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PANTHÉON SORBONNE

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HYBRID VOLATILITY FORECASTING OF A CRYPTOCURRENCY PORTFOLIO: AN ENSEMBLE APPROACH USING CVI, GARCH, AND LSTM MODELS

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PRESENTED BY

Gaétan DUMAS, Pierre LIBERGE, Tonin RIVORY

Hybrid Volatility Forecasting for a Cryptocurrency Portfolio: An Ensemble Approach with CVI, GARCH, and LSTM Models

Gaétan DUMAS, Pierre LIBERGE, Tonin RIVORY
University Paris 1 Panthéon Sorbonne

Abstract

The cryptocurrency market is characterized by high volatility, posing a major challenge for portfolio management and forecasting. This study proposes an innovative framework for predicting the volatilities of a basket of cryptocurrencies in order to construct a portfolio composed of four assets selected for their liquidity. It relies on the adaptation and extension of methodologies from classical time series analysis. We integrate GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models, LSTM (Long Short-Term Memory) neural networks, and hybrid approaches combining these two techniques. Our analysis is based on daily closing prices observed between March 11, 2019, and November 28, 2024, covering the following cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), and Cardano (ADA).

By comparing the performance of the models, we demonstrate that hybrid approaches, combining deep learning (LSTM) and traditional econometric methods (GARCH), outperform individual models in terms of accuracy. This research highlights the importance of combining multiple methodologies to design a robust forecasting framework capable of addressing the specific challenges of cryptocurrency markets.

Contents

1 Introduction

Cryptocurrency markets are notoriously volatile, exhibiting price fluctuations far greater than those observed in traditional stock markets. This heightened volatility reflects both the speculative nature of the market and its sensitivity to macroeconomic events, regulatory developments, and technological advancements. Unlike traditional markets, cryptocurrency prices can react sharply to shifts in market sentiment, technological breakthroughs, or unexpected regulatory announcements.

Accurately predicting volatility is crucial for portfolio risk management, derivative pricing, and strategic asset allocation. This paper explores hybrid models to forecast the daily volatility of a basket of cryptocurrencies to construct an optimal and diversified crypto portfolio. Our study also assesses the added value of integrating sentiment data and examines how these models adapt to the unique characteristics of cryptocurrency markets. By addressing these nuances, this research aims to bridge gaps in the current literature and provide practical solutions for risk management in the emerging digital asset markets.

2 Literature Review

Financial time series forecasting has long relied on econometric models such as GARCH, which excel in modeling volatility clustering. More recently, deep learning models like LSTMs have demonstrated superior performance in capturing nonlinear patterns and long-term dependencies in data. LSTMs are particularly well-suited for analyzing the chaotic and nonlinear dynamics of cryptocurrency prices, offering advantages in adaptability and pattern recognition.

Hybridizing these approaches combines the ability of GARCH models to capture conditional variance with the strength of LSTMs in learning complex temporal dynamics. Research on cryptocurrency volatility forecasting remains limited compared to stock markets, highlighting the novelty and significance of this study. Furthermore, the inclusion of alternative data sources, such as sentiment indicators and blockchain activity, has been shown to enhance predictive performance in recent studies, paving the way for advanced hybrid approaches. Notably, the fusion of machine learning techniques with traditional econometric methods has proven to be a promising avenue for addressing the unique challenges posed by cryptocurrency markets.

3 Methodology

Before presenting the final composition of our portfolio, it is essential to detail the methodology used for its construction. The first step involves estimating the volatility of different crypto assets. Traditionally, the Sharpe ratio uses the historical standard deviation of returns as a measure of risk, assuming constant volatility over the considered horizon. However, cryptocurrency markets are highly volatile and subject to sudden regime shifts. A static approach could therefore lead to an underestimation or overestimation of actual risk.

To better capture this dynamic, we employ conditional volatility models such as GARCH, as well as deep learning models like LSTM, which help anticipate future fluctuations. Two approaches can then be considered for integrating these estimates into portfolio optimization:

- Using the latest estimated volatility as a proxy for future risk, thereby reflecting the current state of the market.
- Calculating an average over a given period (e.g., 30 days).

Furthermore, we explore additional horizons by incorporating the Crypto Volatility Index (CVI). The Crypto Volatility Index is an indicator measuring the implied volatility of the cryptocurrency market, inspired by the well-known VIX used for traditional stock markets. It provides insight into risk perception and uncertainty surrounding digital assets.

In this study, we integrate the CVI as an explanatory variable to enhance cryptocurrency volatility forecasting. To achieve this, we implement a methodology combining data extraction via an API and web scraping from the website: **Crypto Volatility Index - Investing.com**. These two methods ensure extended temporal coverage of the data and maintain availability even in the event of access restrictions.

3.1 Data Preprocessing

Estimation of Logarithmic Returns, Volatility, and Lagged Volatility

The **logarithmic returns** of our assets at date t are calculated as the natural logarithm of the ratio between the closing price on day t and that of the previous day $(t-1)$:

$$\text{Log Returns}_t = \log\left(\frac{\text{Close}_t}{\text{Close}_{t-1}}\right) \tag{1}$$

The estimation of **historical volatility** on day t is obtained by calculating the standard deviation of logarithmic returns over a specified rolling window:

$$\text{Volatility}_t = \sqrt{\frac{1}{N-1} \sum_{i=t-N+1}^{t} (\text{Log Returns}_i - \mu)^2} \tag{2}$$

where:

- N is the size of the rolling window used for estimation,
- Log Returns_i is the logarithmic return on day i,
- μ is the mean of the logarithmic returns over the considered period.

Next, **lagged volatility** for a given day is calculated by shifting the volatility values by a certain number of days. For a one-day lag (lag $= 1$), the calculation is defined as follows:

$$\text{Lagged Volatility}_t = \text{Volatility}_{t-1} \tag{3}$$

where:

- Volatility_{t-1} represents the observed volatility on the previous day $(t-1)$.
- $\text{Lagged Volatility}_t$ corresponds to the lagged volatility value assigned to day t.

3.2 Data Preprocessing: Data Standardization

Standardization is a key step before integrating data into LSTM models. The Min-Max scaling method was used to normalize the raw input data. This normalization process is dynamically applied to each prediction window in our time series, ensuring that our model is trained on data reflecting the most recent conditions. The Min-Max scaling formula is given by:

$$X_{\text{scaled}} = \frac{X - X_{\text{min}}}{X_{\text{max}} - X_{\text{min}}} \tag{4}$$

The normalization scales are adjusted based on the current training data segment. This process recalculates the minimum and maximum values used for scaling, based on the available training data up to the current prediction point. Once adjusted, the training, validation, and test sets are transformed using these dynamically updated scales.

The model is either trained or loaded from previous states, and predictions are then performed on the normalized test data. After prediction, the output is inversely transformed back to the original data scale to ensure accurate error calculation and reliable comparison.

3.3 Model Evaluation: Error Metrics

Error metrics are essential for evaluating the accuracy of predictive models, providing quantitative measures of prediction precision. Two commonly used metrics are the Mean Absolute Percentage Error (MAPE) and the Root Mean Squared Error (RMSE).

The **Mean Absolute Percentage Error (MAPE)** measures the average percentage error between predicted and actual values. Unlike MAE, MAPE expresses the error in relative terms, allowing for comparison of model performance across different data scales. It is defined by the equation:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100 \tag{5}$$

where:

- n : the number of observations,
- y_i : the actual value for the i-th observation,
- $\hat{y}_i$: the predicted value for the i-th observation.

The MAPE is useful for evaluating the accuracy of the model while taking into account the scale of the data. Lower values indicate better model performance. However, MAPE can be biased when y_i is close to zero, which can exaggerate the percentage error.

The **Root Mean Squared Error (RMSE)** is defined as the square root of the mean of the squared differences between predicted and actual values. Unlike MAE, RMSE assigns greater weight to large errors due to the squaring of the error terms. It is defined by the equation:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{6}$$

where the variables are defined as previously.

RMSE is sensitive to outliers and tends to penalize large errors more heavily than MAE. Lower RMSE values indicate better accuracy of the predictive model.

Both metrics, MAE and RMSE, are dependent on the scale of the data, meaning their values are influenced by the range of the data. They are mainly used for comparative purposes, allowing the evaluation of the performance of different models or model configurations on the same dataset.

3.4 Portfolio Construction: Sharpe Ratio Optimization

Once volatility is estimated, we proceed with constructing a portfolio that maximizes the Sharpe ratio. Unlike traditional approaches where historical standard deviation is used as a constant, our methodology relies on a dynamic volatility estimation. This allows for better adaptation of portfolio allocation to current market conditions. Higher recent volatility will prompt a reduction in exposure to riskier assets, while lower volatility will encourage greater risk-taking to maximize risk-adjusted returns.

The objective is to construct an optimized portfolio based on a more responsive and relevant volatility measure, enabling better risk management in a market characterized by significant fluctuations.

The daily closing prices of various crypto assets were collected via Yahoo Finance. To compute returns, we used logarithmic returns, which offer several advantages. Logarithmic returns are preferred as they provide additivity over successive periods, simplifying calculations across different time horizons. Additionally, they better capture the normality of returns and allow for improved volatility modeling, such as with GARCH models.

Data preprocessing incorporated various techniques, including the normalization of logarithmic returns, to reduce noise and optimize model training efficiency. Additionally, data from the Crypto Volatility Index (CVI) was included as a complementary input, highlighting the growing importance of cryptocurrency-specific volatility in price movement analysis.

4 Model Overview

4.1 GARCH Model

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is a fundamental econometric tool for modeling the conditional volatility of financial time series. It is based on the idea that the conditional variance at a given time is influenced by past residuals and past conditional variances. This allows it to capture the volatility clustering phenomenon, characteristic of financial markets, where periods of high volatility alternate with calmer periods.

For a GARCH(p, q) model, the conditional variance equation is expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where:

- σ_t^2 is the variance conditional on past information at time t,
- $\omega > 0$ is a constant,
- $\alpha_i \epsilon_{t-i}^2$ captures the impact of the q past shocks (ϵ_{t-i}^2 being the squared residuals),
- $\beta_j \sigma_{t-j}^2$ measures the persistence effect of the p past conditional variances.

This model captures the variance dynamics by integrating two key components:

- the influence of recent shocks through the sum of terms $\sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$
- the persistence of past volatility via the sum of terms $\sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$.

The coefficients α_i and β_j must satisfy certain constraints (notably $\alpha_i, \beta_j \geq 0$ and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$) to ensure positive variance and stationarity of the process.

Thanks to this formulation, the GARCH model is particularly suited for analyzing and forecasting volatility trends in financial markets, providing a robust foundation for risk assessment and strategy development in uncertain environments.

To determine the optimal parameters p and q of the GARCH model, a systematic approach based on minimizing the Akaike Information Criterion (AIC) was employed. This method involves estimating multiple model specifications by testing different combinations of the orders p (autoregressive) and q (moving average). For each combination, the AIC is computed to evaluate the model's performance.

The Akaike Information Criterion, defined as:

$$\text{AIC} = -2\ln(L) + 2k$$

where L is the model likelihood and k is the number of estimated parameters, balances model fit quality with complexity. The model with the lowest AIC value is selected, as it reflects an optimal trade-off between accuracy and simplicity while reducing the risk of overfitting.

This rigorous strategy ensures that the selected parameters best capture volatility dynamics while remaining econometrically parsimonious.

The GARCH model provides a robust econometric foundation for understanding volatility clustering. However, its limitations in capturing nonlinear patterns have led to the introduction of more advanced models. Additionally, GARCH's sensitivity to parameter settings makes it less adaptable to highly dynamic markets, highlighting the need to explore hybrid solutions.

4.2 LSTM

The Long Short-Term Memory (LSTM) model is an advanced architecture of recurrent neural networks (RNN) designed to capture long-term dependencies in time series. It overcomes the limitations of traditional RNNs, particularly the vanishing or exploding gradient problems, by introducing a memory cell c_t and three main gates.

The **forget gate** (f_t) decides which information from the previous memory state (c_{t-1}) should be forgotten:

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

where f_t indicates which information to retain (values between 0 and 1), W_f and b_f are learned weights and biases, and σ is the sigmoid function that scales values between 0 and 1.

The **input gate** (i_t) determines which new information should be added to memory:

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i), \quad \tilde{c}_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c)$$

where i_t indicates which new information to integrate, and $\tilde{c}_t$ represents candidate new information, compressed between -1 and 1 by the tanh function.

The **output gate** (o_t) controls which part of the current memory c_t should influence the output h_t:

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o), \quad h_t = o_t \odot \tanh(c_t)$$

where o_t determines which information to transmit to the output, and h_t is the hidden state used as the model's output.

The memory cell is updated by combining the information to forget ($f_t \cdot c_{t-1}$) and the new information to add ($i_t \cdot \tilde{c}_t$):

$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t$$

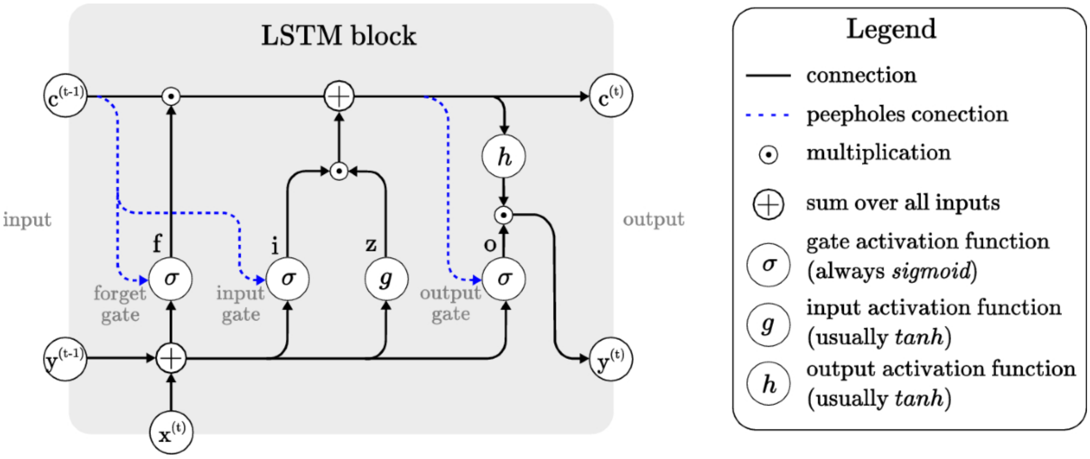

Figure 1: Diagram of an LSTM block with peephole connections

This update mechanism allows LSTM to retain important information while discarding irrelevant details.

The LSTM model is particularly well-suited for complex tasks such as conditional volatility prediction or asset price forecasting. Unlike traditional econometric models such as GARCH, it can capture nonlinear and dynamic relationships thanks to its flexible architecture. The gating mechanisms enable dynamic adjustment of information flow, making this approach powerful for analyzing financial market behaviors.

5 Results

5.1 GARCH Models

In this subsection, we focus on forecasting the conditional volatility of our cryptocurrency basket using GARCH models. Before proceeding, stationarity tests were performed using autocorrelation plots (ACF) on the logarithmic returns. The absence of unit roots, confirmed by a rapid decay of the ACF towards zero, indicates that the series are stationary, a key assumption for the application of GARCH models.

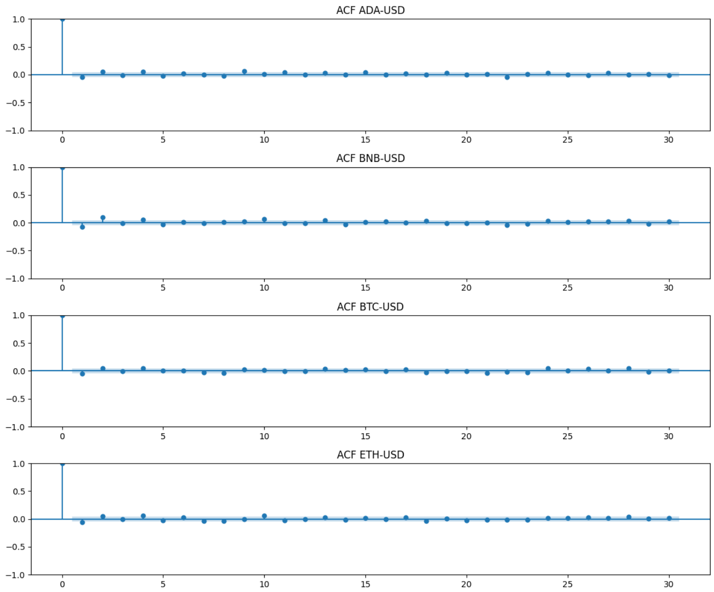

Figure 2: ACF of log returns.

As a result, we modeled the conditional volatility using GARCH models, seeking the optimal p and q parameters for the different models with the goal of minimizing the AIC.

We will also compare the RMSE, an essential metric that allows us to compare the accuracy of our GARCH and LSTM models.

The following table presents the optimal orders as well as the corresponding AIC and RMSE values:

Table 1: Summary of the models' indicators

Crypto-assets	**Best p**	**Best q**	**Constant**	**MSE**	**RMSE**	**MAE**
ADA-USD	4	3	Yes	0.000139	0.011771	0.009060
BNB-USD	2	3	Yes	0.000218	0.014770	0.008849
BTC-USD	4	2	Yes	0.000113	0.010623	0.007601
ETH-USD	4	3	Yes	0.000166	0.012883	0.008780

In our case, BTC-USD has the lowest RMSE (0.010623), suggesting that the model is more accurate in predicting the price of Bitcoin compared to the other cryptocurrencies. Conversely, BNB-USD has the highest RMSE (0.0147700), indicating greater variability in prediction errors for this cryptocurrency.

However, it is important to note that all RMSE values remain relatively low, which reflects the overall good performance of the model. In fact, a low RMSE means that the average gap between predicted and actual values is small, suggesting the model's strong ability to capture price volatility dynamics.

These results show that the model's performance varies slightly depending on the analyzed cryptocurrency. Factors such as market-specific volatility or the quality of historical data used can influence these prediction errors. Nevertheless, the low values indicate that the model used is a good tool for forecasting the volatility of the analyzed crypto-assets.

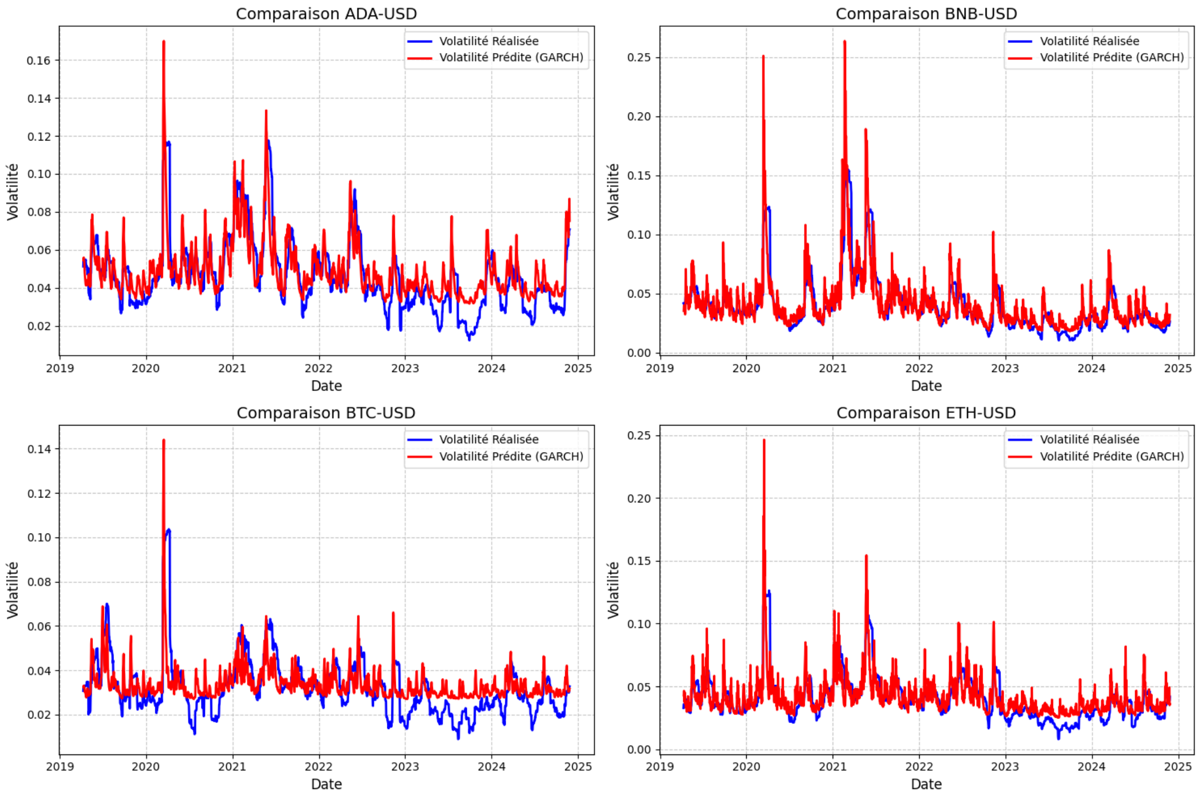

Figure 3: Comparison between conditional volatility forecasts and actual values.

Overall, the GARCH model captures cryptocurrency volatility dynamics effectively, with a close match between the realized and predicted volatility curves. However, some periods of heightened instability, particularly in 2020, show significant deviations. A global trend of reduced volatility following high-instability periods is observed, in line with the volatility clustering effect seen in financial markets.

When analyzed by cryptocurrency, the model performs well for ADA-USD, with only moderate discrepancies. However, during times of heightened instability, predicted volatility is occasionally overestimated. For BNB-USD, several noticeable volatility spikes are observed, especially in 2020 and 2021, where the GARCH model overestimates these extreme movements. During low-volatility periods, the fit is more accurate. For BTC-USD, the model captures overall volatility trends but fails to predict the 2020 volatility spike accurately, overestimating its magnitude. Lastly, for ETH-USD, the model sometimes amplifies volatility peaks, and some high-instability periods are poorly anticipated, though the general trend remains well-followed.

Thus, while the GARCH model proves effective for modeling cryptocurrency volatility during stable periods, it shows limitations during turbulent episodes. Future improvements could involve exploring variants like EGARCH, APARCH, or FIGARCH, which are better suited for capturing asymmetries and long-memory effects. Cryptocurrency volatility remains highly unstable, with calm periods followed by sudden shocks, making models like GARCH valuable for risk forecasting.

The GARCH model proves effective for modeling cryptocurrency volatility, capturing trends and stress periods well. However, slight overestimation of volatility spikes suggests potential improvements with variants like EGARCH, better equipped to handle market asymmetries.

5.2 LSTM Models

We now turn to volatility forecasting for our cryptocurrency basket using the LSTM model. The effectiveness of the LSTM model in forecasting volatility was evaluated on historical data of the daily returns of the crypto-assets. The data were split into two sets: the training set, covering the period from April 10, 2019, to October 13, 2023, representing 80% of the data, and the test set, from October 13, 2023, to November 28, 2024.

Crypto-assets	**RMSE Test**	**MAPE Test (%)**
ADA-USD	0.002919	4.998604
BNB-USD	0.002689	7.865256
BTC-USD	0.001771	4.871594
ETH-USD	0.002877	6.334581

Table 2: Performance Metrics of the LSTM Models

The LSTM models show solid results in forecasting the volatility of the different cryptocurrencies. BTC-USD exhibits the best performance with an RMSE of 0.001771 and a MAPE of 4.87%. The MAPE, which measures the relative error in percentage, indicates that the volatility forecasts for Bitcoin deviate by only 4.87% on average from the actual values, which is very good accuracy. The model for BNB-USD follows closely with an RMSE of 0.002689 and a MAPE of 7.87%, also showing good performance, but the relative error is a bit higher. For ADA-USD, the RMSE is 0.002919 and the MAPE is 4.99%, which remains low, meaning that the forecasts are relatively accurate. Finally, the model for ETH-USD presents an RMSE of 0.002877 and a MAPE of 6.33%, which is a bit higher, but still acceptable. In summary, the volatility forecasts for all cryptocurrencies are of good quality, with BTC-USD achieving the best results, thanks to its low absolute and relative error.

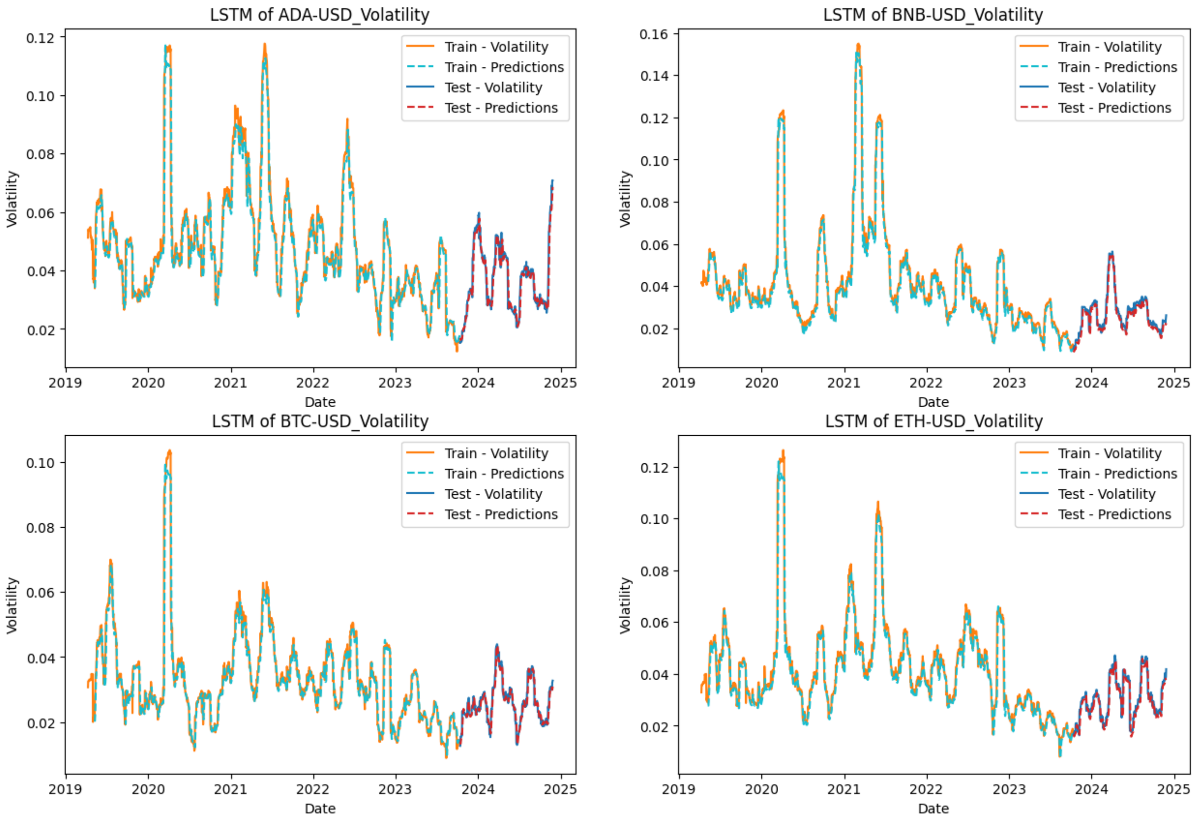

Figure 4: Comparison between volatility forecasts and actual values.

The analysis of the graphs shows that the LSTM model manages to accurately estimate the volatility dynamics of the different cryptocurrencies studied. By comparing the actual volatility curves with the predictions, there is a strong match between the two, both on the training set and on the test set. The volatility spikes, particularly those observed around 2020 and 2021, are well captured by the model, indicating that it is able to grasp major trends and market fluctuations. Additionally, on the test period, the prediction remains aligned with the actual values, further enhancing the reliability of the model for forecasting future volatility. The model thus appears robust and effective for the analysis and forecasting of cryptocurrency volatility.

5.3 Hybrid LSTM-GARCH Models

After analyzing the LSTM and GARCH models separately, we now explore a hybrid LSTM-GARCH approach to estimate crypto-asset volatility. LSTM captures complex time series dynamics, while GARCH models heteroscedasticity and conditional volatility. Their integration aims to enhance forecast accuracy by combining statistical regularities with crypto market complexities.

Crypto-assets	RMSE Test	MAPE Test (%)
ADA-USD	0.003003	5.43
BNB-USD	0.002174	5.99
BTC-USD	0.001860	5.32
ETH-USD	0.003033	7.41

Table 3: Performance Metrics of the LSTM-GARCH Models

The RMSE and MAPE results indicate that the model provides relatively accurate volatility forecasts. The RMSE, which measures the root mean square error, ranges from 0.001860 for BTC-USD to 0.003033 for ETH-USD, indicating that the average error in volatility predictions is quite low. The MAPE, on the other hand, varies between 5.32% for BTC-USD and 7.41% for ETH-USD, meaning the average relative error between predicted and actual values is moderate, with a tendency to be slightly higher for cryptocurrencies like ETH-USD. These findings highlight the model's effectiveness while leaving room for improvement.

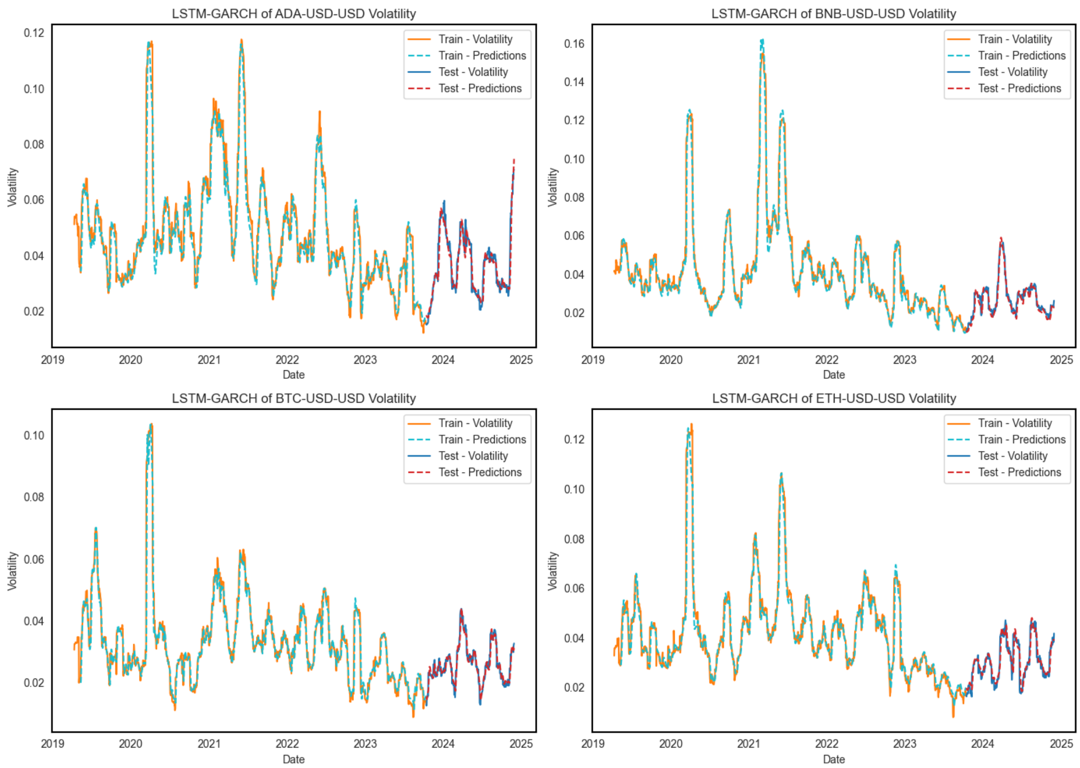

Figure 5: Comparison between volatility forecasts and actual values.

Overall, the prediction curves closely follow the actual volatility curves, indicating the model's good ability to capture the dynamics of fluctuations. There is a strong correlation between predictions and actual values for both training and testing data, although slight differences remain, especially during volatility peaks. Periods of high volatility, visible as sharp peaks on each graph, are well replicated by the model, although some amplitudes may occasionally be slightly under or overestimated. This suggests that the model is effective at capturing the general trend but may have room for improvement in forecasting extremes. In summary, these results suggest that the combined LSTM-GARCH approach is suitable for modeling cryptocurrency volatility, with a good alignment between observed and predicted series.

5.4 Hybrid LSTM-GARCH Models with CVI Input

Building on the previous model, which predicted the volatility of crypto-assets without taking into account the CVI market indicator, we extended the approach by introducing the CVI of crypto-assets, an indicator that measures volatility and uncertainty specific to each crypto-asset based on their past behaviors and market fluctuations. This LSTM-GARCH with CVI input model thus predicts the volatility of a portfolio of crypto-assets by considering both market dynamics captured by the LSTM-GARCH model and additional information provided by the CVI, offering more robust forecasts tailored to the specifics of the crypto markets.

Crypto	**RMSE on Test**	**MAPE on Test (%)**
ADA-USD	0.002778	4.786326
BNB-USD	0.002336	6.802253
BTC-USD	0.001767	5.662220
ETH-USD	0.003185	8.033684

Table 4: Performance Metrics of the LSTM-GARCH models with CVI input.

The LSTM-GARCH model integrated with the CVI shows solid performance in predicting crypto-asset volatility. The fitting errors are relatively low, with a particularly low RMSE for ADA-USD and BTC-USD, indicating a good predictive capacity. The MAPE for these assets also remains within acceptable limits, suggesting that the model is effectively capturing the volatility of these crypto-assets. Although the performance for ETH-USD and BNB-USD is slightly less favorable, the results overall show that integrating the CVI as a market indicator improves the model's accuracy in estimating the volatility of crypto-assets.

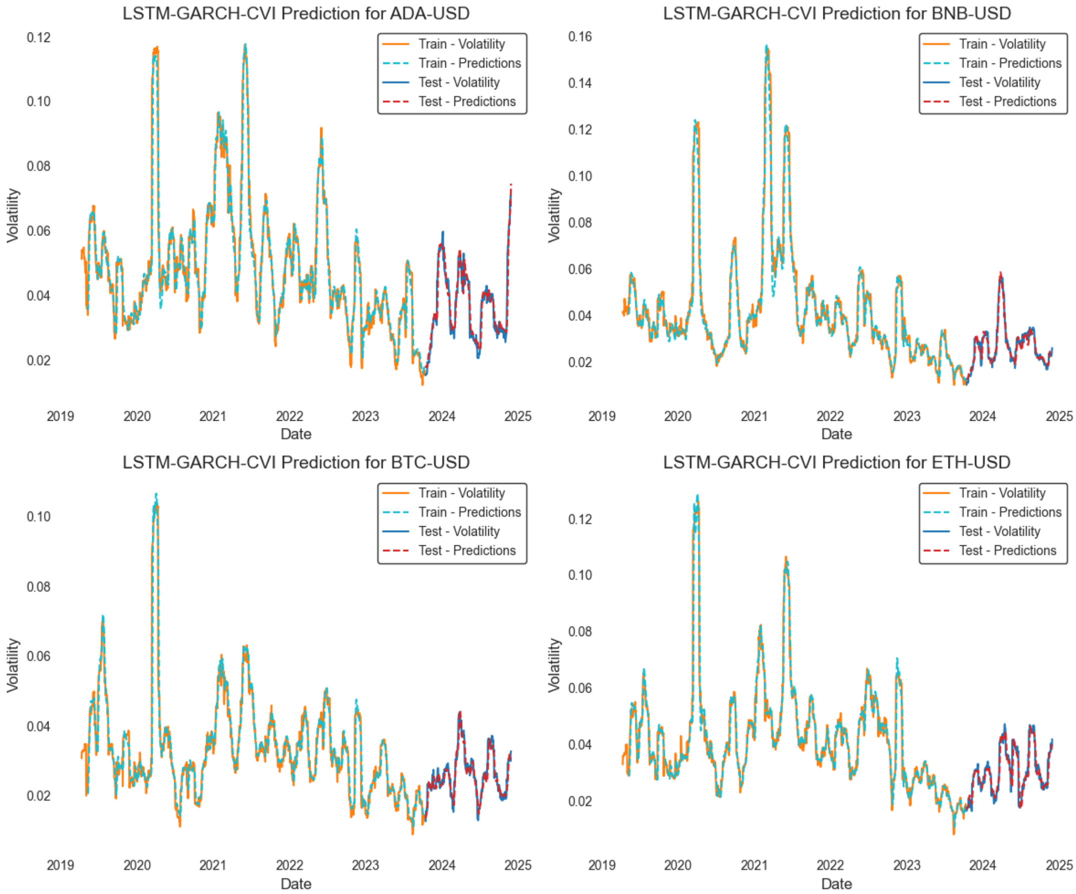

Figure 6: Comparison between volatility forecasts and actual values.

The graphs present the modeling of cryptocurrency volatility incorporating the Crypto Volatility Index (CVI) in a hybrid LSTM-GARCH approach. This combination leverages the long memory of time series through LSTM, captures the persistence of volatility with GARCH, and integrates a market sentiment indicator using the CVI. The results show a strong correlation between the observed volatility and the model predictions, with the ability to closely follow market fluctuations. Volatility spikes, often associated with major events or periods of uncertainty, are well reproduced, indicating that the model effectively captures extreme variations. The addition of the CVI seems to enhance the model's responsiveness to sharp movements, particularly in the post-2023 period, where predictions remain aligned with actual volatility, even during recovery phases. However, while overall accuracy is satisfactory, some misalignments are noticeable, especially in the intensity of the spikes. Some shocks appear slightly attenuated in the predictions, suggesting that the model could still be refined to better anticipate periods of extreme volatility. A comparative analysis with other sentiment indices or optimization of hyper-parameters could further improve predictive performance. The integration of the CVI into cryptocurrency volatility modeling appears relevant, as it allows the model to incorporate market dynamics based on investor sentiment. This approach thus provides a robust framework for anticipating volatility fluctuations, which could be leveraged for risk management or trading strategy optimization.

5.5 Comparison of the Volatility Prediction Model Performances

We first developed simple models, namely GARCH and LSTM, before combining them into the LSTM-GARCH model. Subsequently, we enriched these models with the CVI as a market indicator. In this section, we will compare the performances of these models using RMSE to assess their accuracy in predicting the volatility of crypto-assets.

Crypto-assets	RMSE (GARCH)	RMSE (LSTM)	RMSE (LSTM-GARCH)	RMSE (LSTM-GARCH + CVI)
ADA-USD	0.011771	0.002919	0.003003	0.002778
BNB-USD	0.014770	0.002689	0.002174	0.002336
BTC-USD	0.010623	0.001771	0.001860	0.001767
ETH-USD	0.012883	0.002877	0.003033	0.003185

Table 5: Comparison of RMSE for different models

The results show that the LSTM models outperform traditional GARCH models, with generally lower RMSE for all crypto-assets, indicating better predictive capacity for volatility. Furthermore, integrating the LSTM-GARCH model with the CVI leads to the lowest average RMSE among all the models tested. This improvement suggests that adding the CVI as a market indicator plays a crucial role in refining volatility predictions for crypto-assets, capturing additional market dynamics.

6 Construction of an Optimal Portfolio Based on Volatility Predictions

In this study, we leverage the volatility predictions generated by our hybrid LSTM-GARCH model with CVI input to construct an optimized cryptocurrency portfolio by maximizing the Sharpe ratio. This approach is based on the idea that accurate volatility estimates allow for more effective risk management, leading to optimal asset allocation.

Cryptocurrencies are known for their high volatility and sometimes unpredictable correlations. Using an advanced methodology integrating time series models and deep learning helps improve the robustness of our investment decisions.

6.1 Definition and Importance of the Sharpe Ratio

The Sharpe ratio is an essential measure in portfolio management, introduced by William Sharpe in 1966. It is defined as follows:

$$S = \frac{E[R_p - R_f]}{\sigma_p} \tag{7}$$

where:

- $E[R_p]$ is the expected return of the portfolio.
- R_f is the risk-free rate, typically based on short-term government bond yields.
- σ_p is the standard deviation of the portfolio's returns, representing its overall volatility.

This ratio allows for evaluating the effectiveness of a portfolio by measuring its excess return (over the risk-free rate) adjusted for risk. A high Sharpe ratio indicates that the portfolio provides a higher return for a given level of risk, which is crucial for investors seeking to maximize profits while limiting exposure to losses.

6.2 Portfolio Construction Methodology

Our approach relies on a portfolio weight optimization model based on the volatility forecasts from the LSTM-GARCH model with CVI input. The objective is to maximize the Sharpe ratio under a weight constraint. Thus, we aim to solve the following problem:

$$\max_{w} \frac{E[R_p - R_f]}{\sigma_p} \tag{8}$$

subject to the constraint:

$$\sum_{i=1}^{n} w_i = 1, \quad \forall i, w_i \in [-1, 1] \tag{9}$$

where:

- w_i represents the weight of each asset in the portfolio.
- σ_p is estimated from the volatility predictions of the LSTM-GARCH + CVI model.

The asset weights are dynamically adjusted based on volatility forecasts, rather than using a static approach based on historical volatility. This methodology allows for adapting the portfolio to real-time market conditions and optimizing risk management.

6.3 Numerical Implementation

To implement this optimization, we used the portfolio variance minimization method, combining our volatility forecasts and the recent average returns of the selected cryptocurrencies.
We followed these steps:

1. Extract historical returns over a 30-day period.
2. Estimate future volatility using the LSTM-GARCH-CVI model.
3. Calculate the asset variance-covariance matrix.
4. Optimize portfolio weights by maximizing the Sharpe ratio.

The optimization process was carried out using the Sequential Least Squares Programming (SLSQP) method, which allows finding the optimal weight combination under constraints.

After optimization, we obtained the following results:

Cryptoasset	Optimal Weight
Cardano (ADA)	1
Binance Coin (BNB)	-1
Bitcoin (BTC)	0.5251
Ethereum (ETH)	0.4749
Maximized Sharpe Ratio	**1.0708**

6.4 Performance Analysis of the Optimized Portfolio

The approach developed in this study aims to construct an optimized portfolio based on the maximization of the Sharpe ratio while incorporating advanced risk management through *Value at Risk* (VaR) and *Conditional Value at Risk* (CVaR). The use of the LSTM-GARCH with CVI input model enabled the generation of robust volatility forecasts, which were then used to optimize the asset allocation in our portfolio.

6.4.1 Portfolio Performance Evolution

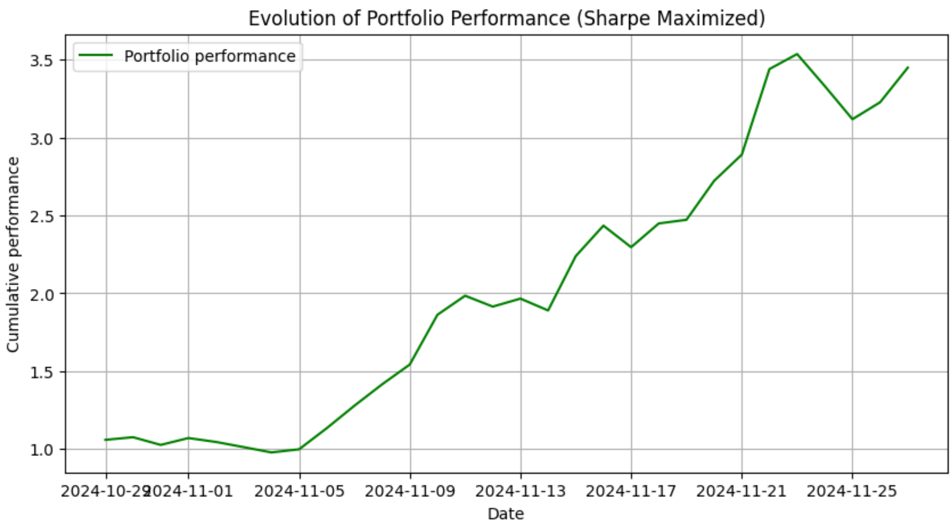

Figure 7: Evolution of the performance of the portfolio optimized based on the Sharpe ratio.

Figure 7 illustrates the evolution of the cumulative performance of the optimized portfolio based on maximizing the Sharpe ratio. The analysis of this curve highlights several key elements:

- **Stable and sustained growth:** From November 5, 2024, the portfolio experiences a period of continuous growth, indicating effective asset allocation and optimized risk management.
- **Progressive performance improvement:** A significant increase in cumulative performance is observed, surpassing a factor of 3.5 by the end of the period. This demonstrates that our volatility-based optimization strategy successfully captured favorable market opportunities.
- **Effective risk management:** Unlike traditional strategies where volatility is estimated solely from historical data, the integration of forecasts from the LSTM-GARCH with CVI input model appears to have played a pivotal role in stabilizing and improving returns.
- **Absence of excessive fluctuations:** Unlike some portfolios that show sharp variations due to poor asset allocation, our optimized portfolio demonstrates a smooth progression, confirming that considering volatility forecasts contributed to better risk management.

These results confirm that using a dynamic approach integrating advanced machine learning models and time series analysis allows for a more effective and market-adjusted portfolio. However, these performances should be tested against periods of economic stress to validate the robustness of this approach in the long term.

6.4.2 Risk Analysis: VaR and CVaR

To complement the portfolio evaluation, a thorough risk analysis was conducted using *Value at Risk* (VaR) and *Conditional Value at Risk* (CVaR). These measures estimate potential losses under different market scenarios and assess the portfolio's exposure to extreme shocks.

Value at Risk (VaR) represents the maximum potential loss of a portfolio at a given confidence level over a specific period. It can be interpreted as the threshold value below which a loss should not occur with a probability higher than α. Formally, it is defined as:

$$\mathrm{VaR}_\alpha = -\inf\{x \in R \mid P(R_p \le x) \ge \alpha\} \tag{10}$$

where R_p represents the portfolio returns, and α is the chosen confidence level.

However, VaR has a major limitation: it provides no information about the magnitude of losses beyond this critical threshold. This is why we also use **Conditional Value at Risk (CVaR)**, which measures the average loss beyond the VaR threshold, providing a more complete estimate of extreme risk. It is defined as:

$$\mathrm{CVaR}_\alpha = E[R_p | R_p \le \mathrm{VaR}_\alpha] \tag{11}$$

Figure 11 shows the distribution of simulated returns for estimating VaR and CVaR using the Monte Carlo method. This approach generates a wide range of market scenarios, assuming that returns follow a normal distribution adjusted according to the previously estimated volatility.

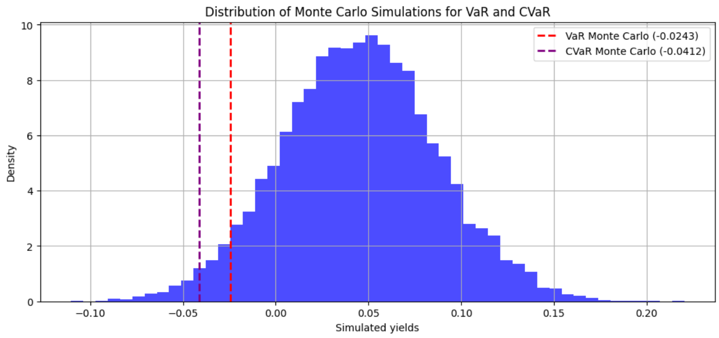

Figure 8: Distribution of Monte Carlo simulations for VaR and CVaR.

The analysis of this distribution highlights several key aspects:

- The **VaR Monte Carlo** is estimated at −2.43%, meaning that with a 95% confidence level, daily losses should not exceed this threshold in 95% of cases.
- The **CVaR Monte Carlo** is estimated at −4.12%, indicating that when losses exceed VaR, the average observed loss is higher.
- A globally symmetric distribution is observed, but with slight asymmetry in the lower part, suggesting that extreme losses may occur more frequently than expected under a purely Gaussian assumption.

In addition to the Monte Carlo approach, we performed an empirical estimate of VaR and CVaR based on the past 30 days' returns. Figure 9 illustrates this analysis.

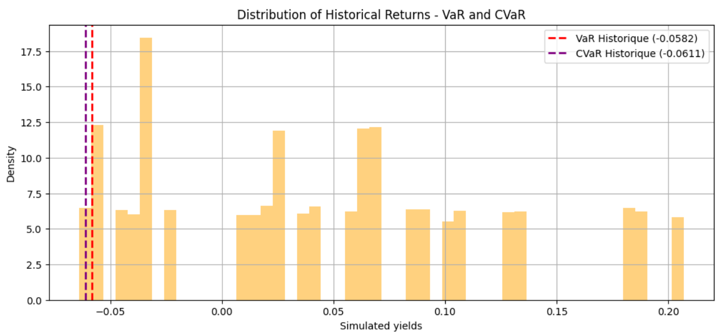

Figure 9: Empirical VaR and CVaR analysis based on historical returns over the last 30 days.

The analysis of historical returns shows that:

- The **Historical VaR** is −5.82%, representing a higher loss level compared to the one estimated by Monte Carlo. This can be explained by the occurrence of recent events that led to extreme losses, which the normal distribution does not fully capture.

- The **Historical CVaR** is -6.11%, indicating that the average losses beyond VaR are even larger than those estimated in the Monte Carlo approach. This observation confirms that the cryptocurrency market may be exposed to extreme risks that are not captured by purely parametric models.

- Unlike the simulated distribution, the historical returns exhibit a more pronounced asymmetry with multiple peaks, indicating irregular volatility.

6.4.3 Summary and Perspectives

The integration of volatility predictions from the LSTM-GARCH with CVI input model into portfolio construction has effectively optimized risk-adjusted returns. The combination of maximizing the Sharpe ratio and risk management through VaR and CVaR provides a balanced approach between performance and control of potential losses.

However, the portfolio's performance remains sensitive to market dynamics and changing volatility regimes. A potential future extension could involve incorporating adaptive approaches, such as dynamic hedging strategies or deep reinforcement learning models, to further enhance the portfolio's resilience to extreme fluctuations in cryptocurrency markets.

7 Portfolio Optimization Based on the Sharpe Ratio with CVaR

Building on our portfolio optimization approach, we have integrated advanced risk measures, namely **Value at Risk** (VaR) and **Conditional Value at Risk** (CVaR). The goal is to improve the management of extreme losses, which are particularly frequent in cryptocurrency markets.

7.1 Definition and Importance of VaR and CVaR

7.1.1 Value at Risk (VaR)

The **Value at Risk** (VaR) is a statistical measure of market risk that estimates the maximum potential loss of a portfolio for a given confidence level. It is defined as:

$$VaR_\alpha = \inf\{x \mid P(R_p \leq x) \geq \alpha\} \tag{12}$$

where:

- R_p represents the portfolio's return.
- α is the chosen confidence level (typically 95% or 99%).

Thus, VaR indicates the maximum likely loss of the portfolio under normal market conditions. However, it provides no information about the magnitude of losses if they exceed this critical threshold, which is why **CVaR** is important.

7.1.2 Conditional Value at Risk (CVaR)

The **Conditional Value at Risk** (CVaR), also known as *Expected Shortfall*, is an extension of VaR that measures the average loss expected when losses exceed the threshold set by VaR. It is defined as:

$$CVaR_\alpha = E[R_p | R_p \leq VaR_\alpha] \tag{13}$$

Unlike VaR, which only captures the critical loss threshold, CVaR accounts for the entire distribution of extreme losses. As such, it is considered a more robust and informative risk measure, particularly useful in volatile markets.

7.2 Sharpe Ratio Optimization with CVaR

The portfolio optimization was adapted by using CVaR as the risk measure instead of traditional volatility. Thus, we redefine the **Sharpe ratio** as follows:

$$S_{CVaR} = \frac{E[R_p - R_f]}{CVaR_\alpha} \tag{14}$$

where:

- $E[R_p]$ is the expected return of the portfolio.
- R_f is the risk-free rate.
- $CVaR_\alpha$ is the *Conditional Value at Risk* at a confidence level α.

The optimization is formulated as:

$$\max_w \frac{E[R_p - R_f]}{CVaR_\alpha} \tag{15}$$

under the constraint:

$$\sum_{i=1}^{n} w_i = 1, \quad \forall i, w_i \in [-1, 1] \tag{16}$$

Thus, the asset weights are dynamically adjusted based on the extreme losses estimated by CVaR, allowing for better risk management during periods of high volatility.

After optimization, we obtain the following results:

Cryptoasset	**Optimal Weight (CVaR)**
Carnado (ADA)	0.7433
Binance Coin (BNB)	-0.9388
Bitcoin (BTC)	0.6139
Ethereum (ETH)	0.5816
Sharpe Ratio (CVaR)	**0.7958**

Our Sharpe ratio is thus lower than that of the previous portfolio, but the weights are slightly more homogeneous.

7.3 Performance of the Optimized Portfolio with CVaR

Optimizing the portfolio by maximizing the Sharpe ratio based on *Conditional Value at Risk* (CVaR) enables a more robust risk management approach that considers not only volatility but also potential extreme losses.

Figure 10 illustrates the cumulative performance of the portfolio optimized using this approach. Unlike the classic Sharpe ratio optimization, which relies solely on volatility as a risk measure, the CVaR approach adjusts asset weights to limit exposure to extreme events.

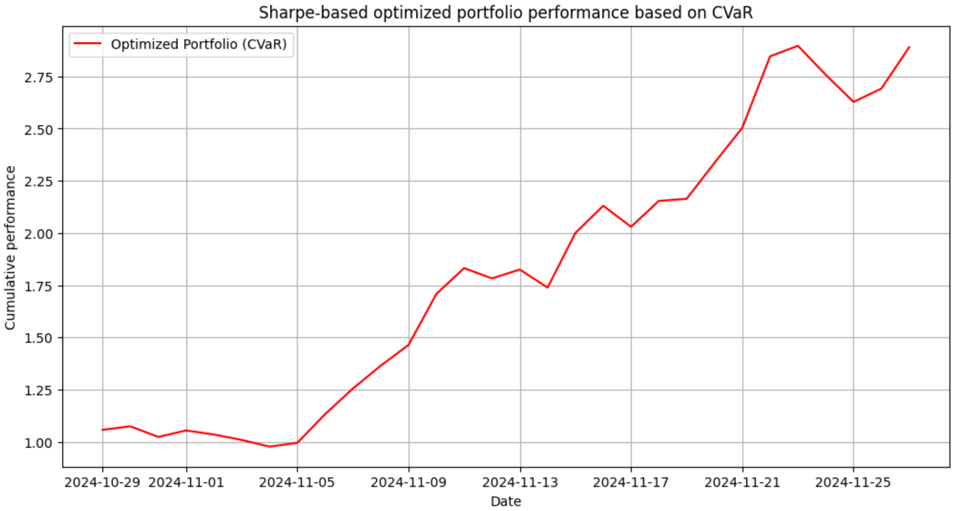

Figure 10: Cumulative performance of the optimized portfolio based on the Sharpe ratio with CVaR.

The analysis of cumulative performance highlights several key differences between the classic optimization and the CVaR-based optimization:

- **Increased stability** : The performance evolution of the CVaR-optimized portfolio shows fewer sharp fluctuations, suggesting better resilience to market variations.
- **More rigorous risk control** : Unlike the standard Sharpe optimization, this approach accounts for the tail distribution of losses, avoiding overexposure to the riskiest assets.
- **More cautious asset allocation** : The asset weights in the portfolio are adjusted more conservatively, thus reducing excessive volatility without compromising portfolio growth.
- **Slightly lower performance** : Compared to our previous portfolio, this one offers marginally lower returns.

The chronological analysis of performance reveals several interesting points:

1. **Initial phase (before November 5, 2024)** : The portfolio shows low growth dynamics, indicating a stabilization phase after the new weightings were applied.
2. **Progressive increase (November 5 - 17, 2024)** : A noticeable acceleration in performance is observed, coinciding with optimal portfolio adjustments in response to market conditions.
3. **Consolidation and risk control (after November 17, 2024)** : Unlike the portfolio optimized only by volatility, the CVaR-based portfolio maintains a smoother progression, with fewer sharp corrections in case of market shocks.

Optimization based on CVaR is a particularly relevant approach for investors aiming to **maximize returns while minimizing extreme losses**. It proves especially effective in highly volatile markets like cryptocurrencies, where unexpected fluctuations can have a significant impact on portfolio performance.

Thus, the joint use of the Sharpe ratio and CVaR in portfolio construction provides an optimal balance between **performance and risk management**, thereby strengthening the robustness of investment strategies in an uncertain environment.

7.4 Interpretation of the Monte Carlo CVaR Results

To evaluate the effectiveness of this optimization, we analyzed the distribution of simulated returns using the Monte Carlo method to estimate *Value at Risk* (VaR) and *Conditional Value at Risk* (CVaR).

The results show that CVaR is always greater in absolute value than VaR, which aligns with its theoretical interpretation. This property is explained by the fact that CVaR accounts for the average of losses beyond the VaR threshold, thus providing a more conservative estimate of extreme risk.

7.4.1 Analysis of Results

- **VaR** represents the loss threshold at a given confidence level (e.g., 95%), meaning that in 95% of cases, losses will not exceed this threshold. It thus sets a lower bound for the risk incurred under normal market conditions.

- **CVaR**, on the other hand, measures the average loss when exceeding the VaR threshold. In other words, it estimates the expected loss when in the worst 5% of market scenarios, providing a more cautious and informative measure of extreme risk.

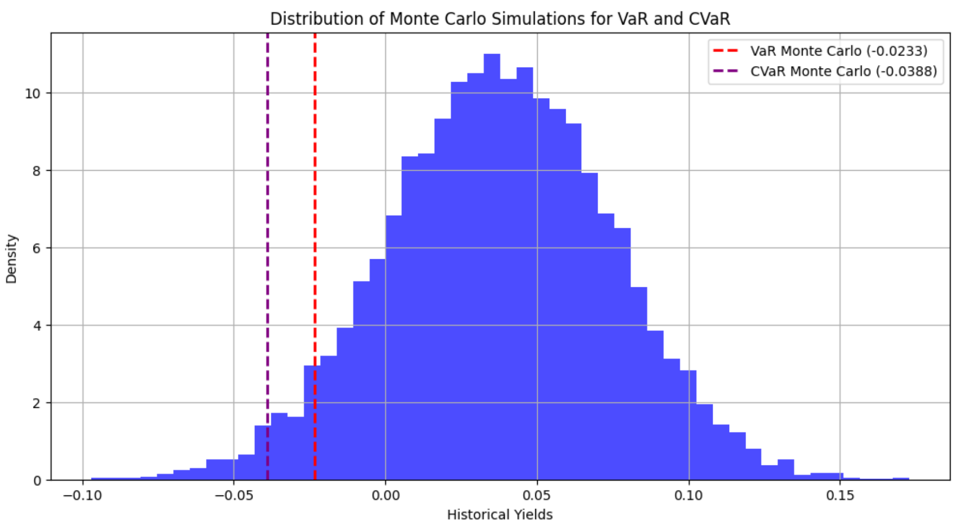

Figure 11: Monte Carlo simulations distribution for VaR and CVaR.

In our case, the Monte Carlo estimate shows that when the loss exceeds VaR, the expected average loss is significantly higher, highlighting the importance of CVaR as a complementary risk management measure. This observation suggests that the model identifies a higher potential risk than that estimated by VaR alone, which is especially relevant in high-volatility contexts such as cryptocurrencies.

7.4.2 Financial Interpretation

Adopting CVaR in portfolio optimization allows for accounting for extreme market scenarios that can significantly impact portfolio performance. By integrating this measure into our optimization approach, we aimed to minimize exposure to severe losses while maximizing risk-adjusted returns. This strategy is particularly useful for investors with high risk aversion or those seeking to stabilize their returns in uncertain market environments.

The results of the Monte Carlo analysis confirm the importance of adopting a cautious approach to asset allocation, considering not only the volatility of returns but also the potential for extreme losses, in order to improve the portfolio's resilience to market fluctuations.

Furthermore, by way of comparison, we observe that the values of VaR and CVaR are slightly lower than those calculated for our previous portfolio. This implies that our portfolio is less risky than the previous one but with slightly lower returns.

7.5 Summary and Perspectives

Portfolio optimization by maximizing the Sharpe ratio adjusted for CVaR allows for a better consideration of extreme risks in the cryptocurrency market. Unlike an approach solely based on volatility, this method incorporates the asymmetric nature of extreme losses, thus offering more robust risk management.

Our results show that the portfolio optimized with CVaR provides a good balance between performance and risk management, with better protection against extreme fluctuations. A potential future improvement would be to integrate real-time CVaR forecasting models, using deep learning techniques to further refine asset allocation.

8 Conclusion

This study explored the modeling and forecasting of cryptocurrency volatility by combining econometric and deep learning approaches. We implemented GARCH models, LSTMs, and a hybrid LSTM-GARCH approach, also incorporating the Crypto Volatility Index (CVI) to enhance the accuracy of volatility estimates. Our results show that the hybridization of models allows for better capturing of the complex dynamics of the cryptocurrency market, providing more robust and market-appropriate forecasts in highly volatile environments.

Based on these forecasts, we constructed two optimized portfolios, each aiming to maximize the Sharpe ratio while adopting distinct risk management strategies. The first portfolio was optimized using a traditional volatility-based approach, while the second incorporated *Conditional Value at Risk* (CVaR) to better account for extreme losses. Our empirical analyses show that the first portfolio offers higher returns but has slightly greater exposure to market risks, while the second, incorporating CVaR, allows for more cautious management of extreme losses with better performance stability.

These results offer interesting perspectives for investors seeking to optimize their exposure to cryptocurrencies. Depending on their risk tolerance, they can choose a portfolio offering higher returns but more volatility or opt for a strategy that incorporates rigorous management of extreme losses. This hybrid approach represents a significant advancement for portfolio management in uncertain environments and paves the way for future research incorporating even more dynamic models, such as reinforcement learning or adaptive risk-hedging strategies.

In conclusion, our approach demonstrates that the use of advanced predictive models, both for volatility forecasting and cryptocurrency portfolio construction, improves risk management and provides investors with effective tools to adapt to a constantly evolving market.