G. Reddy prakash
192365096.

16) Big omega notation prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

$g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

find constant and n_0

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

Divide both Sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

Here $\frac{2}{n}$ and $\frac{4}{n}$ approaches 0

$1 + 2/n + 4/n^2$

Example $c = 1/2$

$1 + 2/n + 4/n^2 \geq 1/2$

$1 + 2/n + 4/n^2 \geq 1$ $\quad (1 \geq 1/2, n \geq 1)$

$1 + 2/n + 4/n^2 \geq 1/2$ $\quad (n \geq 1), n_0 = 1)$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

(7) Big theta notation: Determine whether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not.

$c_1 n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In Lower bound $h(n)$ is $\Omega(n^2)$

$h(n) = 4n^2 + 3n$

$h(n) \leq c_1 n^2$

$4n^2 + 3n \leq c_2 n^2$

$4n^2 + 3n \leq 5n^2$

Lets $c_2 = 5$

Divide both Sides by n^2

$4 + 3/n \leq 5$

$H(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5, n_0 = 1)$

Lower bound :

$h(n) = 4n^2 + 3n$

$h(n) \geq c_1 n^2$

$4n^2 + 3n \geq c_1 n^2$

Let's $c_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

Divide both sides by n^2

$4 + 3/n \geq 4$

$H(n) = 4n^2 + 3n$

$h(n) = 4n^2 + 3n$

$\boxed{c_1 = 4, n_0 = 1 \text{ is } \Theta(n^2)}$

(8) Let's $f(n) = n^3 - 2n^2 + n$ and $g(n) = n$ Show wheather $f(n) = \Omega(g(n))$ is true or false and justify your answer.

$f(n) \geq c \cdot g(n)$

Substituting $f(n)$ and $g(n)$ into this inequality we get

find c and n_0 hols $n \geq n_0$

$n^3 - 2n^2 + n \geq -cn^2$

$n^3 - 2n^2 + n + cn^2 \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$ $(n^3 \geq 0)$

$n^3 + (1-2)n^3 + n \geq 0$

$n^3 + (1-2)n^3 + n = n^3 - n^2 + n \geq 0$ $(c = 2)$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

Therefore, the Statement $f(n) = \Omega(g(n))$ is True.

(9) Determine whether $h(n) = n\log n$ is in $\Theta(n\log n)$ prove a Rigorous proof your conclusion.

$c_1 n\log n \le h(n) \le c_2 n\log n$

upper bound:

$h(n) \le c_2 n\log n$

$h(n) = n\log n + n$

$n\log n + n \le c_2 n\log n$

Divide both sides by $n\log n$

$$1 + \frac{n}{n\log n} \le 2$$

$$1 + \frac{1}{\log n} \le 2$$

$$1 + \frac{1}{\log n} \le 2 \text{ (simplify}$$

then $h(n)$ is $O(n\log n)$ $(c_2 = 2, n_0 = 2)$

lower bound:

$$h(n) \ge c_1 n\log n$$

$$h(n) = n\log n + n$$

$$n\log n + n \ge c_1 n\log n$$

divid both Sides by $n\log n$

$$1 + \frac{n}{n\log n} \ge c_1$$

$$1 + \frac{1}{n\log n} \ge c_1 \text{ (simplify)}$$

$$1 + \frac{1}{\log n} \ge 1 \quad (c_1 = 1)$$

$$\frac{1}{\log n} \ge 0 \text{ for all } n > 1$$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

(10) Solve the following recurrence relations and find the order of growth for Solutions $T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4$, $b = 2$, $f(n) = n^2$

Apply master's theorem

$T(n) = aT(n/b) + f(n)$ $\quad f > 0$

$f(n) = O(n \log_b^{a-1}$ $\quad T(n) = \Theta(n \log_b^a)$

$f(n) = \Theta(n \log_b^a)$ the $T(n) = \Theta(n \log_b^a \log n)$

$f(n) = \Omega(n \log_b^{a+1})$, then $f(n) = f(n)$

calculating $\log_b^a$:

$\log_b^a = \log_2^4 = 2$

$f(n) = n^2 = O(n^2)$

$f(n) = \Theta(n^2) = \Theta(n \log_b^a)$

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n \log_b^a \log n) = \Theta(n^2 \log n)$

order of growth.

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$ is $\Theta(n^2 \log n)$