# Are COMPETants more competent for problem solving? - the case of a multiple objective transportation problem

Karl Doerner, Richard F. Hartl, Marc Reimann [1]

**Abstract**

In this paper we propose a multi-colony Ant System, where the colonies solve a multi-objective optimization problem concerned with goods transportation. The colonies differ from each other by the heuristic information, which guides their search through the solution space. Information exchange occurs as ants from one population observe the pheromone trails of other populations and decide whether or not to utilize this information. Furthermore, population sizes are adapted according to the relative fitness of the populations. The results show the advantages of this approach over common Ant System approaches.

[1]Department of Production and Operations Management, Institute for Management Science, University of Vienna, Bruennerstrasse 72, 1210 Vienna, Austria, Tel.: ++43 1 4277-38096, Fax: ++43 1 4277-38094, e-mail: marc.reimann@univie.ac.at

## 1 Introduction

The Ant System, as a graph based meta-heuristic, was first proposed in the early nineties by Colorni et al. (1991). It is inspired by the behavior of real ants. Ants, when searching for food, mark the traversed paths with a pheromone quantity, which depends on the quality of the food source. Other ants observe these pheromone trails and are attracted to follow them, thus reinforcing the paths. Gradually, paths leading to rich food sources will be used more frequently, while other paths, leading to remote food sources will not be used any more.

This concept can be applied to combinatorial optimization problems, by letting artificial ants search the solution space. The ants are equipped with a local heuristic function which guides their search through the solution space. Furthermore, they use an adaptive memory, which represents the pheromone information. This adaptive memory is biased towards 'good' solutions found in the past, and thus such solutions are more likely to be reinforced.

This approach is incorporated in the meta-heuristic Ant Colony Optimization, which has attracted increasing attention over the last few years. Applications to various hard combinatorial optimization problems can be found in the academic literature ( cf. e.g. Bullnheimer et al. (1999), Costa and Hertz (1997), Dorigo and Gambardella (1997), Stuetzle and Dorigo (1999), Dorigo et al. (1999)). A convergence proof for a generalized Ant System Algorithm is provided in Gutjahr (2000).

A main ingredient of such an algorithm is a heuristic rule which guides the ants through the solution space. For most combinatorial optimization problems more than one heuristic exists. This is due to the complexity of these problems and the fact, that the heuristics are on problem specific knowledge and usually exploit different problem characteristics. Thus, each heuristic solves the problem, but the solution quality usually depends on the actual problem constellation. Especially if the structure of the problem is not a priori obvious it is hard to choose the appropriate heuristic. One could either try a single heuristic and rely on its effectiveness for the problem at hand, or try a number of heuristics and choose the best result. Both approaches have their advantages and drawbacks. The former will generally be fast but may lead to solutions which are far from optimal, while the latter will generally achieve better results using however more computational power.

Apart from that, it may be the case that a problem could be partitioned into smaller subproblems, which call for different heuristics. Here again two difficulties arise. The first one is how to partition the problem, while the second one deals once again with the choice of the appropriate heuristic.

Given these difficulties in determining which heuristic rule to choose we propose the following approach. We utilize two colonies of ants, which solve the problem using different heuristic information. After the two colonies have terminated, ants choose which population they want to belong to in the next iteration. Thus, the population sizes of the colonies will change adaptively according to the solution quality associated with each colony, as the ants strive to belong to the more 'successful' population. Furthermore, when constructing their solutions the ants observe not only the pheromone information from their own population but also from the other population. Again they decide which information to utilize. While the first mechanism leads to an adaptation of the use of the heuristic information, the latter mechanism focuses on the use of the adaptive memory. Thus, the first mechanism will lead to a reinforcement of the heuristic rule appropriate for the problem instance in general, while the second mechanism will guide ants to use the heuristic rule appropriate for different regions of the search space. A similar approach for a related problem, the Vehicle Routing Problem with Time Windows, was proposed in Gambardella et al. (1999). In that approach one colony optimizes fleet size, while the other one minimizes vehicle movements. However, there the populations communicate only if one population improves the global best solution. Thus, both colonies consider fleet size as the main goal.

The problem we applied our new procedure to, is to find an assignment of transportation orders to trucks and a schedule of these orders for each truck, such that the total costs in the transportation network are minimized. These costs consist of fixed costs associated with the fleet size and variable vehicle movement costs. Thus, we have two goals, which generally will call for different heuristics. While for the minimization of vehicle movements idle times of the trucks may be favorable, these have to be avoided if the fleet size is to be minimized. In the latter case some additional vehicle movements may be preferable to a waiting time for a truck.

The remainder of this paper is organized as follows. In section 2 we describe the problem we consider. Our new approach for using the appropriate heuristic rule is proposed in section 3. In section 4 we present our numerical results. We close in section 5 with some final remarks.

## 2 Description of the problem

The problem considered in this paper is common to logistics service providers. We have a network of locations, each of which acts as a depot for the vehicle fleet. Each vehicle is assigned a specific home depot. Transportation orders fill the vehicles completely, we deal with a full truckload problem. Each transportation order requires pickup at one location and delivery to another location. Given the size of the shipments, all orders are transported directly from their source to their destination. Thus, we deal with a special case of the pickup and delivery problem (PDP). This class of problems is described in detail in Savelsbergh and Sol (1995).

Our problem is also characterized and constrained by the following assumptions:

- All order information is known at the time of planning.
- The service provider operates a fleet of homogeneous vehicles.
- Each vehicle can be used repeatedly within the planning horizon.
- The vehicles have to return to their home base after a given time period.

- Customer time windows have to be respected strictly.

The objective we consider is to minimize the total costs in the network. These costs result from the utilized fleet of trucks as well as from the movements this fleet performs to satisfy all orders. Thus, we have a multiple-objective optimization problem. While minimizing empty vehicle movements may lead to smaller fleet sizes, these two goals are conflicting in general. Two obvious reasons for this can be stated. First, if we aim to minimize empty vehicle movements it can be advantageous to accept waiting times for vehicles at certain locations. However, these waiting times have a negative impact on vehicle utilization and thus should be kept at a minimal level if the fleet size is to be minimized. Second, consider the following case: there is only one order left unassigned. We have one vehicle, which could satisfy the order, however this vehicles current position as well as its home depot differ from the pickup and delivery location of the last order. If the empty vehicle movements were to be minimized, the optimal decision were to open a new vehicle, located at either the pickup or delivery location of this last order. Note, that vehicles can be located at any location in the network. On the other hand, if fleet size is to be minimized, the optimal choice is to assign the order to the available vehicle and to accept some additional empty vehicle movements.

Given this structure of the problem, it is also obvious that different heuristics can be applied to the problem, each of which solves the problem with respect to the different goals. For example, if empty vehicle movements are to be minimized a savings type heuristic will perform well, whereas for the minimization of the fleet size a priority rule which takes into account vehicle utilization will be favorable. Based on these problem characteristics we propose our new approach for solving multiple objective combinatorial optimization problems in the next section.

## 3 COMPETants - Competing ant colonies

One main feature of our COMPETants is the use of two ant populations with different priority rules. One ant colony uses a priority rule which puts more emphasis on the goal to maximize utilization, while the other colony puts more emphasis on the empty vehicle movements. It depends on the problem characteristics which of the two populations finds better solutions. In particular, if time windows are wide, which implies that waiting times become negligible, the latter population will be superior, while the reverse effect occurs if time windows are tight. However, as we know that both, a large utilization of the vehicles and little empty vehicle movements are important for the minimization of total costs, our approach is to combine the strength of both colonies. First, the colonies compete with respect to resources. The population sizes are not constant, we assign more computational power to the ant colony which finds solutions with lower average costs $\mu$. Furthermore, information spillovers between the populations occur as ants observe and utilize not only their own pheromone information but also the foreign pheromone information. The ants decision whether or not to utilize the foreign pheromone information is based on the best solution found in each population. Ants, which decide to utilize foreign information are called *spies*. Through these information flows, different patterns of good solutions are communicated between the populations and the overall solution quality should be improved.

Table 1: The COMPETants procedure

```
procedure COMPETants {
    SetParameters;
    InitializeSystem;

    /* execute optimization */
    (ψ^DistPop, μ^DistPop):=ACO (η^DistPop; Γ^DistPop; 0; 0);
    (ψ^TWPop, μ^TWPop):=ACO (η^TWPop, Γ^TWPop; 0; 0);

    for i := 2 to max_Iterations {

        /* adapt population sizes Γ for the two populations
        depending on the average total costs μ */
        AdaptPopSizes(...);

        /* determine the number of spies Ξ
        for each population
        depending on the total costs ψ of the best ant */
        AdaptNumberOfSpies(...);

        /* execute optimization */
        (...):=ACO (...; τ^TWPop; Ξ^DistPop);
        (...):=ACO (...; τ^DistPop; Ξ^TWPop);
```

The procedure `COMPETants`, described in Table 1 initializes the two ant colonies, determines the population sizes, the number of spies for each population and controls the termination.

In detail, for a number of *max_Iterations* the procedure `ACO` is executed for the two ant colonies. Both procedures are called with the parameter settings for the priority rule ($\eta^{DistPop}$, $\eta^{TWPop}$), population size ($\Gamma^{DistPop}$, $\Gamma^{TWPop}$), the number of spies in the population ($\Xi^{DistPop}$, $\Xi^{TWPop}$) and the pheromone information of the foreign population ($\tau^{DistPop}$, $\tau^{TWPop}$). In the first execution of the `ACO` procedure no spies are used, therefore the call parameters $\tau$ and $\Xi$ are set to zero.

After the first execution of the `ACO` procedures the population sizes $(\Gamma^{DistPop}, \Gamma^{TWPop})$ of the two ant populations are adapted according to the average solution quality of the populations $(\mu^{DistPop}, \mu^{TWPop})$. More computational power is assigned to the population with the lower average total costs $\mu$, which means more ants are generated for this populations. The size of the ant population with the higher average total costs is decreased. The procedure `AdaptPopSizes` determines the new values for $\Gamma$.

Afterwards `COMPETants` calls the procedure (`AdaptNumberOfSpies`), which determines the number of spies $\Xi$ for each ant population. The number of spies generated depends on the own as well as on the foreign local best solution $\psi$ found in the previous iteration. The population with the higher total costs of the best ant uses more pheromone information of the population with the lower total costs in the next iteration.

The following calls of the optimization procedure `ACO` use the new values for $\Xi$ and $\Gamma$. After each execution of `ACO` the values $\psi$ and $\mu$ are returned, which are used by the procedures `AdaptPopSizes` and `AdaptNumberOfSpies`.

Let us now turn to a detailed description of these procedures.

## 3.1 Adaptive Population Size

After each execution of the two ACO procedures the population size for each population is adapted. The procedure `AdaptivePopulationSize` determines the two population sizes and returns the values for $\Gamma^{DistPop}, \Gamma^{TWPop}$. The population with the lower average total costs has higher probability to get more ants and has therefore more computational power in the next iteration of the algorithm.

Table 2: The AdaptPopSizes procedure

**procedure** `AdaptPopSizes` (...){

 `for` $i := 1$ `to` $\Gamma = \Gamma^{OwnPop} + \Gamma^{ForeignPop}$ {

  if $random[0,1) < \frac{\mu_{ForeignPop}}{\mu_{ForeignPop} + \mu_{OwnPop}}$

  `then` increment $\Gamma^{OwnPop}$;

 `return` $\Gamma^{OwnPop}, \Gamma^{ForeignPop} = \Gamma - \Gamma^{OwnPop}$;

## 3.2 Adaptive Number of Spies

Another important feature in our approach is the determination of the number of spies in each population, which is controlled by the COMPETants procedure by calling `AdaptiveNumberOfSpies`. The probability to become a spy depends on the solution quality of the best ant $\psi$ in each population. The number of spies is about 20 % under the condition that the best solutions of the two populations are equal.

Table 3: The AdaptNumberOfSpies procedure

**procedure** `AdaptNumberOfSpies` (...){
    `for` $i := 1$ `to` $\Gamma^{OwnPop}$ {
        if $random[0,1) < \frac{\psi_{OwnPop}}{4 \cdot \psi_{ForeignPop} + \psi_{OwnPop}}$
        `then` increment $\Xi$;
    `return` $\Xi$;

In the next section we briefly describe the Ant Colony Algorithm with the construction of a feasible solution (in section 3.3), with the visibility information for the two populations (in section 3.4), the pheromone information (in section 3.5), the decision rules for the ants and the spies (in section 3.6) and the trail update for the pheromone information (in section 3.7).

## 3.3 The Ant Colony Algorithm

The two ant populations use the same ACO algorithm to construct feasible solutions.

Starting at time $t = 0$ a truck is sequentially filled with orders until the end of the planning horizon $T$ is reached, or no more order assignment is feasible. At this point another vehicle is brought into use, $t$ is set to $t = 0$ and the order assignment is continued. This procedure is repeated until all orders are assigned.

For the selection of orders that have not yet been assigned to trucks, two aspects are taken into account: how promising the choice of that order is in general, and how good the choice of that order was in previous iterations of the algorithm. The first information is the visibility, the second is stored in the pheromone information.

The proposed ant system can be described by the algorithm given in Table 4.

In the initialization phase, $\Gamma$ ants are generated and each depot is assigned a number of ants. In this phase also the number of best ants $\Lambda$ is determined depending on the population size $\Gamma$. Then the two basic phases - construction of tours and trail update - are executed for a given number of iterations. In the construction phase the first $\Xi$ ants, the so called spies, use a different decision rule, which depends also on foreign pheromone information.

To improve the solution quality a post optimization procedure will be applied, which seeks to improve a solution by finding the optimal depot for each truck given the orders assigned. For each truck, all possible depot assignments are considered and the one yielding least costs is chosen. Note however, that the sequence of the orders assigned to each vehicle must not be changed.

### 3.4 Visibility

Let $J$ denote the set of orders and $D$ denote the set of depots. The visibility information is stored in a matrix $\eta$, each element in the matrix is denoted by $\eta_{ij}(t)$, where $\eta_{ij}(t)$ is positive, if and only if the assignment of order $j$ after order $i$ is feasible. An assignment of order $j$ is feasible, if the order can be scheduled on the current vehicle without violation of its time window. Hence, it is clear that $\eta$ depends on the time. Note that in each iteration only the row associated with the order assigned in the previous iteration has to be evaluated. The actual value of the visibility of order $j$ depends on the priority rule incorporated in the algorithm. Based on this information we can define the set $\Omega_i(t) = \{j \in J : j$ is an order feasible to assign$\}$.

It is obvious that the choice of the priority rule substantially influences the solution quality. In our problem at hand, we want to minimize total costs, that means a minimization of both empty vehicle movements, as well as number of trucks required. Therefore, the time window population of the COMPETants uses a priority rule, which leads to good solutions with respect to total costs. This priority rule takes into consideration the minimization of the empty vehicle movements only insufficiently. Thus, we introduce a distance population,

which uses a priority rule suitable to minimize this goal.

#### 3.4.1 Visibility for the time window population

The priority rule for the time window population is:

$$\eta_{ij}^{TWPop}(t) = \begin{cases} e^{-4\cdot(EDD_j + 2\cdot EPST_j(i,t))} & \text{if } j \in \Omega_i(t) \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in J. \tag{1}$$

This priority rule aims to maximize truck utilization by avoiding waiting times. It takes into account the due dates ($EDD$), as well as the earliest possible pickup times ($EPST$) of the orders. While the $EDD$ measure exactly represents the due dates, the $EPST$ measure takes into account waiting times and connecting empty vehicle movements.

#### 3.4.2 Visibility for the distance population

The priority rule for the distance population is:

$$\eta_{ij}^{DistPop}(t) = \begin{cases} e^{-16 \cdot DIST(i,j)} & \text{if } j \in \Omega_i(t) \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in J. \tag{2}$$

This priority rule is solely based on the distance traveled to get from the delivery location of the last customer assigned ($i$) to the pickup location of customer $j$, $DIST(i, j)$. It is obvious, that this priority rule is well suited for the minimization of empty vehicle movements.

### 3.5 Pheromone information

The pheromone information is stored in a matrix $\tau$ with $|J| + |D|$ rows and columns, where the first $|J|$ rows and columns correspond to orders, while columns $|J + 1|$ to $|D|$ correspond to depots. Thus, the value $\tau_{ij}$ represents the current pheromone information. For $i \leq |J|, j \leq |J|$, the value $\tau_{ij}$ represents the pheromone information of assigning order $j$ immediately after order $i$. The value $\tau_{ij}, i \leq |J|, j \geq |J| + 1$ represents the pheromone information to begin a new tour with another truck in depot $j$ provided that order $i$ is the last order on the current vehicle. Finally, $\tau_{ij}, i \geq |J| + 1, j \leq |J|$ represents the pheromone information for order $j$ being the first order starting from depot $i$.

### 3.6 Decision rule

#### 3.6.1 Decision rule for the ants

Given the visibility and pheromone information, a feasible order $j$ is selected to be visited immediately after order or depot $i$ according to a random-proportional rule that can be stated as follows:

$$\mathcal{P}_{ij}(t) = \begin{cases} \frac{[\tau_{ij}]^\alpha [\eta_{ij}(t)]^\beta}{\sum_{h \in \Omega_i(t)} [\tau_{ih}]^\alpha [\eta_{ih}(t)]^\beta} & \text{if } j \in \Omega_i(t) \\ 0 & \text{otherwise,} \end{cases} \quad \forall j \in J. \tag{3}$$

This probability distribution is biased by the parameters $\alpha$ and $\beta$ that determine the relative influence of the trails and the visibility, respectively. $\tau_{ij}$, represents the current pheromone information, i.e. the value $\tau_{ij}$ represents the pheromone information of assigning order $j$ immediately after order $i$.

#### 3.6.2 Decision rule for the spies

For the first $\Xi$ ants, the so called spies, the random proportional-rule as stated in (4) is used to construct a feasible solution. These $\Xi$ ants use also foreign pheromone information in their decision process.

$$\mathcal{P}_{ij}(t) = \begin{cases} z & \text{if } j \in \Omega_i(t) \\ & \\ 0 & \text{otherwise,} \end{cases} \quad \forall j \in J, \tag{4}$$

where $z = \frac{[(0.5 \cdot \tau_{ij}^{OwnPop}) + (0.5 \cdot \tau_{ij}^{ForeignPop})]^\alpha [\eta_{ij}(t)]^\beta}{\sum_{h \in \Omega_i(t)} [(0.5 \cdot \tau_{ih}^{OwnPop}) + (0.5 \cdot \tau_{ih}^{ForeignPop})]^\alpha [\eta_{ih}(t)]^\beta}$.

### 3.7 Pheromone update

After each ant of the population has constructed a feasible solution, the pheromone trails are updated. We use a pheromone update procedure, where only a number of the best ants, ranked according to solution quality, contribute to the pheromone trails. Such a procedure was proposed in Bullnheimer (1999). The update rule is as follows:

$$\tau_{ij} = \rho \cdot \tau_{ij} + \sum_{\lambda=1}^{\Lambda} \Delta\tau_{ij}^{\lambda}, \quad \forall i, j \in J, \tag{5}$$

where $\rho$ is the trail persistence (with $0 \leq \rho \leq 1$). Only the $\Lambda$ best ants of the population update the pheromone information. If an order $j$ was performed immediately after an order $i$ in the solution of the $\lambda$-th best ant the pheromone trail is increased by a quantity $\Delta\tau_{ij}^{\lambda}$. This update quantity can be represented as

$$\Delta\tau_{ij}^{\lambda} = \begin{cases} 1 - \frac{\lambda-1}{\Lambda} & \text{if } 1 \leq \lambda \leq \Lambda, \text{where} \\ & \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j \in J. \tag{6}$$

## 4 Numerical analysis

In this section we will present the results of our numerical analysis. We generated a set of test problems with a network of 8 distribution centers, 512 transportation orders and a planning horizon of 8 periods.

Given these data we generated 5 classes of problems, which differ with respect to the pickup and delivery locations for the transportation orders. We will refer to these classes as problem class A, B, C, D and E. Each class consists of 8 problem instances, with different time window strictness. The tightest problem exhibits an average time window length of 1 period (i.e. instances A1, B1, ...), and we generated instances up to an average time window length of 8 periods (instances A8, B8, ...). Note, that time window lengths can only take integer values. Thus, we have 40 different problem instances.

The objective function is given by

$$TC = 20 \cdot FS + 1 \cdot MC,$$

where $TC$ denotes the total costs, $FS$ is the fleet size and $MC$ denotes the vehicle movement costs. An interpretation of the weights used in this objective function can be found in Doerner et al. (2000). The following parameter setting was chosen for the Ant System:

$\alpha = 1$, $\beta = 1$, $\rho = 0.5$, $\tau_0 = 0.1$, $\Gamma^{TWPop} = 80$, $\Gamma^{DistPop} = 80$ and $maxIterations = 30$.

Let us now turn to the analysis of our multiple ant colony approach. We have tested a set of hypothesis using t-tests for paired samples. The results are based on samples consisting of the 40 problem instances. For each instance average values over 5 runs are considered. We generated samples for the following cases:

- Case 1: 2 populations of 80 ants, no communication, no spillovers
- Case 2: 2 populations of initially 80 ants, adaptive population sizes, no spies
- Case 3: COMPETants - 2 populations of initially 80 ants, adaptive population sizes, 20 percent spies on average
- Case 4: 1 population of 160 ants

Case 1 represents a basic ACO algorithm where two equal-sized ant colonies search the solution space independently. In this case, no interaction between the colonies occurs. In Case 2 the population sizes of the two ant colonies adapt as each ant, prior to building a solution, decides which population to join. This decision is based on the solution values of the two populations. Thus, over time the appropriate rule will attract more ants and eventually one rule will not be used anymore. Case 3 represents our new COMPETants approach, where both communication and information spillovers occur. Communication takes place in the same way as described in Case 2. Information spillovers occur, as ants from one population, the so called *spies*, survey and utilize not only their own pheromone information but also the pheromone information of the other population. Finally, Case 4 represents a situation where the user of the system knows the right priority rule for the problem at hand. In this case, all resources are focused on one ant colony using this priority rule.

For reasons of readability we chose the following notation for the statement of the hypotheses: $\bar{TC}$ denotes the average solution value over a number of runs for a given problem. The subscripts denote the choice of the algorithm, where $endo$ stands for Case 2, $ACO$ stands for Case 1, $COMPETants$ denotes Case 3 and $best$ stands for Case 4.

The following hypothesis were tested:

**Hypothesis 1:** An endogenous determination of the appropriate priority rule leads to better solutions than repeated runs of a basic Ant Colony Optimization approach using different priority rules.

$\rightarrow H_0 : \bar{TC}_{endo} = \bar{TC}_{ACO}$

To test this hypothesis we compared Case 1 with Case 2. The idea is that 'wasting' resources for an inappropriate priority rule can be avoided - not completely - by adapting the population sizes according to the associated solution values.

**Hypothesis 2:** Information spillovers between the populations have a positive impact on solution quality.

$\rightarrow H_0 : \bar{TC}_{endo} = \bar{TC}_{COMPETants}$

This hypothesis is tested by comparing Case 2 with Case 3. The idea is, that information spillovers between populations will lead to the transfer of knowledge, concerning patterns of good solutions, from one population to the other. Thus, ants choosing to utilize the foreign pheromone information will thereby incorporate knowledge of this foreign population into their own population thus enhancing this populations knowledge about the search space.

**Hypothesis 3:** Even if the appropriate priority rule were known in advance our COMPETants perform competitive.

$\rightarrow H_0 : \bar{TC}_{best} = \bar{TC}_{COMPETants}$

To test this hypothesis we compared Case 3 with Case 4. The question here is whether the drawback of 'wasting' resources for ants using an inappropriate priority rule is outweighed by the advantages of information spillovers.

The following tables present the results of our analysis. Table 4 shows the sample means for the 4 cases, while Table 4 provides information about 95 % confidence intervals of the mean differences, together with the test statistics and the level of significance.

From Table 4 we see that for both Hypothesis 1 and Hypothesis 2 the null hypothesis can be rejected. Thus, the results suggest that an endogenous choice of the priority rule leads to significantly better results than a 'trial and error' search. Furthermore, information spillovers have a positive impact on solution quality. Together, these results imply, that our COMPETants approach outperforms both the basic ACO algorithm and the approach with multiple ant colonies and an endogenous choice of the priority rule.

Finally, Hypothesis 3 could not be rejected. This implies that our COMPETants approach is competitive even if the appropriate priority rule is known. This finding supports the hypothesis, that different patterns of good solutions are found by different heuristics.

## 5 Conclusion

In this paper we have presented a multi-colony Ant System, where the colonies communicate their solution values. Furthermore, information spillovers occur as ants observe and utilize all pheromone trails, not only the trails of their own population.

We have shown that this approach outperforms traditional approaches significantly. Given a set of heuristic rules our multi-colony Ant System endogenously chose the appropriate one for the problem at hand. Furthermore, the information spillovers between the populations via the pheromone trails proved to be fruitful with respect to the solution quality. Finally, we compared the results achieved with our multi-colony Ant System with the results in cases where the appropriate priority rule is known. This comparison showed that our approach is competitive even if problem-specific knowledge is very sophisticated.

**Acknowledgments**

The authors are grateful for financial support from the Austrian Science Foundation (FWF) under grant SFB #010 'Adaptive Information Systems and Modelling in Economics and Management Science' and from the Oesterreichische Nationalbank (OENB) under grant #8630.

## 6 References

Bullnheimer, B., R. F. Hartl, Ch. Strauss. 1999. An improved ant system algorithm for the vehicle routing problem. *Annals of Operations Research* **89**, 319–328.

Colorni, A., M. Dorigo, V. Maniezzo. 1991. Distributed Optimization by Ant Colonies. F. Varela, P. Bourgine, eds. *Proc. Europ. Conf. Artificial Life*, Elsevier, Amsterdam.

Costa, D., A. Hertz. 1997. Ants can colour graphs. *Journal of the Operational Research Society* **48**(3), 295–305.

Doerner, K. F., M. Gronalt, R. F. Hartl, M. Reimann. 2000. Time Constrained Full Truckload Transportation: Optimizing Fleet Size and Vehicle Movements. POM Working Paper 08/2000, Department of Production and Operations Management, University of Vienna, Vienna, Austria.

Dorigo, M., L. M. Gambardella. 1997. Ant Colony System: A cooperative learning approach to the Travelling Salesman Problem. *IEEE Transactions on Evolutionary Computation* **1**(1), 53–66.

Dorigo, M., G. Di Caro, L. M. Gambardella. 1999. Ant Algorithms for Discrete Optimization. *Artificial Life* **5**(2), 137-172.

Gambardella, L. M., E. Taillard, G. Agazzi. 1999. MACS-VRPTW: A Multiple Ant Colony System for Vehicle Routing Problems with Time Windows. D. Corne, M. Dorigo, F. Glover, eds. *New Ideas in Optimization*, Mc Graw-Hill, London.

Gutjahr, W. J. 2000. A graph-based Ant System and its convergence. *Future Generation Computing Systems* **16**, 873–888.

Savelsbergh, M. W. P., M. Sol. 1995. The general pickup and delivery problem. *Transportation Science* **29**(1), 17–29.

Stützle, T., M. Dorigo. 1999. ACO Algorithms for the Quadratic Assignment Problem. D. Corne, M. Dorigo, F. Glover, eds. *New Ideas in Optimization*, Mc Graw-Hill, London.

Table 4: The ACO procedure

```
procedure ACO (η; Γ; τ; Ξ) {
  Initialization of the ACO;
  set a number of ants on each depot;
  /* determine number of best ants */
  Λ = ⌈Γ · 0.0625⌉
  ξ = Ξ;
  for Ant := 1 to Γ {
    while not all orders are assigned {
      initialize a new truck;
      t = 0;
      select a home base for the truck;
      while ∃ η_ij(t) > 0     ∀i, j ∈ J {
        if ξ > 0 {
         /* spy */
         select an order using formula (4);
        else {
         /* ant */
         select an order using formula (3);
        update t;
    evaluate the objective function;
    decrement ξ;
  for λ := 1 to Λ {
    improve the solution
    using the post optimization procedure;
    evaluate the objective function;
  update local pheromone information;
  determine best solution ψ;
  compute average solution quality μ;
  return ψ;
```

| $Case$ | $SampleMean$ |
|---|---|
| 1 | $666.523$ |
| 2 | $660.832$ |
| 3 | $655.272$ |
| 4 | $654.688$ |

Table 5: Sample Means for the four cases.

| | Paired differences | | | |
|---|---|---|---|---|
| | 95 % conf. interval of the difference | | | |
| $Hyp.$ | $lower$ | $upper$ | $t$ | $sig.$ |
| 1 | $-7.4344$ | $-3.9486$ | $-6.605$ | $0.000$ |
| 2 | $3.2211$ | $7.8989$ | $6.437$ | $0.000$ |
| 3 | $-2.1450$ | $0.9777$ | $-1.012$ | $0.318$ |

Table 6: Results of the statistical tests.