ASSIGNMENT 3

Vaibhav Chhabra AI20BTECH11022

Download all python codes from

https://github.com/vaibhavchhabra25/EE3900-course/blob/main/Assignment-3/codes

and latex-tikz codes from

https://github.com/vaibhavchhabra25/EE3900-course/blob/main/Assignment-3/main.tex

1 Problem

(Ramsey-4.2-Tangent and Normal-Q.21) Verify that the perpendicular bisector of the chord joining two points $\mathbf{x_1}, \mathbf{x_2}$ on the circle

$$\mathbf{x}^{\mathsf{T}}\mathbf{x} + 2(g \quad f)\mathbf{x} + c = 0 \tag{1.0.1}$$

passes through the centre.

2 Solution

Since x_1 and x_2 lie on the circle,

$$\mathbf{x_1}^{\mathsf{T}} \mathbf{x_1} + 2 (g \ f) \mathbf{x_1} + c = 0$$
 (2.0.1)

$$\mathbf{x_2}^{\mathsf{T}} \mathbf{x_2} + 2(g \ f) \mathbf{x_2} + c = 0$$
 (2.0.2)

Subtracting the two, we get,

$$\mathbf{x_2}^{\mathsf{T}} \mathbf{x_2} - \mathbf{x_1}^{\mathsf{T}} \mathbf{x_1} = -2 (g \ f) (\mathbf{x_2} - \mathbf{x_1})$$
 (2.0.3)

The midpoint of the chord x_1x_2 is

$$\mathbf{M} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2} \tag{2.0.4}$$

(2.0.7)

Since perpendicular bisector of x_1x_2 is perpendicular to x_1x_2 and passes through M, any general point P on the perpendicular bisector can be given using the relation

$$(\mathbf{x}_{2} - \mathbf{x}_{1})^{\top} (\mathbf{P} - \mathbf{M}) = 0 \qquad (2.0.5)$$

$$\implies (\mathbf{x}_{2} - \mathbf{x}_{1})^{\top} \mathbf{P} - (\mathbf{x}_{2}^{\top} - \mathbf{x}_{1}^{\top}) \left(\frac{\mathbf{x}_{1} + \mathbf{x}_{2}}{2}\right) = 0$$

$$(2.0.6)$$

$$\implies (\mathbf{x}_{2} - \mathbf{x}_{1})^{\top} \mathbf{P} = \frac{\mathbf{x}_{2}^{\top} \mathbf{x}_{2} - \mathbf{x}_{1}^{\top} \mathbf{x}_{1} + \mathbf{x}_{2}^{\top} \mathbf{x}_{1} - \mathbf{x}_{1}^{\top} \mathbf{x}_{2}}{2}$$

Since $\mathbf{x_1}^{\mathsf{T}}\mathbf{x_2} = \mathbf{x_2}^{\mathsf{T}}\mathbf{x_1}$, the equation reduces to

$$2(\mathbf{x}_2 - \mathbf{x}_1)^{\top} \mathbf{P} = \mathbf{x}_2^{\top} \mathbf{x}_2 - \mathbf{x}_1^{\top} \mathbf{x}_1$$
 (2.0.8)

Using (2.0.3), we get

$$2(\mathbf{x}_2 - \mathbf{x}_1)^{\mathsf{T}} \mathbf{P} = -2(g \quad f)(\mathbf{x}_2 - \mathbf{x}_1) \quad (2.0.9)$$

$$\implies (\mathbf{x}_2 - \mathbf{x}_1)^{\top} \mathbf{P} = (\mathbf{x}_2 - \mathbf{x}_1)^{\top} \begin{pmatrix} -g \\ -f \end{pmatrix} \qquad (2.0.10)$$

Since the centre of the circle is $\mathbf{C} = \begin{pmatrix} -g \\ -f \end{pmatrix}$, we can clearly see that $\mathbf{P} = \mathbf{C}$ satisfies the equation of perpendicular bisector.

Hence, the perpendicular bisector of any chord of a circle passes through the centre of the circle.

For example, consider the circle

$$\mathbf{x}^{\mathsf{T}}\mathbf{x} + 2(-4 \quad -5)\mathbf{x} + 5 = 0$$
 (2.0.11)

and let $\mathbf{x_1} = \begin{pmatrix} 0 \\ 5 - 2\sqrt{5} \end{pmatrix}$ and $\mathbf{x_2} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$. We can clearly see in the figure that the perpendicular bisector the chord $\mathbf{x_1x_2}$ passes through the centre of the circle.

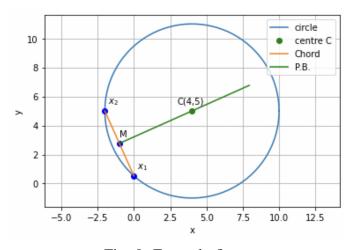


Fig. 0: Example figure