

Physical Layer Design for a Narrow Band Communication System

G V V Sharma

Abstract—This a simple document explaining a question about the concept of similar triangles.

Download all python codes from

svn co <https://github.com/SiddharthPh/Summer2020/trunk/geometry/codes>

and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 SPECIFICATIONS

1.0.1. 8-PSK Consider

$$\mathbf{y} = \mathbf{s} + \mathbf{n} \quad (1.0.1.1)$$

where $\mathbf{s} \in \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$

$$s_0 = \begin{pmatrix} \sqrt{E_s} \\ 0 \end{pmatrix} \quad (1.0.1.2)$$

$$s_1 = \begin{pmatrix} \sqrt{\frac{E_s}{2}} \\ \sqrt{\frac{E_s}{2}} \end{pmatrix} \quad (1.0.1.3)$$

$$s_2 = \begin{pmatrix} 0 \\ \sqrt{E_s} \end{pmatrix} \quad (1.0.1.4)$$

$$s_3 = \begin{pmatrix} -\sqrt{\frac{E_s}{2}} \\ \sqrt{\frac{E_s}{2}} \end{pmatrix} \quad (1.0.1.5)$$

$$s_4 = \begin{pmatrix} -\sqrt{E_s} \\ 0 \end{pmatrix} \quad (1.0.1.6)$$

$$s_5 = \begin{pmatrix} -\sqrt{\frac{E_s}{2}} \\ -\sqrt{\frac{E_s}{2}} \end{pmatrix} \quad (1.0.1.7)$$

$$s_6 = \begin{pmatrix} 0 \\ -\sqrt{E_s} \end{pmatrix} \quad (1.0.1.8)$$

$$s_7 = \begin{pmatrix} \sqrt{\frac{E_s}{2}} \\ -\sqrt{\frac{E_s}{2}} \end{pmatrix} \quad (1.0.1.9)$$

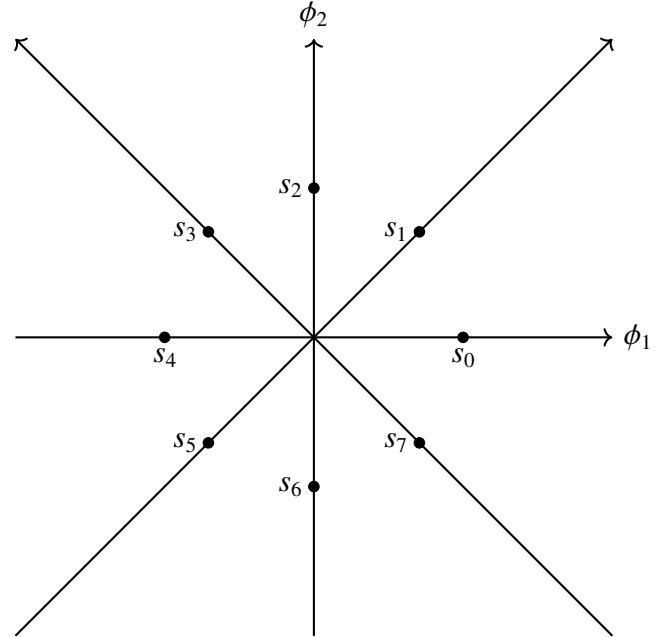


Fig. 1.0.1.1: Constellation diagram

1.0.2. Encoding

s_0 denote bits 000, s_1 denote bits 001, s_2 denote bits 011, s_3 denote bits 010, s_4 denote bits 110, s_5 denote bits 111, s_6 denote bits 101, s_7 denote bits 100.

1.0.3. Decoding

Minimum distance Criterion:

$$\hat{s} = \min \|\mathbf{y} - \mathbf{s}\| \quad (1.0.3.1)$$

where $\mathbf{s} \in \{s_0, s_1, s_2, \dots, s_M\}$

Symbol	Gray Code	Value
s_0	000	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
s_1	001	$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
s_2	011	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
s_3	010	$\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
s_4	110	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
s_5	111	$\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$
s_6	101	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
s_7	100	$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

Fig. 1.0.2.1: Gray coding

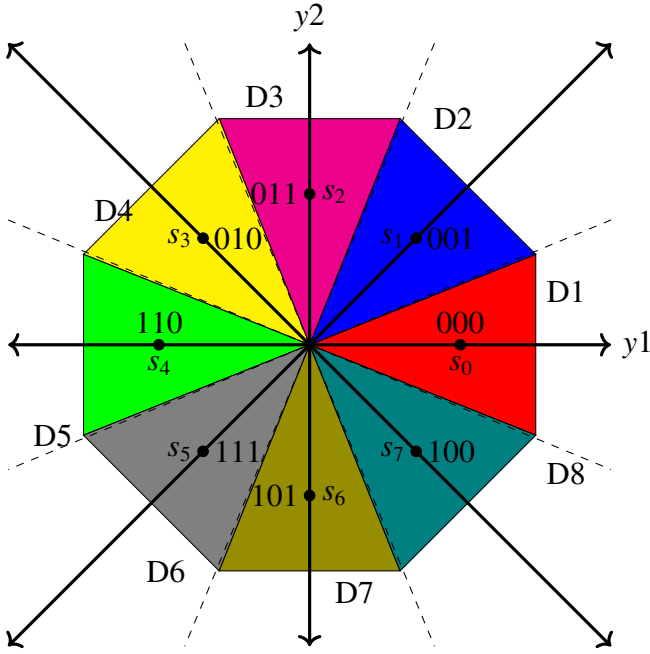


Fig. 1.0.3.1: decision regions

From eq.1.0.3.1, s_0 is chosen if

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_1\|^2 \quad (1.0.3.2)$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_2\|^2 \quad (1.0.3.3)$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_3\|^2 \quad (1.0.3.4)$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_4\|^2 \quad (1.0.3.5)$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_5\|^2 \quad (1.0.3.6)$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_6\|^2 \quad (1.0.3.7)$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_7\|^2 \quad (1.0.3.8)$$

Since $\|s_i\|^2 = E_s$, the above conditions can be simplified to obtain the region

$$(s_0 - s_1)^T \mathbf{y} > 0 \quad (1.0.3.9)$$

$$(s_0 - s_2)^T \mathbf{y} > 0 \quad (1.0.3.10)$$

$$(s_0 - s_3)^T \mathbf{y} > 0 \quad (1.0.3.11)$$

$$(s_0 - s_4)^T \mathbf{y} > 0 \quad (1.0.3.12)$$

$$(s_0 - s_5)^T \mathbf{y} > 0 \quad (1.0.3.13)$$

$$(s_0 - s_6)^T \mathbf{y} > 0 \quad (1.0.3.14)$$

$$(s_0 - s_7)^T \mathbf{y} > 0 \quad (1.0.3.15)$$

Substituting the values of s_0, s_1, \dots, s_7 in the above and eliminating $\sqrt{E_s}$, the desired region is

$$\begin{pmatrix} 1 - \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \quad (1.0.3.16)$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \quad (1.0.3.17)$$

$$\begin{pmatrix} 1 + \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \quad (1.0.3.18)$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \quad (1.0.3.19)$$

$$\begin{pmatrix} 1 + \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \quad (1.0.3.20)$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \quad (1.0.3.21)$$

$$\begin{pmatrix} (1 - \frac{1}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}} \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \quad (1.0.3.22)$$

yielding $|y_2| < (\sqrt{2} - 1)y_1$

i.e., Red region(D1) is detected at the receiver.

Similarly,

From eq:1.0.3.1

For detecting s_1 , $y_2 - (\sqrt{2} + 1)y_1 < 0$ and $y_2 - (\sqrt{2} - 1)y_1 > 0$.

\iff D2(blue) region is detected at receiver

For detecting s_2 , $y_2 - (\sqrt{2} + 1)y_1 > 0$ and $y_2 + (\sqrt{2} + 1)y_1 > 0$

\iff D3(pink) region at receiver.

For detecting s_3 , $y_2 + (\sqrt{2} - 1)y_1 > 0$ and $y_2 + (\sqrt{2} + 1)y_1 < 0$.

\iff D4(yellow) region at receiver.

For detecting s_4 , $y_2 + (\sqrt{2} - 1)y_1 < 0$ and $y_2 - (\sqrt{2} - 1)y_1 > 0$.

\iff D5(green) region at receiver.

For detecting s_5 , $y_2 - (\sqrt{2} + 1)y_1 > 0$ and $y_2 - (\sqrt{2} - 1)y_1 < 0$.

\iff D6(grey) region at receiver.

For detecting s_6 , $y_2 - (\sqrt{2} + 1)y_1 < 0$ and $y_2 + (\sqrt{2} + 1)y_1 < 0$.

\iff D7(olive) region at receiver.

For detecting s_7 , $y_2 + (\sqrt{2} - 1)y_1 < 0$ and $y_2 + (\sqrt{2} + 1)y_1 > 0$.

\iff D8(teal) region at receiver.

1.0.4. The following code has simulation of 8PSK.

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codes/ee18btech11012.py
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