Physical Layer Design for a Narrow Band Communication System

G V V Sharma

Abstract—**The design and implementation of a simple narrow band communication system is provided in this document. TCM-8PSK is used for modulation, followed by channel estimation and equalization in the presence of Rayleigh fading. Through simulation results, it is shown that the proposed system is robust.**

1 Specifications

The specifications for the communication system to be designed are listed in Table 1.

Parameter	**Value**
Hardware	FPGA based baseband
MODEM	8PSK-TCM
Modem Rate	555Kbps
SNR	7.6 db at 1e5
Channel (V/UHF)	30Mhz - 512Mhz
Bandwidth	250khz
Bit Duration	2.7us
Throughput	100kbps (Throughput at application Layer)
Ramp up time	116 us (Junk symbols will be sent)
Propagation Delay	100 us (Junk symbols will be sent)
Training sequence	421.2us(provided time for training sequence)
Frame Slot	2 ms
Frame SOM	8 bytes
Payload	32 bytes (692 us)

TABLE 1: Specifications

2 Frame Design

The specifications for the communication system to be designed are listed in Table 1.

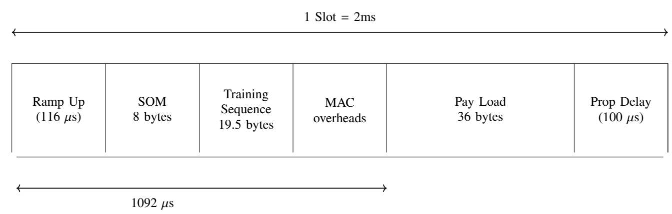

Fig. 2: Physical Layer Frame

3 8-PSK

3.1 Modulation

3.1.1. See Fig. 3.1.1.1 for the constellation diagram. The transmitted symbol set is given by

$$\mathbf{s}_m = \begin{pmatrix} \cos\frac{2m\pi}{8} \\ \sin\frac{2m\pi}{8} \end{pmatrix}, \quad m \in \{0, 1, \ldots, 7\}. \quad (3.1.1.1)$$

The numerical values for $\mathbf{s}_m$ are listed in Table 3.1.2.1

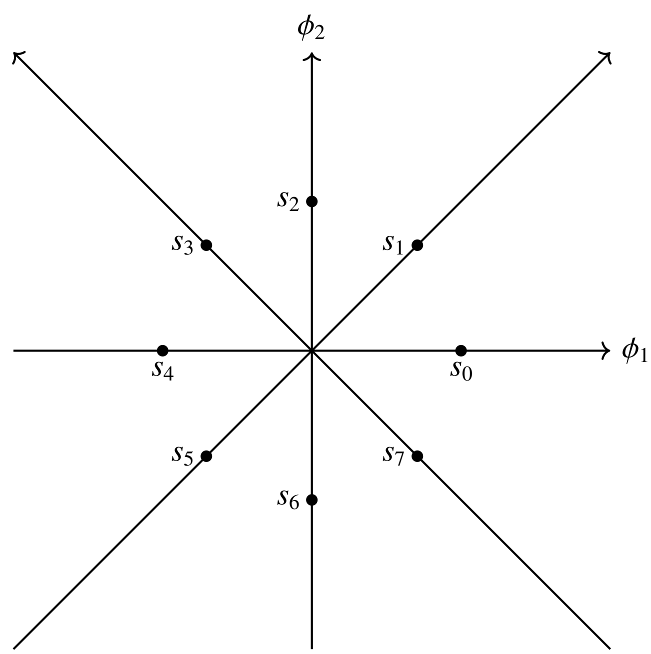

Fig. 3.1.1.1: Constellation diagram

3.1.2. The gray code shown in Table 3.1.2.1 is used for encoding the 8-PSK symbols.

Symbol	Gray Code	Value
s_0	000	$\begin{pmatrix}1\\0\end{pmatrix}$
s_1	001	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_2	011	$\begin{pmatrix}0\\1\end{pmatrix}$
s_3	010	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_4	110	$\begin{pmatrix}-1\\0\end{pmatrix}$
s_5	111	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$
s_6	101	$\begin{pmatrix}0\\-1\end{pmatrix}$
s_7	100	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$

TABLE 3.1.2.1: Gray coding

3.1.3. The received symbol is then obtained as

$$\mathbf{y} = \sqrt{E_s}\mathbf{s} + \mathbf{n} \tag{3.1.3.1}$$

where E_s is the symbol energy and

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \frac{N_0}{2}\mathbf{I}\right) \tag{3.1.3.2}$$

$$\mathbf{s} \in \{\mathbf{s}_m\}_{m=0}^{7} \tag{3.1.3.3}$$

3.1.4. Using the ML criterion, the decision rule for each symbol is given by Fig. 3.1.4.1. For $\mathbf{s}_0$, this can be expressed as

$$\|\mathbf{y} - s_0\|^2 \le \|\mathbf{y} - s_i\|^2, \quad i = 1, \dots, 7 \tag{3.1.4.1}$$

$$\implies (\mathbf{s}_0 - \mathbf{s}_i)^T \mathbf{y} \ge 0 \tag{3.1.4.2}$$

$$\tag{3.1.4.3}$$

which can be simplified to obtain the matrix inequality

$$\begin{pmatrix}(\mathbf{s}_0 - \mathbf{s}_1)^T\\(\mathbf{s}_0 - \mathbf{s}_2)^T\\(\mathbf{s}_0 - \mathbf{s}_3)^T\\(\mathbf{s}_0 - \mathbf{s}_4)^T\\(\mathbf{s}_0 - \mathbf{s}_5)^T\\(\mathbf{s}_0 - \mathbf{s}_6)^T\\(\mathbf{s}_0 - \mathbf{s}_7)^T\end{pmatrix}\mathbf{y} \ge \mathbf{0} \tag{3.1.4.4}$$

resulting in

$$\begin{pmatrix}\sqrt{2}-1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0} \tag{3.1.4.5}$$

after considering the intersection of all the regions and simplifying.

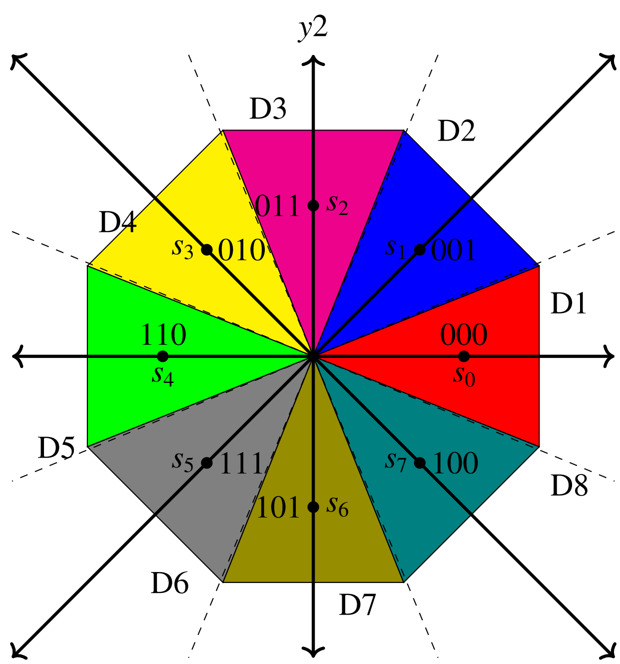

Fig. 3.1.4.1: decision regions

Similarly the decisions for all symbols are available in Table 3.1.4.1

Symbol	Decision region	Inequality	Matrix Inequality
$\mathbf{s}_0$	D1	$y_2 + (\sqrt{2}-1)y_1 > 0,\ y_2 - (\sqrt{2}-1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}-1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_1$	D2	$y_2 - (\sqrt{2}+1)y_1 < 0,\ y_2 - (\sqrt{2}-1)y_1 > 0$	$\begin{pmatrix}\sqrt{2}+1 & -1\\-(\sqrt{2}-1) & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_2$	D3	$y_2 - (\sqrt{2}+1)y_1 > 0,\ y_2 + (\sqrt{2}+1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}+1) & -1\\\sqrt{2}+1 & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_3$	D4	$y_2 + (\sqrt{2}-1)y_1 > 0,\ y_2 + (\sqrt{2}+1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}-1 & 1\\-(\sqrt{2}+1) & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_4$	D5	$y_2 + (\sqrt{2}-1)y_1 < 0,\ y_2 - (\sqrt{2}-1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}-1) & -1\\-(\sqrt{2}-1) & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_5$	D6	$y_2 - (\sqrt{2}+1)y_1 > 0,\ y_2 - (\sqrt{2}-1)y_1 < 0$	$\begin{pmatrix}-(\sqrt{2}+1 & 1\\\sqrt{2}-1 & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_6$	D7	$y_2 - (\sqrt{2}+1)y_1 < 0,\ y_2 + (\sqrt{2}+1)y_1 < 0$	$\begin{pmatrix}\sqrt{2}+1 & -1\\-(\sqrt{2}+1) & -1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$
$\mathbf{s}_7$	D8	$y_2 + (\sqrt{2}-1)y_1 < 0,\ y_2 + (\sqrt{2}+1)y_1 > 0$	$\begin{pmatrix}-(\sqrt{2}-1) & -1\\\sqrt{2}+1 & 1\end{pmatrix}\mathbf{y} \ge \mathbf{0}$

TABLE 3.1.4.1: Decision rules

3.2 Simulation

3.2.1. Fig. 3.2.1.1 shows the comparison of the SER for 8-PSK for simulation as well as anlysis.

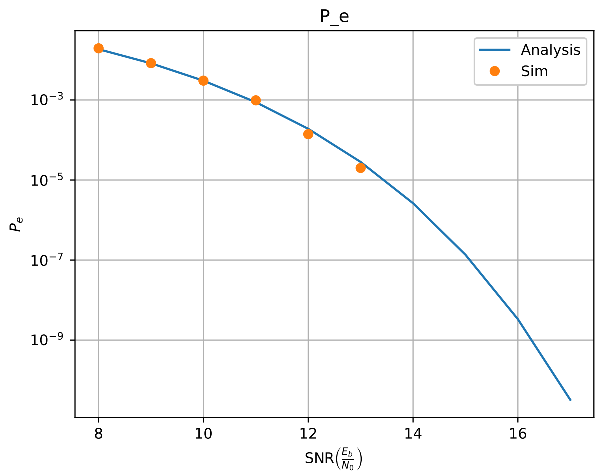

Fig. 3.2.1.1: Constellation diagram

Parameter	Length	Description
$\mathbf{x}_p$	$P = 10$	Transmitted Pilot Vector
$\mathbf{h}$	$L = 5$	Channel Vector
$\mathbf{n}$	$P = 10$	AWGN
$\mathbf{y}_p$	$L-1+P = 14$	Received Pilot Symbol Vector
$\mathbf{I}$		Identiy Matrix
$\mathbf{R}$	$L+P-1 \times L+P-1 = 14 \times 14$	Flips vector
$\mathbf{y}$	$P = 10$	Circular Received Pilot Vector
$\mathbf{1}$		Ones Vector
$\mathbf{0}$		Zeros Vector

TABLE 4.1.1.1

4 Channel

4.1 Estimation

The pilot symbols are used for channel estimation.

4.1.1. $P = 10$ pilot symbols are used at a time for channel estimation. The channel filter has length $L = 5$. See Table 4.1.1.1 for details. The consequent model is

$$\mathbf{y}_p = \mathbf{x}_p * \mathbf{h} + \mathbf{n}_p \tag{4.1.1.1}$$

4.1.2. Let

$$\mathbf{x}_p \overset{\mathcal{F}}{\rightleftharpoons} \mathbf{X}_p, \mathbf{y}_p \overset{\mathcal{F}}{\rightleftharpoons} \mathbf{Y}_p, \mathbf{h} \overset{\mathcal{F}}{\rightleftharpoons} \mathbf{H}_p, \tag{4.1.2.1}$$

be the DFTs of the signals. Then,

$$\mathbf{H}_p = \frac{\mathbf{Y}_p}{\mathbf{X}_p} \tag{4.1.2.2}$$

$$\text{and } \mathbf{H}_p \overset{\mathcal{F}}{\rightleftharpoons} \mathbf{h} \tag{4.1.2.3}$$

This is how channel estimation is done $\because$ both $\mathbf{x}_p$ and $\mathbf{y}_p$ are known at the receiver.

4.1.3. While (4.1.2.3) cannot be applied directly, since the lengths of $\mathbf{x}_p(P)$ and $\mathbf{h}(L)$ are different, resulting in a circular convolution. To address this, we do the following operations

$$\mathbf{y} = \begin{pmatrix} \mathbf{I}_P & \mathbf{0}_{P\times L-1} \end{pmatrix} \Big[(\mathbf{I} + \mathbf{R}) \begin{pmatrix} \mathbf{1}_{L-1}^T & \mathbf{0}_P^T \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{L-1}^T & \mathbf{1}_{P-L+1}^T & \mathbf{0}_{L-1}^T \end{pmatrix} \mathbf{y}_p \Big] \tag{4.1.3.1}$$

where

$$\mathbf{R} = \begin{pmatrix} 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 1 & 0 \\ & & \vdots & & \\ 1 & 0 & \dots & 0 & 0 \end{pmatrix} \tag{4.1.3.2}$$

is a reflection matrix. The channel is now estiamted as

$$\mathbf{y} \overset{\mathcal{F}}{\rightleftharpoons} \mathbf{Y} \tag{4.1.3.3}$$

$$\frac{\mathbf{Y}}{\mathbf{X}_p} \overset{\mathcal{F}}{\rightleftharpoons} \mathbf{h} \tag{4.1.3.4}$$

4.2 SNR Estimation

MMSE require estimation of the SNR. The signal power is estimated using $\mathbf{h}$ in the pilot duration. The noise power is estimated in the rampup time in Fig. 2.

4.2.1. Let $\mathbf{h}_p$ be the channel estimate using the pth pilot block. Then, the average symbol SNR at the receiver is computed using

$$\hat{E}_s = \frac{1}{2\left\lfloor \frac{P_f}{P} \right\rfloor} \sum_{p=1}^{\left\lfloor \frac{P_f}{P} \right\rfloor} \left\|\mathbf{h}_p\right\|^2 \tag{4.2.1.1}$$

were P_f is the total number of pilot symbols in a frame.

4.2.2. The noise variance is computed as

$$\hat{\sigma}^2 = \frac{1}{2K} \sum_{i=1}^{K} |Y_k|^2 \tag{4.2.2.1}$$

where K is the maximum number of symbols that could possibly be transmitted during ram-pup time.

4.2.3. From the above,

$$SNR = \frac{\hat{E}_s}{\hat{\sigma}^2} \tag{4.2.3.1}$$

4.3 Equalization

4.3.1. From Fig. 2, the number of payload symbols per frame is $N = \frac{36\times 8}{3} = 96$. The received symbols are then given by

$$\mathbf{y}_n = \mathbf{x}_n * \mathbf{h} + \mathbf{n}_n, \quad n = 1, \dots, N \tag{4.3.1.1}$$

In (4.3.1.1), $\mathbf{h}, \mathbf{y}$ are known and we wish to estimate $\mathbf{x}$.

4.3.2. The MMSE estimate is obtained as

$$\mathbf{x} = \mathbf{W}^T \mathbf{y} \tag{4.3.2.1}$$

where

$$\mathbf{W} = \left(\mathbf{H}^* \mathbf{H} + \frac{\mathbf{I}}{SNR}\right)^{-1} \mathbf{H}^* \tag{4.3.2.2}$$

$$\mathbf{H} = \text{Toeplitz}\,(\mathbf{h}) \tag{4.3.2.3}$$

5 Frequency Synchronization

5.1 Algorithm

5.1.1. See the parameters listed in Table **??**.

5.1.2. Let

$$r_k = e^{j2\pi \Delta f T_s + \theta} + v_k, 1 \le k \le N \tag{5.1.2.1}$$

5.1.3.

$$\Delta f = \frac{1}{2\pi T_s} \frac{\sum_{k=1}^{M} Im\left\{R(k)\right\}}{\sum_{k=1}^{M} kRe\left\{R(k)\right\}} \tag{5.1.3.1}$$

where

$$R(k) = \frac{1}{N-K} \sum_{i=k+1}^{M} r_i r_{i-k}^* \tag{5.1.3.2}$$

$$\sum_{k=1}^{M} ImR(k) = Marg \sum_{k=1}^{M} R(k) \tag{5.1.3.3}$$

$$\sum_{k=1}^{M} kReR(k) = M\frac{M+1}{2} \tag{5.1.3.4}$$