(1) Solve the following recurrence relations

(a) $x(n) = x(n-1) + 5$ for $x(1) = 0$

$x(n) = x(1) + 5(n-1)$

let $x(1) = 0$

$x(n) = 5(n-1)$

$\boxed{\therefore x(n) = 5n-5}$

(b) $x(n) = 3x(n-1)$ for x $n>1$ $x(1) = 4$

Substituting:

$x(n) = 3^{n-1}(x)(1)$

$\Rightarrow x(1) = 4$:

$x(n) = 4 \cdot 3^{n-1}$

$\boxed{\therefore x(n) = 4 \cdot 3^{n-1}}$

(c) $x(n) = x(n/2) + n$ for $n>1$, $x(1) = 1$ (solve for $n = 2^k$)

$x(n) = n + n/2 + n/4 + \cdots + 1$

here $1 + n/2 + n/4 + \cdots + n/2 \log n$ simplifies to $2n-1$

$\boxed{\therefore x(n) = 2n-1}$

(d) $x(n) = x(n/3) + 1$ for $n>1$, $x(1) = 1$ (solve for $n = 3^k$)

$x(n) = 1+1+1+\cdots$ (for $\log_3 n$ times)

$x(n) = \log_3 n$

$\boxed{\therefore x(n) = \log_3 n}$

(2) Evaluate the following recurrences completely

$T(n) = T(n/2) + 1$, where $n = 2k$ for all $k \geq 0$

here

$T(n) = T(n/2) + 1$

$T(n/2) = T(n/4) + 1$

$T(n/4) = T(n/8) + 1$

$\Rightarrow T(n) = 1 + 1 + 1 + \ldots$ for $\log_2 n$ times

$\therefore T(n) = T(n/2) + 1$ for $n = 2^k$ is

$T(n) = \log_2 n$

$\therefore T(n) = \Theta(\log n)$

(ii) $T(n) = T(n/3) + T(2n/3) + cn$, where c is a constant and 'n' is the input size.

$\Rightarrow T(n) = aT(n/b) + f(n)$

$a = 1,\ b = 3,\ f(n) = cn$

(i) calculate $n^{\log_b a}$

$n^{\log_3 1} = n^0 = 1$

(ii) compare $f(n)$ with $n^{\log_b a}$;

$f(n) = cn$

$f(n) = O(n^{0+1})$

(iii) Apply case 3 for master theorem.

if $f(n) = O(n^{\log_b a} \log^k n)$ for some $k \geq 0$, then

$T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$

Since $f(n) = O(n)$

$T(n) = \Theta(n \log n)$

$T(n) = T(n/3) + T(2n/3) + cn$ is: $T(n) = \Theta(n \log n)$

(3) Consider the following recursion Algorithm

min1 (A[0 ... n-1])

if n=1 return A[0]

else temp = min [[A[0 .. n-2]]

if temp <= A[n-1] return temp

Else

return A[n-1]

(a) what does this algorithm compute.

this algorithm is designed to find the minimum element in an array 'A' of size(n)

(b) Set up a recurrence relation for the algorithm basic operation count and solve it

$T(n) = T(n-1) + 2$

* $T(1) = 1$

* $T(n) = T(n-1) + 2$

$T(n) = T(n-2) + 2 + 2$

$T(n) = T(n-3) + 2 + 2 + 2$

continue the pattern

$T(n) = 1 + 2(n-1)$

$\boxed{T(n) = (2n-1)}$ => Best case.

(5) Analyze the order of growth

$f(n): 2n^2+5$ and $g(n) = 7$

use the Ω $g(n)$ notation

compute the limit:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2n^2+5}{7n}$$

Simplify the fraction.

$$\lim_{n\to\infty} \frac{2n^2+5}{7n} = \lim_{n\to\infty} \left(\frac{2n^2}{7n} + \frac{5}{7n}\right) = \lim_{n\to\infty} \left(\frac{2}{7}n + \frac{5}{7n}\right)$$

Evaluate the limit

$$\lim_{n\to\infty} \left(\frac{2}{7}n + \frac{5}{7n}\right) = \infty$$

Conclusion:

$$f(n) = \Omega\, g(n)$$

$f(n)$ is asymtotically bounded below by $g(n)$, meaning $f(n)$ grows at least as fast as $g(n)$. In simpler terms $f(n)$ is a asymptotically quadratic.