OPERATIONS RESEARCH, Vol. 25, No. 1, January-February 1977

A Survey of Scheduling Rules

S. S. PANWALKAR

Texas Tech University, Lubbock, Texas

WAFIK ISKANDER

C. B. M. Inc., Atlanta, Georgia

(Received original June 13, 1974; final, April 5, 1976)

In the past two decades researchers in the field of sequencing and scheduling have analyzed several priority dispatching rules through simulation techniques. This paper presents a summary of over 100 such rules, a list of many references that analyze them, and a classification scheme.

IN THE PAST two decades there has been a substantial growth in the field of sequencing and scheduling research. This research can be divided mainly into two categories: theoretical research dealing with optimizing procedures limited to the static problems and experimental research dealing with scheduling (dispatching) rules in both static and dynamic cases. Jackson [22] and Moore and Wilson [29] have presented a survey of simulation research. Our main interest here is to provide the reader with a more exhaustive survey of scheduling rules in a form that can be readily used by both practitioners and researchers. We will list over 100 scheduling rules according to different categories. Many of these rules were obtained from some of the pioneering work, such as Conway [7–9], Jackson [22], and Baker and Dzielinski [3]. Other surveyed articles include Gere [18] for his classification of heuristics, some recent work such as Holloway and Nelson [21] and Jones [23], and survey articles such as Moore and Wilson [29].

In Section 1 we present various definitions and the proposed classification of scheduling rules. Section 2 consists of scheduling rules and their description. For each rule, reference numbers of all surveyed articles that use the particular rule will be provided. Section 3 describes the type of problem and measures of performance used in each of the surveyed articles. We provide a cross-indexing, given by listing appropriate rules for each of the surveyed articles.

1. SCHEDULING RULES AND THEIR CLASSIFICATION

In the sequencing/scheduling literature, terms such as scheduling rule, dispatching rule, priority rule, or heuristic are often used synonymously. Gere [18] has made an attempt to distinguish between priority rules, heuristics, and scheduling rules. He considers priority rules as simply a technique by which a number (or value) is assigned to each waiting job according to some method and the job with minimum "value" is selected. Gere defines a heuristic to be simply some "rule of thumb," whereas a scheduling rule can consist of a combination of one or more priority rules and/or one or more heuristics.

Jackson [22] makes a distinction between static and dynamic dispatching rules, the static being the ones in which the job priority values does not change as a function of the passage of time. Conway and Maxwell [11] describe "local" dispatching rules as those that require information only about those jobs that are waiting at a machine, while "global" rules require additional information about jobs or machine states at other machines. Moore and Wilson [29] have categorized a few dispatching rules by a two-dimensional classification showing whether a specific rule is static or dynamic and whether it is local or global.

In this paper we have classified the scheduling rules into the following categories:

I(a). Simple Priority Rules. These are usually based on information related to a specific job such as its due date, processing time, remaining number of operations, etc. In some instances information such as the queue length at the machine where the job will go next is considered simple enough so that rules based on such information are also included in this category. Rules such as random selection that are not dependent on information related to a specific job are also considered to be simple. Subclassification is based on information related to (i) processing times, (ii) due dates, (iii) number of operations, (iv) costs, (v) setup times, (vi) arrival times (and random), (vii) slack (based on processing times and due dates), (viii) machines (machine-oriented rules), and (ix) miscellaneous information.

I(b). Combination of Simple Priority Rules. In many cases these work by dividing a queue into two or more priority groups with different rules applied to different groups. In many instances two rules apply to the same queue under different circumstances.

I(c). Weighted Priority Indexes. These rules are I(a) or I(b) by combining them with different weights. Many research studies involve parameterization of these weights over a specified range.

II. Heuristic Scheduling Rules. These rules involve a more complex consideration such as anticipated machine loading, the effect of alternate routing, scheduling alternate operation, etc. These rules are usually used in conjunction with the rules in Category I. In some cases a heuristic rule may involve nonmathematical aspects of human intelligence, such as inserting a job in an idle time slot by visual inspection of a schedule.

III. Other Rules. These may involve rules designed for a specific shop, combination of priority indexes based on mathematical functions of job parameters, or those rules not categorized earlier.

2. SCHEDULING RULES

Code names used for different rules in this section may or may not coincide with the names used by many researchers. This is due to the fact that a given rule may be known by two or more names. In some instances we have coined an entirely new code name. Categorization presented here is not unique and some rules can be put into different categories.

I(a). Simple Priority Rules (related to)

(*i*) *Processing Time*

1. *SI:* Select the job with the "shortest imminent operation time" (also called SIO, SPT). References 1–5, 7–13, 15, 16, 18, 20, 21, 23, 24, 27, 29, 32, 36.

2. *SIS:* Select the job with the "shortest gross imminent operation time" (which includes processing plus setup). Reference 36.

3. *LI:* Select the job with the "largest imminent operation time." References 2, 3, 7–10, 12, 13, 29, 32, 36.

4. *LIS:* Select the job with the "largest gross imminent operation time." Reference 36.

5. *SR:* Select the job with the "shortest remaining processing time" (for operations not performed). References 3, 7, 8, 10, 12, 29.

6. *LR:* Select the job with the "largest remaining processing time." References 3, 7, 8, 10, 12, 29, 33.

7. *LRM:* Select job with the "longest remaining processing time" excluding the operation under consideration. Reference 33.

8. *SIRIP:* Select the job with the shortest imminent operation time where the processing time is not known a priori and is determined by using normally distributed value of error in estimation. Reference 11.

9. *LSPON:* Select the job with the "longest subsequent operation." Reference 7.

(*ii*) *Due Date*

10. *DD:* Select the job with the earliest due date. References 4, 7, 9, 10, 12, 13, 20, 21, 27, 29, 30, 36.

11. *OPNDD:* Select the job with the earliest operation due date, where equally spaced due dates are assigned to each operation when the job enters the shop. References 7, 9.

(*iii*) *Number of Operations*

12. *FOPNR:* Select the job that has fewest operations remaining. References 3, 7, 8, 10, 29.

13. *MOPNR:* Select the job that has most operations remaining. References 3, 7, 10, 20.

14. *LHALF:* Give a higher priority to the job for which less than one-half of the total number of operations remain to be performed ("last half preference"). Reference 7.

15. *FHALF:* Give a higher priority to the job for which more than half of the total number of operations remain to be performed ("first half preference"). Reference 7.

(*iv*) *Cost Rules*

16. *Value:* Select the job with highest "dollar" value. Reference 10.

17. *1/C:* Select the job that has the highest per-unit penalty cost for tardiness. Reference 30.

(*v*) *Setup Time*

18. *NSUT:* Select the job that requires no setup time. References 1, 20·

19. *MINSEQ:* Select the job or a class of jobs for which the setup time is minumum. Reference 36.

20. *NB:* Select the unassigned job that has the least setup time relative to the job just completed. References 17, 19.

21. *NB′:* Use NB rule but start with a different job as the first job in schedule. References 17, 19.

22. *NB″:* Apply NB rule to the setup matrix after the minimum setup value for each column is subtracted from all values in the column. References 17, 19.

(*vi*) *Arrival Times and Random*

23. *FIFO:* First in, first out. References 3, 4, 7–10, 12, 13, 15, 16, 18, 20, 24, 27, 29, 32.

24. *Random:* Select in random order. References 2, 3, 7–11, 13, 18, 20, 29, 36.

25. *FASFO:* First at shop, first out (the first job arriving in shop goes first). References 7–9, 20, 29.

26. *LIFO:* Last in, first out. Reference 29.

27. *S-1:* Select the job with the least amount of slack (available time before due date-time for remaining operations). References 7, 9, 10, 18, 21, 24, 27, 29, 31.

28. *S-2:* Select the job with the least "static" slack (difference between due date and arrival time). Reference 29.

29. *S-1/OP:* Select the job with the least ratio of slack time to the number of remaining operations. References 1, 4, 5, 18, 23, 24, 26, 29, 31.

30. *S-2/OP:* Select the job with the least ratio of static slack to the number of remaining operations. Reference 29.
31. *JSR:* Select the job with the "least job slack ratio" (ratio of job slack time to the total time available until due date). Reference 18, 24, 29.
32. *RSPT1:* Select the job with the least ratio of the job slack time to the remaining processing time. Reference 24.

(*viii*) *Machine*

These rules may also be considered in category III because of their similarity with "look ahead" type heuristics.

33. *NINQ:* Select the job that will go on to its next operation where the machine has the shortest queue. References 7, 8, 20.
34. *WINQ:* Select the job that will go on to its next operation where the machine has the least work. References 7, 8.

(*ix*) *Miscellaneous Information*

35. *ESD:* Select the job that has the earliest planned start date for its operation (earliest start date). Reference 32.

I(b). Combination of Simple Priority Rules

36. *FIFO/SI:* From jobs waiting for more than a specified time, select according to FIFO (#23); if all waiting jobs are in the queue for a smaller duration, select according to SI (#1). References 7, 11, 12, 29.
37. *$:* Divide jobs in two classes; high and low dollar value. Select the job from the high dollar value with FIFO, then from the other class (used with parameter p, the proportion of jobs in the low dollar value class). Reference 10.
38. *SMOVE:* Select the job that will go on to the next operation where the machine has the shortest "critical" queue (meaning least processing time). If there are no critical queues, use FIFO (used with a parameter Q, which changes the critical status of any machine). Reference 10.
39. *ASIFIFO:* Use SI (#1) for certain time, then FIFO (#23). Repeat cycle. References 11, 29.
40. *SI + JSR:* If at least one job has a negative slack, the priority of all jobs is equal to their slack value and we use S-1 (#27). If all jobs have positive slack, jobs having a job slack ratio less than twice the minimum ratio get the priority by SI (#1); other jobs in queue are not considered. Reference 18.
41. *3CL-FIFO:* Divide jobs into three priority classes and use FIFO (#23). Reference 32.

42. *SEQ:* Considers in-process value of the job, elasped waiting time and the number of operations (can be modified with different proportions). Reference 32.
43. *OPNDDP:* Select job with SI unless the job is behind its (equally spaced) operation due date. Reference 7.
44. *DDNINQ:* Jobs leaving shop after the operation or going to a machine with a small queue ($<Q$) get higher priority. Within this, select according to DD (#10). Reference 7.
45. *FCFS*S:* Use FIFO (#23) if the number of jobs waiting is fewer than Q; otherwise, use SI (#1). Reference 7.
46. *LOPN*S:* Same as rule 45, except use LI (#3) in place of FIFO. References 2, 7.
47. *NINQ*S:* Select the job that will join the shortest queue, if all jobs will join queues with more than Q jobs, use SI rule. Reference 7.
48. *2 CLASS:* Divide jobs in two queues randomly such that, depending upon the time required for the imminent operation, a certain percentage of jobs will join the high-priority queue; then use FIFO (#23). Reference 7.
49. *2C-SI:* Form two priority queues based on the time for the imminent operation; then use FIFO. Reference 29.
50. *2C-TSI:* Prefer a job with negative slack; then apply SI (#1). Reference 29.
51. *Cost/Time:* First select any "critical" job using SI, and then select a job that is late (but not critically late) with the largest ratio of the cost of lateness to operation time; then select a job among the remaining with SI (#1). (This rule is similar to COVERT in [29].) Reference 29.
52. *DDSU:* Select the jobs with the earliest due date; in case of a tie, select the job that requires no setup time. Reference 20.
53. *SI/Q:* Use SI (#1), but if the selected job will join the next machine with a queue of certain length (or greater), try the next job in queue with SI (leave the previously selected job in the queue). Reference 21.
54. *SI/Q_1Q_2:* Form three queues, one with jobs going to a "short" queue for their next operation, one to a "medium," and the third to a "long" queue. Select with SI rule with priority to the first, then the second, and then the third queue. Reference 12.
55. *SOR:* Use SI rule, but give preference to those jobs that will go to "critical" queues. (Queue with only small amount of work waiting.) References 11, 19.
56. *SI/S-1:* Use SI (#1) if an immediate consequence is not a negative slack for jobs in queue. Otherwise, choose the job with the smallest value of imminent operation processing time and slack. Reference 21.

I(c). Weighted Priority Indexes

Generally, in the rules the job with the smallest value of priority index is chosen.

57. *PI1:* Same as S-1 (#27), except attach a weight α to the time available until due date. Reference 12.
58. *P + S-1/OP:* Select the job with the smallest weighted sum of the next processing time and slack time per operation remaining. References 7, 9.
59. *P + WKR:* Select the job with smallest weighted sum of the next processing time and work remaining. References 7, 8.
60. *P/WKR:* Select the job with the smallest weighted ratio of the next processing time to work remaining. References 7, 8.
61. *P/TWK:* Select the job that has the smallest ratio of the next processing time to the total work. References 7, 8.
62. *P + WQ:* Select the job with the smallest weighted sum of the next processing time and work in the next queue. Reference 8.
63. *MSR:* Select the job with the smallest value of the job due date plus the weighted value of the next operation minus the weighted value of the work remaining. Reference 33.
64. *RPT/RT:* Select the job with the minimum ratio of the remaining processing time to the current job slack. Reference 26.
65. *PI3:* Select the job with the highest value of the ratio of the difference between the current time and the time the operation is planned minus a factor based on the flow allowance to the weighted value of the number of remaining operations. Reference 32.
66. *PI4:* A weighted index based on the parameters in Rule 60 and also using (the parameter) remaining work. Reference 32.
67. *P/OPNR:* Select the job with the lowest ratio of the processing time of the imminent operation to the weighted value of the remaining number of operations. Reference 7.
68. *P-SP:* Select the job with the lowest value of the difference between the weighted processing time of the imminent operation and the weighted value for the next operation. Reference 7.
69. *P + NINQ:* Select the job with the smallest value of the sum of the weighted value of the imminent operation and the weighted value of the number of jobs in the next queue. Reference 7.
70. *(P + WQ)/P:* Select the job with the smallest index calculated as the sum of the weighted values of the imminent operation time and work in the next queue divided by weighted processing time of the next operation. Reference 7.
71. *SHOPNE:* Similar to Rule 1, except the time for the imminent operation is estimated from the average value by a weighted function. Reference 7.

72. *PI6:* Index based on due date, transportation time, weighted processing time for the remaining operations, and weighted time for the imminent operation. Reference 22.
73. *PI7:* Index based on time available, mean, and the variance in the processing time for the imminent operation. Reference 14.
74. *PI8:* Index based on due date, transportation time, and the weighted value of the remaining processing time. Reference 34.
75. *PI11:* Select the job with the highest value of the expected penalty (jobs finishing early have negative penalty or bonus). Reference 35.
76. *PI12:* Index based on PI6 (#72) and the modified value of the index in PI6 after the job has been processed. Reference 35.
77. *PI13:* (For parallel processors.) Index based on production requirement in a given period, processing time, and cost. Reference 28.
78. *PI14:* Index based on in-process inventory cost, processing cost, lateness cost, setup cost and time, and external priority. Reference 1.

II. Heuristic Scheduling Rules

79. *Alternate Operation:* If selection of a job according to some simple rule makes another job "critical" (such as negative slack), see the effect of the job already selected. Repeat, if some other job(s) is affected. Reference 18.
80. *Alternate Routing:* Study the effect of routing a job through another (predetermined) set of alternate routing facilities.
    (a) Alternate routing with SI (#1);
    (b) Alternate routing with LI (#3);
    (c) Alternate routing with Random (#24). Reference 2.
81. *Look Ahead:* Study the effect of scheduling a job (determined by a simple rule) on another job that may arrive in the queue before the scheduled job is completed. (All simple rules consider only jobs that are already in queue.) Reference 18.
82. *P*S-1:* Apply SI rule, except when the job is in its final operation and lateness can be avoided by taking this job out of order. Reference 7.
83. *SHOPNH:* Select the job with SI rule, but hold if there are a few jobs in the queue and another job with smaller processing time is expected soon. (Keep machine idle until this job arrives.) Reference 7.
84. *DSIJ:* Use S-1 (#27) for the jobs in a queue and those that will join this specific queue from their current operations. Reference 29.
85. *Insert:* Use Look Ahead (#81); if an idle time slot is observed at the next machine, insert another job from the corresponding queue that can be completed before the critical job arrives at the machine. Reference 18.
86. *Subset:* Determine critical jobs, schedule these first (in their en-

tirety), and then schedule other jobs around the critical jobs. Reference 18.

87. Re-do: If one or more jobs are found to be late with some schedule, re-do the scheduling with adjusted due dates for an improved schedule. Reference 18.

88. Flexibility: Similar to Insert (#85) except "squeezing in" is done after the Gantt chart is laid out to make improvements. Reference 18.

89. Manipulation: Try to improve the schedule by manipulating different operations while the Gantt chart is being laid out. Reference 18.

90. Time Transcending Schedule: Determine priority rating for each job. Schedule the next operation of the job with top priority. Re-evaluate priority and repeat, always scheduling the next operation of the job with top priority. Reference 18.

III. Other Rules

The following rules (91 through 103) can be included in either of the first two categories. However, because of the term "estimated" (anticipated) waiting time or expected work, they are listed below in a separate category.

91. S-3: Same as S-1 (#27), except anticipated waiting time for all operations is subtracted from the slack for each job. Reference 24.

92. S-3/OP: Same as S-1/OP (#29), except subtract the anticipated waiting time from the slack value (numerator). References 6, 24.

93. MJSR: Similar to JSR (#31), except expected delay time is added to each operation time. Reference 18.

94. RSPT2: Same as RSPT1 (#32), except subtract anticipated waiting time for all operations of a job from the slack value. Reference 24.

95. RSPWT1: Same as RSPT1 (#32), except add the anticipated waiting time to the denominator. Reference 24.

96. RSPWT2: Same as RSPT1 (#32), except add the anticipated waiting time for all operations of a job to the denominator and subtract the same quantity from the numerator. Reference 24.

97. RSWT1: Select the job that has the smallest ratio of the job slack time to the expected future waiting time. Reference 24.

98. RSWT2: Same as RSWT1 (#97), except subtract the expected future waiting time from the slack time. Reference 24.

99. RSMWT: Select the job with the least ratio of the job slack time to the total of standard move time plus expected wait time. Reference 31.

100. XWINQ: Select the job that will go on for its next operation to the queue with the least work, both present and expected. References 7, 8.

101. P + XWQ: Select the job that has the smallest weighted sum of the processing time of the imminent operation and the expected work in the next queue. References 7, 8.

102. PI9: Index based on due date, weighted sum of the processing time for the remaining operation, weighted sum of the expected waiting, and the weighted value of the processing time of the next operation. Reference 34.

103. PI10: Select the job with the smallest ratio of the time available until the due date to the lead time remaining (which includes processing times plus expected wait time). Reference 31.

The following four rules were used for a special shop where a job can be simultaneously started on all machines. For convenience the rules are stated as the reciprocals of the actual one with minimum value changed to maximum value.

104. PI14: Select the job with the maximum value of the product of per unit penalty cost and the difference between the completion time of all jobs and the due date of the job. Reference 30.

105. PI15: Same as PI14 (# 104), except the product is divided by the total work load for the machine. Reference 30.

106. PI16: Same as PI14 (# 104), except the product is divided by the maximum processing time for the given job among all machines. Reference 30.

107. PI17: Same as PI14 (# 104), except the product is divided by the minimum processing time for the given job among all machines. Reference 30.

108. OSF: Operation slack factor—an index based on operation due date, processing time of operation, and arrival time of the job. Reference 25.

109. PTF: Processing time factor—an index based on the weighted product of the processing time of ith job and the sum of processing times of jobs at certain machine groups. Reference 25.

110. OUF: Operating urgency factor—an index based on a function of the due date, number of operations, and number of degrees of all operations at a certain level. Reference 25.

111. PCF: Precedent constraint factor—an index based on precedence constraints of a job. Reference 25.

Reference 25 contains several other rules that represent a combination of Rules 108, 109, 110, and 111 with different weights.

112. SI/SI(F): Modify the slack value by subtracting a control parameter to allow for the delays in the system. Schedule jobs with modified slack value less than zero first, with SI rule. Then schedule other jobs with SI rule (# 1). Reference 12.

113. F/SI(F): Calculate the modified slack as in # 112. Schedule the job with the least modified slack value if less than zero; otherwise, schedule according to the SI rule (#1). Reference 12.

3. INFORMATION ON SURVEYED ARTICLES

In this section we will present information on the type of problem, measures of performance, and scheduling rules used or discussed in all surveyed articles. The terms objective criteria, performance criteria, or measures of performance are often employed with similar meanings—we have adopted measures of performance. The number in brackets corresponds to the reference numbers of the articles. For each reference we use the format:

a. Type of problem,
b. Measures of performance, and
c. Rules.

[1] a. Simulation of a hypothetical job shop with 6 machine groups and 1000 jobs per run.
b. Total cost per job, number of late jobs, percent of machine utilization, jobs in queue, number of late jobs in queue.
c. 1, 18, 29, 78.

[2] a. Simulation of a hypothetical job shop with 78 machines (18 groups) and a labor force of 40.
b. Percent of labor utilization.
c. 1, 3, 24, 46, 80.

[3] a. Simulation of a hypothetical job shop involving 9 to 30 machines.
b. Average of the jobs' total manufacturing time, predictability of jobs' completion time.
c. 1, 3, 5, 6, 12, 13, 23, 24.

[4] a. Simulation of a job shop problem with 25 facilities and 60 individual machines.
b. Work-in-process, finished good investment, delays, machine and labor utilization.
c. 1, 10, 23, 29.

[5] a. Analysis of a computerized scheduling system at the El Sugundo Division of Hughes Aircraft Company.
b. Percentage of orders meeting due dates.
c. 1, 29.

[6] a. This article presents a discusson on implementing a job shop scheduling system as well as the operating of the system.
b. None.
c. 92.

[7] a. Simulation of a hypothetical job shop involving 9 machines and 8700 jobs per run.

b. Job lateness, mean lateness, variance of lateness, number of jobs late, total time of jobs in shop, mean shop times, variance of shop time, total number of jobs in queue, total processing time, work remaining, sum of imminent operation processing times of jobs in queue.
c. 1, 3, 5, 6, 9–15, 23–25, 27, 33, 34, 36, 43–48, 58–61, 67–71, 82, 83, 100, 101. (The article lists a total of 92 rules by changing weights in some rules.)

[8] a. Simulation of a hypothetical job shop with 9 machines and 8700 jobs per run. (This work is based on reference 7.)
b. Work remaining, total work content, work completed, imminent operation work content.
c. 1, 3, 5, 6, 12, 23–25, 33, 34, 59–62, 100, 101.

[9] a. Simulation of a hypothetical job shop with 9 machines and 8700 jobs per run. (This work is based on reference 7).
b. Average job lateness, number of jobs late.
c. 1, 3, 10, 11, 23–25, 27, 58.

[10] a. Simulation of a hypothetical job shop with 5 machines and 100 jobs per run.
b. Average lateness, percentage of jobs late, machine utilization, average dollar days of queue inventory.
c. 1, 3, 5, 6, 10, 12, 13, 16, 23, 24, 27, 37, 38.

[11] a. Simulation of a hypothetical job shop and a flow shop with 3 to 9 machines, 2000 jobs per run, and the number of jobs in the shop at any given time held constant to number equal to KN ($2 \leqq K \leqq 6$, N = number of machines).
b. Mean idle time, mean and variance of flow time, mean and variance of flow time per operation.
c. 1, 8, 24, 36, 39, 55.

[12] a. Simulation of a hypothetical job shop with 4 machines and 5000 jobs per run.
b. Job waiting time, queue length, throughput time, delay factor, machine idle time, missed due date time, penalty cost.
c. 1, 3, 5, 6, 10, 23, 36, 53, 54, 57, 112, 113.

[13] a. Simulation of a hypothetical job shop with two identical machines operating in parallel with 1800 jobs per run.
b. Job waiting time, queue length, flow time, delay factor, facility idle time, missed due date time, total number of jobs late, early (or late) completion penalty cost.
c. 1, 3, 10, 23, 24.

[14] a. Simulation of a hypothetical job shop with 10 machines and a few jobs (<100).
b. Job lateness.
c. 73.

[15] a. Simulation of a hypothetical job shop with 3 divisions of 4 work centers each (with 2 identical machines in every center), 12 workers, and 40,000 jobs.
b. Mean flow time, variance of flow time, number of labor transfers between divisions, number of labor transfers between the work centers of a specified division.
c. 1, 23.

[16] a. Same as in reference 15 except additional results are analyzed.
b. Same as in reference 15.
c. 1, 23.

[17] a. One machine, 5 to 20 job problems with setup times.
b. Setup time.
c. 20–22.

[18] a. Simulation of a hypothetical job shop (static and dynamic) with 4 to 6 machines, 6 to 60 jobs, and 1 to 16 operations per job.
b. Sum of tardiness.
c. 1, 23, 24, 27, 29, 31, 40, 79, 81, 85–90, 93.

[19] a. One machine, 5 to 11 job problems with setup times.
b. Setup time.
c. 20–22.

[20] a. Simulation of a hypothetical batch production shop with 5 different parts requiring 2 to 6 operations.
b. Remaining work content, machine idle time, job lateness, individual delay, variance of remaining work content, variance of individual delay.
c. 1, 10, 13, 18, 23–25, 33, 52.

[21] a. Job shop problems with 5 to 7 machines and 6 to 14 jobs.
b. Fraction of tardy jobs, mean tardiness, variance of tardiness, maximum tardiness.
c. 1, 10, 27, 56.

[22] a. Simulation of a real job shop with 8 machines and 70 jobs.
b. On time completion of orders.
c. 72.

[23] a. Simulation of a hypothetical job shop with 4 machines and 5 to 46 jobs per run.
b. Cost of idle machines, cost of carrying work-in-process inventory, cost of long promises, cost of missed due dates.
c. 1, 29.

[24] a. This article describes in detail steps that the authors planned in formulating the model. No simulation results are given.
b. Job flow time, congestion in the shop, job tardiness distribution.
c. 1, 23, 27, 29, 31, 32, 91, 92, 94–98.

[25] a. Simulation of a hypothetical job shop in an assembly environment

in which the jobs have a tree structure of operations (1000 jobs per run).

b. Mean flow time of jobs in shop, mean tardiness, percent of jobs tardy.

c. 108–111, and several combinations of these four rules.

[26] a. This article briefly describes some scheduling procedures used in Hughes Tool Co., Texas Instruments, Fairfield Manufacturing Co., and Western Electric Co.

b. Not specified. A table provides highlights of all four systems.

c. 29, 64 (and minor modification of rule #29).

[27] a. Simulation of an actual shop with parallel processors.

b. Maximum contents, average contents, average utilization, average time of transaction.

c. 1, 10, 23, 27.

[28] a. Scheduling 5 parallel machines with different production rates and setup costs.

b. Primary—total cost; secondary—not required for the type of problems attempted but recommended in general.

c. 77.

[29] a. This article is a review of simulation research.

b. None.

c. 1, 3, 5, 6, 10, 12, 23–31, 36, 39, 49, 50, 51, 55, 84.

[30] a. A special type of shop with 7 jobs and 3 machines, where a job can be processed on three machines simultaneously.

b. Percent of deviation in penalty cost from the penalty cost associated with the optimal schedule.

c. 10, 17, 104–107.

[31] a. This article compares two commonly used priority rules in industry and describes some modifications of these rules used by different companies.

b. None.

c. 27, 29, 95, 99, 103.

[32] a. Simulation of a job shop with 17 machine groups (85 machines). Simulation based on a model of an actual job shop.

b. Mean wait time, utilized capacity, carrying cost, cost ratio, percentage of jobs late.

c. 1, 3, 23, 35, 41, 42, 65, 66.

[33] a. This article describes the state of the art as of 1955.

b. None.

c. 6, 7, 63.

[34] a. This paper presents an informal review of job shop research as of 1958.

b. None.

c. 74, 102.

[35] a. Simulation of flow shop and job shop problems involving 1 to 6 machines and 3 to 20 jobs.
b. Penalty cost.
c. 75, 76.

[36] a. A hypothetical simulation job shop with 9 machines and 2700 processes per run.
b. Value of work-in-progress, number of processes completed in a week, number of jobs out in a week, number of processes late in a week, distribution of completion times, queue wait time of a job, number of jobs waiting, shop utilization, number of waiting jobs for more than a week, size of job waiting more than a week.
c. 1–4, 10, 19, 24.

4. DISCUSSION

We have classified over 100 scheduling rules and made an attempt to explain the general idea behind different rules. Originally, we tried to analyze the results of various researchers and found that the task was quite a formidable one because of conflicting results in many cases. However, the following generalizations and recommendations can be made.

Industrial Applications

1. To use the information from articles described in this paper, it is important to determine the performance criteria. For specified performance criteria, one can then obtain results on the usefulness of different rules.

2. One of the main reasons for obtaining conflicting results in various simulation experiments involving a given rule is the difference in operating conditions. It is therefore essential to know the type of assumptions and shop parameters used before scheduling rules can be selected for proper applications.

3. Many rules described in this paper (Nos. 1 through 20, 27 through 35) are ambiguous in the sense that two more jobs can be chosen with the same priority. Such rules must be used in conjunction with other rules. Rules described in category I(b), for example, are designed to break ties that occur with the simple rules.

4. A consensus among researchers appears to be that a combination of simple priority rules, or a combination of heuristics with a simple priority rule, works better than individual priority rules.

Simulation Research

1. Most research in scheduling involving simulation is based on hypothetical problems. While there is a definite need for such an effort, we feel that it is desirable to have more research based on real problems.

2. There is a need to perform studies involving the application of certain scheduling rules under a wide range of operating conditions to arrive at general conclusions.

3. Results need to be analyzed with the aim of developing guidelines for practitioners.

Finally, we would like to invite readers to send comments and papers describing additional rules and the results of simulations using these rules.

ACKNOWLEDGMENT

Partial support for this work was provided by NSF Grant GK2869.

REFERENCES

1. S. C. Aggarwal, F. P. Wayman, and B. A. McCarl, "An Investigation of a Cost-based Rule for Job-shop Scheduling," *Int. J. Prod. Res.* **11**, 247–261 (1968).
2. M. Allen, "Efficient Utilization of Labor," in J. F. Muth and G. J. Thompson (eds.), *Industrial Scheduling*, pp. 252–276, Prentice-Hall, Englewood Cliffs, N.J., 1963.
3. C. T. Baker and B. P. Dzielinski, "Simulation of a Simplified Job Shop," *Management Sci.* **6**, 311–323 (1960).
4. R. G. Brown, "Simulation to Explore Alternative Sequencing Rules," *Naval Res. Log. Quart.* **15**, 281–286 (1968).
5. M. H. Bulkin, J. L. Colley, and H. W. Steinhoff, Jr., "Load Forecasting, Priority Sequencing, and Simulation in a Job Shop Control System," *Management Sci.* **13**, B29–B51 (1966).
6. J. L. Colley, Jr., "Implementing a Job Shop Scheduling System," *Systems Procedures J.* **19**, 28–33 (1968).
7. R. W. Conway, "An Experimental Investigation of Priority Assignment in a Job Shop," RM-3789-PR, The Rand Corporation, Santa Monica, Calif., 1964.
8. R. W. Conway, "Priority Dispatching and Work-in-Process Inventory in a Job Shop," *J. Indust. Engr.* **16**, 123–130 (1965).
9. R. W. Conway, "Priority Dispatching and Job Lateness in a Job Shop," *J. Indust. Engr.* **16**, 228–237 (1965).
10. R. W. Conway, B. M. Johnson, and W. L. Maxwell, "An Experimental Investigation of Priority Dispatching," *J. Indust. Engr.* **11**, 221–229 (1960).
11. R. W. Conway and W. L. Maxwell, "Network Dispatching by the Shortest Operation Discipline," *Opns. Res.* **10**, 51–73 (1962).
12. S. Eilon and D. J. Cotterill, "A Modified SI Rule in Job Shop Scheduling," *Int. J. Prod. Res.* **7**, 135–145 (1968).
13. S. Eilon and R. M. Hodgeson, "Job Shop Scheduling with Due Dates," *Int. J. Prod. Res.* **6**, 1–13 (1967).
14. W. J. Fabrycky and J. E. Shamblin, "A Probability Based Sequencing Algorithm," *J. Indust. Engr.* **17**, 308–312 (1966).
15. J. S. Fryer, "Operating Policies in Multiechelon Dual-Constraint Job Shop," *Management Sci.* **19**, 1001–1012 (1973).

16. J. S. FRYER, "Labor Flexibility in Multiechelon Dual-Constraint Job Shop," *Management Sci.* **20,** 1073–1080 (1974).
17. J. W. GAVETT, "Three Heuristic Rules for Sequencing Jobs to a Single Production Facility," *Management Sci.* **11,** B166–B176 (1965).
18. W. S. GERE, JR., "Heuristics in Job Shop Scheduling," *Management Sci.* **13,** 167–190 (1966).
19. R. D. HAYNES, C. A. KUMAR, AND J. BYRD, JR., "The Effectiveness of Three Heuristic Rules for Job Sequencing in a Single Production Facility," *Management Sci.* **19,** 575–580 (1973).
20. R. H. HOLLIER, "A Simulation Study of Sequencing in Batch Production," *Opnl. Res. Quart.* **19,** 389–407 (1968).
21. C. A. HOLLOWAY AND R. T. NELSON, "Job Shop Scheduling with Due Dates and Variable Processing Times," *Management Sci.* **20,** 1264–1275 (1974).
22. J. R. JACKSON, "Simulation Research on Job Shop Production," *Naval Res. Log. Quart.* **4,** 287–295 (1957).
23. C. H. JONES, "An Economic Evaluation of Job Shop Dispatching Rules," *Management Sci.* **20,** 293–307 (1973).
24. Y. KURATANI AND R. T. NELSON, "A Pre-Computational Report on Job Shop Simulation Research," *J. Opns. Res. Soc. Japan* **2,** 145–183 (1960).
25. W. L. MAXWELL AND M. MEHRA, "Multiple-Factor Rules for Sequencing with Assembly Constraints," *Naval Res. Log. Quart.* **15,** 241–254 (1968).
26. C. L. MOODIE AND D. J. NOVOTNY, "Computer Scheduling and Control Systems for Discrete Part Production," *J. Indust. Engr.* **19,** 336–341 (1968).
27. C. L. MOODIE AND S. D. ROBERTS, "Experiments with Priority Dispatching Rules in a Parallel Processor Shop," *Int. J. Prof. Res.* **6,** 303–312 (1968).
28. C. L. MOODIE AND J. M. WALTER, "A Scheduling Procedure for Parallel Processors in the Paper Industry," *Technical Papers, Twenty-First Institute Conference and Convention, AIIE*, pp. 339–345, 1970.
29. J. M. MOORE AND R. G. WILSON, "A Review of Simulation Research in Job Shop Scheduling," *J. Prod. Inv. Management* **8,** 1–10 (1967).
30. A. A. B. PRITSKER, L. W. MILLER, AND R. J. ZINKL, "Sequencing n Products Involving m Independent Jobs on m Machines," *AIIE Trans.* **3,** 49–60 (1971).
31. A. O. PUTNAM, R. EVERDELL, G. H. DORMAN, R. R. CROMAN, AND L. H. LINDGREN, "Updating Critical Ratio and Slack-Time Priority Scheduling Rules," *Proceedings of the 14th Annual International Conference of the American Production and Inventory Control Society*, pp. 137–158, 1971.
32. A. J. ROWE, "Toward a Theory of Scheduling," *J. Indust. Engr.* **11,** 125–136 (1960).
33. A. J. ROWE AND J. R. JACKSON, "Research Problems in Production Routing and Scheduling," *J. Indust. Engr.* **7,** 116–121 (1956).
34. R. L. SISSON, "Sequencing in Job Shops—A Review," *Opns. Res.* **7,** 10–29 (1959).
35. A. G. SULLENBERGER AND B. L. FOOTE, "Minimization of Penalty Costs in a Job Shop with Uncertain Processing Times," College of Business Administration, University of Arkansas, Fayetteville, Ark., November 1972.
36. J. K. WILBRECHT AND W. B. PRESCOTT, "The Influence of Setup Time on Job Shop Performance," *Management Sci.* **16,** B274–B289 (1969).