Lunes 24/10/22

En la clase de Hoy se consideró el sistema No Homogéneo:

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} e^{t} \\ e^{-t} \end{pmatrix} \quad (S_{NH})$$

Se dijo que el vector

$$\bar{Y}(t) = \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \frac{1}{4} \begin{pmatrix} (2t+1)(e^{t} - e^{-t}) \\ (2t-1)(e^{t} + e^{-t}) \end{pmatrix}$$

es solución de (S_{NH}).

Para comprobar lo anterior, debemos verificar que: $\bar{Y}(t) = \begin{pmatrix} u(t) \\ v(t) \end{pmatrix}$ es tal que:

$$\bar{Y}'(t) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \bar{Y}(t) + \begin{pmatrix} e^{t} \\ e^{-t} \end{pmatrix}$$

(Recordemos que si $\bar{g}(t) = (g_1(t), g_2(t))$

$\bar{g} : [a,b] \subseteq \mathbb{R} \longrightarrow \mathbb{R}^2$, derivable

entonces $\frac{d}{dt}\bar{g}(t) = (g_1'(t), g_2'(t))$

si $\bar{g}(t) = (e^{t^2}, t^3 - 2t)$,

entonces $\bar{g}'(t) = (2t\,e^{t^2}, 3t^2 - 2)$

Así,

$$\bar{\gamma}'(t) = \frac{1}{4}\frac{d}{dt}\left[\begin{pmatrix}(2t+1)(e^t - e^{-t})\\(2t-1)(e^t + e^{-t})\end{pmatrix}\right]$$

$$= \frac{1}{4}\begin{pmatrix}2(e^t - e^{-t}) + (2t+1)(e^t + e^{-t})\\2(e^t + e^{-t}) - (2t-1)(e^t - e^{-t})\end{pmatrix}$$

$$= \left(\frac{1}{4}\right)\begin{pmatrix}(2t+3)e^t + (2t-1)e^{-t}\\(3-2t)e^t + (2t+1)e^{-t}\end{pmatrix} \quad (*)$$

Por otra parte, veamos el valor de

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \bar{Y}(t) + \begin{pmatrix} e^{t} \\ e^{-t} \end{pmatrix} = \begin{pmatrix} v(t) + e^{t} \\ u(t) + e^{-t} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{4}(2t-1)(e^{t}+e^{-t}) + e^{t} \\ \frac{1}{4}(2t+1)(e^{t}-e^{-t}) + e^{-t} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{4}(2t-1)e^{-t} + \left(\frac{2t}{4}+\frac{3}{4}\right)e^{t} \\ \frac{1}{4}(2t+1)e^{t} - \left(\frac{2t}{4}-\frac{5}{4}\right)e^{-t} \end{pmatrix}$$

Este último término es igual a $\bar{Y}'(t)$ (como se ve de $(*)$.)