Ficha de trabalho nº 2

1.

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 1 \\ 1 & 1 & 1 & -1 & 0 \\ 2 & 2 & 2 & -1 & 0 \\ 1 & 0 & 2 & -1 & -1 \end{array}\right]$$

a)

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & 2 & -1 \\ 1 & 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

Logo, $(-1,1,0,0)$ é solução do sistema

b)

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & 2 & -1 \\ 1 & 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 4 \end{bmatrix}$$

Logo, $(0,2,4,4)$ não é solução do sistema.

c)

$$A \cdot \left(\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right) = A \cdot \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} - A \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & -1 \\ 1 & 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Logo, $(-3,2,1,0)-(-1,1,0,0)$ é solução do sistema homogéneo associado

d)

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & -1 \\ 1 & 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

Logo, $(-3,2,1,0)$ é solução do sistema.

e)

$$A \cdot \left(a \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right) = a \times A \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + b \times A \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} =$$

$$= a \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} + b \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$a(-1,1,0,0)+b(-3,2,1,0)$ seria solução do sistema ssc $a=1$, $b=0$ ou $a=0$, $b=1$, logo esta afirmação é falsa.

f)

$$A \cdot \left(\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \alpha \left(\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right) \right) =$$

$$= A \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \alpha \times \left(A \cdot \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} - A \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} + \alpha \times \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} + \alpha \times 0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

Logo, a afirmação é verdadeira.

g)

O conjunto de soluções é um conjunto infinito, uma vez que para qualquer que seja o valor de α em $(-1,1,0,0)+\alpha((-3,2,1,0)-(-1,1,0,0))$ esta será sempre solução do sistema.

Logo, a afirmação é falsa.

2.

a)

$$\left[\begin{array}{cccc|c} -1 & 0 & -1 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 3 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 \end{array}\right] \xrightarrow[L_2 \leftarrow L_1 + L_2]{} \left[\begin{array}{cccc|c} -1 & 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 1 \\ 3 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 \end{array}\right]$$

$$\xrightarrow[L_3 \leftarrow 3L_1 + L_3]{} \left[\begin{array}{cccc|c} -1 & 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 1 & -3 & -2 & 3 \\ 0 & 0 & -2 & -1 & -1 \end{array}\right] \xrightarrow[L_2 \leftrightarrow L_3]{} \left[\begin{array}{cccc|c} -1 & 0 & -1 & -1 & 1 \\ 0 & 1 & -3 & -2 & 3 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & -2 & -1 & -1 \end{array}\right]$$

$$\xrightarrow[L_4 \to -2L_3 + L_4]{} \left[\begin{array}{cccc|c} -1 & 0 & -1 & -1 & 1 \\ 0 & 1 & -3 & -2 & 3 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 3 & -3 \end{array}\right]$$

- $3x_4 = -3 \Rightarrow x_4 = -1$
- $-x_3 - 2x_4 = 1 \Leftrightarrow -x_3 - 2\times(-1) = 1$
  $\Leftrightarrow -x_3 = 1 - 2 \Leftrightarrow x_3 = 1$
- $x_2 - 3x_3 - 2x_4 = 3 \Leftrightarrow$
  $\Leftrightarrow x_2 - 3\times 1 - 2\times(-1) = 3$
  $\Leftrightarrow x_2 = 4$
- $-x_1 - x_3 - x_4 = 1 \Leftrightarrow$
  $\Leftrightarrow -x_1 - 1 + 1 = 1$
  $\Leftrightarrow x_1 = -1$

$$\begin{cases} x_1 = -1 \\ x_2 = 4 \\ x_3 = 1 \\ x_4 = -1 \end{cases} \qquad (x_1, x_2, x_3, x_4) = (-1, 4, 1, -1)$$

Sistema possível determinado

b)

$$\left[\begin{array}{cccc|c} -1 & 0 & -3 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 3 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right] \xrightarrow[\substack{L_2 \leftarrow L_1 + L_2 \\ L_3 \leftarrow 3L_1 + L_3}]{} \left[\begin{array}{cccc|c} -1 & 0 & -3 & -1 & 0 \\ 0 & 0 & -3 & -2 & 0 \\ 0 & 1 & -9 & -2 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[L_2 \leftrightarrow L_3]{} \left[\begin{array}{cccc|c} -1 & 0 & -3 & -1 & 0 \\ 0 & 1 & -9 & -2 & 0 \\ 0 & 0 & -3 & -2 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right] \xrightarrow[\substack{L_3 \to -2L_3 \\ L_4 \to 3L_4}]{} \left[\begin{array}{cccc|c} -1 & 0 & -3 & -1 & 0 \\ 0 & 1 & -9 & -2 & 0 \\ 0 & 0 & 6 & 4 & 0 \\ 0 & 0 & -6 & -3 & 0 \end{array}\right]$$

$$\xrightarrow[L_4 \leftarrow L_3 + L_4]{} \left[\begin{array}{cccc|c} -1 & 0 & -3 & -1 & 0 \\ 0 & 1 & -9 & -2 & 0 \\ 0 & 0 & 6 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \qquad \begin{cases} x_4 = 0 \\ x_3 = 0 \\ x_2 = 0 \\ x_1 = 0 \end{cases}$$

sistema possível e determinado

c)

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 2 \\ -2 & 0 & 0 & -1 & 0 \\ 1 & 0 & 3 & -1 & -1 \end{array}\right] \xrightarrow[L_3 \leftarrow -L_1 + L_3]{L_2 \leftarrow 2L_1 + L_2} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 2 \\ 0 & 4 & 2 & 1 & 4 \\ 0 & -2 & 2 & -2 & -3 \end{array}\right]$$

$$\xrightarrow[L_3 \leftarrow 2L_3 + L_2]{} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 2 \\ 0 & 4 & 2 & 1 & 4 \\ 0 & 0 & 6 & -3 & -2 \end{array}\right]$$

$6x_3 - 3x_4 = -2 \Leftrightarrow$
$\Leftrightarrow 6x_3 = -2 + 3x_4 \Leftrightarrow$
$\Leftrightarrow x_3 = \frac{-2 + 3x_4}{6} \Leftrightarrow x_3 = -\frac{1}{3} + \frac{1}{2}x_4$

$4x_2 + 2x_3 + x_4 = 4 \Leftrightarrow$
$\Leftrightarrow 4x_2 - \frac{2}{3} + x_4 + x_4 = 4$
$\Leftrightarrow 4x_2 = 4 + \frac{2}{3} - 2x_4$
$\Leftrightarrow x_2 = \frac{12 + 2}{12} - \frac{2x_4}{4} \Leftrightarrow$
$\Leftrightarrow x_2 = \frac{7}{6} - \frac{1}{2}x_4$

$x_1 + 2x_2 + x_3 + x_4 = 2 \Leftrightarrow$
$\Leftrightarrow x_1 + \frac{7}{3} - x_4 - \frac{1}{3} + \frac{1}{2}x_4 + x_4 = 2$
$\Leftrightarrow x_1 + \frac{6}{3} + \frac{1}{2}x_4 = 2$
$\Leftrightarrow x_1 = -\frac{1}{2}x_4$

Sistema possível indeterminado
$(x_1, x_2, x_3, x_4) = \left(-\frac{1}{2}x_4, \frac{7}{6} - \frac{1}{2}x_4, -\frac{1}{3} + \frac{1}{2}x_4, x_4\right)$

d)

$$\left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 & 3 \\ -2 & 1 & 1 & 0 & -1 \\ -1 & 0 & -3 & -2 & -2 \end{array}\right] \xrightarrow[\substack{L_2 \leftarrow -2L_2 + L_1 \\ L_3 \leftarrow L_3 + L_1 \\ L_4 \leftarrow 2L_4 + L_1}]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & -6 \\ 0 & 1 & 3 & 2 & -1 \\ 0 & 0 & -4 & -2 & -4 \end{array}\right]$$

$$\xrightarrow[L_3 \leftarrow -2L_3 + L_2]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & -6 \\ 0 & 0 & -4 & -2 & -4 \\ 0 & 0 & -4 & -2 & -4 \end{array}\right] \xrightarrow[L_4 \leftarrow -L_3 + L_4]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & -6 \\ 0 & 0 & -4 & -2 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

- $-4x_3 - 2x_4 = -4 \Leftrightarrow -4x_3 = -4 + 2x_4 \Leftrightarrow x_3 = 1 - \frac{1}{2}x_4$
- $2x_2 + 2x_3 + 2x_4 = -6 \Leftrightarrow 2x_2 + 2 - x_4 + 2x_4 = -6 \Leftrightarrow 2x_2 = -6 - 2 - x_4$
$\Leftrightarrow x_2 = -4 - \frac{x_4}{2}$
- $2x_1 + 2x_2 + 2x_4 = 0 \Leftrightarrow 2x_1 + 2 - x_4 + 2x_4 = 0 \Leftrightarrow 2x_1 = -2 - x_4$
$\Leftrightarrow x_1 = -1 - \frac{1}{2}x_4$

Sistema possível indeterminado
$(x_1, x_2, x_3, x_4) = \left(-1 - \frac{1}{2}x_4, -4 - \frac{x_4}{2}, 1 - \frac{1}{2}x_4, x_4\right)$

e)

$$\left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 1 \\ 1 & -1 & 0 & 0 & 3 \\ -2 & 1 & 1 & 0 & -1 \\ -1 & 0 & -3 & -2 & -2 \end{array}\right] \xrightarrow[\substack{L_3 \leftarrow L_3+L_1 \\ L_2 \leftarrow L_2+L_4 \\ L_4 \leftarrow 2L_4+L_1}]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 1 \\ 0 & -1 & -3 & -2 & 1 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & -4 & -2 & -3 \end{array}\right]$$

$$\xrightarrow[L_3 \leftarrow L_2+L_3]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 1 \\ 0 & -1 & -3 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -4 & -2 & -3 \end{array}\right] \xrightarrow[L_3 \leftrightarrow L_4]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 1 \\ 0 & -1 & -3 & -2 & 1 \\ 0 & 0 & -4 & -2 & -3 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Sistema impossível

f)

$$\left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ -2 & 1 & 1 & 0 & 0 \\ -1 & 0 & -3 & -2 & 0 \end{array}\right] \xrightarrow[\substack{L_3 \leftarrow L_3+L_1 \\ L_2 \leftarrow L_2+L_4 \\ L_4 \leftarrow 2L_4+L_1}]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 0 \\ 0 & -1 & -3 & -2 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & -4 & -2 & 0 \end{array}\right]$$

$$\xrightarrow[L_3 \leftarrow L_3+L_2]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 0 \\ 0 & -1 & -3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & -2 & 0 \end{array}\right] \xrightarrow[L_3 \leftrightarrow L_4]{} \left[\begin{array}{cccc|c} 2 & 0 & 2 & 2 & 0 \\ 0 & -1 & -3 & -2 & 0 \\ 0 & 0 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

- $-4x_3 - 2x_4 = 0 \Leftrightarrow x_3 = -\frac{1}{2}x_4$
- $-x_2 - 3x_3 - 2x_4 = 0 \Leftrightarrow -x_2 + \frac{3}{2}x_4 - 2x_4 = 0 \Leftrightarrow$
  $\Leftrightarrow -x_2 - \frac{1}{2}x_4 = 0 \Leftrightarrow x_2 = -\frac{1}{2}x_4$
- $2x_1 + 2x_3 + 2x_4 = 0 \Leftrightarrow 2x_1 - x_4 + 2x_4 = 0$
  $\Leftrightarrow 2x_1 = -x_4 \Leftrightarrow x_1 = -\frac{1}{2}x_4$

$$(x_1, x_2, x_3, x_4) = \left(-\frac{x_4}{2}, -\frac{x_4}{2}, -\frac{x_4}{2}, x_4\right)$$

3.

a)

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & 1 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ -1 & 2 & -1 & -1 & -1 \\ 1 & 1 & 0 & 2 & 0 \\ 1 & -1 & -1 & 2 & 0 \end{array}\right] \xrightarrow[\substack{L_2 \leftarrow -L_1+L_2 \\ L_3 \leftarrow L_1+L_3 \\ L_4 \leftarrow L_1+L_4 \\ L_5 \leftarrow -L_1+L_5 \\ L_6 \leftarrow -L_1+L_6}]{} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 2 & -2 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[\substack{L_3 \leftarrow -L_2+L_3 \\ L_4 \leftarrow -2L_2+L_4 \\ L_5 \leftarrow -L_2+L_5 \\ L_6 \leftarrow L_2+L_6}]{} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -2 & 0 & -3 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[\substack{L_4 \leftarrow -2L_3+L_4 \\ L_5 \leftarrow L_3+L_5}]{} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -4 & -1 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[L_5 \leftarrow 3L_4+4L_5]{} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -4 & -1 \\ 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[L_6 \leftarrow 4L_6+L_4]{} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -4 & -1 \\ 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Sistema Impossível

b)

$$\left[\begin{array}{cccc|c} 2 & -2 & 3 & 0 & 0 \\ 6 & 5 & -3 & 8 & 0 \\ 0 & 2 & 2 & 6 & 0 \end{array}\right] \xrightarrow[L_2 \leftarrow -3L_1+L_2]{} \left[\begin{array}{cccc|c} 2 & -2 & 3 & 0 & 0 \\ 0 & 11 & -12 & 8 & 0 \\ 0 & 2 & 2 & 6 & 0 \end{array}\right]$$

$$\xrightarrow[L_3 \leftarrow 2L_2-11L_3]{} \left[\begin{array}{cccc|c} 2 & -2 & 3 & 0 & 0 \\ 0 & 11 & -12 & 8 & 0 \\ 0 & 0 & -46 & 50 & 0 \end{array}\right]$$

Sistema possível e indeterminado

4.

Se $Ax_0 = 0$ é um sistema determinado, então, $r(A) = n$.
Se $r(A) = r(A|B) = n$ então este é um sistema possível e determinado.
Se $r(A) < r(A|B)$ então este é um sistema impossível