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Competition-based neural network for the multiple travelling salesmen problem with minmax objective

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Scope and purpose

Recently, the neural networks have been one of much interest to OR community; in particular to those dealing with the combinatorial optimization problem such as travelling salesman problem (TSP). Nevertheless, the neural approaches are difficult to apply to more complex problems as additional constraints must be satisfactorily considered. In addition, the comparison between the heuristic approaches and the neural approaches has never been reported before. In this paper, an adaptive neural network approach has been proposed for solving the minmax multiple travelling salesmen problem, which is an extension of the TSP, and it is compares with previous heuristics: elastic net, generalized 2-opt exchange heuristic and adaptive tabu search heuristic.

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Abstract

In this paper, a new algorithm in competition-based network has been introduced to solve the minmax multiple travelling salesmen problem (MTSP) which needs the maximum distance among all salesmen to be minimum. As in the previous approaches, the generalized 2-opt exchange heuristic algorithms and the elastic net algorithm are reviewed and applied to the minmax MTSP problem solution. Furthermore, a comprehensive empirical study has been provided in order to investigate the performance of the algorithms. The adaptive approach can obtain the superior solution in all instances, compared to the generalized 2-opt exchange heuristic and the elastic net. In additional evaluation, the adaptive algorithm is combined with a simple improvement heuristic and compared with a recently adaptive tabu search. As a result, the adaptive approach can obtain the appropriate solutions 3% in average of the best solution of the adaptive tabu search heuristic accompanied with the higher speed of 31% in average.

Keywords: Competition-based neural network; Multiple travelling salesmen problem; Optimization

1. Introduction

Routing problems are one of the extensively studied problems in Operation Research literature. A number of meaningful variants and extensions have been analyzed [1]. Traditionally, the TSP [2] is the most famous and well-studied problem of routing problems because many vehicle routing applications are its variants. Several exact and approximate algorithms have been designed to this problem, for a recent overview of algorithms see Laporte [3]. Similar to most of the routing problems, the multiple travelling salesmen problem (MTSP) is an extension of TSP. This problem relates to accommodating real world problems where there is a need to account for more than one salesman. The MTSP can be generalized to a wide variety of routing and scheduling problems; for example, the School Bus Routing Problem [4, 5] and the Pickup and Delivery Problem [6, 7]. In [8], it has shown to be an appropriate model for the problem of bank messenger scheduling, where a crew of messengers pick up deposits at branch banks and returns them to the central office for processing. Therefore, finding a good optimal solution method for the MTSP is important and induces to improve the solution of any other complex routing problems. However, the MTSP is NP-complete as it includes the TSP as a special case.

The MTSP can be stated as follows: There are m salesmen who must visit a set of n cities, and each salesman is defined to start and end at the same depot. In this problem, each city must be visited exactly once by only one salesman and its objective is to find the minimum of total distances travelled by all the salesmen. Several authors [8, 9] suggested transforming the MTSP with m salesmen and n cities into a TSP with $n+m-1$ cities by the introduction of $m-1$ artificial depots $(n+1, \ldots, n+m-1)$. However, the resulting TSP is highly degenerate [10], when an MTSP is transformed to a single TSP since the resulting problem is more arduous to solve than an ordinary TSP with the same number of cities. Using a constraint relaxation approach to solve the MTSP directly, which is more efficient, has been previously reported by Laporte and Nobert [11].

While the general objective of the MTSP is to minimize the total distance which can be called minsum criterion, França et al. [12] have argued that the minmax criterion that minimizes the cost of the most expensive route among all salesmen is more appropriate in the real world problems; for instance, the distribution of gas in the province of Namur, Belgium, is described to be such a one by Giust [13]. In this problem, the equitable distribution of workload for each vehicle is important. Therefore, the m salesmen should be assigned delivery routes of similar lengths. In this research, we will consider the MTSP problem with minmax constraint and use the name "the minmax MTSP" throughout the paper.

On the other hand, the recent study of artificial neural networks for the TSP has been extensively considered by several researchers [14–19] as a standard problem in the operation research literature. While the accomplishments of adaptive neural networks (NN) for the TSP solution (in particular, the self-organizing feature maps (SOFM) of Kohonen [20]) are broadly introduced in [16, 17, 21, 22], they have not been used for solving the MTSP and such possibility has not received wide attention. Accordingly, the propose of this study is to apply SOFM (which is competition-based neural network) to solve the minmax MTSP. In this paper continuing our work in [23], simple adaptation of SOFM has been proposed to solve the minmax MTSP. The results of simulation are satisfactory, compared to other heuristic algorithms. Additionally, this proposed algorithm has been combined with, a simple improvement heuristic, the 2-opt in order to advance in comparison with the adaptive tabu search as a high effective method in recent.

This paper is organized along the following lines. In this section, an introduction is presented. The descriptions of heuristic algorithms and self-organizing networks are given in Sections 2 and 3, respectively. In Section 4, our algorithm is explained and the simulation results of the proposed method in comparison with other algorithms are reported. Comparison with adaptive tabu search is also presented. In the last section, conclusions and further research are discussed.

2. Heuristic algorithms

Potvin et al. [24] generalized a classical 2-opt exchange heuristic to solve a minsum MTSP by considering exchanges leading to the partition of a single tour into a number of smaller subtours. This procedure is named "a generalized 2-opt exchange". The property of the classical 2-opt exchange heuristic [25, 26], as improvement heuristic, is to modify the current solution by replacing 2 arcs in the tour by 2 new arcs, so as to generate a new improved tour. For the TSP, any subtour leads to infeasible solutions, but for the MTSP, solutions with multiple subtours can be feasible. A solution with multiple subtours is feasible if and only if at least one copy of the depot is included in each subtour and it is then said to be "a new feasible solution". Thus, the generalized 2-opt exchange procedure has considered all possible 2-changes leading to new feasible solutions.

The generalized 2-opt exchange procedure has been defined as follows:

Step 1 Generate an initial solution.

Step 2 Try to decrease the length of the tour using a classical or a generalized 2-change.

Step 3 If the exchange leads to multiple subtours, then regenerate an equivalent unique tour by an exchange of "equivalent edges" as shown in Fig. 1. However, this step can be neglected since for the MTSP, the complexity is in the order of m which is usually very small as compared with the number of edges in the tour.

Step 4 Repeat step 1 and 2 until no more improvement is possible.

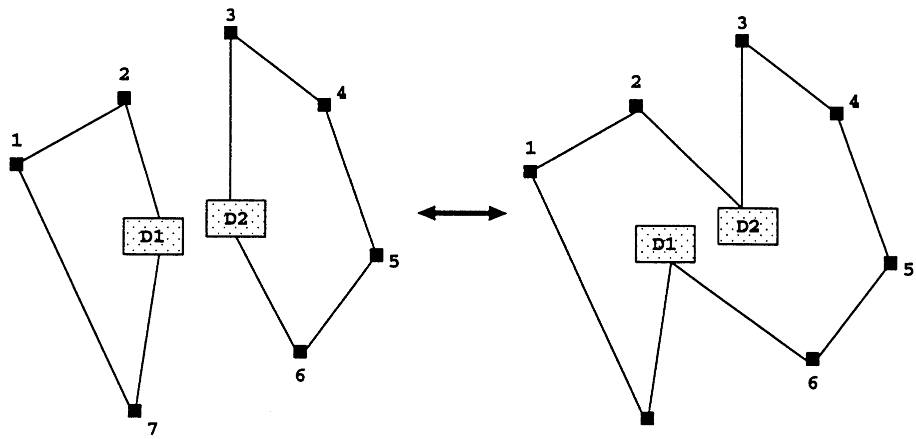

Fig. 1. Regenerating an equivalent unique tour.

Potvin et al. suggested that generalized 2-opt exchange procedure is suitable for constrained problems. Accordingly, this procedure was applied in their research on both "constraint-free" MTSP (minsum MTSP) problems and MTSP problems with time windows. In order to generate the initial solution, they introduced a farthest insertion heuristic. Before this work, Frederickson et al. [27] provided approximate algorithms for the minmax MTSP. They proposed two basically different methods for constructing m-tour. The first method is to construct m tours simultaneously by using a least cost insertion criterion or a nearest neighbour criterion. The second is to construct an m-tour by splitting a good TSP solution. In the first method, the construction procedures such as nearest insertion and nearest neighbour heuristics have been considered.

As in previous researches, we apply the generalized 2-opt exchange procedure to minmax MTSP problem. For selecting the construction procedure of the initial solution, we consider the generalizations of the farthest insertion procedure as Potvin et al.'s work and the nearest insertion procedure to handle m salesmen. As in the generalization of the nearest insertion of Frederickson et al., the farthest insertion replaced the minimum distance between a node not in route and any node in route with the farthest distance in order to select the node to insert the route.

3. Self-organizing networks

The first artificial neural network (NN) was applied to the TSP by Hopfield and Tank [14]. Afterwards, this network was extended to the MTSP by Wacholder et al. [28]. Unfortunately, the complexity of the model and its inability to guarantee feasible solutions have been noted as two major weak points [15, 18] of this approach.

In contrast to the Hopfield and Tank network which is not trained on problem data but configured from it, the self-organizing network is trained on repetitive presentations of the problem data by itself. The basic self-organizing network processes which are somewhat similar to the simpler phenomena of human brain occur in the retinotectal mappings. It is related to an interconnection neural structure. Based on the inputs that a neuron receives, a certain output is computed which is propagated to other neurons. The determination of outputs and propagation of signals in the network is given by some kind of learning and functioning properties. In the next section, an elastic net and competition-based neural networks have been discussed. Both of them are classified into the self-organizing networks, while the way by which the conditions of nodes are updated differs.

3.1. Elastic net

The elastic net was firstly introduced by Durbin and Willshaw [29] as a new approach to solve the TSP. They proposed a network which adaptively forms a tour of the city in response to problem data. Initially, a small tour(ring) on which several nodes lie is situated in the centre of the cities. In order to form the route, the nodes on the ring gradually and non-uniformly move until all cities are claimed by its corresponding node in the appropriate distance. Similar to the Hopfield and Tank network, the elastic net utilizes the property of the convergent energy function in order to stretch the ring towards each city.

Furthermore, Vakhutinsky and Golden [30] have proposed the elastic net to solve the MTSP with m salesmen by constructing initially, m small rings going out of the depot. Thus, the starting node of each ring is the depot. As with the TSP, by using an iterative procedure the rings are gradually elongated according to a convergent energy function. The update of the positions of the nodes on each ring depends on two forces: the first one advances the node towards the nearest city and the second one, that favors the shortest tour, pulls the node towards its neighbours. At each complete cycle [when all the cities have been presented], the starting node of each ring will be reset to be the depot. This iterative procedure is implemented until all the cities are matched by the ring nodes. However, Vakhutinsky and Golden reported that the computational time was worse although their obtained results were approximately 10% better than Ghaziri [31].

3.2. Competition-based neural networks

As in the elastic net's concept, the competition-based neural network views a tour as a mapping from a circle to the plane so that each city in the plane is mapped to by some node on the circle. Nevertheless, the property of updating nodes on the circle is different as discussed previously.

While the elastic net approach realizes in a gradient descent energy function, the competition-based approach realizes in a competitive learning mechanism in order to enlarge the ring. This competitive learning is known as Kohonen's work [20] on the self-organizing feature maps (SOFM) which is a self-organization training principle (as opposed to a supervised training principle). Recently, Goldstein [32] extended Angeniol's TSP algorithm to solve the MTSP. In the Goldstein's process, at the beginning, m rings where only one node exists for each ring have been defined. The number of nodes grows subsequently according to a node creation and deletion process similar to the Angeniol's algorithm. Through an iterative procedure, the nodes on all m rings are freely moved in the plane according to the update function, eventually an actual solution is obtained when every city has caught one node of a ring.

To distinguish from solving a TSP problem, the competition's rule for selecting the winner node has been extended by the deviation of a corresponding salesman's distance which is defined in [32]. Although Goldstein did not apply this algorithm to the minmax MTSP problem, the additional constraint of his algorithm may be considered to be appropriate for solving the minmax MTSP. Because the additional constraint in the competition's rule implicitly considers the probability of generating the same distance for each route, this algorithm is more appropriate to the minmax MTSP than the minsum MTSP. Goldstein performed simulations on small sets of cities problems and reported that in all such cases satisfactory solutions were obtained.

4. The proposed method

In this paper, we slightly extend our previous research [23] to solve the minmax MTSP by the additional condition in the competition's rule. To consider the efficiency of the method, the real problem with various sizes of cities have been simulated. However, the facility of parameters determination and problems adaptation is on importance in our consideration. Furthermore, the update function is considered to obtain a faster convergence.

4.1. Algorithm

We propose a competition-based approach to solve the minmax MTSP by considering two main points. Firstly, the consideration of the selection winner node under the competition's rule with an additional minmax constraint. Secondly, the consideration of the neighbourhood function to update the coordinates of nodes.

To confer stability on the algorithm and reduce its dependence on the initial configuration and the order of input data, the order of cities is randomly redefined after one complete cycle of iterations in our algorithm. Additionally, the neighbourhood function which determines the influence of the presented city on the neighbour nodes of the winner is limited to update only 40% of the nodes in our algorithm. Because the effect of this force lessens as the proximity of nodes decreases, this function has practically no meaningful effect on some nodes which are far from the presented city. Therefore, decreasing the domain of influence for this force will diminish the computation time without any meaningful effect on the behaviour of the network. The value of the neighbourhood function for nodes with a cardinal distance of more than 15 is indicated when there is no meaningful effect, as has been described in [23]. In view of this fact, the effective neighbourhood length in our algorithm which refers to the number of nodes which are updated in one presentation of the input data, is limited to 40% of the nodes on one ring. According to our practical experience, increasing the number of update nodes is more than 40% has no effect on the quality of results, but saves substantial computation time.

Followed by other previous methods in a TSP problem, determination of the appropriate number of nodes is one of the realizations of the effectiveness of method. If it is too large, it needs the computational process. On the contrary, the process of node separation will be more delicate and the results will be sensitive to the initial configurations when the number of nodes equal to the number of cities. These conditions appear to be the same as in applying this approach to solve the minmax MTSP problem. In our algorithm, m small rings which depart from the same depot are initiated and each ring consists of $2n/m$ nodes.

The proposed algorithm may be described as follows:

Step 0 (Initialization): Let $n =$ the number of cities, $m =$ the number of salesman and $M =$ the number of nodes on one ring ($M = 2n/m$). At the beginning, m rings are initially defined by M nodes that are positioned on one small ring, where the starting node and the ending node is the depot. Let $\alpha = 0.1$; learning rate, $\mu = 0.6$ and the initial value of gain parameter, $G_0 = 10$ (for example).

Step 1 (Randomizing): Randomize the order of cities and label the cities $1, \ldots, n$. Let i be the index of the city in presentation. Set $i = 1$.

Step 2 (Winner selection): Through a competitive procedure, select the closest node to city i based on Euclidean distance to be the winner node, $(vk)^*$, according to the following competition's rule.

$$(vk)^* = \mathrm{Argmin}_{vk}\{|Y_{vk} - X_i| * [1 + (\mathrm{dist}_v - \mathrm{AVDIST})/\mathrm{AVDIST}]\}, \tag{1}$$

where dist_v is the total distance of tour v, AVDIST is the average distance of a salesman and $|\cdot|$ represents Euclidean distance. This function comprises two components: the Euclidean distance between node k on the ring v and city i, and the deviation of the total distance of salesman v from the average distance. When considering the salesman v, if $\mathrm{dist}_v < \mathrm{AVDIST}$ the potential of the nodes on the ring v is smaller than the corresponding Euclidean distance; consequently, the opportunity that nodes on the ring v will be chosen and moved towards the presented city is higher. Otherwise, if $\mathrm{dist}_v > \mathrm{AVDIST}$ then the opportunity of selecting nodes on the ring v will be decreased. This deviation may be considered as the minmax constraint which needs to adjust each ring to obtain a similar distance.

Step 3 (Adaptation). Using the following update function, update the winner node k^* and its neighbours only on the ring (v^*).

The update function:

$$Y_{v^*k}(t+1) = Y_{v^*k}(t) + \mu f(G, d)(X_i - Y_{v^*k}(t)), \tag{2}$$

where μ is a learning rate.

The neighbourhood function, $f(G, d)$, can be stated as follows:

$$f(G, d) = \begin{cases} \exp(-d^2/G^2) & d < 0.2M, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

$$d = \min[\|k - k^*\|, M - \|k - k^*\|],$$

where d is the cardinal distance measured along the ring between nodes k and k^* on the v^*th ring, $\|\cdot\|$ represents absolute value, and G is the gain parameter.

Step 4 Set $i = i + 1$. If $i \leqslant N$, go to Step 2. Otherwise, set $G = (1 - \alpha)\,G$; $i = 1$; Go to Step 5.

Step 5 (Convergence test): For each city, if the position of the corresponding node is within an acceptable distance (e.g. 0.001), then Stop. Otherwise, go to Step 1.

4.2. *Empirical results*

In the simulation, we used a series of standard data of VRP problem in TSPLIB [33] without consideration of the capacity constraint. Additionally, the number of salesmen in all instances was defined equal to 2, 3 and 4, respectively. To evaluate the effectiveness of the algorithm, the proposed algorithm was compared with the elastic net and the generalized 2-opt heuristic algorithm. For the generalized 2-opt heuristic algorithm, two classical insertion heuristics that are the farthest insertion and the nearest neighbour insertion were applied to generate the initial solution. All algorithms in this simulation were coded in C language and processed on the Sun Sparc 10 workstation.

The evolution of the results by the proposed algorithm is depicted in Fig. 2. The figure illustrates the initial status of rings, after 10 and 20 iterations as well as the final configuration. As shown here, m small rings on which the starting and ending points are defined to be the depot are formed in the same manners the elastic net's property. Furthermore, the rings are elongated by an iterative process until settlement.

The parameters for the elastic net algorithm and the proposed algorithm in our simulation were set as follows. For the elastic net algorithm, in order to define the parameter more easily, the data were normalized by projection of the city positions into the unit square. Following Vakhutinsky and Golden's work [30], the simulation had the following parameters: $\alpha = 0.2$, $\beta = 2.0$, and $K = 2.0$. For our proposed algorithm, we attempt to reduce the number of parameters and simplify their adjustment in order to facilitate the practical application of the algorithm. As in our previous work in for a TSP problem, there are three defined parameters in our algorithm. Two of them have no crucial effect on the quality of results. In all the examined data, the learning rate, μ, and the decreasing rate parameter for the gain parameter in the neighbourhood function, α, were set to 0.6 and 0.1, respectively. The initial value of the gain parameter, G_0, which settles upon the circumstances has a role on the quality of results. Although the characteristics of problems vary considerably, the simulation shows that the initial value of G_0 related to the problem size. In accordance with the

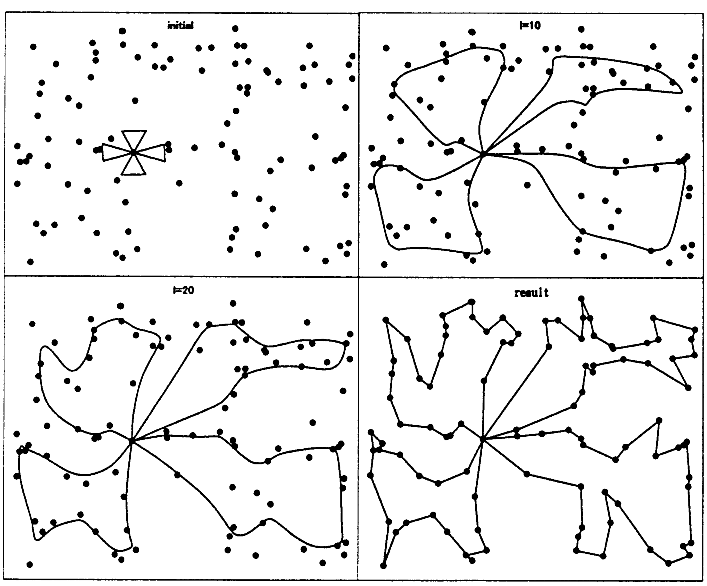

Fig. 2. The evolution of the problem by the proposed algorithm.

practical results, the relation between the value of G_0 and the size of the problem could be formed in the correlation function of the linear regression as follow:

$$G_0 = 0.06 + 12.41n$$

where n is the number of cities.

The results of this simulation are given in Table 1. In this table the best and average of solution over 10 runs for our proposed algorithm, and the solution on a single run and the deviation from

Table 1
The computational results of algorithms

Problem	n	m	Proposed			Farthest + 2opt			Nearest + 2opt			Elastic net		
			Best	Avg.	CPU	dist.	PDP$^+$	CPU	dist.	PDP$^+$	CPU	dist.	PDP$^+$	CPU
eil22	22	2	157	158.50	0.33	160	0.95	0.02	183	15.46	0.03	204	28.71	12.03
		3	117	119.42	0.33	133	11.37	0.02	137	14.72	0.03	208	74.18	13.10
		4	111	112.50	0.22	119	5.78	0.02	131	16.44	0.03	150	33.33	12.73
eil30	30	2	230	232.00	0.55	250	7.76	0.03	237	2.16	0.10	264	13.79	24.07
		3	174	180.00	0.38	186	3.33	0.05	220	22.22	0.07	302	67.78	25.05
		4	171	174.75	0.35	175	0.14	0.05	175	0.14	0.08	203	16.17	23.70
eil51	51	2	247	248.67	1.90	254	2.14	0.17	271	8.98	0.47	250	0.53	78.70
		3	170	172.00	1.87	175	1.74	0.15	213	23.84	0.33	175	1.74	76.65
		4	136	137.33	1.97	147	7.04	0.15	166	20.88	0.33	164	19.42	84.53
eil76	76	2	289	292.00	4.83	310	6.16	0.53	353	20.89	1.15	327	11.97	200.02
		3	205	210.50	4.85	227	7.84	0.47	258	22.57	1.30	219	4.04	194.72
		4	159	162.75	5.08	182	11.83	0.52	206	26.57	1.35	190	16.74	196.27
eil101	101	2	340	344.67	15.12	350	1.55	1.38	393	14.02	3.53	360	4.45	402.38
		3	232	236.00	13.60	243	2.97	1.18	315	33.48	2.43	289	22.46	390.50
		4	187	189.67	15.53	203	7.03	1.33	240	26.54	1.60	218	14.94	379.50
kroA100	100	2	11484	11532.80	17.25	12718	10.28	1.67	14599	26.59	4.03	13280	15.15	432.22
		3	9062	9276.25	15.85	9862	6.32	1.03	9458	1.96	7.88	11439	23.32	411.82
		4	7497	7516.75	14.80	8695	15.67	1.38	8878	18.11	8.22	9444	25.64	433.28
kroA150	150	2	14885	15076.00	24.53	15561	3.22	8.47	19460	29.08	9.78	16456	9.15	673.50
		3	10527	10756.00	23.14	12182	13.26	3.38	12147	12.93	23.52	15252	4.80	678.98
		4	8571	8625.60	22.65	10363	20.14	5.18	12077	40.01	10.15	10866	25.97	659.95
kroA200	200	2	17353	17547.50	37.33	17824	1.58	9.80	20317	15.78	39.37	23539	34.14	1156.67
		3	11502	11722.00	37.93	12930	6.77	14.50	14124	20.49	28.79	17541	49.64	1215.23
		4	10433	10776.33	33.33	11807	9.56	10.38	13123	21.78	24.05	15648	45.21	1685.00
fl417	417	2	7207	7266.75	193.41	7730	6.38	122.18	8224	13.17	157.90	18626	156.32	5423.90
		3	5618	5902.50	255.37	6721	13.87	103.47	6887	16.68	248.57	16615	181.49	8823.58
		4	5032	5109.50	323.18	5934	16.14	134.57	5878	15.04	311.53	14972	193.02	8807.57

dist.: distance.
n: the number of cities, m: the number of salesmen.
CPU (s): for the proposed algorithm, the average of time over 10 runs; for other algorithms, the time of a single run.
PDP$^+$: percent deviation from the average solutions of the proposed algorithm.

the average solution of our proposed algorithm for other algorithms is reported. The table shows that the proposed algorithm is superior to all other algorithms. In comparison with the generalized 2-opt exchange heuristic algorithms, the proposed algorithm is better in quality than both of the insertion heuristics in particular the nearest neighbour insertion heuristic. Regarding the computational time, the proposed algorithm is quite slower than two heuristics while it is far superior to the elastic net. In addition, the computational time of the generalized 2-opt exchange algorithm of which the initial solutions are generated by nearest neighbour insertion, is quite similar to the proposed algorithm when the problem size is large. Because most of the initial solutions produced by the nearest neighbour insertion heuristic were worse than that produced by the farthest insertion heuristic, it leads to a time consuming application of the 2-opt. The proposed algorithm, however, attains good solutions within reasonable time frame in comparison with the heuristic algorithms. For the elastic net algorithm, it obviously requires more computing time due to a heavy interconnection between nodes.

Since the NN produces the results by using a constructive strategy, combination of this method with an improvement procedure which results in a composite algorithm is a promising strategy. We examined this idea by combination of a simple post-improvement routine; namely, 2-exchange procedure, with the proposed NN algorithm. To evaluate the efficacy of the algorithm we considered tabu search algorithm as benchmark. Among the several significant developments of general local search methods [34] such as simulated annealing and tabu search, tabu search appears to be the most powerful method [35, 36]. This method is a metaheuristic one which explores the solution space repeatedly to approach the best admissible neighbour, even if the objective has deteriorated. With the highly-developed solution, tabu search can achieve to find the optimal or near-optimal solution. In a recent work of Golden, Laporte and Taillard [37], an Adaptive Memory Procedure(AMP) has been used in conjunction with tabu search to solve a class of VRPs with minmax objective. In the adaptive tabu search, incorporation of AMP which enriches the search would increase the possibility of producing near-optimal or optimal solutions. We have compared the NN algorithm with this work.

Table 2 shows the simulation results for the composite NN and Adaptive tabu search. In this table the best and average results of both algorithms over 5 runs as well as computation time is given. The simulation results show that the post-improvement procedure will enhance the quality of solutions about 3% with negligible computational efforts. Since both algorithms are examined on Sun Sparc 10 workstation, it is reasonable to compare computation times directly. As may be seen, the NN can attain results within about 3% of that of the tabu search with quite low computational efforts. As mentioned above, in this simulation we considered a very simple exchange routine as post-improvement procedure. Obviously, using sophisticated improvement routines would produce superior results. This point indicates that hybrid NN and improvement algorithms would be a promising solution strategy for combinatorial problems.

5. Conclusion

Recently, a competition-based neural network has been broadly indicated as being the potential for the OR analyst. In this paper, the solution of the minmax MTSP which is an extension of the well-known TSP problem by competition-based neural network has been proposed. To evaluate its

Table 2
The computational results of algorithms compared with adaptive tabu search method

Problem	n	m	ATS[a]			Composite NN[b]			
			Best	Avg.	CPU	Best	Avg.	PDB$^+$	CPU
1	50	5	110.20	110.57	210	112.70	113.80	2.27	3.53
		6	99.26	100.93	190	102.23	103.88	2.99	5.87
		7	91.62	92.03	160	94.34	95.65	2.97	10.24
2	75	10	91.21	91.34	210	94.14	96.55	3.21	5.32
		11	88.72	88.95	210	93.84	94.19	5.77	10.72
		12	88.08	88.08	210	90.80	92.22	3.09	13.34
3	100	8	111.12	112.35	610	115.18	116.93	3.65	11.52
		9	105.39	106.65	610	107.34	110.37	1.85	17.48
		10	100.37	101.33	550	105.68	106.86	5.29	25.67
4	150	12	100.12	100.27	1100	104.30	106.15	4.17	36.54
5	199	15	99.86	99.86	51	103.87	104.63	4.02	40.37
6	120	7	199.39	199.49	1400	202.71	204.17	1.67	24.62
7	100	10	117.05	117.05	7	117.05	118.83	0	6.05
8	71	4	65.10	65.10	960	65.46	66.08	0.55	4.53
		5	59.32	59.38	790	61.79	62.37	4.16	6.14
		6	55.19	55.30	700	58.25	58.84	5.54	8.99
9	134	7	293.54	293.54	41	296.02	297.25	0.84	20.35

PDB$^+$: percent deviation from the best solution of tabu search based method.
ATS: adaptive tabu search method from Ref. [37].
CPU: the CPU time in seconds.
[a]: The instances were run on a Sun Spare 10 workstation over 5 runs.
[b]: The time is the average time over 5 runs.

effectiveness, we compared the algorithm with the elastic net algorithm and the generalized 2-opt exchange heuristic algorithms which utilized the nearest insertion heuristic and the farthest insertion heuristic to construct the initial solution. This algorithm attains the satisfactory solutions for all problems with reasonable computational efforts. In addition, the idea of combination of this algorithm with an improvement procedure has been proposed and broadly investigated.

In summary, the contribution of this research can be specified as follows. Firstly, this paper reviewed and evaluated the heuristic algorithms for the minmax MTSP by using the standard data of VRP in TSPLIB without consideration of capacity constraint. Secondly, the paper proposed the adaptive neural network algorithm to solve the complex routing problems and compared its effectiveness with other previous heuristics. Finally, the paper presented the possibility of the advancement of neural network algorithm in conjunction with a improvement heuristic.

In this research, the encouraging results may induce further attempts in using the combination of adaptive neural network with some of effective improvement heuristics to attain the optimal and near-optimal results efficiently.

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