Physical Layer Design for a Narrow Band Communication System

G V V Sharma

Abstract—**The design and implementation of a simple narrow band communication system is provided in this document. TCM-8PSK is used for modulation, followed by channel estimation and equalization in the presence of Rayleigh fading. Through simulation results, it is shown that the proposed system is robust.**

1 Specifications

The specifications for the communication system to be designed are listed in Table 1.

Parameter	Value
Hardware	FPGA based baseband
MODEM	8PSK-TCM
Modem Rate	555Kbps
SNR	7.6 db at 1e5
Channel (V/UHF)	30Mhz - 512Mhz
Bandwidth	250khz
Bit Duration	2.7us
Throughput	100kbps (Throughput at application Layer)
Ramp up time	116 us (Junk symbols will be sent)
Propagation Delay	100 us (Junk symbols will be sent)
Training sequence	421.2us(provided time for training sequence)
Frame Slot	2 ms
Frame SOM	8 bytes
Payload	32 bytes (692 us)

TABLE 1: Specifications

2 Frame Design

The specifications for the communication system to be designed are listed in Table 1.

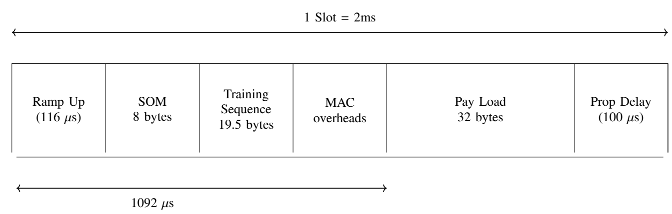

Fig. 2: Physical Layer Frame

3 8-PSK

3.1 Modulation

3.1.1. See Fig. 3.1.1.1 for the constellation diagram. The transmitted symbol set is given by

$$\mathbf{s}_m = \begin{pmatrix} \cos\frac{2m\pi}{8} \\ \sin\frac{2m\pi}{8} \end{pmatrix}, \quad m \in \{0, 1, \dots, 7\}. \tag{3.1.1.1}$$

The numerical values for $\mathbf{s}_m$ are listed in Table 3.1.2.1

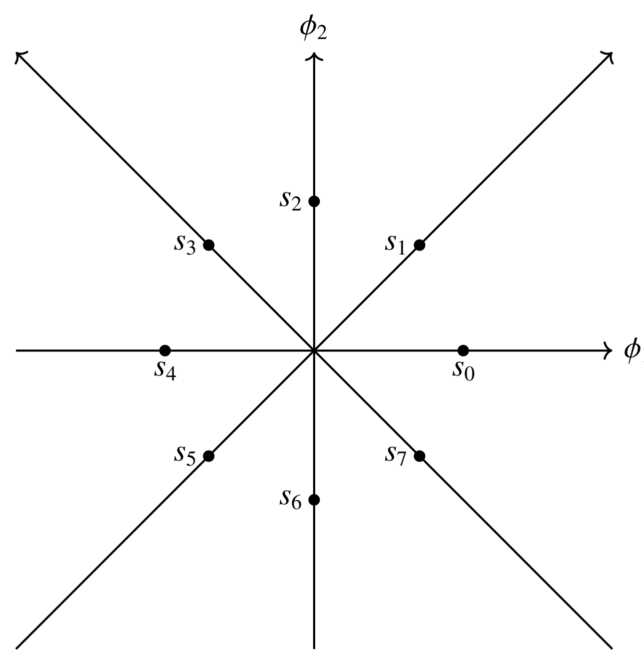

Fig. 3.1.1.1: Constellation diagram

3.1.2. The gray code shown in Table 3.1.2.1 is used for encoding the 8-PSK symbols.

Symbol	Gray Code	Value
s_0	000	$\begin{pmatrix}1\\0\end{pmatrix}$
s_1	001	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_2	011	$\begin{pmatrix}0\\1\end{pmatrix}$
s_3	010	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_4	110	$\begin{pmatrix}-1\\0\end{pmatrix}$
s_5	111	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$
s_6	101	$\begin{pmatrix}0\\-1\end{pmatrix}$
s_7	100	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$

TABLE 3.1.2.1: Gray coding

3.1.3. The received symbol is then obtained as

$$\mathbf{y} = \sqrt{E_s}\mathbf{s} + \mathbf{n} \tag{3.1.3.1}$$

where E_s is the symbol energy and

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \frac{N_0}{2}\mathbf{I}\right) \tag{3.1.3.2}$$

$$\mathbf{s} \in \{\mathbf{s}_m\}_{m=0}^{7} \tag{3.1.3.3}$$

3.1.4. Using the ML criterion, the decision rule for each symbol is given by Fig. 3.1.4.1. For $\mathbf{s}_0$, this can be expressed as

$$\|\mathbf{y} - s_0\|^2 \le \|\mathbf{y} - s_i\|^2, \quad i = 1, \ldots, 7 \tag{3.1.4.1}$$

$$\implies (\mathbf{s}_0 - \mathbf{s}_i)^T \mathbf{y} \ge 0 \tag{3.1.4.2}$$

(3.1.4.3)

which can be simplified to obtain the matrix inequality

$$\begin{pmatrix}(\mathbf{s}_0 - \mathbf{s}_1)^T\\(\mathbf{s}_0 - \mathbf{s}_2)^T\\(\mathbf{s}_0 - \mathbf{s}_3)^T\\(\mathbf{s}_0 - \mathbf{s}_4)^T\\(\mathbf{s}_0 - \mathbf{s}_5)^T\\(\mathbf{s}_0 - \mathbf{s}_6)^T\\(\mathbf{s}_0 - \mathbf{s}_7)^T\end{pmatrix}\mathbf{y} \ge \mathbf{0} \tag{3.1.4.4}$$

resulting in

$$\begin{pmatrix}\sqrt{2}-1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0} \tag{3.1.4.5}$$

after considering the intersection of all the regions and simplifying.

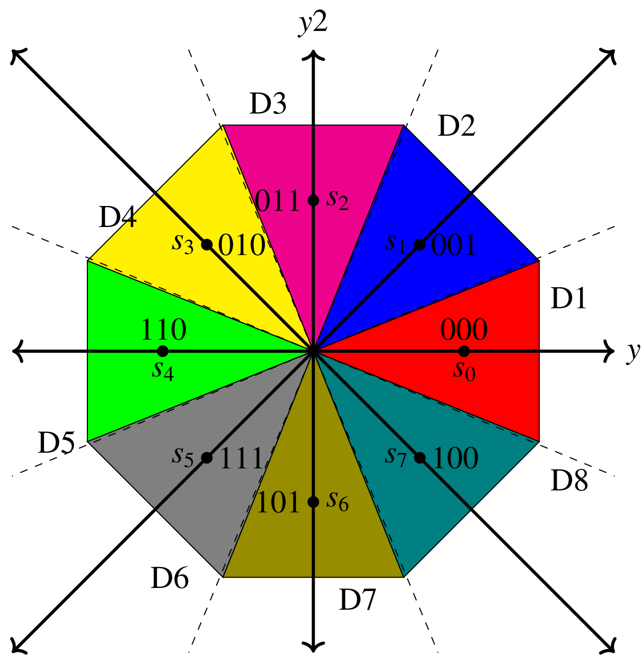

Fig. 3.1.4.1: decision regions

Similarly the decisions for all symbols are available in Table 3.1.4.1

Symbol	Decision region	Inequality	Matrix Inequality
$\mathbf{s}_0$	D1	$y_2 + (\sqrt{2}-1)y_1 > 0,\ y_2 - (\sqrt{2}-1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}-1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_1$	D2	$y_2 - (\sqrt{2}+1)y_1 < 0,\ y_2 - (\sqrt{2}-1)y_1 > 0$	$\begin{pmatrix}\sqrt{2}+1 & -1\\-(\sqrt{2}-1) & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_2$	D3	$y_2 - (\sqrt{2}+1)y_1 > 0,\ y_2 + (\sqrt{2}+1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}+1) & -1\\\sqrt{2}+1 & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_3$	D4	$y_2 + (\sqrt{2}-1)y_1 > 0,\ y_2 + (\sqrt{2}+1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}-1 & 1\\-(\sqrt{2}+1) & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_4$	D5	$y_2 + (\sqrt{2}-1)y_1 < 0,\ y_2 - (\sqrt{2}-1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}-1) & -1\\-(\sqrt{2}-1) & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_5$	D6	$y_2 - (\sqrt{2}+1)y_1 > 0,\ y_2 - (\sqrt{2}-1)y_1 < 0$	$\begin{pmatrix}-(\sqrt{2}+1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_6$	D7	$y_2 - (\sqrt{2}+1)y_1 < 0,\ y_2 + (\sqrt{2}+1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}+1 & -1\\-(\sqrt{2}+1) & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_7$	D8	$y_2 + (\sqrt{2}-1)y_1 < 0,\ y_2 + (\sqrt{2}+1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}-1) & -1\\\sqrt{2}+1 & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$

TABLE 3.1.4.1: Decision rules

3.2 Simulation

3.2.1. Fig. 3.2.1.1 shows the comparison of the SER for 8-PSK for simulation as well as anlysis.

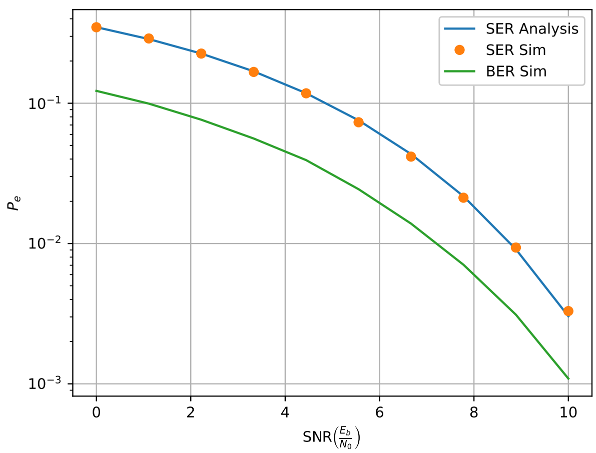

Fig. 3.2.1.1: Constellation diagram

4 Equalization

4.1 Introduction

Rician fading channel is one of the useful models of real-world phenomena in wireless communication.These phenomena include multipath scattering effects , time dispesion and doppler shifts that arise from relative motion between transmitter and reciever.
A channel filter applies path gains to the input signal. Path gains are configured based on settings chosen in fading channel object or block which are used to model fading channels.
The path gains are found using the matlab function ricianchan by running the code

```
./codes/LMS_equalizer_MATLAB.m
```

The channel specifications considered in the above code are as follows:

$$t_s = \frac{1}{185000} \tag{1.1}$$

$$fd = 0.1 \tag{1.2}$$

$$k = 0.87/0.13 \tag{1.3}$$

$$\tau = \begin{pmatrix} 0.0 & 0.2 & 0.4 & 0.6 & 0.8 \end{pmatrix} \times 10^{-5} \tag{1.4}$$

$$pdb = \begin{pmatrix} 0 & -2 & -10 & -20 & -22 \end{pmatrix} \tag{1.5}$$

$$fdLos = 0.7 * fd = 0.07 \tag{1.6}$$

Where t_s is the sample time of the input signal, fd is the maximum doppler shift in hertz, k is the rician K-factor in linear scale,fdLos is the doppler shift of line of sight component.
τ is vector of path delays specified in seconds and pdb is the vector of average path gains specified in dB.

The path gains thus found are stored in the file

```
./codes/path_gains.dat
```

4.2 Channel Model

The multipath fading cahnnel is modelled as a linear finite impulse-reponse filter. Let $\mathbf{x}_n \in \{\mathbf{s}_k\}_0^7$ be transmitted 8-PSK symbols. The received symbols can then be expressed as

$$\mathbf{r} = \mathbf{x} * \mathbf{h} \tag{4.2.1}$$

where

$$h_n = \sum_{k=0}^{K-1} \mathbf{a}_k \operatorname{sinc}\left(\frac{\tau_k}{t_s} - n\right) \tag{4.2.2}$$

$$k = 0, 1, \dots K - 1 \tag{4.2.3}$$

$$-N_1 \leq n \leq N_2 \tag{4.2.4}$$

and

1) t_s is the input sample period to the channel
2) τ_k where $1 \leq k \leq K$is the set of path delays(pd).
3) K is the total number of paths in the multiple fading channel. Here, K=5
4) a_k where $1 \leq k \leq K$is the set of complex path gains (pg). N_1 and N_2 are chosen so that g_n is small when n is less than$-N_1$ and greater than N_2.In the given code,

$$N_1 = N_2 = 800 \tag{4.2.1}$$

(4.2.2) can be expressed as

$$\mathbf{h} = \mathbf{Ta} \tag{4.2.2}$$

where $\mathbf{T}$ is a $(N_1 + N_2 + 1) \times K$ matrix with entries

$$T_{ij} = \operatorname{sinc}\left(\frac{\tau_j}{t_s} - i\right) \tag{4.2.3}$$

$$j = 0, 1, \dots K - 1 \tag{4.2.4}$$

$$-N_1 \leq i \leq N_2 \tag{4.2.5}$$

and $\mathbf{a}$ is a $K \times 1$ vector with entries a_k.

4.3 Estimation

4.3.1. See Table for description of the various parameters

4.3.2. Let $\mathbf{x}_p$ be the sequence of 8-PSK pilot symbols. The received pilot symbols can then be expressed as

$$\mathbf{y} = \mathbf{x}_p * \mathbf{h} + \mathbf{n}, \tag{4.3.2.1}$$

where L is the channel response length. If P be the number of pilot symbols, then $\mathbf{y}$ is of length $P + L - 1$.

4.3.3. Let the FFT of $\mathbf{x}_p$ and $\mathbf{y}$ be $\mathbf{X}_p$ and $\mathbf{Y}$ respectively. Then, the estimated channel vector is

$$\hat{\mathbf{h}} = \text{IFFT}\left(\mathbf{Y} \oslash \mathbf{X}_p\right) \tag{4.3.3.1}$$

where $\oslash$ represents the Hadamard division or elementwise division.

4.3.4. Select the first L elements of $\hat{\mathbf{h}}$ as the channel estimate.

4.4 Equalization

4.4.1. See Table for the description of variables.

4.4.2. Add $N-L$ zeros to $\mathbf{h}$ to get the Toeplitz matrix $\mathbf{H}$ of size N.

4.4.3. The MMSE matrix

$$\mathbf{W} = \left(\mathbf{H}^*\mathbf{H} + \frac{\mathbf{I}}{\mathbf{SNR}}\right)^{-1} \mathbf{H}^* \tag{4.4.1}$$

4.4.4. The payload is designed so that after every 4 unknown symbols there are 4 known symbols $\mathbf{s}_0$.This is to account for the channel spread.

4.4.5. Taking 8 received symbols at a time from the payload,

$$\mathbf{y} = \mathbf{x} * \mathbf{h} + \mathbf{n}, \tag{4.4.1}$$

4.4.6. The estimated symbol is then obtained

$$\hat{\mathbf{x}} = \mathbf{W}\mathbf{y} \tag{4.4.1}$$

4.4.7. 4 symbols are discarded to get the desired estimates.

4.5 Results

A path gain must be choosen by modifying the value of r in the command

```
pg = dlmread('path_gains.dat',',',[r,0,r,4])
```

Where r can be any value from 0 to 4.
For r=0, the following figures are obtained
Hence the code has been executed in octave.

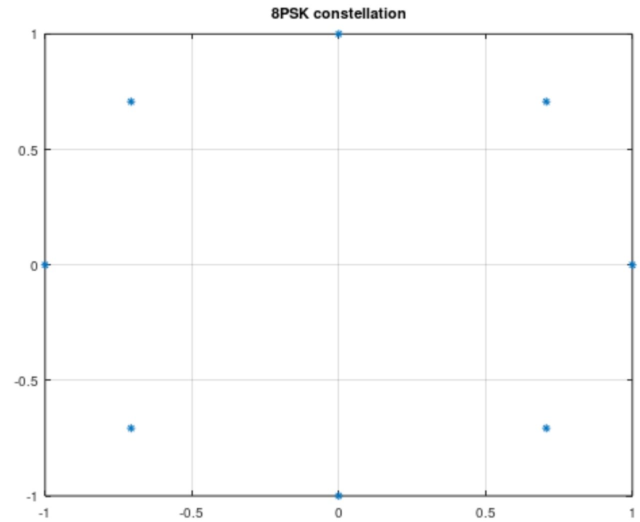

Fig. 4.5.1: 8-PSK constellation

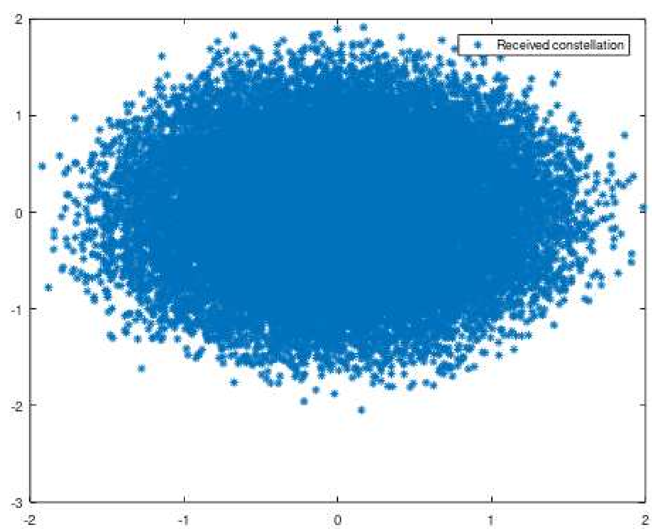

Fig. 4.5.2: Recieved constellation from the channel

5 Frequency Offset: LR Technique

Let the frequency offset be Δf [1] . Then

$$\mathbf{y}_k = \mathbf{x}_k e^{j2\pi\Delta f k T_s} + \mathbf{n}_k, \quad k = 1, \ldots, N \tag{5.0.1}$$

where $T_s \le \frac{1}{2\Delta f}$ is the sampling time.

$$Y_k = X_k e^{j2\pi\Delta f k M} + V_k, \quad k = 1, \ldots, N \tag{5.0.2}$$

From (5.0.2),

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f k M} + X_k^* V_k \tag{5.0.3}$$

$$\implies r_k = e^{j2\pi\Delta f k M} + \bar{V}_k \tag{5.0.4}$$

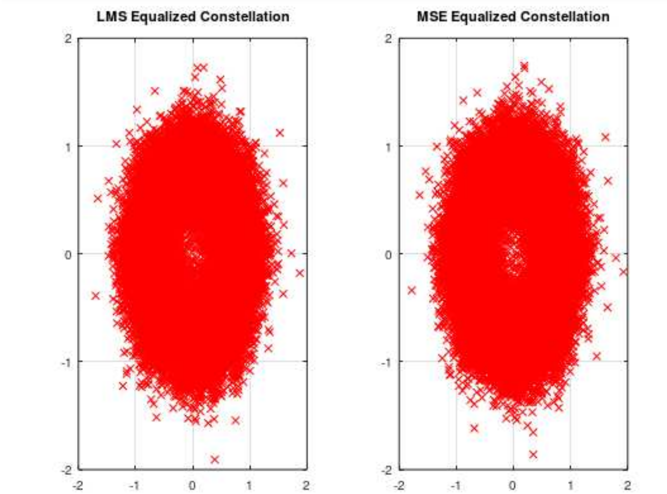

Fig. 4.5.3: LMS and MSE equalized constellation

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{5.0.5}$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \leq k \leq N-1 \tag{5.0.6}$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta\hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \mathrm{Im}(R(k))}{\sum_{k=1}^{P} k\mathrm{Re}(R(k))}, \quad P\Delta f M << 1 \tag{5.0.7}$$

where P is the number of pilot symbols.

5.1 Plots

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. **??** shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. **??** shows the variation of the error with respect to the SNR for $\Delta f = 5$MHz.

References

[1] M. Luise and R. Reggiannini, "Carrier frequency recovery in all-digital modems for burst-mode transmissions," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 1169–1178, Feb 1995. [Online]. Available: https://doi.org/10.1109/26.380149