Assignment 7

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https://github.com/Ananthoju−Pranav−Sai/AI1103/tree/main/Assignment_7_1/Codes

and latex codes from

https://github.com/Ananthoju−Pranav−Sai/AI1103/blob/main/Assignment_7_1/main.tex

STATS P1 IESISS19 Q 31

Let X be a random variable with p.d.f

$$f_X(x) = \begin{cases} \frac{2x}{\pi^2} & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases} \tag{0.0.1}$$

Let Y = $\sin X$, then for $0 < y < 1$, the p.d.f of Y is given by,

(A) $\frac{2\pi}{\sqrt{1-y^2}}$

(B) $\frac{\pi}{2}\sqrt{1-y^2}$

(C) $\frac{2}{\pi}\sqrt{1-y^2}$

(D) $\frac{2}{\pi\sqrt{1-y^2}}$

Solution

Given p.d.f of X as

$$f_X(x) = \begin{cases} \frac{2x}{\pi^2} & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases} \tag{0.0.2}$$

$$F_X(x) = \Pr(X \le x) \tag{0.0.3}$$

$$\implies F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt \tag{0.0.4}$$

which can be written as

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ \frac{x^2}{\pi^2} & 0 < x < \pi \\ 1 & x \ge \pi \end{cases} \tag{0.0.5}$$

Now c.d.f of Y can be written as

$$F_Y(y) = \Pr(Y \le y) \tag{0.0.6}$$

$$\implies F_Y(y) = \Pr(\sin X \le y) \tag{0.0.7}$$

As $X \in (0, \pi)$, $\sin X \le y$ has two solutions i.e., either $X \le \sin^{-1} y$ or $X \ge \pi - \sin^{-1} y$ as shown in 4

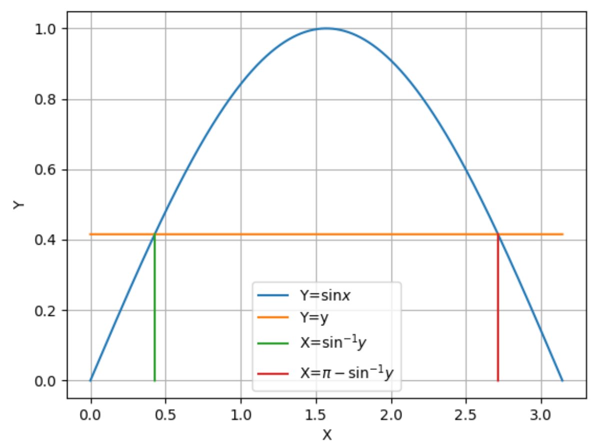

Fig. 4: $Y = \sin X$ plot

$$\implies F_Y(y) = \Pr\left(X \le \sin^{-1} y\right) + \Pr\left(X \ge \pi - \sin^{-1} y\right) \tag{0.0.8}$$

$$\implies F_Y(y) = F_X(\sin^{-1} y) + 1 - \Pr\left(X \le \pi - \sin^{-1} y\right) \tag{0.0.9}$$

$$\implies F_Y(y) = F_X(\sin^{-1} y) + 1 - F_X(\pi - \sin^{-1} y) \tag{0.0.10}$$

using (0.0.5) in (0.0.10)

$$\implies F_Y(y) = \frac{\left(\sin^{-1} y\right)^2}{\pi^2} + 1 - \frac{\left(\pi - \sin^{-1} y\right)^2}{\pi^2} \tag{0.0.11}$$

$$\implies F_Y(y) = \frac{2\sin^{-1} y}{\pi} \tag{0.0.12}$$

Now pdf of Y can be written as

$$f_Y(y) = \frac{\mathrm{d}F_Y(y)}{\mathrm{d}y} \tag{0.0.13}$$

$$\implies f_Y(y) = \frac{2}{\pi}\frac{\mathrm{d}\left(\sin^{-1} y\right)}{\mathrm{d}y} \tag{0.0.14}$$

$$\implies f_Y(y) = \frac{2}{\pi\sqrt{1-y^2}} \tag{0.0.15}$$

Hence, option(D) is correct.