Physical Layer Design for a Narrow Band Communication System

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***Abstract*—The design and implementation of a simple narrow band communication system is provided in this document. TCM-8PSK is used for modulation, followed by channel estimation and equalization in the presence of Rayleigh fading. Through simulation results, it is shown that the proposed system is robust.**

1 Specifications

The specifications for the communication system to be designed are listed in Table 1.

Parameter	**Value**
Hardware	FPGA based baseband
MODEM	8PSK-TCM
Modem Rate	555Kbps
SNR	7.6 db at 1e5
Channel (V/UHF)	30Mhz - 512Mhz
Bandwidth	250khz
Bit Duration	2.7us
Throughput	100kbps (Throughput at application Layer)
Ramp up time	116 us (Junk symbols will be sent)
Propagation Delay	100 us (Junk symbols will be sent)
Training sequence	421.2us(provided time for training sequence)
Frame Slot	2 ms
Frame SOM	8 bytes
Payload	32 bytes (692 us)

TABLE 1: Specifications

2 Frame Design

The specifications for the communication system to be designed are listed in Table 1.

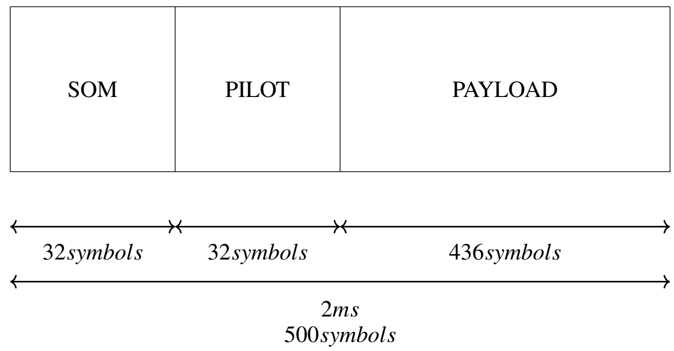

Fig. 2: Physical Layer Frame

3 8-PSK

3.1 Modulation

3.1.1. See Fig. 3.1.1.1 for the constellation diagram. The transmitted symbol set is given by

$$\mathbf{s}_m = \begin{pmatrix} \cos\frac{2m\pi}{8} \\ \sin\frac{2m\pi}{8} \end{pmatrix}, \quad m \in \{0, 1, \dots, 7\}. \quad (3.1.1.1)$$

The numerical values for $\mathbf{s}_m$ are listed in Table 3.1.2.1

3.1.2. The gray code shown in Table 3.1.2.1 is used for encoding the 8-PSK symbols.

3.1.3. The received symbol is then obtained as

$$\mathbf{y} = \sqrt{E_s}\mathbf{s} + \mathbf{n} \quad (3.1.3.1)$$

where E_s is the symbol energy and

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \frac{N_0}{2}\mathbf{I}\right) \quad (3.1.3.2)$$

$$\mathbf{s} \in \{\mathbf{s}_m\}_{m=0}^{7} \quad (3.1.3.3)$$

3.1.4. Using the ML criterion, the decision rule for each symbol is given by Fig. 3.1.4.1. For $\mathbf{s}_0$,

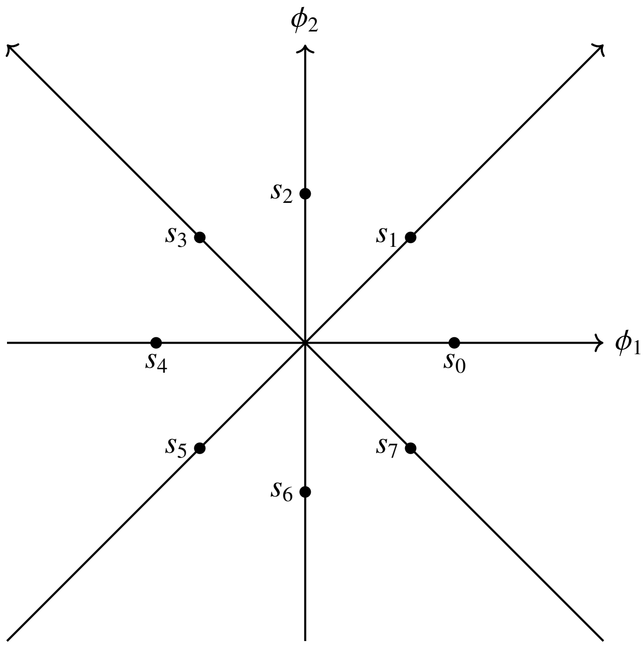

Fig. 3.1.1.1: Constellation diagram

Symbol	Gray Code	Value
s_0	000	$\begin{pmatrix}1\\0\end{pmatrix}$
s_1	001	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_2	011	$\begin{pmatrix}0\\1\end{pmatrix}$
s_3	010	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_4	110	$\begin{pmatrix}-1\\0\end{pmatrix}$
s_5	111	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$
s_6	101	$\begin{pmatrix}0\\-1\end{pmatrix}$
s_7	100	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$

TABLE 3.1.2.1: Gray coding

this can be expressed as

$$\|\mathbf{y} - s_0\|^2 \le \|\mathbf{y} - s_i\|^2, \quad i = 1, \dots, 7 \tag{3.1.4.1}$$

$$\implies (\mathbf{s}_0 - \mathbf{s}_i)^T \mathbf{y} \ge 0 \tag{3.1.4.2}$$

$$\tag{3.1.4.3}$$

which can be simplified to obtain the matrix inequality

$$\begin{pmatrix}(\mathbf{s}_0 - \mathbf{s}_1)^T\\(\mathbf{s}_0 - \mathbf{s}_2)^T\\(\mathbf{s}_0 - \mathbf{s}_3)^T\\(\mathbf{s}_0 - \mathbf{s}_4)^T\\(\mathbf{s}_0 - \mathbf{s}_5)^T\\(\mathbf{s}_0 - \mathbf{s}_6)^T\\(\mathbf{s}_0 - \mathbf{s}_7)^T\end{pmatrix}\mathbf{y} \ge \mathbf{0} \tag{3.1.4.4}$$

resulting in

$$\begin{pmatrix}\sqrt{2}-1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0} \tag{3.1.4.5}$$

after considering the intersection of all the regions and simplifying.

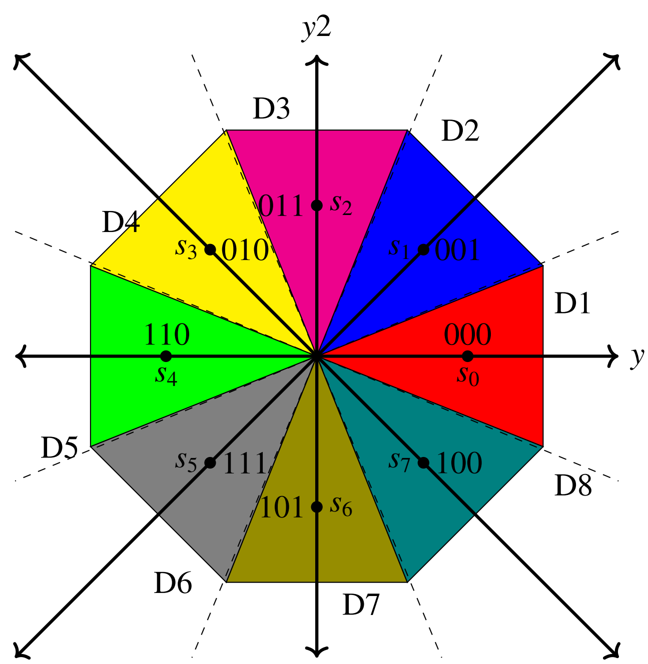

Fig. 3.1.4.1: decision regions

Similarly the decisions for all symbols are available in Table 3.1.4.1

Symbol	Decision region	Inequality	Matrix Inequality
$\mathbf{s}_0$	D1	$y_2 + (\sqrt{2}-1)y_1 > 0,\ y_2 - (\sqrt{2}-1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}-1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_1$	D2	$y_2 - (\sqrt{2}+1)y_1 < 0,\ y_2 - (\sqrt{2}-1)y_1 > 0$	$\begin{pmatrix}\sqrt{2}+1 & -1\\-(\sqrt{2}-1) & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_2$	D3	$y_2 - (\sqrt{2}+1)y_1 > 0,\ y_2 + (\sqrt{2}+1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}+1) & -1\\\sqrt{2}+1 & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_3$	D4	$y_2 + (\sqrt{2}-1)y_1 > 0,\ y_2 + (\sqrt{2}+1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}-1 & 1\\-(\sqrt{2}+1) & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_4$	D5	$y_2 + (\sqrt{2}-1)y_1 < 0,\ y_2 - (\sqrt{2}-1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}-1) & -1\\-(\sqrt{2}-1) & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_5$	D6	$y_2 - (\sqrt{2}+1)y_1 > 0,\ y_2 - (\sqrt{2}-1)y_1 < 0$	$\begin{pmatrix}-(\sqrt{2}+1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_6$	D7	$y_2 - (\sqrt{2}+1)y_1 < 0,\ y_2 + (\sqrt{2}+1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}+1 & -1\\-(\sqrt{2}+1) & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_7$	D8	$y_2 + (\sqrt{2}-1)y_1 < 0,\ y_2 + (\sqrt{2}+1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}-1) & -1\\\sqrt{2}+1 & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$

TABLE 3.1.4.1: Decision rules

3.2 Simulation

3.2.1. Fig. 3.2.1.1 shows the comparison of the SER for 8-PSK for simulation as well as anlysis.

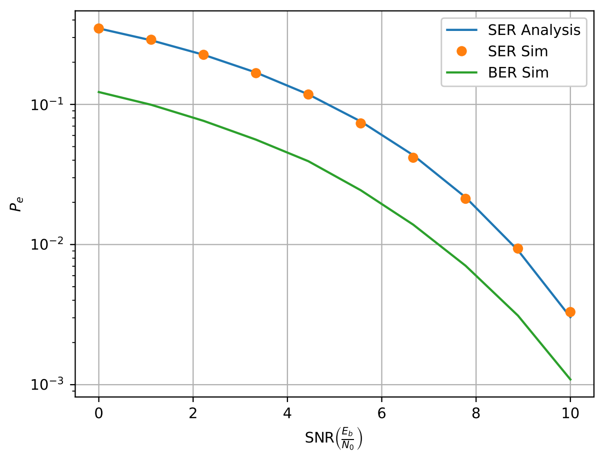

Fig. 3.2.1.1: Constellation diagram

4 Frequency Offset: LR Technique

Let the frequency offset be Δf [1] . Then

$$\mathbf{y}_k = \mathbf{x}_k e^{j2\pi\Delta f k T_s} + \mathbf{n}_k, \quad k = 1, \dots, N \tag{1.1}$$

where $T_s \leq \frac{1}{2\Delta f}$ is the sampling time.

$$Y_k = X_k e^{j2\pi\Delta f k M} + V_k, \quad k = 1, \dots, N \tag{1.2}$$

From (1.2),

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f k M} + X_k^* V_k \tag{1.3}$$

$$\implies r_k = e^{j2\pi\Delta f k M} + \bar{V}_k \tag{1.4}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{1.5}$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \leq k \leq N-1 \tag{1.6}$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\hat{\Delta f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \mathrm{Im}(R(k))}{\sum_{k=1}^{P} k \mathrm{Re}(R(k))}, \quad P\Delta f M << 1 \tag{1.7}$$

where P is the number of pilot symbols.

4.1 Plots

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. **??** shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. **??** shows the variation of the error with respect to the SNR for Δf = 5MHz.

References

[1] M. Luise and R. Reggiannini, "Carrier frequency recovery in all-digital modems for burst-mode transmissions," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 1169–1178, Feb 1995. [Online]. Available: https://doi.org/10.1109/26.380149