CSYE 7245 – Big Data Systems & Intelligence Analytics

Assignment 02

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Q1 (5 Points) Give a brief definition for the following:

i. Tree graph

"A tree is an undirected graph G that satisfies any of the following equivalent conditions:

- G is connected and has no cycles.
- G is acyclic, and a simple cycle is formed if any edge is added to G.
- G is connected, but is not connected if any single edge is removed from G.
- G is connected and the 3-vertex complete graph K3 is not a minor of G.
- Any two vertices in G can be connected by a unique simple path." - Wikipedia

ii. Adjacency List

"An adjacency list representation for a graph associates each vertex in the graph with the collection of its neighboring vertices or edges" - Wikipedia

iii. Spanning Tree

"In the mathematical field of graph theory, a spanning tree T of a connected, undirected graph G is a tree that includes all of the vertices and some or all of the edges of G." - Wikipedia

iv. Breadth-first search (BFS)

"In graph theory, breadth-first search (BFS) is a strategy for searching in a graph when search is limited to essentially two operations: (a) visit and inspect a node of a graph; (b) gain access to visit the nodes that neighbor the currently visited node. The BFS begins at a root node and inspects all the neighboring nodes. Then for each of those neighbor nodes in turn, it inspects their neighbor nodes which were unvisited, and so on. The time complexity can be expressed as $O(|V|+|E|)$ since every vertex and every edge will be explored in the worst case.:" - Wikipedia

v. Admissible heuristic

An admissible heuristic is used to estimate the cost of reaching the goal state in an informed search algorithm. In order for a heuristic to be admissible to the search problem, the estimated cost must always be lower than or equal to the actual cost of reaching the goal state. The search algorithm uses the admissible heuristic to find an estimated optimal path to the goal state from the current node. For example, in A* search the evaluation function

Q2 (5 Points)

Arrange the following functions in increasing order of asymptotic growth: •

5n^5

0.33^n

5n^3

n^2 √n

5n

log n

√n

Ans:

1. log n
2. √n = $n^{0.5}$
3. n^2 √n = $n^{2.5}$
4. $5n^3$
5. $5n^5$
6. 0.33^n (Exponentially increasing)
7. 5^n (Exponentially increasing)

Q3 (5 Points) Master Theorem: For the following recurrence, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply.

T(n) = 8T (n/2)+ n

Ans.

a=8 b=2 f(n) = n c=1

log of 8 to the base 2 = 3 > c

Case 1 is applicable

T(n) = $\theta(n^3)$

Q4 (5 Points) Master Theorem: For the following recurrence, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply.

T(n) = n2 T (n/2) + log n

Ans.

It does not apply as a is not constant

Q5 (5 Points) Master Theorem: For the following recurrence, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply.

T(n) = 4T (n/2)+ n2

Ans.

a=4 b=2 $f(n)=n^2$ c=2

$\log_b a = \log_2 4 = 2 = c$

k=0

Case 2 is applicable

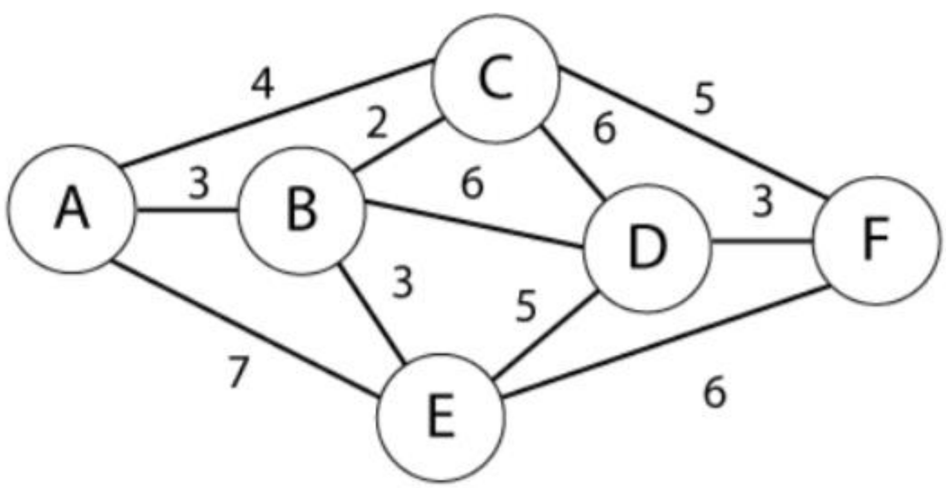
$T(n) = \theta(n^1 \log^1 n) = \theta(n\log n)$

Q6 (5 Points) Sort the list of integers below using Merge sort. Show your work. Write a recurrence relation for Merge sort. (22, 13, 26, 1, 12, 27, 33, 15)

Original	22	13	26	1	12	27	33	15							
Divide2	22	13	26	1		12	27	33	15						
Divide4	22	13		26	1		12	27		33	15				
Divide8	22		13		26		1		12		27		33		15
Merge1	13	22		1	26		12	27		15	33				
Merge2	1	13	22	26		12	15	27	33						
Merge3	1	12	13	15	22	26	27	33							

Recurrence relation: T(n) = 2 T(n/2) + n

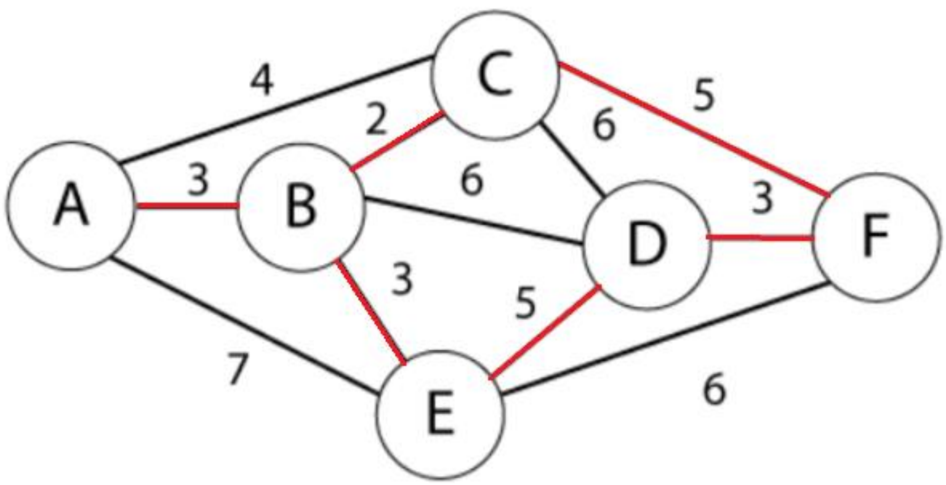
Q7 (5 Points) Use Kruskal's algorithm to find a minimum spanning tree for the connected weighted graph below



Steps:

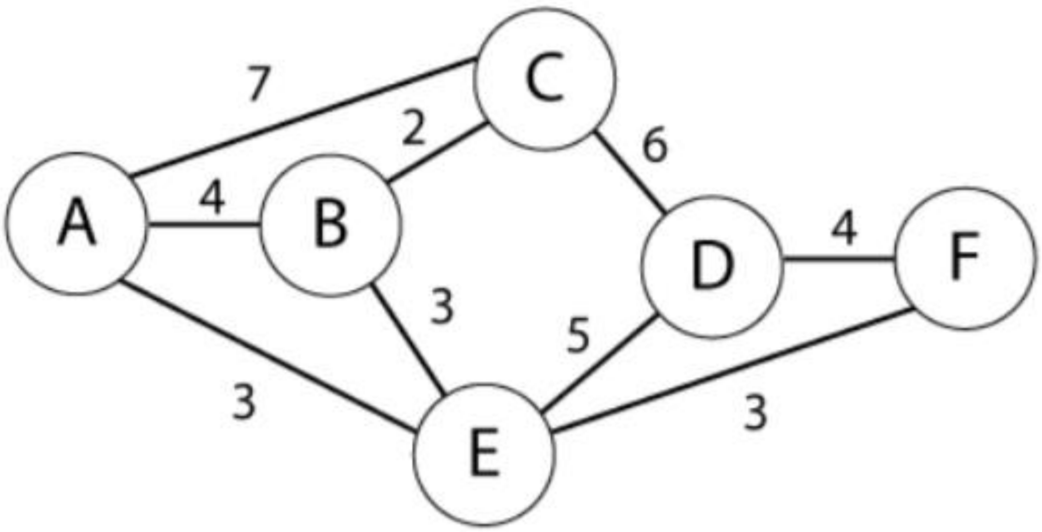
- Connect B-C (2)
- Connect A-B (3)
- Connect B-E (3)
- Connect D-F (3)
- Skip A-C (4) forms cycles
- Connect C-F (5)
- Connect E-D (5)
- Skip B-D (6) forms cycles
- Skip C-D (6) forms cycles
- Skip E-F (6) forms cycles
- Skip A-E (7) forms cycles

Stop. MST formed (6 Edges, 6 Vertices)



MST = {A-B , B-C , B-E, E-D, D-F, C-F}

Q8 (5 Points) Use Prim's algorithm to find a minimum spanning tree for the connected weighted graph below. Show your work



Ans.

Take the minimum edge of the cut-set each time.

0: A S={A}

1: A-E(3) is min-cut take A-E S={A,E}

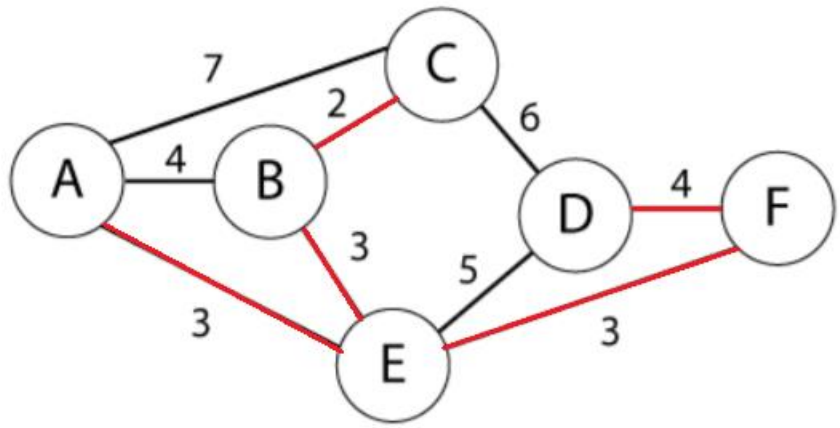
2: A-B(3) or E-F(3) is min-cut take A-B S={A,E,B}

3: B-C(2) is min-cut take B-C S={A,E,B,C}

4: E-F(3) is min-cut take E-F S={A,E,B,C,F}

5: F-D(4) is min-cut take F-D S={A,E,B,C,F,D}

Done n-1 edges. (5)

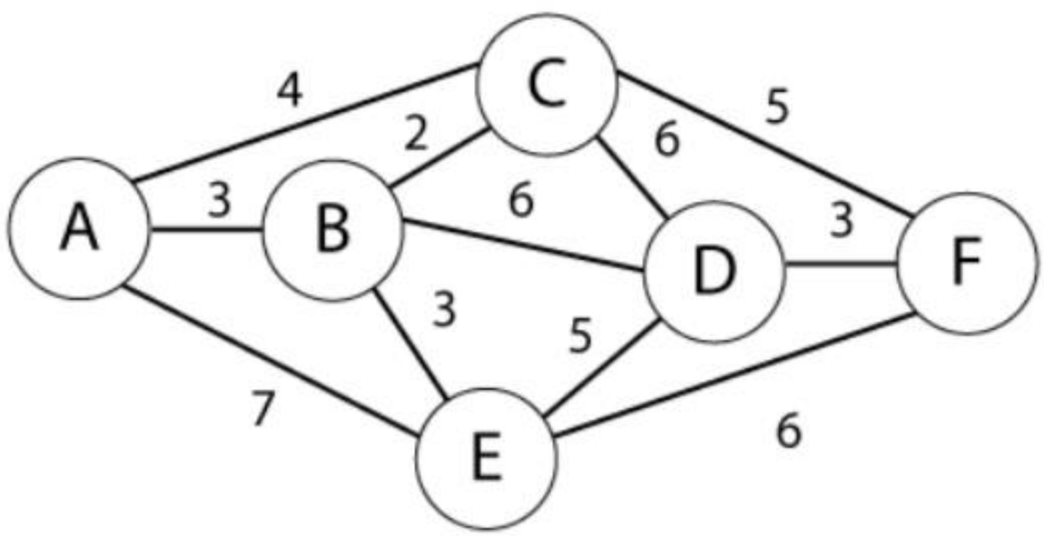


MST = {A-E, E-B, B-C, E-F, F-D}

MST weight = 3+3+2+3+4 = 15

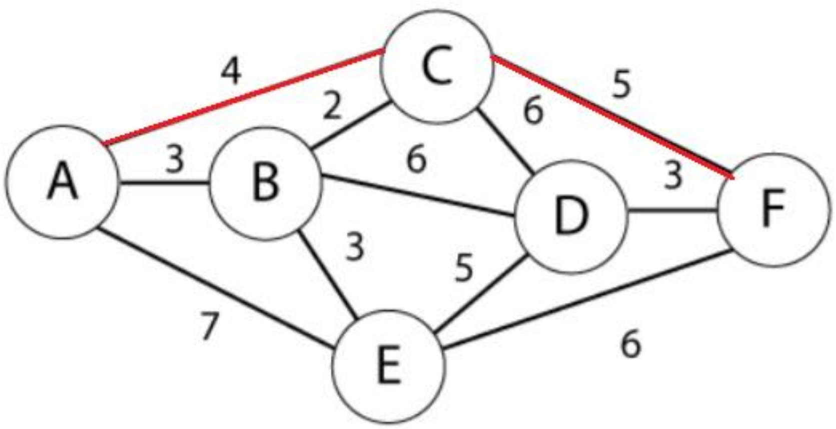
Q9 (5 Points)

Find shortest path from A to F in the graph below using Dijkstra's algorithm. Show your steps.



Source A

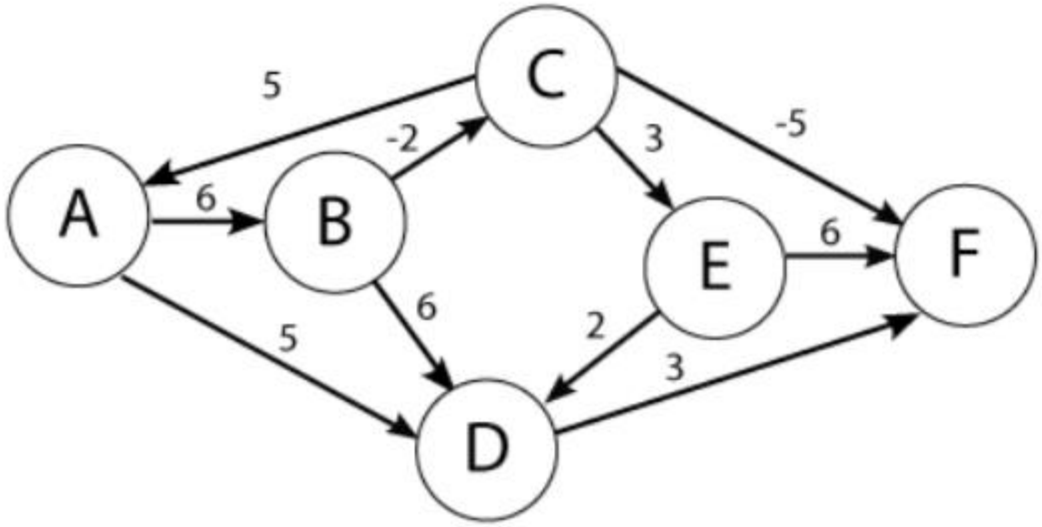
		A	B	C	D	E	F
1: A {A}	A	0	(3,A)**	(4,A)	INF	(7, A)	INF
2: B {A,B}	B	0	(3,A)	(4,A)**	(9,B)	(6,B)	INF
3: C {A,B,C}	C	0	(3,A)	(4,A)	(9,B)	(6,B)**	(9,C)
4: E {A,B,C,E}	E	0	(3,A)	(4,A)	(9,B)**	(6,B)	(9,C)
5: D {A,B,C,E,D}	D	0	(3,A)	(4,A)	(9,B)	(6,B)	(9,C) **
4: F {A,B,C,E,D,F}	F	0	(3,A)	(4,A)	(9,B)	(6,B)	(9,C)



Shortest Path: A -> C -> F Cost(4+5 = 9)

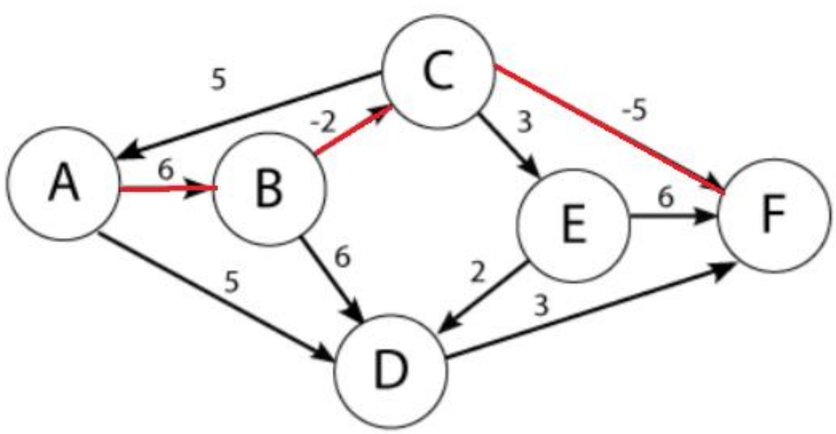
Q10 (5 Points)

Use the Bellman-Ford algorithm to find the shortest path from node A to F in the weighted directed graph above. Show your work.

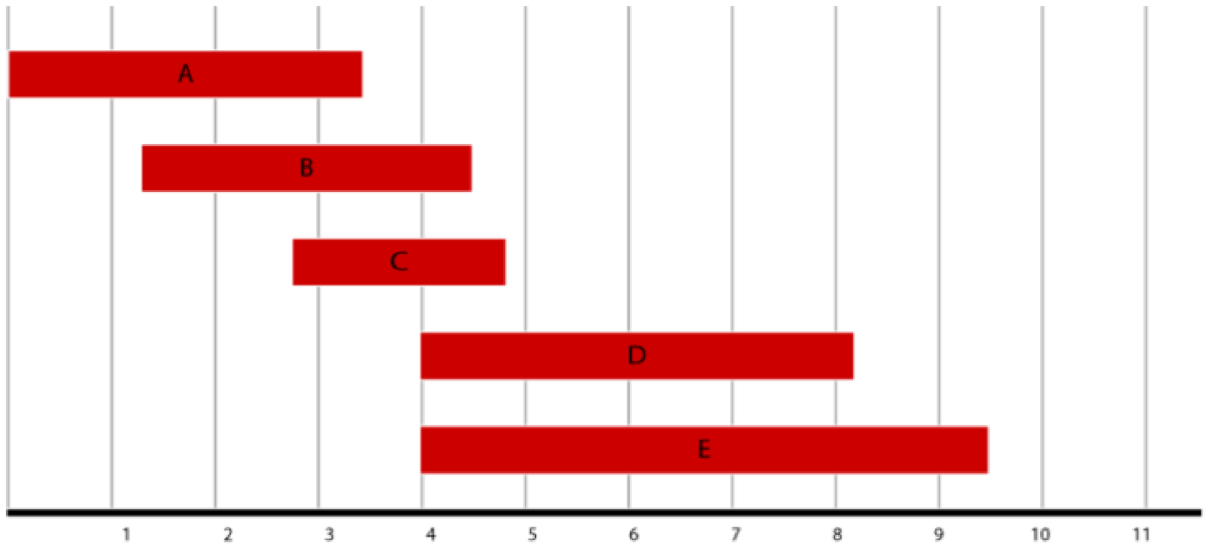


Shortest Path: A->B->C->F at the cost -1

	A	B	C	D	E	F
0	0	INF	INF	INF	INF	INF
1	0	6	INF	5	INF	INF
2	0	6	4	5	INF	8
3	0	6	4	5	7	-1



Q11 (5 Points) Given the five intervals below, and their associated values; select a subset of nonoverlapping intervals with the maximum combined value. Use dynamic programming. Show your work



Interval	Value
A	2
B	3
C	2
D	3
E	2

Interval	Value	Previous	Max
A	2	n/a	Max(2, 0) = 2
B	3	n/a	Max(3, 2) = 3
C	2	n/a	Max(2, 3) = 3
D	3	A	Max(5, 3) = 5
E	2	A	Max(4, 5) = 5

Interval	Trace(i)	S
E	5 > 2+1, jump to A	{E}
D	3+2 > 3	{E}

C	3 > 2+0	{E}
B	3+0 > 2	{E}
A	2=2	{E,A}

S= {A,E}

Q12 (5 Points) Given the weights and values of the four items in the table below, select a subset of items with the maximum combined value that will fit in a knapsack with a weight limit, W, of 6. Use dynamic programming. Show your work.

Capacity of knapsack W=6

Item $_i$	Value v_i	Weight w_i
1	3	4
2	2	3
3	4	2
4	4	3

Ans

			0	1	2	3	4	5	6
Item	Value	Weight							
1	3	4	0	0←	0	0	3	3	3
2	2	3	0	0←	0	2	3	3	3
3	4	2	0	0	4	4←	4	6	7
4	4	3	0	0	4	4	4	8	8←

We used items 3 and 4 for a combined value of 8 in the knapsack

S= {3,4}