Probability

G V V Sharma*

Contents

***Abstract*—This book provides solved examples on Probability**

1 Axioms of Probability

1.1.

2 Markov Chain

2.1. **Step 1.** Flip a coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (upto two decimal places)......
Solution: The given problem can be repre-

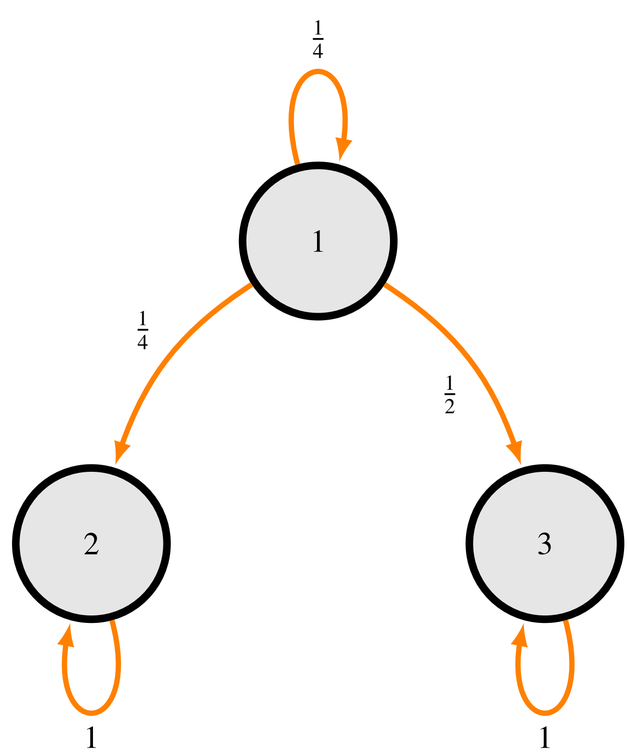

Fig. 2.1.1

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

sented using Table 2.1.1 and the Markov chain in Fig. 2.1.1. The state transition matrix for the

State	Description
1	$\{T, T\}$
2	$Y = \{T, H\}$
3	$N = \{\{H, H\}, \{H, T\}\}$

TABLE 2.1.1: States and their notations

Markov chain can be expressed as

$$P = \begin{array}{c c} & \begin{array}{ccc} 2 & 3 & 1 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 1 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.25 & 0.5 & 0.25 \end{bmatrix} \end{array} \tag{2.1.1}$$

Clearly, the state 1 is transient, while $2, 3$ are absorbing. Comparing (2.1.1) with the standard form of the state transition matrix

$$P = \begin{array}{c c} & \begin{array}{cc} A & N \end{array} \\ \begin{array}{c} A \\ N \end{array} & \begin{bmatrix} I & O \\ R & Q \end{bmatrix} \end{array} \tag{2.1.2}$$

where, From (2.1.1) and (2.1.2),

TABLE 2.1.2: Notations and their meanings

Notation	Meaning
A	All absorbing states
N	All non-absorbing states
I	Identity matrix
O	Zero matrix
R, Q	Other submatices

$$R = \begin{pmatrix} 0.25 & 0.5 \end{pmatrix}, Q = \begin{pmatrix} 0.25 \end{pmatrix} \tag{2.1.3}$$

The limiting matrix for absorbing Markov chain is

$$\bar{P} = \begin{pmatrix} I & O \\ FR & O \end{pmatrix} \tag{2.1.4}$$

where

$$F = (I - Q)^{-1} = (1 - 0.25)^{-1} = \frac{4}{3} \tag{2.1.5}$$

is called the fundamental matrix of P. Upon substituting from (2.1.3) in (2.1.5),

$$F = (1 - 0.25)^{-1} = \frac{4}{3} \tag{2.1.6}$$

and

$$FR = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix} \tag{2.1.7}$$

which, upon substituting in (2.1.4) yields

$$\bar{P} = \begin{array}{c c} & \begin{array}{ccc} 2 & 3 & 1 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 1 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix} \end{array} \tag{2.1.8}$$

$$\therefore \bar{p}_{12} = \frac{1}{3} \tag{2.1.9}$$