About change of units

Question 8 from HW1

Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. If we measure temperature in degree Celsius and time in minute, the constant of proportionality k equals 0.3. Suppose the ambient temperature $T_A(t)$ is equal to a constant 22 degree Celsius. Write the differential equation that describes the time evolution of the temperature T of the object.

Approach 1: T, t are with units

$$k = 0.3 \ \text{min}^{-1}$$

$$T_A(t) = 22 \cos\left(\frac{\pi t}{30\,\text{min}}\right) \ \text{celsius}$$

$$\frac{dT}{dt} = \frac{0.3}{\text{min}} \times \left(22 \cos\frac{\pi t}{30\,\text{min}} \ \text{celsius} - T\right)$$

$$\frac{dT}{dt} = \frac{0.3}{\text{hour}} \times \left(22 \cos\left(\frac{\pi t}{30\,\text{hour}} \cdot \frac{\text{hour}}{\text{min}}\right) \text{celsius} - T\right) \times \frac{\text{hour}}{\text{min}}$$

$$= \frac{0.3}{\text{hour}} \times 60 \left(22 \cos\left(\frac{\pi t}{30\,\text{hour}} \cdot 60\right) \text{celsius} - T\right)$$

$$\frac{dT}{dt} = \frac{18}{\text{hour}}\left(22\cos\frac{2\pi t}{\text{hour}}\ \text{celsius} - T\right)$$

(use A celsius $= \left(\frac{9}{5}A + 32\right)$ Fahrenheit)

$$\frac{dT}{dt} = \frac{18}{\text{hour}}\left(\frac{9}{5}\times 22\cos\frac{2\pi t}{\text{hour}}\ \text{Fahrenheit} + 32\ \text{Fahrenheit} - T\right)$$

$$= \frac{18}{\text{hour}}\left(\frac{198}{5}\cos\frac{2\pi t}{\text{hour}}\ \text{Fahrenheit} + 32\ \text{Fahrenheit} - T\right)$$

Can just ignore all the units when writing down DE

Approach 2. T, t are without units

The temperature of the object is T celsius
$\tilde{T}$ Fahrenheit

$$\tilde{T} = \frac{9}{5}T + 32$$

$$\frac{dT}{dt} = 0.3\left(22\cos\left(\frac{\pi}{30}t\right) - T\right)$$

t min $= \tilde{t}$ hour

$$t = 60\tilde{t}$$

$$\frac{dT}{d\tilde{t}} = \frac{dT}{dt}\cdot\frac{dt}{d\tilde{t}} = 0.3\left(22\cos\frac{\pi}{30}\tilde{t}\,\frac{t}{\tilde{t}} - T\right)\frac{dt}{d\tilde{t}}$$
$$= 18\left(22\cos 2\pi\tilde{t} - T\right)$$

$$\frac{d\tilde{T}}{d\tilde{t}} = \frac{9}{5}\frac{dT}{d\tilde{t}} = \frac{9}{5}\times 18\left(22\cos 2\pi\tilde{t} - \frac{5}{9}(\tilde{T}-32)\right)$$
$$= 18\left(\frac{198}{5}\cos 2\pi\tilde{t} + 32 - \tilde{T}\right)$$