1D Rocket Wall Thermal Response Using FTCS and Crank-Nicolson

Objective

The goal of this project was to extend a homework FTCS heat-conduction solver into a more structured, testable, and verifiable project. The system modeled is a 1 cm thick rocket combustion chamber wall exposed to a sequence of transient heat flux pulses. The project required:

1. Refactoring the code into a modular package
2. Implementing a more realistic physical boundary condition
3. Running a full multi-second simulation and visualizing a temperature field
4. Adding a second numerical method for verification (Crank-Nicolson)
5. Creating automated tests using pytest
6. Documenting the reasoning, verification results, and development choices

Problem

A rocket combustion chamber wall experiences rapid and repeated heat flux spikes caused by combustion instabilities. The objective is to estimate how the temperature of the wall evolves and how deeply each pulse penetrates into the material. This was done by solving the 1D transient heat equation. The boundary conditions were as follows:

- Hot side (x = 0): Gaussian heat flux pulses applied as Neumann BC
- Cold side (x = L): Convective boundary with gas temperature T_g and heat transfer coefficient h_g.

Software Structure

rocket-wall-heat-mini-project/
|
| – src/
| | – main.py
| | – schemes.py
| | – grid.py
| | – config.py
|
| – tests/
| | – test_schemes.py
|
| – figures/
| – README.md
| – report/

Main: runs the full simulation, plotting, and verification

Schemes: FTCS and CN solvers and boundary conditions
Grid: Generates space time grids
Config: Contains all physical constants and simulation settings
Test_schemes: pytest verification tests
Figures: saved plots
README: written instructions, structure, and dependencies

Key Inputs –

- Material properties: alpha, kappa
- Boundary condition parameters: hg, Tg
- Pulse function: pulse_is_on(t)

Key Outputs –

- Temperature field array T(t,x)
- Line plots at selected times
- Contour (time v space)
- FTCS and CN verification metrics
- Pytest results

Numerical Methods

The primary solver is FTCS or Forward Time Centered Space where $Fo = \alpha (dt/dx^2)$ with the selected grid $dx = 1.01 \times 10^{-4}$ and $Fo \approx 0.0166$ which is well below the stability limit of $Fo < 0.5$ meaning FTCS is stable and accurate.

Crank-Nicolson or CN was implemented to verify the correctness of FTCS because it is unconditionally stable and second-order in both time and space. CN requires solving a tridiagonal system each step. I thought this method would provide a trusted comparison to measure FTCS accuracy.

The boundary conditions were set so Hot wall (left, x = 0) used Neumann flux BC: $-k (\partial T/\partial x) = q''(t)$ approximated with a ghost node update and cold wall (right, x = L) was convective BC: $-k (\partial T/\partial x) = h_g(T(L,t) = T_g)$. Both BCs were implemented consistently in FTCS and CN.

Testing and Verification

A standard diffusion test was run with initial condition 300 K everywhere and 1000 K bump in center with no flux boundaries or simple fixed BCs depending on implementation. The problem has no physical meaning but was rather used as just a verification case. The results were as follows:

- FTCS final center temperature: 334.207 K
- CN final center temperature: 334.272 K
- Max difference across domain/time = 46.9 K

- RMS difference = 0.8 K

When interpreted, this means the RMS error is extremely small, so the FTCS behaves correctly. Maximum error occurs at steep gradients which is to be expected. Both solvers diffuse the bump identically in shape and rate so this was a meaningful verification demonstrating FTCS accuracy.

In addition, a dedicated test file runs the diffusion test for FTCS, the same for CN, and assertions that FTCS and CN both remain within tolerance. All tests passed (2 passed in 0.05s).

When analyzing physical behavior, the final rocket-wall simulation shows that hot-wall temperature rises from 300 K to 1523 K, cold-wall temperature rises to 566 K, pulse pattern is visible as horizontal bands in the contour plot, and temperature profiles decay smoothly into the wall, so everything is generally physically consistent.

Results

Temperature:

At selected times, the wall near the hot side heats rapidly, heat penetrates progressively deeper over time, later times reach a smoother long time diffusion shape, and the cold wall stays much cooler due to convection

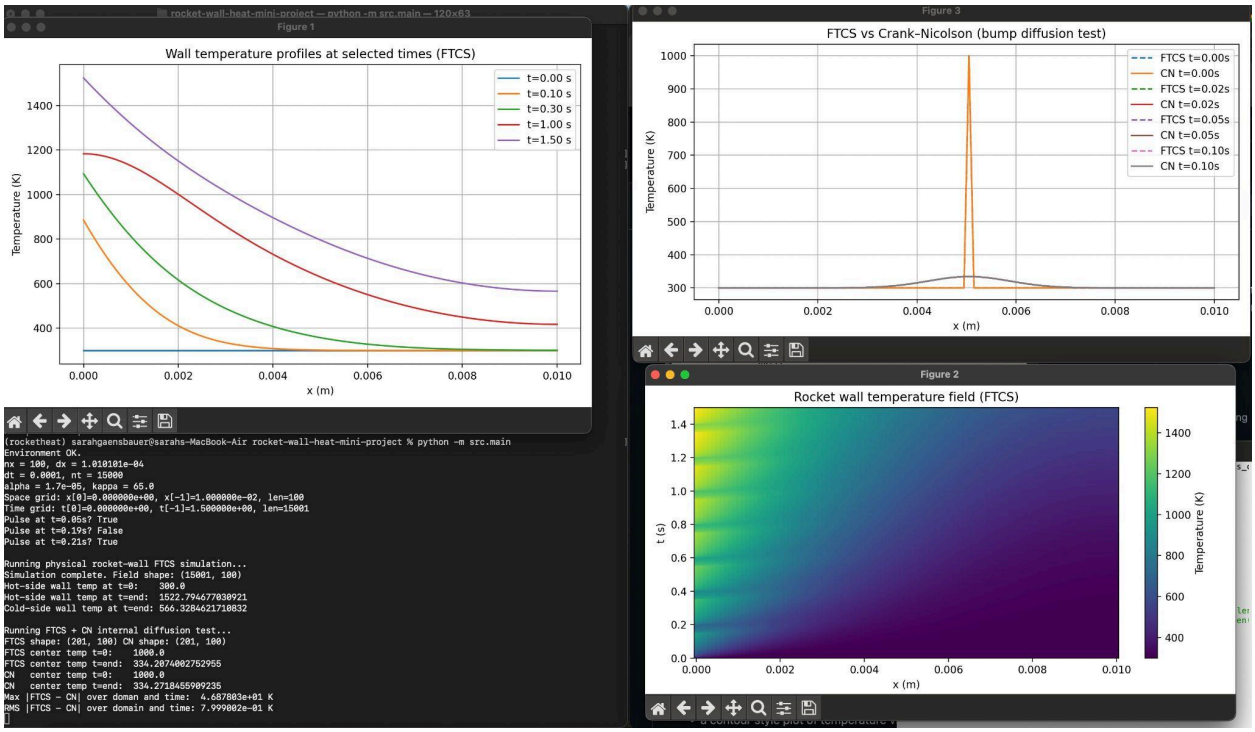
Field Plot:

The contour plot shows that each heat pulse is visible at early times, there are strong gradients near the hot side, and there is an expected diffusion trend from left to right or rather hot to cold.

Stability:

FTCS remained stable for 15000 steps and the CN comparison proved its accuracy. No numerical oscillations or non-physical behavior was observed.

Image of results from the third iteration.



Decisions and Obstacles

Why FTCS?

This method was simple to implement and debug, stable for the chosen grid, and a solid match with CN during verification.

Why a separate CN solver?

This method was ideal for a formal verification, and upon researching, seemed to be the standard for 1D diffusion. This solver ensured FTCS was not hiding numerical errors.

Some obstacles that were encountered include getting BCs implemented consistently, numerical stability tuning, organizing import paths and package structure, refactoring while keeping everything runnable, and writing meaningful automated tests. All of these issues were resolved through restructuring and debugging. LLMs were somewhat useful in the process of debugging, though occasionally adding its own errors.

Engineering Quality

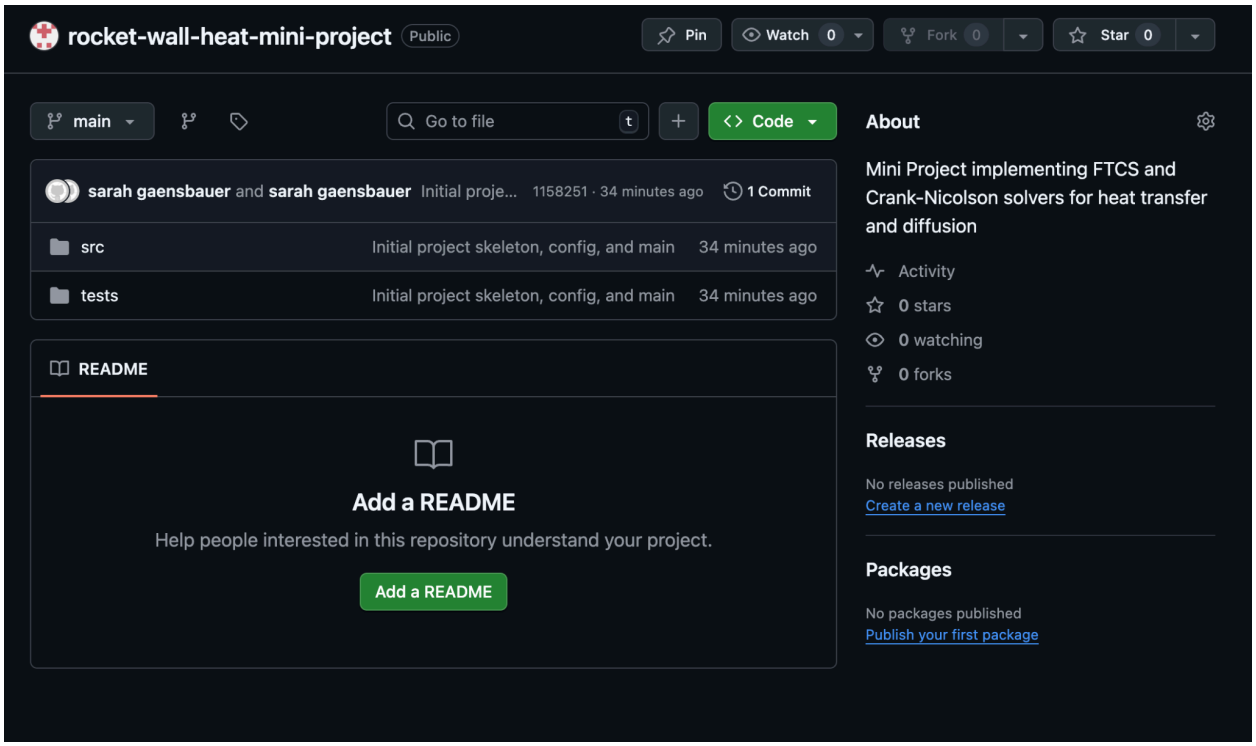
This project makes use of modular code design, folders for grids, schemes, and config, pytest for automated testing, matplotlib plots saved to figures, a README with the overall structure to aid in running, and a stable environment using Conda.

Reflection and Next Steps

Some items that worked well: verification gave confidence in the results, the solver is numerically stable and appears to be physically correct, and the codebase is clean and modular. Some improvements that could be made: using adaptive time stepping, adding spatial refinement and running a grid convergence study, implementing a 2D solver, or adding material layer modeling as in thermal barrier coatings. If I were to continue the project, I would add performance optimizations, parallelize with NumPy vectorization or Numba, implement unit-aware inputs, and add CLI arguments for configuration.

Conclusion

Overall, this project transformed the homework solver into a structured, tested, and verified numerical simulation tool. The combination of FTCS and Crank-Nicolson produced consistent results and the physical rocket wall simulation behaved as expected. Some images that were taken throughout the process can be seen below.



```
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
Traceback (most recent call last):
  File "/Users/sarahgaensbauer/miniconda3/envs/rocketheat/lib/python3.10/runpy.py", line 196, in _run_module_as_main
    return _run_code(code, main_globals, None,
  File "/Users/sarahgaensbauer/miniconda3/envs/rocketheat/lib/python3.10/runpy.py", line 86, in _run_code
    exec(code, run_globals)
  File "/Users/sarahgaensbauer/rocket-wall-heat-mini-project/src/main.py", line 1, in <module>
    from config import nx, dx, dt, nt, kappa, alpha, hg, Tg, pulse_is_on
ModuleNotFoundError: No module named 'config'
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % open .
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
Traceback (most recent call last):
  File "/Users/sarahgaensbauer/miniconda3/envs/rocketheat/lib/python3.10/runpy.py", line 196, in _run_module_as_main
    return _run_code(code, main_globals, None,
  File "/Users/sarahgaensbauer/miniconda3/envs/rocketheat/lib/python3.10/runpy.py", line 86, in _run_code
    exec(code, run_globals)
  File "/Users/sarahgaensbauer/rocket-wall-heat-mini-project/src/main.py", line 1, in <module>
    from .config import nx, dx, dt, nt, kappa, alpha, hg, Tg, pulse_is_on
ImportError: cannot import name 'nx' from 'src.config' (/Users/sarahgaensbauer/rocket-wall-heat-mini-project/src/config.
py)
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % open .
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
Environment OK.
nx = 100, dx = 1.010101e-04
dt = 0.0001, nt = 15000
alpha = 1.7e-05, kappa = 65.0
Space grid: x[0]=0.000000e+00, x[-1]=1.000000e-02, len=100
Time grid: t[0]=0.000000e+00, t[-1]=1.500000e+00, len=15001
Pulse at t=0.05s? True
Pulse at t=0.19s? False
Pulse at t=0.21s? True
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project %
```

```
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git status
On branch main
Your branch is up to date with 'origin/main'.

Changes not staged for commit:
  (use "git add <file>..." to update what will be committed)
  (use "git restore <file>..." to discard changes in working directory)
        modified:   src/config.py
        modified:   src/grid.py
        modified:   src/main.py

Untracked files:
  (use "git add <file>..." to include in what will be committed)
        .DS_Store
        src/__pycache__/

no changes added to commit (use "git add" and/or "git commit -a")
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git add src/config.py src/main.py src/gr
id.py
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git status
On branch main
Your branch is up to date with 'origin/main'.

Changes to be committed:
  (use "git restore --staged <file>..." to unstage)
        modified:   src/config.py
        modified:   src/grid.py
        modified:   src/main.py

Untracked files:
  (use "git add <file>..." to include in what will be committed)
        .DS_Store
        src/__pycache__/

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(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
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Pulse at t=0.05s? True
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    return _run_code(code, main_globals, None,
  File "/Users/sarahgaensbauer/miniconda3/envs/rocketheat/lib/python3.10/runpy.py", line 86, in _run_code
    exec(code, run_globals)
  File "/Users/sarahgaensbauer/rocket-wall-heat-mini-project/src/main.py", line 25, in <module>
    from schemes import ftcs
ModuleNotFoundError: No module named 'schemes'
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % ls src
__init__.py     bc.py           diagnostics.py  main.py         schemes.py
__pycache__     config.py       grid.py         problems.py     simulate.py
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % open .
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
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Pulse at t=0.05s? True
Pulse at t=0.19s? False
Pulse at t=0.21s? True

Running FTCS test...
FTCS Okay: shape = (201, 100)
Center temp t=0: 1000.0
Center temp t=end: 334.2074002752955
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project %
```

```
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
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Time grid: t[0]=0.000000e+00, t[-1]=1.500000e+00, len=15001
Pulse at t=0.05s? True
Pulse at t=0.19s? False
Pulse at t=0.21s? True

Running physical rocket-wall FTCS simulation...
Simulation complete. Field shape: (15001, 100)
Hot-side wall temp at t=0:    300.0
Hot-side wall temp at t=end:  -6926.619384087068
Cold-side wall temp at t=end: -512.1028351235017

Running FTCS internal diffusion test...
FTCS Okay: shape = (201, 100)
Center temp t=0:    1000.0
Center temp t=end:  334.2074002752955
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project %
```

```
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
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Time grid: t[0]=0.000000e+00, t[-1]=1.500000e+00, len=15001
Pulse at t=0.05s? True
Pulse at t=0.19s? False
Pulse at t=0.21s? True

Running physical rocket-wall FTCS simulation...
Simulation complete. Field shape: (15001, 100)
Hot-side wall temp at t=0:    300.0
Hot-side wall temp at t=end:  1522.794677030921
Cold-side wall temp at t=end: 566.3284621710832

Running FTCS internal diffusion test...
FTCS Okay: shape = (201, 100)
Center temp t=0:    1000.0
Center temp t=end:  334.2074002752955
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project %
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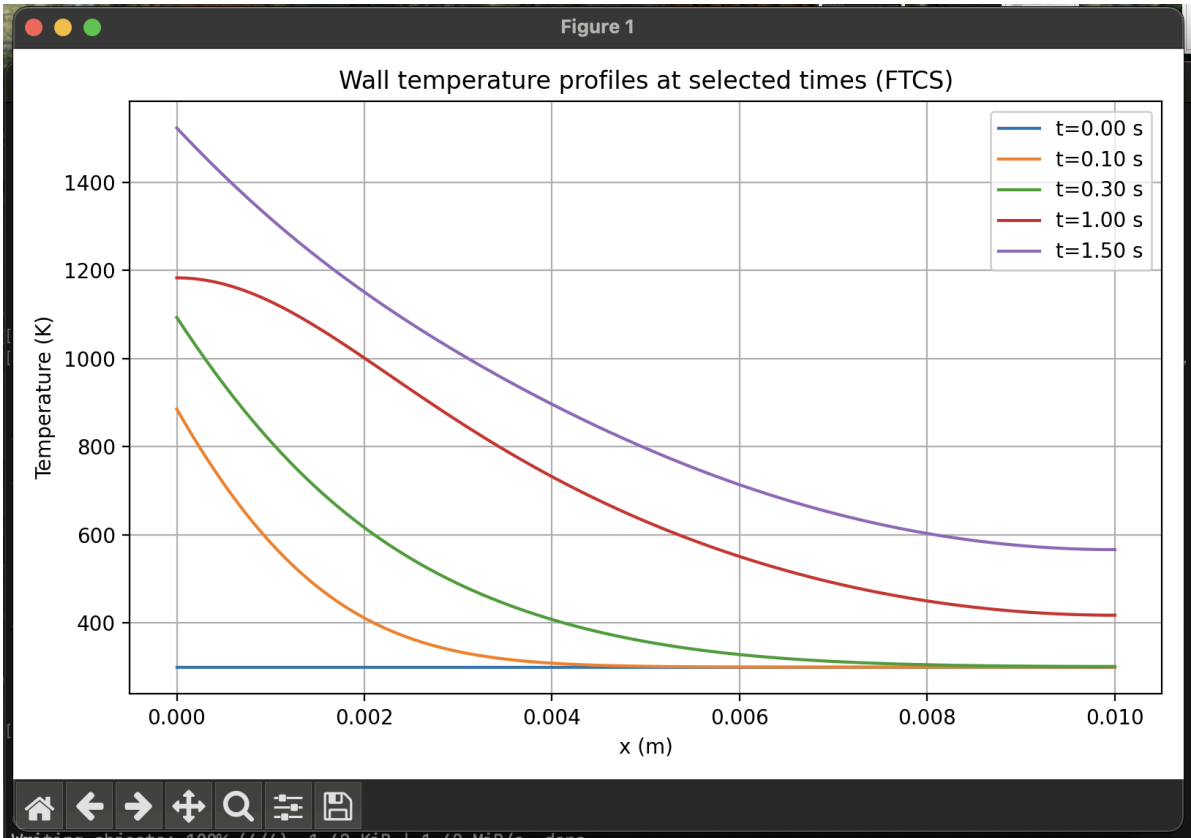
FTCS Okay: shape = (201, 100)
Center temp t=0:    1000.0
Center temp t=end:  334.2074002752955
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project %
```




```
Writing objects: 100% (4/4), 1.42 KiB | 1.42 MiB/s, done.
Total 4 (delta 1), reused 0 (delta 0), pack-reused 0 (from 0)
remote: Resolving deltas: 100% (1/1), completed with 1 local object.
To https://github.com/gaensbau-droid/rocket-wall-heat-mini-project.git
   e9fe88a..33b37fe  main -> main
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % open .
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
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Time grid: t[0]=0.000000e+00, t[-1]=1.500000e+00, len=15001
Pulse at t=0.05s? True
Pulse at t=0.19s? False
Pulse at t=0.21s? True

Running physical rocket-wall FTCS simulation...
Simulation complete. Field shape: (15001, 100)
Hot-side wall temp at t=0:    300.0
Hot-side wall temp at t=end:  1522.794677030921
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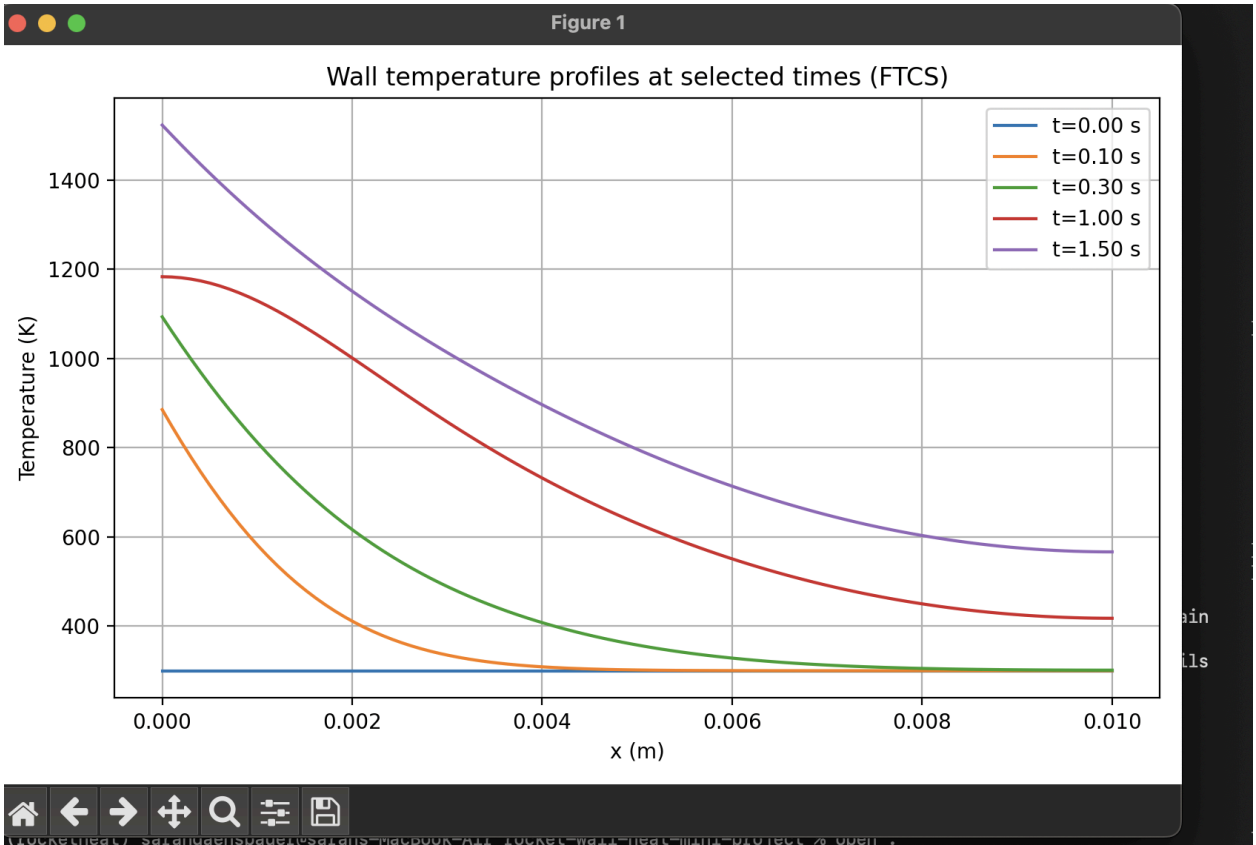


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Time grid: t[0]=0.000000e+00, t[-1]=1.500000e+00, len=15001
Pulse at t=0.05s? True
Pulse at t=0.19s? False
Pulse at t=0.21s? True

Running physical rocket-wall FTCS simulation...
Simulation complete. Field shape: (15001, 100)
Hot-side wall temp at t=0:    300.0
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Cold-side wall temp at t=end: 566.3284621710832
```

```
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git add src/main.py
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git commit -m "made all plots display si
multaneously, moved show to end of all plotting"
[main 77abaf0] made all plots display simultaneously, moved show to end of all plotting
 Committer: sarah gaensbauer <sarahgaensbauer@sarahs-MacBook-Air.local>
Your name and email address were configured automatically based
on your username and hostname. Please check that they are accurate.
You can suppress this message by setting them explicitly. Run the
following command and follow the instructions in your editor to edit
your configuration file:

    git config --global --edit

After doing this, you may fix the identity used for this commit with:

    git commit --amend --reset-author

 1 file changed, 42 insertions(+), 8 deletions(-)
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git push
Enumerating objects: 7, done.
Counting objects: 100% (7/7), done.
Delta compression using up to 10 threads
Compressing objects: 100% (4/4), done.
Writing objects: 100% (4/4), 901 bytes | 901.00 KiB/s, done.
Total 4 (delta 2), reused 0 (delta 0), pack-reused 0 (from 0)
remote: Resolving deltas: 100% (2/2), completed with 2 local objects.
To https://github.com/gaensbau-droid/rocket-wall-heat-mini-project.git
   f7590d2..77abaf0  main -> main
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project %
```

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(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % python -m src.main
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Time grid: t[0]=0.000000e+00, t[-1]=1.500000e+00, len=15001
Pulse at t=0.05s? True
Pulse at t=0.19s? False
Pulse at t=0.21s? True

Running physical rocket-wall FTCS simulation...
Simulation complete. Field shape: (15001, 100)
Hot-side wall temp at t=0:    300.0
Hot-side wall temp at t=end:  1522.794677030921
Cold-side wall temp at t=end: 566.3284621710832

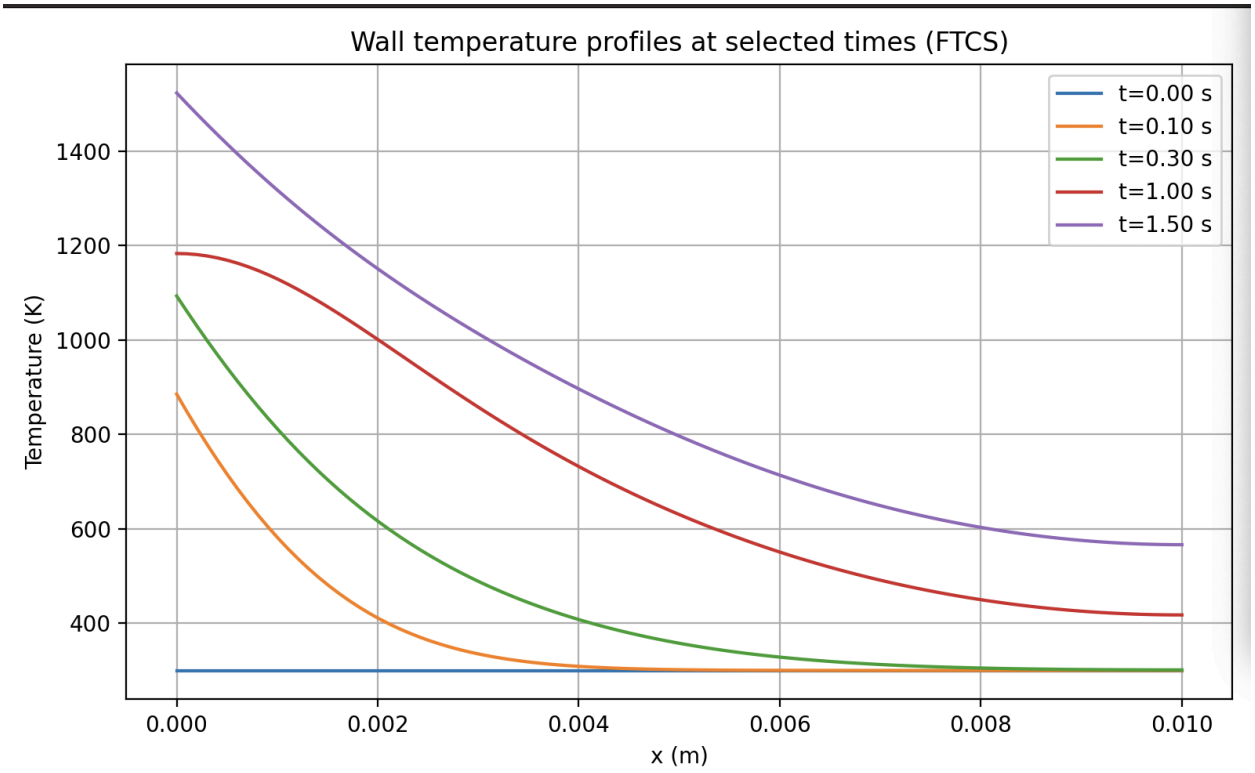
Running FTCS + CN internal diffusion test...
FTCS shape: (201, 100) CN shape: (201, 100)
FTCS center temp t=0:    1000.0
FTCS center temp t=end:  334.2074002752955
CN   center temp t=0:    1000.0
CN   center temp t=end:  334.2718455909235
Max |FTCS - CN| over doman and time:  4.687803e+01 K
RMS |FTCS - CN| over domain and time: 7.999002e-01 K
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git add src/main.py
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git commit -m "added ftcs vs cn err
trics (quantitative)"
[main 5f9a91d] added ftcs vs cn error metrics (quantitative)
 Committer: sarah gaensbauer <sarahgaensbauer@sarahs-MacBook-Air.local>
Your name and email address were configured automatically based
on your username and hostname. Please check that they are accurate.
You can suppress this message by setting them explicitly. Run the
following command and follow the instructions in your editor to edit
your configuration file:

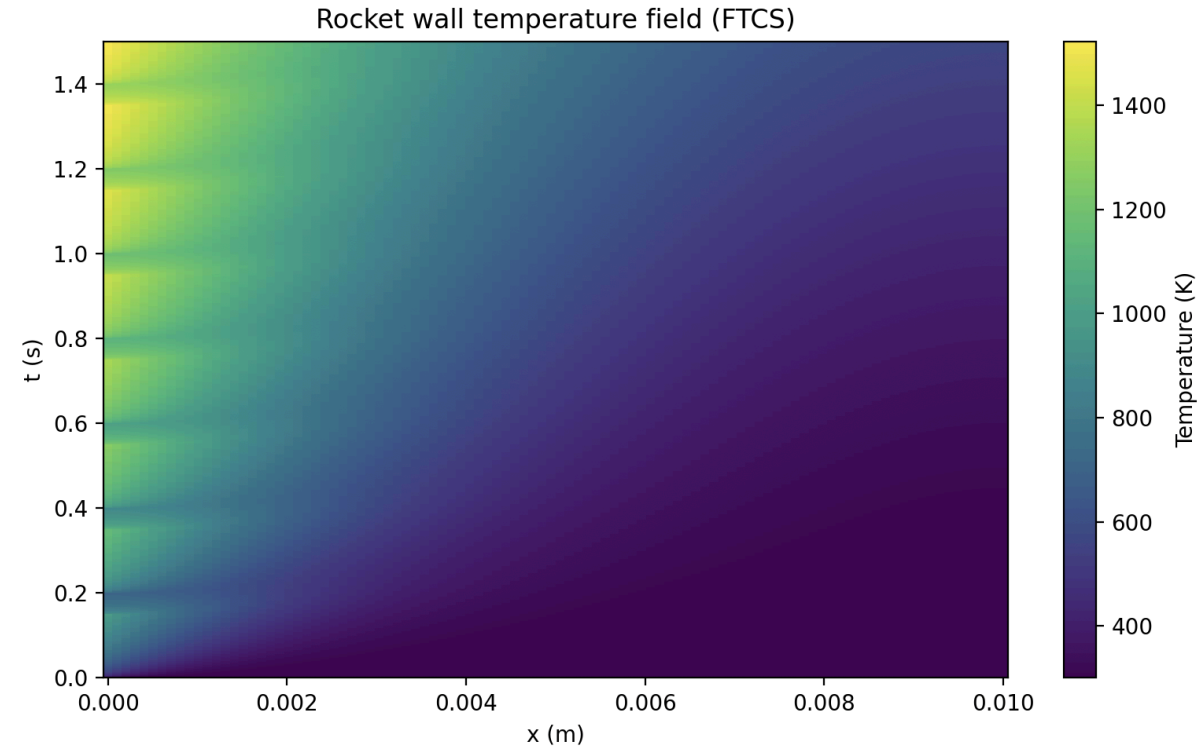
    git config --global --edit

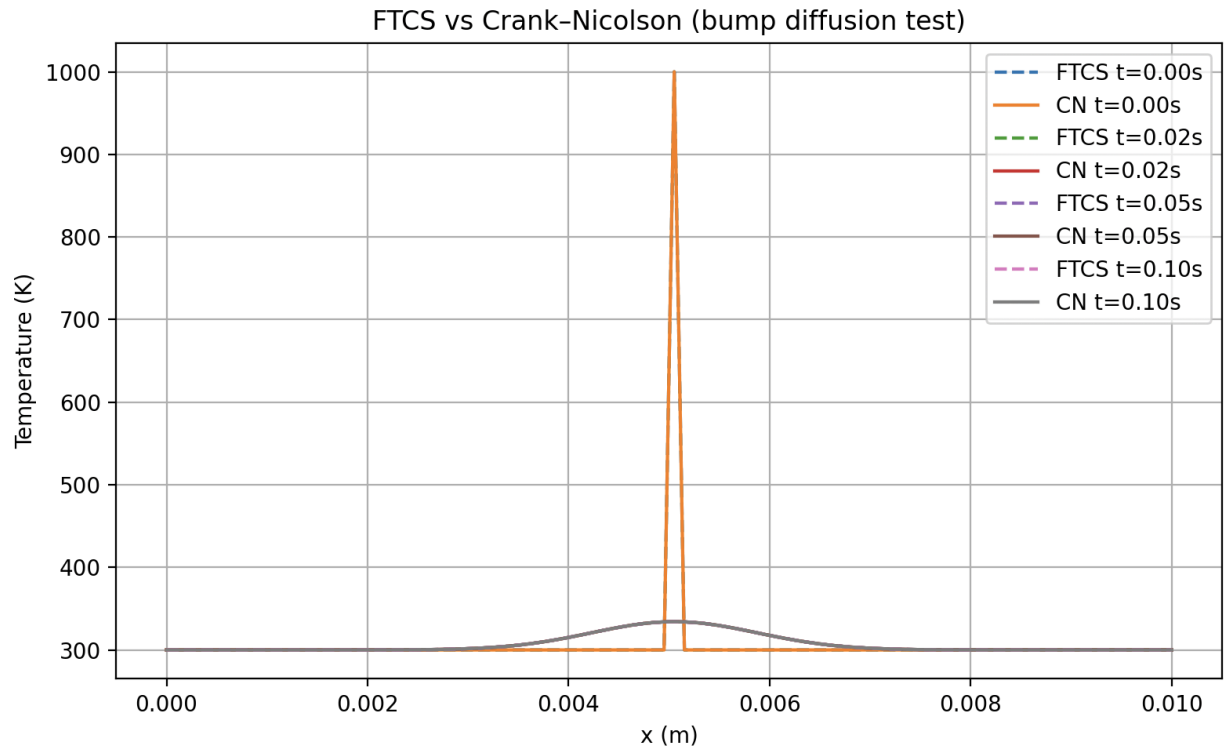
After doing this, you may fix the identity used for this commit with:

    git commit --amend --reset-author

 1 file changed, 7 insertions(+)
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % git push
Enumerating objects: 7, done.
Counting objects: 100% (7/7), done.
Delta compression using up to 10 threads
Compressing objects: 100% (4/4), done.
Writing objects: 100% (4/4), 605 bytes | 605.00 KiB/s, done.
Total 4 (delta 2), reused 0 (delta 0), pack-reused 0 (from 0)
remote: Resolving deltas: 100% (2/2), completed with 2 local objects.
To https://github.com/gaensbau-droid/rocket-wall-heat-mini-project.git
   77abaf0..5f9a91d  main -> main
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project %
```


Wall temperature profiles at selected times (FTCS)
t=0.00 s
t=0.10 s
t=0.30 s
t=1.00 s
t=1.50 s
Temperature (K)
1400
1200
1000
800
600
400
0.000
0.002
0.004
0.006
0.008
0.010
x (m)


Rocket wall temperature field (FTCS)
t (s)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.000
0.002
0.004
0.006
0.008
0.010
x (m)
Temperature (K)
1400
1200
1000
800
600
400




```
========================================== ERRORS ==========================================
_____________________ ERROR collecting tests/test_schemes.py _____________________
ImportError while importing test module '/Users/sarahgaensbauer/rocket-wall-heat-mini-project/
tests/test_schemes.py'.
Hint: make sure your test modules/packages have valid Python names.
Traceback:
../miniconda3/envs/rocketheat/lib/python3.10/importlib/__init__.py:126: in import_module
    return _bootstrap._gcd_import(name[level:], package, level)
tests/test_schemes.py:2: in <module>
    from src.schemes import ftcs, crank_nicolson
E   ModuleNotFoundError: No module named 'src'
=================================== short test summary info ====================================
ERROR tests/test_schemes.py
!!!!!!!!!!!!!!!!!!!!!!!!!! Interrupted: 1 error during collection !!!!!!!!!!!!!!!!!!!!!!!!!!
====================================== 1 error in 0.09s ======================================
```

```
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % open .
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project % pytest
=================================== test session starts ====================================
platform darwin -- Python 3.10.19, pytest-9.0.2, pluggy-1.6.0
rootdir: /Users/sarahgaensbauer/rocket-wall-heat-mini-project
collected 2 items

tests/test_schemes.py ..                                                            [100%]

==================================== 2 passed in 0.05s =====================================
(rocketheat) sarahgaensbauer@sarahs-MacBook-Air rocket-wall-heat-mini-project %
```