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Vehicle Scheduling Problems with Time-Varying Speed

Yang-Byung Park[1], Sung-Hun Song[2]

[1]Department of Industrial Engineering, Kyung Hee University, Korea and
[2] Department of Industrial Engineering, Hong Ik University, Korea

Abstract

The vehicle scheduling problem with time-varying speed (VSPTS) is a generalization of the vehicle scheduling problem in which the travel speed between two locations depends on the passing areas and time of day. We propose a simple model for estimating time-varying travel speeds in the VSPTS that relieves much burden for the data-related problems. We present three heuristics for the VSPTS, developed by extending and modifying existing heuristics for conventional vehicle scheduling problems. The results of computational experiments demonstrate that the proposed estimation model performs well and the saving method is the best among the three heuristics.

Keywords : Vehicle scheduling problems, Time-varying speed

1. Introduction

The vehicle scheduling problem (VSP) can be described as the problem of designing optimal delivery or collection schedules of vehicles from a central depot to a number of geographically scattered customers, subject to side constraints. The vehicle travel speed is assumed to be constant in most VSP. If travel speed is constant, travel time is directly proportional to distance. However, in nearly all metropolitan areas, speed is subject to rapid change during a day on account of intricate road networks, heavy traffic congestion and some other random situations (accident, road construction, weather, etc.).

The VSP which considers the varying speed brings up two important issues :

(i) Database for the travel times (speeds) between pairs of locations - When N locations including a depot and M time intervals are given, a total of $MN(N-1)$ travel time data are needed. If N and M are augmented beyond a certain point, it will be almost impossible to collect such a great number of data and to store them in memory for computer application. Even if the establishment of the database is possible, the working of computer can be extremely slow on account of the bulkiness of data.

(ii) Scheduling algorithms - The fluctuation of speed and other time-related restrictions (upper bound for travel time, time windows, etc) make it difficult to use the existing algorithms for the conventional VSP and to develop the solutions based on mathematical models.

Research on solving the VSP under varying travel speed has been conducted very recently by a few scholars. Some important papers are by Hill and Benton[1], Malandraki and Daskin[2], and Ahn and Shin[3].

This paper discusses the vehicle scheduling problems with time-varying speed (VSPTS) where the customers to serve in urban area are changed every day. First, we propose a simple model for estimating time-varying travel speeds. Second, we present three heuristic algorithms for the VSPTS that makes use of the proposed estimation model for time-varying travel speeds. Finally, the results of computational experiments are demonstrated.

2. An Estimation Model for Time-Varying Travel Speeds

The average travel speeds between pairs of locations can be computed by using the estimated travel speeds in divided areas and discrete time intervals. This approach reduces dramatically the data and its storage requirements to solve the VSPTS.

Hill and Benton[1] proposed for the first time a model to measure the average travel speed between two locations, but their model has some important weaknesses. We propose a new model for estimating area and time-dependent travel speed between two locations that resolves these problems. On the basis of the significance of the variation in vehicle travel speed, the whole service area is divided into D areas and the daily service time into M discrete time intervals.

A simple formulation for computing the average travel speed from location i to location j starting in time interval T, S_{ijT}, is presented :

$$S_{ijT} = \alpha_{ij} S_{[i]T} + (1 - \alpha_{ij}) S_{[j]T} \quad (1)$$

In the equation, $[i]$ means the area of customer i, $S_{[i]T}$ the average travel speed at $[i]$ during time interval T, and α_{ij} the extent to which the travel speed in $[i]$, in comparing with the travel speed in $[j]$, affects the determination of S_{ijT} while a vehicle travels from location i to j. It is a constant between 0 and 1.

The average travel speed in area Q during time interval T, S_{QT}, can be estimated by minimizing the sum of the weighted squared differences between the actual historical speed R_{ijT} and the S_{ijT} computed from (1). The sum of the weighted squared deviations is a convex function of S_{QT}.

$$\text{Min } Z = \sum_{I=1}^{D} \sum_{i\in r(I)} \sum_{J=1}^{D} \sum_{\substack{j\in r(J)\\ j\neq i}} \sum_{T=1}^{M} L_{ijT}[R_{ijT} - \{\alpha_{ij} S_{IT} + (1-\alpha_{ij}) S_{JT}\}]^2 \quad (2)$$

Here, $r(I)$ represents the customer locations belonging to the area I and L_{ijT} means the weights on the squared differences.

Two types of L_{ijT} are proposed :

(i) $L_{ijT} = R_{ijT} / d_{ij}$ (3)

This means a reciprocal number of the travel time from location i to j during time interval T, which is more sensitive to errors between the locations with shorter travel times.

(ii) $L_{ijT} = (R_{ijT} / d_{ij})^2 / \sum_{J=1}^{D} \sum_{\substack{k\in r(J)\\ k\neq i}} (R_{ikT} / d_{ik})^2$ (4)

This denotes the probability that a vehicle travels from location i to j starting in time interval T. The equation is based on Huff[4]'s idea.

(i) and (ii) are formulated on the fact that the vehicle routes in the conventional VSP are constructed in the petal shape. If there exist many historical observations of speeds (R_{ijTl}, l = 1,2,...,L), the average is used as R_{ijT}.

The optimal S_{QT} can be found by taking the derivatives of (2) with respect to S_{QW} for two cases of $I=Q$, $T=W$ and $I\neq Q$, $J=Q$, $T=W$, and setting the derivatives to zero.

$$S^*_{QT} = \{2 \sum_{i\in r(Q)} \sum_{J=1}^{D} \sum_{\substack{j\in r(J)\\ j\neq i}} L_{ijT}\alpha_{ij}R_{ijT} + \sum_{i\in r(Q)} \sum_{\substack{j\in r(Q)\\ j\neq i}} L_{ijT}R_{ijT} - 2 \sum_{i\in r(Q)} \sum_{\substack{J=1\\ J\neq Q}}^{D} \sum_{j\in r(J)} L_{ijT}\alpha_{ij}(1-\alpha_{ij})S_{JT} + 2\sum_{\substack{I=1\\ I\neq Q}}^{D} \sum_{i\in r(I)} \sum_{j\in r(Q)} L_{ijT}(1-\alpha_{ij})R_{ijT} - 2\sum_{\substack{I=1\\ I\neq Q}}^{D} \sum_{i\in r(I)} \sum_{j\in r(Q)} L_{ijT}\alpha_{ij}(1-\alpha_{ij})S_{IT}\} /$$

$$\{2 \sum_{i\in r(Q)} \sum_{J=1}^{D} \sum_{\substack{j\in r(J)\\ j\neq i}} L_{ijT}\alpha_{ij}^2 + 1.5 \sum_{i\in r(Q)} \sum_{\substack{j\in r(Q)\\ j\neq i}} L_{ijT} + 2\sum_{\substack{I=1\\ I\neq Q}}^{D} \sum_{i\in r(I)} \sum_{j\in r(Q)} L_{ijT}(1-\alpha_{ij})^2\} \quad (5)$$

S_{ijT} is then computed by applying the estimated $S^*_{[i]T}$ and $S^*_{[j]T}$ to the formula (1). A minimum of MD travel speed data between pairs of locations are only required to estimate all S_{IT} (I = 1,2,...,D; T = 1,2,...,M) using (5). The number of values which need to be stored in database are only MD that is much less than $MN(N-1)$ required for the case of using R_{ijT}.

3. Algorithms for the Vehicle Scheduling Problems with Time-Varying Speed

The mathematical model of the VSPTS is different from that of the general VSP because, in the VSPTS, the travel time between two locations varies depending on the departure time interval. The VSPTS may be formulated as the mixed integer nonlinear programming model.

It is extremely difficult to get an optimal solution since the mathematical model belongs to NP-hard problem. We suggest simple three heuristic algorithms for the VSPTS that makes use of the proposed estimation model for time-varying travel speeds. These have been developed by extending and modifying well-known route-first building heuristics that were originally developed for conventional VSP.

Discussion is focused on extensions and modifications required to accommodate area and time-dependent travel speeds, rather than on the repetition of details for each heuristic.

① Saving technique (SAT)

The saving technique developed by Clarke and Wright [5] is extended and modified. The saving which can be obtained from the combination of two routes by connecting the last location i on one route and the first location j on the other route is a function of departure time from location i as follow:

$$SV_{ij}(\omega_i) = t_{i1}(\omega_i) + t_{1j}(\omega_1) - t_{ij}(\omega_i) + \beta$$

if $\omega_j{}' = \omega_j$, $\beta = 0$ and

if $\omega_j{}' \neq \omega_j$, $\beta = [TT(j) - \omega_j] - [t_{jk}(\omega_j{}') + t_{kl}(\omega_k{}') + \ldots\ldots + t_{r1}(\omega_r{}')]$ (6)

where location 1 = depot ;

ω_i = departure time from location i ;

ω_j' = departure time from location j which is newly determined by combining two routes ;

$t_{ij}(\omega_i)$ = travel time from location i to j when a vehicle departs at ω_i ;

$TT(j)$ = total travel time of the route which includes location j before combining two routes.

② Proximity priority searching technique (PRT)

The heuristic developed by Williams[6] is modified. This method connects a location furthest from the depot at the very first and joins continuously the closest feasible locations within the immediate proximity in terms of travel time. That is, the next location j to visit after the location i is determined by $j = \arg\min_k t_{ik}(\omega_i)$. k means the feasible locations which are not included in the routes yet and satisfy the vehicle capacity and travel time restrictions.

③ Insertion technique (INT)

The saving method suggested by Mole and Jameson [7] is extended and modified. The saving from inserting a new location c between the locations i and j on the route is obtained as follow:

$$SV_c(i,j) = t_{1c}(\omega_1) + t_{c1}(\omega_c) + t_{ij}(\omega_i) - t_{ic}(\omega_i) - t_{cj}(\omega_c) + \beta$$

$$\text{if } \omega_j' = \omega_j,\ \beta = 0 \text{ and}$$

$$\text{if } \omega_j' \neq \omega_j,\ \beta = [TT(j) - \omega_j] - [t_{jk}(\omega_j') + t_{kl}(\omega_k') + \ldots\ldots + t_{r1}(\omega_r')] \qquad (7)$$

where ω_j' = departure time from location j newly determined by inserting location c in the route;

$TT(j)$ = total travel time of the route before inserting location c.

4. Computational Experiments

Three test problems were randomly constructed for evaluating the estimation model for time-varying travel speeds and the scheduling heuristics for the VSPTS proposed in this paper. Table 1 is a summary of their characteristics. In each test problem, the whole service area was divided into several regular squares. The daily service time was evenly divided at the interval of 2 hours.

Table 1. Summary of Three Test Problems

Problem Number	Number of Locations	Number of Areas	Number of Time Intervals
1	8	6	4
2	15	9	5
3	22	9	5

First, the proposed model and the modification of the Hill and Benton[1]'s model for estimating time-varying travel speeds were applied to each of the three problems in order to evaluate their performances. In the proposed estimation model, both equations of (3) and (4) were applied for L_{ijT} . We call the estimation model differently as the proposed model 1 and the proposed model 2, depending on the application of equations of (3) and (4).

For the evaluation of the estimation models, the average errors between the artificial observations and the estimated values for travel speeds were calculated. The average errors of the travel speeds between the locations in the same and adjacent areas were separately calculated and used for the evaluation.

Table 2 shows the average errors in the whole area and the average errors in the same and adjacent areas for the three estimation models. The proposed model 1 is the most accurate with the whole average error rate of about 7%.

Table 2. Comparison of Average Errors of Three Estimation Models in Three Test Problems

(km/hr)

Prob-blems	Proposed Model 1		Proposed Model 2		Modification of Hill & Benton's Model	
	a	b	a	b	a	b
1	2.01	1.97	2.07	2.00	2.67	2.38
2	2.19	2.14	2.21	2.15	2.81	2.68
3	2.17	2.16	2.20	2.18	2.87	2.69

(note) a: the whole average error

b: the average error in the same and adjacent areas

Next, the three proposed heuristics were applied to the three test problems in order to evaluate their performances. The programs were written in Quick Basic and implemented on an IBM compatible 586 PC. A subroutine was constructed for

Table 3. Performance Comparison of Three Heuristics in Three Test Problems

Problem	SAT Routes	SAT Total travel time(hr)	SAT Computation time(sec)	PRT Routes	PRT Total travel time(hr)	PRT Computation time(sec)	INT Routes	INT Total travel time(hr)	INT Computation time(sec)
1	1-2-6-1 1-3-1 1-8-4-5-7-1	12.67	0.11	1-2-1 1-5-4-3-1 1-8-7-6-1	13.14	0.06	1-2-6-1 1-3-1 1-8-4-5-7-1	12.67	0.11
2	1-6-5-4-1 1-13-12-11-1 1-14-9-3-2-7-8-1 1-15-10-1	24.73	1.05	1-2-4-3-9-12-1 1-5-6-1 1-11-7-8-14-1 1-15-10-13-1	33.31	0.11	1-6-5-4-1 1-10-9-3-2-7-8-1 1-13-12-11-1 1-15-14-1	27.65	0.60
3	1-5-7-8-4-6-1 1-9-2-3-11-10-1 1-14-15-1 1-17-16-18-19-1 1-20-22-21-13-12-1	27.28	4.45	1-2-3-4-6-12-8-1 1-16-17-19-10-9-11-1 1-7-5-13-21-22-20-1 1-18-15-14-1	38.92	0.16	1-5-7-8-6-3-2-9-1 1-10-1 1-13-12-1 1-15-14-22-21-20-1 1-17-16-18-19-4-11-1	29.60	1.82

computing travel speeds between locations based on the proposed estimation model 1 and called whenever needed in applying the heuristics. Table 3 summarizes the performance comparison of the three heuristics. The total travel time was calculated by using the artificially generated data for R_{ijT} instead of the estimated travel speeds. The computation time covers all the computer working time excluding the time for input and output.

It is found from the Table 3 that the SAT produces the routes with the shortest total travel time in all the test problems, followed by INT and PRT. But the SAT requires longer computation time than the other two heuristics.

5. Conclusion

This paper discussed the vehicle scheduling problems with time-varying speed where the customers to serve in urban area are changed every day. From the computational experiments, the proposed model estimated the travel speeds between locations more accurately than the modification of the Hill and Benton's model. It was also found that the saving method(SAT) performed best among the three proposed heuristics with respect to the minimization of total travel time. More extensive computational experiments need to be performed on real data collected from congested urban networks

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