SNR Estimation in Generalized Fading Channels and its Application to Turbo Decoding

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SNR Estimation in Generalized Fading Channels and its Application to Turbo Decoding

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Abstract—**In this paper, we propose an online SNR estimation scheme for generalized fading channels. We derive the SNR estimate based on a statistical ratio of observables over a block of data, when the channel undergoes Nakagami fading. An online SNR estimator for AWGN channels has been derived recently by Summers and Wilson. Our SNR estimation scheme in this paper is for a general Nakagami fading channel, where the SNR estimates in Rayleigh fading and AWGN can be obtained as special cases corresponding to the Nakagami parameter $m = 1$ and $m = \infty$, respectively. As an example, we use our SNR estimate in the iterative decoding of turbo codes on both i.i.d and correlated Rayleigh fading channels. We show that the turbo decoder performance on Rayleigh fading channels using our SNR estimate is close to the performance using perfect knowledge of the fade amplitudes and the SNR.**

Keywords – SNR estimate, Nakagami fading, Turbo codes.

I. INTRODUCTION

Turbo codes have been shown to offer near-capacity performance on AWGN channels and significantly good performance on fully-interleaved flat Rayleigh fading channels [1], [2]. The optimum decoding of turbo codes (and concatenated coding schemes of similar nature) requires knowledge of the channel signal-to-noise ratio (SNR). For the AWGN case, Summers and Wilson [3] have recently addressed the issue of the sensitivity of the turbo decoder performance to imperfect knowledge of the channel SNR, and proposed an online SNR estimation scheme. It was shown that a simple estimator of SNR, based on both the sum of the squared receiver output values and square of the sum of their absolute values, can provide accurate estimates.

Performance of turbo codes on flat Rayleigh fading has been addressed in [2],[4],[5]. In the performance evaluation of turbo codes in [2], perfect knowledge of both the fade amplitudes of each symbol and E_s/N_o (perfect side information) are assumed to be available at the decoder. But in practice, however, the decoder has to estimate this information from the received symbols. In this paper, we are interested in estimating the received average SNR without requiring the transmission of known training symbols, particularly when the channel undergoes Nakagami fading.

A channel estimation technique suitable for decoding turbo codes on flat Rayleigh fading channels is presented in [4]. But the technique is based on sending known pilot symbols at regular intervals in the transmit symbol sequence. In [5], a channel estimator based on a low pass FIR filter is presented for flat Rayleigh and Rician fading channels. However, none of these and other studies in the literature have considered channel estimation schemes suitable for decoding turbo codes without training bits (pilot symbols) on generalized Nakagami fading channels. Our contribution in this paper fills this gap. We derive an SNR estimate based on a statistical ratio of observables over a block of data, when the channel undergoes Nakagami fading. Interestingly, our general results encompass the AWGN results of Summers and Wilson in [3] as a special case when the Nakagami parameter $m \to \infty$. Likewise SNR estimation results for Rayleigh and Rician fading, respectively, can also be obtained as special cases when $m = 1$ and when a one-to-one mapping between m and the Rice K-factor is used.

The rest of the paper is organized as follows. In Section II, the system model is described. The proposed online SNR estimation procedure is presented in Section III. The detailed derivations of the statistical parameters of interest are moved to the Appendix. Section IV presents the application of the proposed online estimation procedure to the decoding of turbo codes. For the special case when $m = 1$ (i.e., Rayleigh fading), we show that the turbo decoder performance using our SNR estimate is close to the performance using perfect knowledge of the fade amplitudes and the SNR. Conclusions are provided in Section V.

II. SYSTEM MODEL

We assume that the encoded data symbols at the transmitter are BPSK modulated and the receiver employs coherent demodulation. Assuming perfect synchronization, the demodulated symbols r_n can be represented by

$$r_n = \pm\alpha_n \cdot \sqrt{E_s} + n_n, \tag{1}$$

where α_n is the random fade experienced by n^{th} symbol, E_s is the symbol energy, and n_n is a Gaussian random variable having zero mean and variance $\sigma^2 = N_o/2$, and the two-sided power spectral density of the channel noise process is $N_0/2$ W/Hz. We assume that the α_n's are Nakagami m-distributed and independent of the n_n's. Specifically,

$$p_\alpha(a) = \frac{2m^m a^{2m-1}}{\Gamma(m)} e^{-ma^2}. \tag{2}$$

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We have normalized the second moment of the fade, $E(\alpha^2)$, to unity. The Nakagami m-distribution spans, via the parameter m, the widest range of multipath fading distributions. For instance, it includes the one sided Gaussian distribution (m=1/2) and the Rayleigh distribution (m=1) as special cases. In the limit as $m \rightarrow +\infty$, the Nakagami fading channel converges to a non-fading AWGN channel, i.e., as $m \rightarrow \infty$ the pdf approaches $\delta(a-1)$. When $m \geq 1$, a one-to-one mapping between the parameter m and the Rician factor K allows the Nakagami m-distribution to closely approximate the Rice distribution. The Nakagami m-distribution often gives the best fit to land-mobile and indoor-mobile multipath propagation, as well as to scintillating ionospheric radio links. In this paper, we assume that $m \geq 1$.

III. SNR Estimation

We want to estimate the average received SNR, $\gamma = \frac{E_s}{2\sigma^2}E(\alpha^2) = \frac{E_s}{2\sigma^2}$. In Eqn. (1), the actual data polarity is unknown. Our interest is to devise a blind algorithm which does not require the transmission of known training symbols to estimate the SNR. Accordingly, we formulate an estimator for the SNR based on a block observation of the r_n's. As in [3], we define a parameter z to be the ratio of two statistical computations on the block observation of the r_n's as

$$z = \frac{E(r_n^2)}{[E(|r_n|)]^2}. \quad (3)$$

In the following, we derive z as a function $f(.)$ of the received SNR, γ. Thus, the ratio of the two statistical computations and the known function $f(\gamma)$ provide a means to estimate γ.

The derivation of $E(r_n^2)$ in Eqn. (3) is straightforward. To derive $E(|r_n|)$, we first derive $E(|r_n| \mid \alpha)$ and then take its expectation over α to get $E(|r_n|)$. The detailed derivations of $E(r_n^2)$ and $E(|r_n|)$ are given in Appendix I. Using the expressions for $E(r_n^2)$ and $E(|r_n|)$ in Equations (11) and (30) derived in Appendix I, the parameter z can be obtained as

$$z = \frac{1+2\gamma}{\left(\sqrt{\frac{2}{\pi}}\left(\frac{m}{m+\gamma}\right)^m + \sqrt{\frac{2\gamma}{m}} \cdot \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)} \cdot \left(1 - \frac{2I(m)}{\pi}\right)\right)^2}, \quad (4)$$

where $\gamma = \frac{E_s}{2\sigma^2}$, $I(m)$ is given by Eqn. (29), and $\Gamma(\,.\,)$ is the Gamma function [6]. Note that Eqn. (4) assumes the knowledge of the Nakagami parameter m in the SNR estimation process, which can be computed accurately using the method given in [7].

For $m = 1$, the Nakagami m-distribution becomes the Rayleigh distribution with the pdf $p_\alpha(a) = 2ae^{-a^2}$. The corresponding z for Rayleigh fading can be derived from (4) by substituting $m = 1$. It is obtained as (see Appendix II for the derivation)

$$z_{rayleigh} = \frac{\pi}{2} \cdot \frac{1+2\gamma}{\left[1+\sqrt{\gamma}\left(\frac{\pi}{2} - \cos^{-1}\left(\sqrt{\frac{\gamma}{1+\gamma}}\right)\right)\right]^2}. \quad (5)$$

For a given value of z (computed from a block observation of the r_n's), the corresponding estimate of γ can be found from Eqn. (5). For easy implementation, an approximate relation between z and γ can be obtained through polynomial curve fitting for Eqn. (5). A second order (quadratic), a third order (cubic), and a fourth order (bi-quad) polynomial fit are done to approximate the relation of z with γ. The quadratic fit is given by

$$\gamma_{quad} = a_0 z^2 + a_1 z + a_2, \quad (6)$$

where $a_0 = 209.459174179830$, $a_1 = -632.893088576470$, and $a_2 = 478.443659510047$. The cubic fit is given by

$$\gamma_{cubic} = b_0 z^3 + b_1 z^2 + b_2 z + b_3, \quad (7)$$

where $b_0 = -1566.0419834643, b_1 = 6973.4026101590, b_2 = -10357.0288229892$, and $b_3 = 5131.6668286643$. The bi-quadratic fit is given by

$$\gamma_{bi-quad} = c_0 z^4 + c_1 z^3 + c_2 z^2 + c_3 z + c_4, \quad (8)$$

where $c_0 = 10971.3670508672$, $c_1 = -64731.6367893422$, $c_2 = 143212.237224577$, $c_3 = -140825.801468004$, and $c_4 = 51938.6459401357$.

In order to obtain an estimate for z, we replace the expectations in Eqn. (3) with the corresponding block averages, yielding

$$\hat{z} = \frac{\overline{r^2}}{\overline{|r|}^2}. \quad (9)$$

Substituting (9) into (6), (7), and (8) we get the SNR estimates, $\widehat{\gamma}$. We tested the accuracy of the polynomial approximations in Eqns. (6), (7), and (8) by evaluating the mean and standard deviation of the SNR estimates $\widehat{\gamma}$, determined by over 20000 trials. The block sizes considered are 1000 and 5000 bits (3000 and 15000 code symbols). Tables 1 and 2 give these results. Note that the true SNR value ($\gamma = E_s/N_o$) ranges from -4.77 dB to 3.23 dB in Tables 1 and 2, and corresponds to E_b/N_o values in the range 0 to 8 dB for a rate-1/3 code. From Tables I and II, it can be seen that the mean SNR estimates $\widehat{\gamma}$, obtained through the bi-quad fit, are quite close to the true value of SNR γ, and the standard deviation of the estimate is reduced as the block size is increased. We have also verified that the coefficients for the quadratic fit obtained through our general expression in Eqn. (4) for the Nakagami parameter $m \geq 27$ are the same as the coefficients obtained by Summers and Wilson in [3] for the AWGN case.

True SNR, γ (dB)	BI-QUAD		CUBIC	
	E[$\widehat{\gamma}$], dB	SD [$\widehat{\gamma}$], dB	E[$\widehat{\gamma}$], dB	SD[$\widehat{\gamma}$], dB
-4.77	-4.22	0.82	-6.06	3.63
-3.77	-3.99	0.88	-4.14	3.21
-2.77	-3.20	1.13	-2.26	1.96
-1.77	-2.00	1.15	-1.03	0.90
-0.77	-0.70	0.94	-0.29	0.47
0.23	0.46	0.71	0.28	0.38
1.23	1.42	0.56	0.91	0.45
2.23	2.29	0.50	1.76	0.58
3.23	3.15	0.52	2.85	0.68

TABLE I

MEAN AND STANDARD DEVIATION OF THE SNR ESTIMATE, $\widehat{\gamma}$, FOR DIFFERENT VALUES OF THE TRUE SNR, γ. BLOCK SIZE = 1000 BITS.

True SNR, γ (dB)	BI-QUAD		CUBIC	
	$E[\hat{\gamma}]$, dB	SD $[\hat{\gamma}]$, dB	$E[\hat{\gamma}]$, dB	SD$[\hat{\gamma}]$, dB
-4.77	-4.73	0.22	-6.20	2.92
-3.77	-4.32	0.44	-3.41	1.27
-2.77	-3.31	0.58	-1.84	0.57
-1.77	-1.97	0.54	-0.89	0.30
-0.77	-0.64	0.41	-0.25	0.18
0.23	0.50	0.31	0.27	0.16
1.23	1.45	0.24	0.88	0.20
2.23	2.28	0.21	1.72	0.26
3.23	3.13	0.23	2.82	0.30

TABLE II

MEAN AND STANDARD DEVIATION OF THE SNR ESTIMATE, $\hat{\gamma}$, FOR DIFFERENT VALUES OF THE TRUE SNR, γ. BLOCK SIZE = 5000 BITS.

IV. TURBO DECODER PERFORMANCE RESULTS

Simulations were performed using the proposed online estimator to provide $\hat{\gamma}$ for the iterative decoding of turbo codes on flat Rayleigh fading channels ($m = 1$). For details regarding turbo coding and decoding, the reader is referred to reference [8]. In this paper, we consider a rate-1/3 turbo code using two 16-state (constraint length = 5) recursive systematic code (RSC) encoders with generator $(21/37)_8$, which is the encoder used in the first paper on turbo codes [1]. A random turbo interleaver is employed. The number of information bits per frame is 5000. The transmitted symbols are corrupted by flat Rayleigh fading and AWGN. Both i.i.d. and correlated Rayleigh fading are considered. The correlated Rayleigh fading samples are simulated using the Jakes' model [9] for a carrier frequency of 900 MHz and vehicle speeds of 1, 10, 100 km/h. This carrier frequency and a vehicle speed of 1 km/h correspond to a Doppler frequency f_d of 0.8 Hz. The correlation in the fading process is characterized by the Bessel function of zeroth order and first kind, $J_o(2\pi f_d T)$. Here, T is the bit duration and is fixed at 0.1 msec (i.e., a data rate of 10 kbps).

We used the Log-MAP algorithm in the iterative decoder [10]. The number of decoding iterations is eight. The decoder performance is evaluated for four different cases: *a*) assuming perfect knowledge of the fade amplitudes of each symbol and E_s/N_o at the receiver (i.e., perfect side information), *b*) using the SNR estimate from the *bi-quadratic* fit in Eqn. (8), *c*) using the SNR estimate from the *cubic* fit in Eqn. (7), and *d*) using the SNR estimate from the *quadratic* fit in Eqn. (6). In cases *b*), *c*) and *d*), the *average* SNR estimate is computed from the r_n's over each frame (i.e., 15000 symbol observations) according to Eqns. (8), (7) and (6), respectively. This average SNR estimate is then given as the channel information to the turbo decoder.

Figure 1 shows the simulated performance of the turbo decoder when the proposed SNR estimates are used, relative to the performance when perfect side information (SI) is used. In evaluating the performance in Figure 1, we have considered the Rayleigh fading to be i.i.d (i.e., infinite channel interleaving). With perfect SI, it is seen that a bit error rate of 10^{-5} is achieved at an E_b/N_o of 1.8 dB, which illustrates the ability of turbo codes to provide excellent coding gains even on fading channels. From Figure 1, we further observe that the turbo decoder using the proposed online SNR estimator (bi-quad fit) on i.i.d fading performs close to the perfect SI case (to within about 1 dB). The cubic fit performs poorer than the bi-quad fit by less than one-fourth of a dB. The quadratic fit performs poorer than the cubic fit by about one-fourth of a dB.

Next, in Figure 2, we illustrate the turbo decoder performance when a finite channel interleaver is employed. A 125×120 size channel interleaver is used. The effect of varying the correlation in the fading process (equivalently, different vehicle speeds) on the turbo decoder performance is evaluated. Note that the correlation in the fading process decreases with increasing vehicle speeds [9]. Figure 2 shows the BER plots for three different vehicle speeds – 1, 10, 100 km/h. We observe that the turbo decoder yields increasingly better performance as the fading process becomes less and less correlated. This observation is in line with the results reported in [2] for the perfect SI case. In addition, we see that our online estimator performs quite close to the perfect SI case to within less than a dB. Thus, the results in Figures 1 and 2 illustrate that our online SNR estimator can be used in turbo decoding without much loss in bit error performance.

V. CONCLUSION

We proposed an online SNR estimation scheme for Nakagami fading channels and derived the SNR estimate based on a statistical ratio of observables over a block of data. We showed that our generalized fading results provided the SNR estimates on AWGN and Rayleigh fading channels as special cases when the Nakagami parameter $m = \infty$ and $m = 1$, respectively. We then applied our online SNR estimation technique to the iterative decoding of turbo codes for both i.i.d and correlated Rayleigh fading channels. We showed that the turbo decoder performance using our SNR estimate is close to the performance using perfect knowledge of the fade amplitudes and E_s/N_o. In addition to its application in turbo decoding on fading channels, the proposed online SNR estimation technique could as well be applied in other problems where knowledge of the fading channel SNR is necessary.

APPENDIX

I. DERIVATION OF $E(r_n^2)$ AND $E(|r_n|)$

The expressions for $E(r_n^2)$ and $E(|r_n|)$ are derived here. $E(r_n^2)$ is derived as

$$\begin{aligned} E(r_n^2) &= E[(\pm\alpha_n\sqrt{E_s} + n_n)^2] \\ &= E_s E[(\alpha_n)^2] \pm 2\sqrt{E_s}E[\alpha_n]E[n_n] + E[n_n^2]. \end{aligned} \quad (10)$$

Since $E[n_n]$ is zero and $E[(\alpha_n)^2]$ is normalized to unity, we get

$$E(r_n^2) = E_s + \sigma^2. \quad (11)$$

Next, to derive $E(|r_n|)$, we proceed as follows. We have

$$r_n = \alpha_n X_n + n_n, \quad (12)$$

where X_n is a binary random variable taking values $\pm\sqrt{E_s}$ with equal probability. X_n is independent of α_n and n_n. In (12), r_n depends on the random variables X_n, α_n, and n_n. To derive the expected value of $|r_n|$, we first average $|r_n|$ over random variable X_n, then average over n_n conditioned on α_n, and finally average over α_n.

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First, averaging $|r_n|$ over X_n, we get

$$E(|r_n|\,|n_n,\alpha_n) = \frac{1}{2}E(|-\alpha_n\sqrt{E_s}+n_n|)+\frac{1}{2}E(|\alpha_n\sqrt{E_s}+n_n|). \quad (13)$$

Denoting $A = |-\alpha_n\sqrt{E_s}+n_n|$ and $B = |\alpha_n\sqrt{E_s}+n_n|$ in the above equation, and averaging over n_n, conditioned on $\alpha_n = \alpha$, we get

$$\begin{aligned} E(A|\alpha_n=\alpha) &= \frac{1}{\sigma\sqrt{2\pi}}\int_{x=-\infty}^{\infty} |-\alpha\sqrt{E_s}+x|\cdot e^{-\frac{x^2}{2\sigma^2}}\,dx \\ &= \sigma\sqrt{\frac{2}{\pi}}e^{\frac{-\alpha^2E_s}{2\sigma^2}}+\alpha\sqrt{E_s}-2\alpha\sqrt{E_s}Q\left(\frac{\alpha\sqrt{E_s}}{\sigma}\right), \end{aligned} \quad (14)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_{u=x}^{\infty} e^{-\frac{u^2}{2}}\,du$. Similarly, $E(B|\alpha_n=\alpha)$ can be obtained as

$$E(B|\alpha_n=\alpha) = \sigma\sqrt{\frac{2}{\pi}}e^{\frac{-\alpha^2E_s}{2\sigma^2}}-\alpha\sqrt{E_s}+2\alpha\sqrt{E_s}Q(\frac{-\alpha\sqrt{E_s}}{\sigma}). \quad (15)$$

By noting that $Q(-x) = 1-Q(x)$ and substituting Eqns. (14) and (15) in Eqn. (13), we get

$$E(|r_n|\,|\alpha) = \sigma\sqrt{\frac{2}{\pi}}e^{\frac{-\alpha^2E_s}{2\sigma^2}}+\alpha\sqrt{E_s}-2\alpha\sqrt{E_s}Q\left(\frac{\alpha\sqrt{E_s}}{\sigma}\right). \quad (16)$$

Now, we take the expectation over α to get $E(|r_n|)$. The expectation of $e^{\frac{-\alpha^2E_s}{2\sigma^2}}$ in the first term in (16) is obtained as

$$E\left[e^{\frac{-\alpha^2E_s}{2\sigma^2}}\right] = \int_0^{\infty} 2\frac{m^m}{\Gamma(m)}a^{2m-1}e^{-ma^2}e^{\frac{-a^2E_s}{2\sigma^2}}\,da, \quad (17)$$

where $\Gamma(.)$ is the Gamma function [6]. Letting $a^2(m+\frac{E_s}{2\sigma^2}) = u$, Eqn. (17) becomes

$$\begin{aligned} E[e^{\frac{-\alpha^2E_s}{2\sigma^2}}] &= \frac{m^m}{\Gamma(m)(m+\frac{E_s}{2\sigma^2})^m}\int_0^{\infty} e^{-u}u^{m-1}\,du \quad (18) \\ &= \frac{m^m}{(m+\frac{E_s}{2\sigma^2})^m}. \end{aligned}$$

The expectation of the second term in Eqn. (16) is obtained as

$$E(\alpha\sqrt{E_s}) = \sqrt{E_s}\frac{2m^m}{\Gamma(m)}\int_0^{\infty} a^{2m}e^{-ma^2}\,da. \quad (19)$$

Letting $ma^2 = u$ in Eqn. (19), we get

$$\begin{aligned} E(\alpha\sqrt{E_s}) &= \sqrt{E_s}\frac{1}{\sqrt{m}\Gamma(m)}\int_0^{\infty} e^{-u}u^{m-\frac{1}{2}}\,du \quad (20) \\ &= \sqrt{E_s}\frac{\Gamma(m+\frac{1}{2})}{\sqrt{m}\,\Gamma(m)}. \end{aligned}$$

To derive $E\left[\alpha Q\left(\frac{\alpha\sqrt{E_s}}{\sigma}\right)\right]$ for the third term in Eqn. (16), we use the alternative expression for $Q(x)$, which is given by [11],[12]

$$Q(x) = \frac{1}{\pi}\int_{\phi=0}^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\phi}}\,d\phi\ ;\ x \geq 0. \quad (21)$$

This gives

$$\begin{aligned} E\left[\alpha Q\left(\frac{\alpha\sqrt{E_s}}{\sigma}\right)\right] &= \frac{2m^m}{\Gamma(m)\pi}\int_{a=0}^{\infty}\int_{\phi=0}^{\frac{\pi}{2}} a^{2m}e^{-ma^2} \\ &\quad e^{-\frac{a^2E_s}{2\sigma^2\sin^2\phi}}\,d\phi\,da. \end{aligned} \quad (22)$$

Letting $\left(m+\frac{E_s}{2\sin^2\phi\sigma^2}\right)a^2 = u$, the double integration in Eqn. (22) becomes

$$\begin{aligned} E\left[\alpha Q\left(\frac{\alpha\sqrt{E_s}}{\sigma}\right)\right] &= \frac{m^m}{\pi\Gamma(m)}\int_{\phi=0}^{\frac{\pi}{2}} \frac{d\phi}{(m+\frac{E_s}{2\sin^2\phi\sigma^2})^{m+\frac{1}{2}}} \\ &\quad \cdot\int_{u=0}^{\infty} e^{-u}u^{m-\frac{1}{2}}\,du \\ &= \frac{\Gamma(m+\frac{1}{2})}{\pi\Gamma(m)\sqrt{m}}\int_{\phi=0}^{\frac{\pi}{2}} \frac{\sin^{2m+1}\phi}{(\sin^2\phi+\beta)^{m+\frac{1}{2}}}\,d\phi \end{aligned} \quad (23)$$

where $\beta = \frac{E_s}{2m\sigma^2}$. Let us define

$$I(m) = \int_{\phi=0}^{\frac{\pi}{2}} \frac{\sin^{2m+1}\phi}{(\sin^2\phi+\beta)^{m+\frac{1}{2}}}\,d\phi. \quad (24)$$

Substituting $\sin^2\phi+\beta = t^2$, Eqn. (24) becomes

$$\begin{aligned} I(m) &= \int_{t=\sqrt{\beta}}^{\sqrt{1+\beta}} \frac{(t^2-\beta)^m}{t^{2m}\sqrt{1+\beta-t^2}}\,dt \\ &= \sum_{k=0}^{m-1}\binom{m}{k}(-\beta)^{m-k}\cdot\int_{t=\sqrt{\beta}}^{\sqrt{1+\beta}} \frac{t^{2k}}{t^{2m}\sqrt{1+\beta-t^2}}\,dt + \\ &\quad \int_{t=\sqrt{\beta}}^{\sqrt{1+\beta}} \frac{1}{\sqrt{1+\beta-t^2}}\,dt \\ &= \sum_{k=0}^{m-1}\binom{m}{k}(-\beta)^{m-k}\cdot\int_{t=\sqrt{\beta}}^{\sqrt{1+\beta}} \frac{1}{t^{2(m-k)}\sqrt{1+\beta-t^2}}\,dt + \\ &\quad \cos^{-1}\left(\sqrt{\frac{\beta}{1+\beta}}\right). \end{aligned} \quad (25)$$

In order to further simplify Eqn. (25), let us define $I_1(m)$ as

$$I_1(m) = \int_{t=\sqrt{\beta}}^{\sqrt{1+\beta}} \frac{1}{t^{2(p)}\sqrt{1+\beta-t^2}}\,dt\ ;\ p = m-k,\ p > 0. \quad (26)$$

Substituting $t = \frac{1}{u}$ in Eqn. (26), we get

$$I_1(m) = \frac{1}{\sqrt{1+\beta}}\int_{u=\frac{1}{\sqrt{1+\beta}}}^{\frac{1}{\sqrt{\beta}}} \frac{u^{2p-1}}{\sqrt{u^2-\frac{1}{1+\beta}}}\,du. \quad (27)$$

Substituting $u = \frac{1}{\sqrt{1+\beta}} \cosh\theta$ in (27), Eqn. (27) becomes

$$I_1(m) = \frac{1}{(1+\beta)^p} \int_{\theta=0}^{\theta_1} \cosh^{2p-1}\theta \, d\theta \;\; ; \theta_1 = \cosh^{-1}\sqrt{\frac{1+\beta}{\beta}}$$

$$= \frac{1}{(1+\beta)^p} \frac{1}{2^{2p-1}} \sum_{l=0}^{2p-1} \binom{2p-1}{l} \frac{e^{\theta_1(2l-2p+1)} - 1}{(2l-2p+1)}. \quad (28)$$

Plugging the above expression for $I_1(m)$ in Eqn. (25), we get $I(m)$ as

$$I(m) = \cos^{-1}\left(\sqrt{\frac{\beta}{1+\beta}}\right) + \sum_{k=0}^{m-1} \sum_{l=0}^{2(m-k)-1} \binom{m}{k} \binom{2(m-k)-1}{l} \left(\frac{-\beta}{1+\beta}\right)^{m-k} \cdot \frac{1}{2^{2(m-k)-1}} \frac{e^{\theta_1(2l-2(m-k)+1)} - 1}{(2l-2(m-k)+1)}. \quad (29)$$

Finally, combining Eqns. (29), (23), (20), (18) and (16), we get the expression for $E(|r_n|)$ as

$$E(|r_n|) = \sigma\sqrt{\frac{2}{\pi}}\left(\frac{m}{m+\frac{E_s}{2\sigma^2}}\right)^m + \sqrt{E_s}\frac{\Gamma(m+\frac{1}{2})}{\sqrt{m}\Gamma(m)} - 2\sqrt{E_s}\frac{\Gamma(m+\frac{1}{2})}{\pi\sqrt{m}\Gamma(m)} I(m). \quad (30)$$

II. Derivation of Eqn. (5)

To derive z for the Rayleigh distribution, we substitute $m = 1$ in Eqn. (4). Substituting $m = 1$ in Eqn. (29), and observing that $\beta = \frac{\gamma}{m} = \gamma$ and $\Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}$, $I(1)$ in the denominator of Eqn. (4) can be obtained as

$$I(1) = \cos^{-1}\left(\sqrt{\frac{\gamma}{1+\gamma}}\right) - \frac{\gamma}{1+\gamma}\sinh\theta_1. \quad (31)$$

By substituting for θ_1 from Eqn. (28), we get

$$I(1) = \cos^{-1}\left(\sqrt{\frac{\gamma}{1+\gamma}}\right) - \frac{\sqrt{\gamma}}{1+\gamma}. \quad (32)$$

Upon substituting Eqn. (32) in Eqn. (4), we get Eqn. (5).

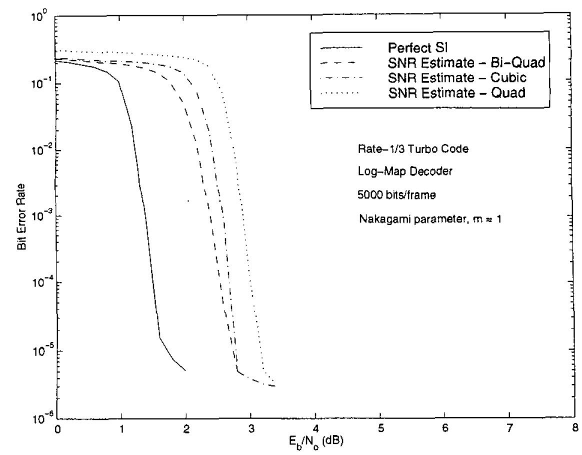

Fig. 1. Comparison of turbo decoder performance using perfect side information vs SNR estimates on i.i.d Rayleigh fading (i.e., Nakagami parameter, $m = 1$).

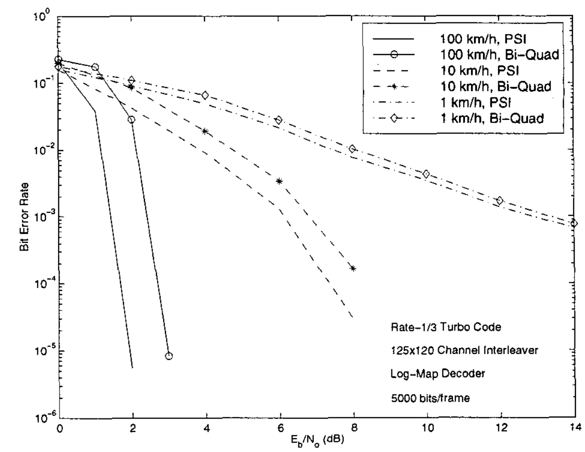

Fig. 2. Comparison of turbo decoder performance using perfect side information vs SNR estimates on correlated Rayleigh fading. 125×120 channel interleaver. Carrier frequency = 900 MHz. Vehicle speed = 1, 10, 100 km/h. $T = 0.1$ msec.

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