of $\mathbf{x}_p(P)$ and $\mathbf{h}(L)$ are dif-

Parameter	Length	Description
\mathbf{x}_p	$P = 10$	Transmitted Pilot Vector
\mathbf{h}	$L = 5$	Channel Vector
\mathbf{n}	$P = 10$	AWGN
\mathbf{y}_p	$L - 1 + P = 14$	Received Pilot Symbol Vector
\mathbf{I}		Identity Matrix
\mathbf{R}	$L + P - 1 \times L + P - 1 = 14 \times 14$	Flips vector
\mathbf{y}	$P = 10$	Circular Received Pilot Vector
$\mathbf{1}$		Ones Vector
$\mathbf{0}$		Zeros Vector

TABLE 4.1.1.1

ferent, resulting in a circular convolution. To address this, we do the following operations

$$\mathbf{y} = \begin{pmatrix} \mathbf{I}_P & \mathbf{0}_{P \times L-1} \end{pmatrix} \left[\begin{pmatrix} \mathbf{I} + \mathbf{R} \end{pmatrix} \begin{pmatrix} \mathbf{1}_{L-1}^T & \mathbf{0}_P^T \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{L-1}^T & \mathbf{1}_{P-L+1}^T & \mathbf{0}_{L-1}^T \end{pmatrix} \mathbf{y}_p \right] \quad (4.1.3.1)$$

where

$$\mathbf{R} = \begin{pmatrix} 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 1 & 0 \\ & & \vdots & & \\ 1 & 0 & \dots & 0 & 0 \end{pmatrix} \quad (4.1.3.2)$$

is a reflection matrix. The channel is now estimated as

$$\mathbf{y} \xrightarrow{\mathcal{F}} \mathbf{Y} \quad (4.1.3.3)$$

$$\frac{\mathbf{Y}}{\mathbf{X}_p} \xrightarrow{\mathcal{F}} \mathbf{h} \quad (4.1.3.4)$$

4.2 Equalization

4.2.1. From Fig. 2, the number of payload symbols per frame is $N = \frac{36 \times 8}{3} = 96$. The received symbols are then given by

$$\mathbf{y}_n = \mathbf{x}_n * \mathbf{h} + \mathbf{n}_n, \quad n = 1, \dots, N \quad (4.2.1.1)$$

In (4.2.1.1), \mathbf{h}, \mathbf{y} are known and we wish to estimate \mathbf{x} .

4.2.2. The MMSE estimate is obtained as

$$\mathbf{x} = \mathbf{W}^T \mathbf{y} \quad (4.2.2.1)$$

where

$$\mathbf{W} = \left(\mathbf{H}^* \mathbf{H} + \frac{\mathbf{I}}{SNR} \right)^{-1} \mathbf{H}^* \quad (4.2.2.2)$$

$$\mathbf{H} = \text{Toeplitz}(\mathbf{h}) \quad (4.2.2.3)$$