An Ant Colony Optimization Algorithm for Multiple Travelling Salesman Problem

Pan Junjie[1] and Wang Dingwei[2]
[1]*School of Information Science and Engineering, Northeastern University,Shenyang,110004, China*
junjiepan@126.com
[2]*School of Information Science and Engineering, Northeastern University ,Shenyang ,110004, China*
dwwang@mail.neu.edu.cn

Abstract

Multiple travelling salesman problem (MTSP) is a typical computationally complex combinatorial optimization problem,which is an extension of the famous travelling salesman problem (TSP). The MTSP can be generalized to a wide variety of routing and scheduling problems.It is known that classical optimization procedures are not adequate for this problem.The paper makes the attempt to show how the ant colony optimization(ACO) can be applied to the MTSP with ability constraint. In this paper,we compare it with MGA by testing several standard problems from TSPLIB.The computational results show that the proposed algorithm can find competitive solutions even not all of the best solutions within rational time, especially for large scale problems.

1. Introduction

Travelling Salesman Problem(TSP) is the most famous and well-studied problem in the combinatorial optimization area[1].And the mutiple travelling problem(MTSP) is an extension of TSP.This problem relates to accommodating real world problems where there is a need to account for more than one saleman.The MTSP can be generalized to a wide variety of routing and scheduling problems,for example,the School Bus Routing Problem[2,3]and the Pickup and Delivery Problem[4,5].Therefore,finding a good optimal solution method for the MTSP is important and induces to improve the solution of any other complex routing problems.However,MTSP is a NP-complete problem for which optimal solutions can only be found for small size problems. It is known that classical optimization procedures are not adequate for this problem. Good heuristic techniques are necessary for solving MTSP due to its high computational complexity. Modern heuristic techniques, namely genetic algorithms and ant colony optimization,can be good candidates for this problem.

In this research, ant colony optimization (ACO) heuristic is used for solving ability limited MTSP. The main research contribution of the present paper is to make the first attempt in the published literature to show how the ACO algorithm can be applied to MTSP with the limited ability,and the results are encouraging.

In the following sections of this paper, the algorithm based ACO for MTSP is explained then the computational results are reported.

2. The MTSP with ability constraint

The MTSP can be stated as follows:There are m salesman who must visit a set of n cities,and each salesman is defined to start and end at the same depot.In this problem,each city must be visited exactly once by only one salesman and its objective is to find the minimum of total distances travelled by all the salesmen.An example is depicted in Fig.1,where m=3,n=7.Several authors[6,7] suggested transforming the MTSP with m salesmen and n cities into a TSP with $n+m$-1 cities by the introduction of m-1 artificial depots(n+1,…,$n+m$-1).The tranformation of the previous example is depicted in Fig.2.

Several authors have some researches on MTSP.Samerkae Somhom et.al. have used Competition-based neural network to solve MTSP with minmax objective[8],and LinxinTang et.al. have used the modified genetic algorithm to solve hot rolling scheduling problem,which is an example of MTSP[9].

Proceedings of the First International Conference on Innovative Computing, Information and Control (ICICIC'06)
0-7695-2616-0/06 $20.00 © 2006 IEEE

However,the resulting TSP is highly degenerate,when an MTSP is transformed to a single TSP since the resulting problem is more arduous to solve than an ordinary TSP with the same number of cities. While the general objective of the MTSP is to minimize the total distance which can be called minimum criterion,generally,there are m-1 cities always to select the nearest cities as their round trip.As a result,TSP which is made up of the left n-m+1cities is left.During the m salesmen,there are m-1 salesmen travelling only one city,and one saleman needs to travel the left n+m-1 cities. This is not up to the mustard.In practice,every saleman has the similar ability,and the limit in ability.So the MTSP with ability constraint is more appropriate in the real world problems.In this paper,we suppose the number of cities which are travelled by every salesman is limited.

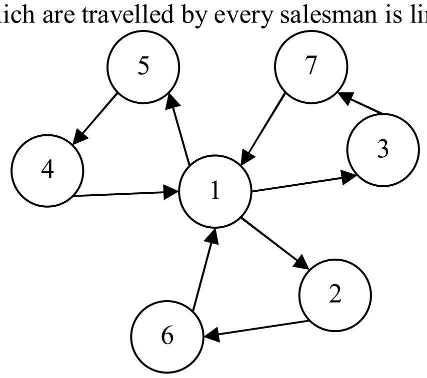

Fig 1. Example solution of MTSP

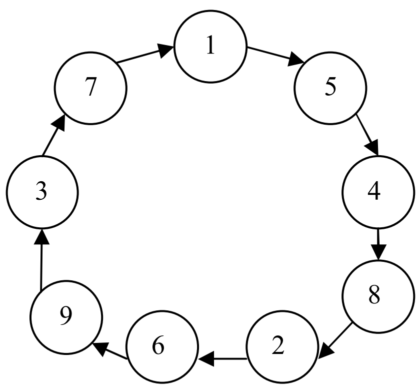

Fig 2. Transformation from MTSP to TSP

We define the following variables and parameters:
Distance from city i to city j:
$D_{ij}, i \in \{1,...n+m-1\}, j \in \{1,...,n+m-1\}$
l_i is maximal number of cities salesman $i(1 \le i \le M)$ can travel,n_i is the real number of cities the salesman $i(1 \le i \le M)$ has travelled,
td_i is the total distance salesman $i(1 \le i \le M)$ has travelled,define the penalty function is

$$if\ n_i > l_i \quad td_i = (n_i - l_i + 1) * td_i \qquad (1)$$

Based the general MTSP,the target function can be described as follows:

$$\text{Min} \sum_{i=1}^{m} td_i \qquad (2)$$

In this way,we can minimize the total distance on the premise of the limited ability of salesmen.

3. Ant Colony Optimization

Ant colony algorithms are becoming popular approaches for solving combinatorial optimization problems in the literature.They were first introduced by Dorigo et al[11].The fundamental idea of ant heuristics is based on the behabiour of natural ants that succeed in finding the shortest paths from their nest to food sources by communicating via a collective memory that consists of pheromone trails.Due to ant's weak global perception of its environment,an ant moves essentially at random when no pheromone is available.However,it tents to follow a path with a high pheromone level when many ants move in a common area,which leads to an autocatalytic process.Finally,the ant does not choose its direction based on the level of pheromone exclusively,but also takes the proximity of the nest and of the food source,respectively,into account.This allows the discovery of new and potentially shorter paths.

4. ACO for the MTSP with ability constraint

4.1 Solution construction

Some authors used ACO to solute TSP,however, MTSP is different from TSP.In thepaper,we improve on ACO according to the characteristic of the MTSP.One salesman should firstly travel an amount of cities,and the next salesman travels an amount of unvisited cities,in this way,all the salemen travell the total cities. The travelled number of salesman is generated randomly in a certain range.Define the number of cities salesman i travell is tn_i ,then

$$\begin{cases} 2 \le tn_i \le l_i \\ \sum_{i}^{m} tn_i = n-1 \end{cases} (i = 1,2,...,m) \qquad (3)$$

Where m is the number of salesmen,n is the numer of cities, l_i is the max number of cities saleman i can travel.

In the ACO algorithm,we assign a tour list to every ant.Evey ant tours from the start city,and then select unvisited cities which is not start city or artificial cities.We suppose that the saleman i is visiting when ants start from the start city or artificial cities the ith time.While the salesman i is visiting,if the number of cities the ants have visited is equal to tn_i,then the ants select a unvisited artificial city,it means that the next salesman starts to visit.The cities which have been visited are recorded in the tour list ,until all the cities are visited by all the salesmen.In this way,a valid solution is constructed.In the algorithm,we used k ants,so there are k solutions constructed.

4.2 Route selection

Every ant selects the next city independently. Suppose that the probability of ant k moves from city i to city j is $P_{ij}^k(t)$,uses the following probabilistic formula:

$$P_{ij}^{k}(t)=\begin{cases}\dfrac{\tau_{ij}(t)^{\alpha}\eta_{ij}^{\beta}}{\sum_{l\in A_k}\left[\tau_{ij}\right]^{\alpha}\eta_{il}^{\beta}}, & j\in A\\ 0 & \text{, otherwise}\end{cases} \tag{4}$$

Where η_{ij} is an optional weighting function, The values that are given by the weighting function are commonly called the heuristic information,here, suppose $\eta_{ij}=(d_{ij})^{-1}$, d_{ij} is the distance from city i to city j. $A_k = N\backslash tabu_k(t)$, A_k denotes the cities that ant k has not visited,and $tabu_k(t)$ denotes that cities that ant k has visited. Furthermore, the exponents α and β are positive parameters whose values determine the relation between pheromone information and heuristic information.

4.3 Trail updating

In order to improve future solutions, the pheromone trails of the ants must be updated to reflect the ant's performance and the quality of the solutions found.This updating is a key element to the adaptive learning technique of ACO and helps to ensure improvement of subsequent solutions. Trail updating includes local updating of trails after individual solutions have been generated and global updating of the best solution route after a predetermined number of solutions m has been accomplished. This is done with the following local trail updating equation,

$$\tau_{ij}(t+n)=\rho\tau_{ij}(t)+\Delta\tau_{ij} \tag{5}$$

where ρ is a parameter that controls the speed of evaporation, $\Delta\tau_{ij}$ is the adding pheromone to all the arcs in this tour.

$$\Delta\tau_{ij}=\sum_{k=1}^{M}\Delta\tau_{ij}^{k} \tag{6}$$

Where $\Delta\tau_{ij}^k$ is the adding pheromone to the arcs in the tour ant k has visited. if i,j are at the tour ant k has visited.

$$\Delta\tau_{ij}^{k}=Q/L_k\,, \tag{7}$$

Where Q is a constant, L_k is the distance ant k has toured.

This updating encourages the use of shorter routes and increases the probability that future routes will use the arcs contained in the best solutions. This process is repeated for a predetermined number of iterations and the best solution from all of the iterations is presented as an output of the model and should represent a good approximation of the optimal solution for the problem.

5. Computational experiments

To demonstrate the efectiveness of the algorithm computational experiment is conducted on a series of standard data. The data set which is selected from theTSPLIB ,and we suppose the number of salesmen is 5. The algorithms are coded by C++ language and implemented on a Pentium 4 PC at 2.66GHZ(512MB RAM).

In order to evaluate the efficacy of the ACO algorithm for the MTSP with ability constraint, we simulated the other algorithm available in the literature, namely modified genetic algorithm(MGA)[9]. The algorithms are applied to 6 standard problems, for most of which the optimal solutions are available. The results of this simulation are given in Table 1. In this table, for each algorithm the average and the best of obtained solutions through 10 replications as well as the computation time is reported.The table shows that MGA is little superior to the proposed ACO algorithm in pr76 and pr152,which are not large scale problems.And in following larger scale problems,such as pr226,pr299,pr439 and pr1002,the solutions of the proposed ACO algorithm are superior to MGA.

Table 1. The computational results

Problem	n	m	l	Proposed			MGA		
				Best	Avg	Time	Best	Avg	Time
pr76	76	5	20	178597	180690	51s	157444	160574	43s
pr152	152	5	40	130953	136341	128s	127839	133337	91s
pr226	226	5	50	167646	170877	143s	166827	178501	165s
pr299	299	5	70	82106	83845	288s	82176	85796	363s
pr439	439	5	100	161955	165035	563s	173839	183698	623s
pr1002	1002	5	220	382198	387205	2620s	427269	459179	2892s

n: the number of cities.

m: the number of salesmen.

l: the max number of cities a salesman can visit.

6. Conclusion

ACO is a promising optimization technique for solving complex combinatorial optimization problems like the MTSP. In the past, ACO has been applied to several combinatorial optimization problems successfully. In this study its application to the MTSP with ability constraint which is an extension of the well-known TSP is presented. To evaluate its effectiveness,we compare the algorithm with MGA by testing problems from standard TSPLIB. In these tests the proposed algorithm found not all of the best solutions but competitive solutions within rational time,especially in large scale problems.

Ant colony optimization clearly has the ability to find good results within 1% of the known optimum for small problems. However, consistent with past research, the ACO methods used in this research are not as efficient in finding solutions for larger problems.In the research,a algorithm based ACO is applied to larger problems successly,and the solution is encouraging,

7. References

[1] Lawler EL,Lenstra JK,Kan Rinnooy AHG,Shmoys DB, The traveling salesman problem:a guided tour of combinatorial optimization,*Chichester*,Wiley,1985, 11.201-209.

[2] Angel RD,Caudle WL,Noonan R,Whinson A,Computer assisted school bus scheduling,*Management Science*,1972, 18.279-88.

[3] Orloff DS,Routing a fleet of M vehicles to/from a central facility,*Networks*,1974,4.147-162.

[4] Christofides N,Eilon S,An algorithm for the vehicle dispatching problem,*Operations Research Quarterly*,1969, 20.309-318.

[5] Savelsbergh MWP,The general pickup and delivery problem,*Transactions of Science* ,1995,29.17-29.

[6] Sveska JA,Huckfeldt VE,Computational experience with an *m*-salesman traveling salesman algorithm.*Management Science*,1973,19.790-799.

[7] Bellmore M,Hong S,Transformation of multisalesmen problem to the standard traveling salesman problem,*Jurnal of the Association Computer Machinery*,1974,21.500-504.

[8] Samerkae Somhom,Competition-based neural network for the multiple travelling salesmen problem with minmax objiective,*Computers&Operations*,1999,395-407.

[9] Lixin Tang,Jiyin Liu,multiple traveling salesman problem model for hot rolling scheduling in Shanghai Baoshan Iron & Steel Complex,*European Journal of Operational Research* , 2000,24.267- 282.

[10]Lenstra JK,Rinnooy Kan AHG,Some simple applications of the travelling salesma *n* problem,*Operations Research Quarterly*,1975,26.717-734.

[11] Dorigo M,Maniezzo V,Colomi A,Positive feedback as a search strategy,*Technical Report,*Politecnico idi Milano, 1991,91-106.