II. LIKELIHOOD

The quantity that is maximized is the likelihood L, which is equivalent to minimizing the χ^2 function: $\chi^2 = -2\ln L$. We assume Gaussian likelihoods, and thus, the contribution to the χ^2 function from the N signals is given by

$$\chi^2_{\text{signals}} = \sum_{i=1}^{N} \chi^2_i(\mathcal{O}^i_{\text{th}}) = \sum_{i=1}^{N} \left(\frac{\mu^i_{\text{th}} - \mu^i_{\text{obs}}}{\Sigma^i} \right)^2 , \qquad \Sigma_i = \sqrt{(\Sigma^i_{\text{th}})^2 + (\Sigma^i_{\text{obs}})^2}. \tag{1}$$

1. Relic density.

   The measurement of the relic density reported by Planck reads

   $$\Omega h^2 = 0.120 \pm 0.001. \Longrightarrow \Omega_{\text{obs}} = 0.120. \tag{2}$$

   The relic density Ω_{th} is calculated via `MicrOmegas`. We assume a theoretical uncertanty on the calculation with the aim of taking care of quantum corrections, variation of the renormalization scheme and scale, and modifications to the QCD equations of state. Thus

   $$\Sigma_\Omega = \sqrt{(0.1\Omega_{\text{th}})^2 + 0.001^2} \approx 0.012 \tag{3}$$

2. Direct detection.

   The DD spin independent cross section σ^{SI}_{th} is calculated via `MicrOmegas`. Here we use a half-Gaussian functions with $\sigma^{SI}_{\text{obs}} = 0$ based on the null signal.

   Typically an upper bound at some confidence level on the elastic cross section is reported. For LZ experiment [2] (see Fig 5) https://arxiv.org/pdf/2207.03764.pdf, the solid black line corresponds to the 90% confidence limit for the spin-independent WIMP cross section vs. WIMP mass. The green and yellow bands are the 1σ and 2σ sensitivity bands.

   The upper bound of the 2σ band can be cast as a function of the DM mass normalized to 1.64:

   $$\Sigma^{DD}_{obs}(M) = f_{DD}(M)/1.64. \tag{4}$$

   We assume a theoretical uncertanty on the calculation σ^{SI}_{DD} with the aim of taking care of quantum corrections, variation of the renormalization scheme and scale, and modifications to the QCD equations of state. Hence

   $$\Sigma^{DD}_{th} = 0.2\sigma^{SI}_{\text{th}}. \tag{5}$$

   Thus

   $$\Sigma_{DD} = \sqrt{(\Sigma^{DD}_{\text{th}})^2 + (\Sigma^{DD}_{\text{obs}})^2}. \tag{6}$$