Recent Developments in Dynamic Vehicle Routing Systems

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Summary. This chapter examines the evolution of research on dynamic vehicle routing problems (DVRP). We define the DVRP and show how it is different from the traditional static vehicle routing problem. We then illustrate the technological environment required. Next, we discuss important characteristics of the problem, including the degree of dynamism, elements relevant for the system objective, and evaluation methods for the performance of algorithms.The chapter then summarizes research prior to 2000 and focuses on developments from 2000 to present. Finally, we offer our conclusions and suggest directions for future research.

Key words: Networks; transportation; dynamic vehicle routing problems.

1 Introduction

Supply chains have become a competitive weapon in the global economy. The remarkable advances in telecommunications and information technology have enabled companies to focus on velocity and timeliness throughout the supply chain. To achieve these competitive advantages, they must be able to make effective use of the vast amount of real-time information now available to them. The Dynamic Vehicle Routing Problem (DVRP) is a prime example of a distribution context where intelligent use of real-time information can differentiate one company from another by means of superior on-time service.

The DVRP is the dynamic counterpart of the generic vehicle routing problem (VRP). In the latter problem the objective is generally to minimize the travel cost for several vehicles that must visit and service a number of

B. Golden et al. (eds.), *The Vehicle Routing Problem*,
doi: 10.1007/978-0-387-77778-8_9,

customers. Constraints specifying capacity restrictions, time windows within which to start service at customers, and additional requirements on the drivers and vehicles restrict the optimization space. In the VRP all routing and demand information is known with certainty prior to the day of operations, so routes can be planned ahead. In contrast, in the DVRP part or all of the necessary information becomes available only during the day of operation. In other words, not all information relevant to the planning of the routes is known by the planner when the routing process begins and information can change after the initial routes have been constructed.

The practical significance of the DVRP is highlighted by the variety of environments it can model. An important application is the pickup and delivery of overnight mail. Other scenarios include the distribution of heating oil or liquid gas to private households, residential utility repair services, such as cable and telephone, and appliance repair. Additional settings are the transportation of the elderly and physically disabled, taxi cab services, and emergency services, such as police, fire, and ambulance dispatching.

This chapter is organized as follows. In section 2 we illustrate the technological environment required for the DVRP. Next, section 3 discusses different characteristics of the DVRP, including the degree of dynamism, elements relevant for the system objective, and evaluation methods for the performance of algorithms. Then, in section 4 we review the most important research conducted until the year 2000. In the following section, section 5, we focus on developments from 2000 to present. Finally, in section 6 we offer our conclusions and suggest directions for future research.

2 Technological Environment

In this section we present some of the essential technologies needed for dynamic vehicle routing environments.

2.1 Communication and Positioning Equipment

The communication between the vehicle drivers and the dispatching center is essential in order to feed the most up-to-date information into the routing system. We briefly describe below the equipment for determining the current position of the vehicles and the communication equipment for transferring information between the dispatching center and the vehicle drivers.

A simple vehicle positioning strategy is to have the driver report back to the dispatching center every time a customer has been serviced. To give the planner much more information, more sophisticated alternatives make use of *positioning equipment* like the GPS (Global Positioning System). The GPS is a constellation of more than two dozen GPS satellites orbiting the Earth that constantly send out signals giving their positions and time. Signals from a number (usually three) of different satellites at any given time can provide receivers on the ground with enough information to calculate their precise location within a few meters depending on which version of the GPS system is used. For further reading on the GPS system the reader should refer to Collins [18]. Over the last decade GPS receiver prices has dropped considerably and today literally all high-end passenger cars as well as all service vehicles such as trucks, delivery vans and pick-ups have GPS equipment installed from the manufacturer.

The *communication equipment* between the vehicle and the dispatching center is another essential element for the routing system. Mobile telephone communication systems are one example of a technology capable of providing this information. Another technology is a dedicated radio based communications system. The main difference between these technologies is in terms of initial and operating costs. Although the competition is fierce within the mobile telephone communication market, operating costs are still considerable. Usually these systems are based on text messaging or newer technologies such as GPRS (General Packet Radio Service) and 3G (third generation) cell phones. For a text messaging based system, a message should be sent every time a position update is required. In case the day of operation is eight hours and the position should be updated every 10th second this means that almost 3000 text messages per vehicle should be sent every day. On the other hand, the initial costs of implementing a radio based communications system are very high because transmission masts will have to be put up and relatively expensive radio equipment must be installed in every vehicle. In all, a radio based communication system has very high initial costs, while the operating costs are almost negligible. However, the radio based system does not offer the same flexibility as does its mobile telephone communications system counterpart. This factor may be important in cases where service providers are considering servicing a new area where transmission masts have not been installed.

In Figure 1 the basic information flows between the vehicle and the dispatching center are shown. Ideally, the dispatching center will know in which state the vehicle and the driver are at any given point in time. However, as the above discussion indicated, this may prove to be unreasonable for some applications due to the operating costs of this method. Generally, within a real-world setting the positioning information is transmitted at fixed intervals and an interpolation scheme is employed in order to estimate the positions of the vehicles. Alternatively, the driver sends a message about his current status and position to the dispatch center each time he/she finishes the service at a customer. Obviously, this approach does not offer the same level of information for the dispatcher to support his/her decision as to which vehicle to dispatch to the next customer to be served. If the new information provided by the now idle driver/vehicle makes the dispatcher alter the current planned routes, he/she will have to call the other drivers directly to inform them about the changes in the current routes. Overall a thorough analysis must be performed to determine which approach to choose when designing a system. As we have witnessed, the cost of communication has decreased rapidly over the years.

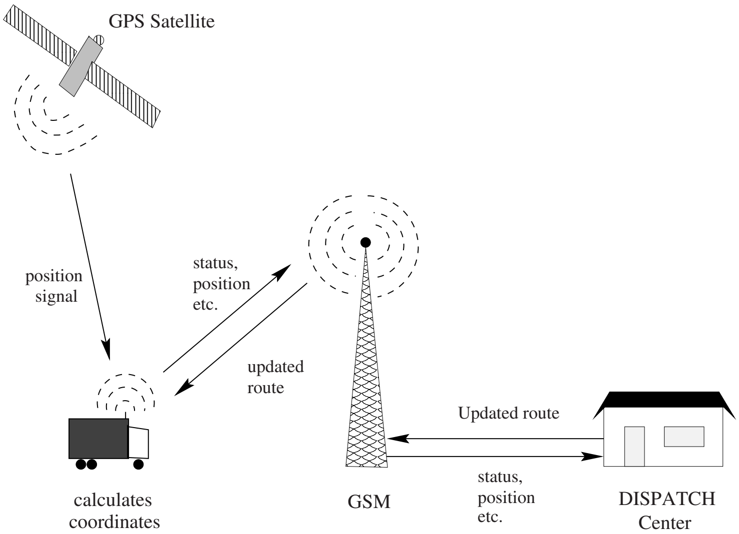

Fig. 1. Sketch of the information flow in a GPS based vehicle routing system.

It is not surprising that this has been the motivation for most companies to deploy more and more sophisticated telecommunications systems.

2.2 Geographical Information Systems

The advances in digital road maps and geographical information systems (GIS) have also been considerable over the last decade. Most industrialized countries now have almost fully detailed road network databases. In Denmark, DAV (Dansk Adresse og Vejdatabase - Danish Road and Address Database) offers a digital road database which is connected to detailed information on every address in the country. The DAV database includes information on the zip codes, official street names, route numbers, road classification, highest and lowest street number on both sides of the streets. However, for the time being DAV still needs to include restrictions on turning and information on one-way streets.

Naturally, in a real-world application it is vital that the chosen solution algorithm is capable of processing large amounts of geographical information fast enough to solve the problem online. Issues related to the computation of the shortest paths in a road network become extremely important when implementing end user online routing applications.

The advanced GPS/GIS systems discussed above enable companies to keep track of the position and status of their fleet of vehicles at any given time. Such advanced distribution planning systems based on the DVRP are beginning to be embedded in Enterprise Resource Planning (ERP) systems allowing to link their routing data with inventory and other important information.

3 The Dynamic Vehicle Routing Problem

As discussed above the DVRP is the dynamic counterpart of the classic VRP. Psaraftis [48], [49] discusses a number of dimensions that makes the DVRP different from the VRP. Psaraftis [49] also differentiates the two problem classes in terms of attributes of the information used as input for the respective problem types. In addition, Powell *et al.* [46] distinguish between dynamism within a problem, a model and the application of a model.

In the DVRP, vehicles must service two types of requests: *advance requests* and *immediate requests*. The former are requests of static customers that have placed them before the routing process was begun. The latter requests are received from dynamic customers and arise in real-time during the day of operations. The insertion of immediate request customers into already planned routes is usually a complicated task that leads to either partial or full re-planning of the non-visited parts of the routes. The complexity of a routing problem directly affects the difficulty of inserting dynamic customers. For example the presence of time windows will usually increase the insertion difficulty. This can lead to immediate request customers being denied service.

3.1 The Degree of Dynamism

The complexity of a dynamic vehicle routing system is a function of the number of customers and their spatial distribution, just like the VRP, but more importantly it also depends on the number of dynamic events and their temporal distribution.

Dynamism without Time Windows

In a system without time windows three important parameters are: the number of static customers, the number of dynamic customers and the arrival times of the dynamic customers. The first two dimensions are captured by the *degree of dynamism* concept introduced by Lund et al. [39]. It is the ratio, denoted *dod*, of the number of immediate requests, n_{imm}, relative to the total number of requests, n_{tot}. Formally:

$$dod = \frac{n_{imm}}{n_{tot}} \tag{1}$$

For example in a system with ten customers, if two customers arrive while the system is on-line, the degree of dynamism is 20 % . The degree of dynamism mirrors many practical environments. Generally, some information is available before the day of operations begins while the rest is received in real-time during the day. The extent of the information received in real-time relative to the total system information provides insights into how dynamic the routing system really is.

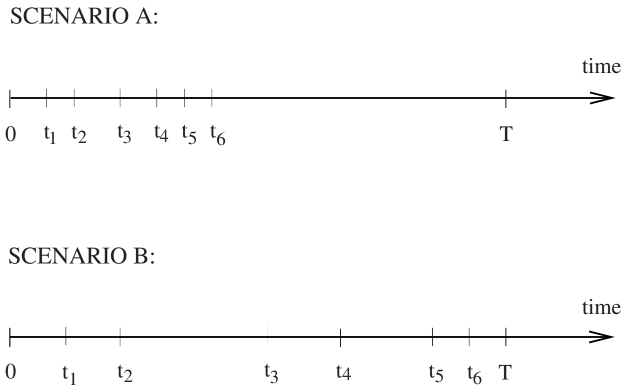

Fig. 2. Arrival time of immediate requests.

However, this basic measure does not take the arrival times of the immediate requests into account. This means that two systems, one in which the immediate requests are received at the beginning of the planning horizon and the other in which they occur late during the day, are perceived as equivalent. Naturally, in real-life routing settings these two scenarios are however very different. Figure 2 illustrates two DVRP scenarios in which the times for receiving immediate requests differ considerably.

In `Scenario A` all six immediate requests are received relatively early during the planning horizon. In `Scenario B` the requests are distributed almost evenly throughout the planning horizon. We suggest that the planner would prefer the former scenario to the latter, since having the highest number of requests in the pool of waiting requests improves the solution quality with respect to the objective of minimizing the total distance driven. Hence, the expected length of the route should be shorter in `Scenario A` than in `Scenario B` since time t_6 occurs much earlier in the former scenario than in the latter. Formally, this is captured by the *effective degree of dynamism* introduced by Larsen *et al.* [36]. Let the time the i'th immediate request is received be denoted by t_i and the entire planning horizon be denoted by $[0, T]$. Then the *effective degree of dynamism*, denoted by *edod*, is defined as:

$$edod = \frac{\sum_{i=1}^{n_{imm}} (\frac{t_i}{T})}{n_{tot}} \tag{2}$$

This measure takes the arrival times of the immediate requests into account and is a natural extension of the *dod*. It is apparent that `Scenario A` has a smaller *edod* than `Scenario B`.

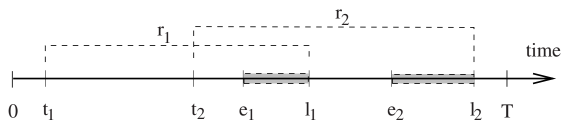

Fig. 3. The reaction times of two dynamic customers in a DVRP with time windows.

Dynamism and Time Windows

The measures defined above can be refined to allow for time windows to be taken into consideration. We do this by means of the *reaction time*. The reaction time is defined as the temporal distance between the time the request is received and the latest possible time at which the service of the requests could begin. Formally, the earliest time that service can begin (i.e., the start of the time window) is denoted by e_i while the latest possible time that service could begin is denoted by l_i. The reaction time of the i'th immediate request is denoted by r_i and $r_i = l_i - t_i$. Figure 3 shows this graphically.

The *effective degree of dynamism* when customers impose time windows (Larsen *et al.* [36]), denoted by $edod_{tw}$, can be defined as follows:

$$edod_{tw} = \frac{1}{n_{tot}} \sum_{i=1}^{n_{imm}} (1 - \frac{r_i}{T}) \qquad (3)$$

In a static system $edod_{tw} = 0$ and in general $0 \leq edod_{tw} \leq 1$. When there are no time windows the $edod_{tw}$ simplifies to the *edod*. According to this measure, the smaller the reaction times, the more dynamic the system is.

3.2 Determining the Objectives

The traditional objective for the static VRP has been to minimize the overall distribution costs. For the DVRP additional measures come into play. In particular, the level of service offered to the customers is important for the overall performance of the system. Often the multiple objectives encountered in the DVRP may be conflicting. Naturally, objectives may differ from one application to the other. Nevertheless, a few measures are almost always relevant to consider. These are: travel costs, service level, and throughput maximization. As for the static VRP, the distribution costs should be considered since they represent a true cost for the distribution company. The service level measure of system performance is generally in conflict with the objective of minimizing the distribution costs since a fast response to a new immediate request for service may imply that the vehicles will have to travel longer distances. Throughput optimization considers the ability to serve as many customers as possible. For some dynamic problems this is the most important objective. As an example, the maximization of the expected number of serviced requests is the main objective in the taxi cab business.

3.3 Measuring Algorithmic Performance

The most accepted framework for measuring the performance of on-line algorithms is *competitive analysis* introduced by Sleator and Tarjan [50]. For a minimization problem the *competitive ratio*, cr_A, can be defined as:

$$cr_A = \sup_I \frac{z(A, I)}{z^*(I)} \tag{4}$$

where $z(I)$ is the cost of the solution found by algorithm A for instance I and z^* is the optimal cost found by an (ideal) offline algorithm which had access to the entire instance I, including dynamic requests, beforehand. The competitive analysis framework offers a measure for evaluating the performance of a certain on-line routing policy based on the worst-case ratio between this policy and the optimal offline policy. In other words, for each policy examined this ratio quantifies the loss of cost-efficiency stemming from the lack of full information.

The competitive analysis framework provides a strong basis for studies of the performance of on-line algorithms which have produced interesting analytical results and insights. However, only very simple versions of the DVRP can be treated using this framework. Important real-world constraints such as time windows have so far proved to be too complex to be considered in this framework. Furthermore, in most real-world situations it is indeed possible to achieve an average performance which is considerably better than the one suggested by the competitive ratio.

For more complex versions of the problem algorithmic performance has been evaluated through empirical studies. This has usually been done by discrete-time simulation. This type of analysis can be extended so that the performance of a certain algorithm is evaluated by running the algorithm on both the original dynamic instances and on the instances in which the immediate requests are changed into static data. This provides an estimate of the competitive ratio of the algorithm to go along with its average performance.

4 Research Prior to 2000

Early research has considered a number of dynamic and/or stochastic elements in vehicle routing. The problems addressed include the Probabilistic Traveling Salesman Problem (PTSP) and the Probabilistic Vehicle Routing Problem (PVRP), the Stochastic Vehicle Routing Problem (SVRP), the Dynamic Traveling Salesman Problem (DTSP), the Dynamic Traveling Repairman Problem (DTRP), the Dynamic Vehicle Routing Problem (DTSP), the Dynamic Dial-A-Ride Problem (DARP), the Dynamic Pick-Up and Delivery Problem (DPDP), and the dynamic version of the Vehicle Routing Problem with Time

Windows (DVRPTW). Other variants include the Time-Dependent Traveling Salesman Problem (TD-TSP) and the Time-Dependent Vehicle Routing Problem (TD-VRP). In contrast to the DVRP defined above, the PVRP is a VRP where each customer has a given probability of requiring a visit. In the SVRP customers are present at locations with some probabilities and their demands are random. In the TD-VRP the travel time between two customers is a function of the departure time from the first one.

Figure 4 gives a chronological overview of some of the most important work on dynamic and stochastic vehicle routing problems prior to the year 2000. A virtual line through the middle of the illustration divides the literature in two sides: the left-hand one embracing a-priori optimization based models and the right-hand one embracing real-time optimization based models. Within the vehicle routing context a-priori based solutions mean that the planner determines one or more routes based on probabilistic information on future requests for service, customers demands, travel times or other parameters. Within this setting routes will be planned before the vehicles leave the depot. Real-time optimization models construct routes during the day of operation while the vehicle is in-route. Horizontally, the illustration is organized so that the basic models, as for instance the DTRP and PTSP, are located in the middle of the illustration corresponding to the axis of complexity shown at the bottom of the figure. Similarly the DVRPTW is placed at the very right to represent the high complexity of this problem.

The a-priori optimization based methods have been used for the PVRP and SVRP which are static by nature. They are generally not applicable in a real-world on-line DVRP context since the dynamic environment leads to very high computation times. Furthermore, it may not be worth the computational effort to try to find an optimal or near-optimal solution in a real-time setting, because new requests may render the solution sub-optimal. Using re-optimization each time a new request appears only seems computationally tractable in cases where the degree of dynamism is quite low. Finally, these algorithms require extensive a-priori information such as the probability of a certain customer requiring service upon a certain day and time. In most dynamic cases such detailed information will not be available.

Real-time optimization based methods have been used for problems that are dynamic by nature, such as the DTSP and the DTRP. Extensive theoretical work on the DTRP has been conducted by Bertsimas and Van Ryzin ([12], [13] and [14]). More complex DVRP problems have been tackled empirically. The parallel implementation of the tabu search proposed by Gendreau *et al.* [23] exemplifies this type of approaches.

5 Developments from 2000 to Present

Recently we have witness an ever growing body of research on the DVRP and many of its variants. To maintain focus we will only consider advances for the

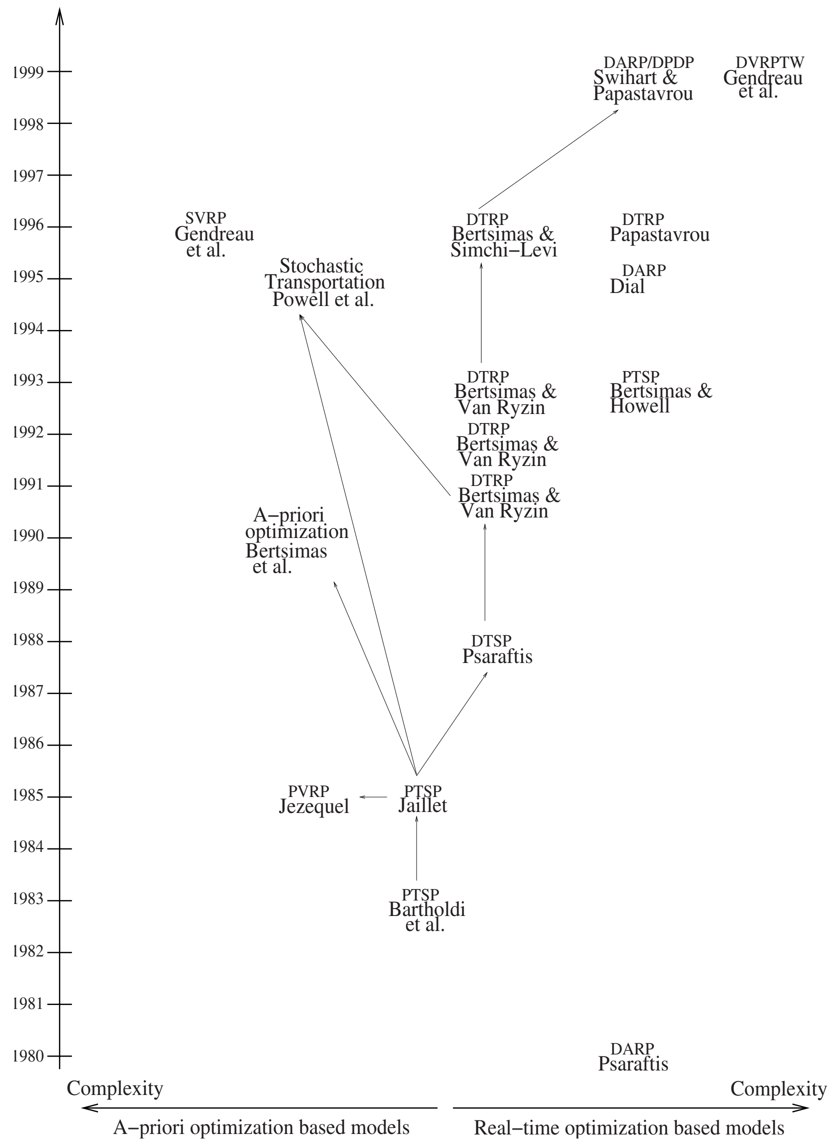

Fig. 4. Chronological overview of important literature prior to 2000.

DVRP. Ghiani *et al.* [27] provide and excellent earlier review of the field and address the role of parallel computing strategies in this context.

5.1 A Priori Knowledge, Waiting and Relocation in Partially Dynamic Problems

One stream of research has addressed the DVRP with various degrees of dynamism. Larsen *et al.* [36] proposed a framework for dynamic routing systems based on their *degree of dynamism*. Systems are partitioned in weakly, moderately, and strongly dynamic depending on whether their degree of dynamism is below 20-30 %, between 30 and 80 % or over 80 %, respectively. Recently, Larsen *et al.* [38] have refined their three-echelon classification of dynamic vehicle routing systems based on the degree of dynamism and the system objective. They also discuss methods for evaluation of the performance of algorithms that solve on-line routing problems and list some of the most important issues to include in the system objective. Larsen *et al.* [36] then describe and test several dynamic policies to minimize routing costs for the Partially Dynamic Traveling Repairman Problem (PDTRP) with various degrees of dynamism. Later, Bent and Van Hentenryck [7] considered moderately dynamic DVRPTW with stochastic customers. They proposed a multiple scenario approach that continuously generates routing plans for scenarios including advanced and immediate requests to maximize the number of serviced customers. Computational results on problems adapted from the Solomon benchmarks highlight the effectiveness of their approach which increases with the degree of dynamism.

Larsen *et al.* [37] proposed real-time solution methods for the Partially Dynamic Traveling Salesman Problem with Time Windows (PDTSPTW) that minimize lateness. One method requires the vehicle, when idle, to wait at the current customer location until it can service another customer without being early. Other algorithms, called relocation policies, may reposition the vehicle at a location different from that of the current customer based on a priori information on future requests. The results obtained on both randomly generated data and on a real-world case study indicated that all policies significantly reduced lateness at the expense of only small distance increases. The basic policy outperformed the other methods primarily when lateness and distance were equally minimized and proved very robust in all environments studied. When only lateness was considered, the policy to reposition the vehicle at a location near the current customer generally provided the largest reductions in average lateness and the number of late customers. It also produced the least extra distance to be traveled among the relocation policies. where waiting and relocation points are defined a priori using knowledge of the distribution, clustering of the customers, and heuristics.

While Larsen *et al.* [37] defined waiting and relocation points a priori using heuristics that exploit knowledge of the distribution and clustering of the customers, Bent and Van Hentenryck [8] allowed the vehicle to wait or relocate anywhere and at any time during the algorithm execution. Computational results indicate that these two strategies were very effective in maximizing the number of customer served, particularly for high *dod* problems that contain many late requests. Vehicle relocation is also addressed by van Hemert and La Poutre [55] in a DVRP context where loads generated throughout the day must be picked up and returned to a depot within a specified time interval. The authors analyze the benefit of anticipatory vehicle moves within regions that have a high potential of generating loads. They propose a self-adaptive evolutionary algorithm and examine under what conditions such moves improve its effectiveness in terms of the ratio of the loads successfully delivered to the total number of loads made available for transport.

Ichoua *et al.* [32] also propose a strategy that exploits probabilistic knowledge about future request arrivals. Forecasted requests are incorporated as dummy customers in vehicle routes. The routes are constructed by a modified version of the tabu heuristic of Gendreau *et al.* [23]. Earlier, Ichoua *et al.* [31] extended the method proposed by Gendreau *et al.* [23] to incorporate diversion. That involves allowing a vehicle to be diverted from its predetermined destination. Rather than using local or myopic strategies where a new request for service impacts only one vehicle, the authors took a global view where several vehicles may be affected. The above papers have been motivated in part by the DVRP faced by courier service companies. The same environment is examined in Angelelli *et al.* [1] where customer requests with service time windows have to be serviced in real time by a fleet of vehicles. The authors consider both pick-up and delivery requests and assume that customer requests cannot be refused but can be postponed to future shifts. A heuristic algorithm based on local search is proposed.

Branke *et al.* [15] have further analyzed waiting policies that maximize the probability of being able to service an additional immediate request customer. For the single vehicle case, a "no wait" or drive without waiting policy was shown to be optimal. However, for the multiple vehicle case, it proved to be the worst among eight waiting policies for a vehicle starting or restarting from the depot. A "variable" policy proved the best. Specifically, each vehicle drives without waiting until the time to drive the remaining distance to the depot is equal to the slack time. Then the available waiting time is distributed to the remaining customers in proportion to the remaining driving distances. Independently, Mitrovic-Minic and Laporte [41] have examined similar policies for the pickup and delivery problem with time windows. Whether to wait and for how long is also addressed by Potvin *et al.* [45]. The authors consider the impact of dynamic events in the form of travel time and customers on the partially dynamic DVRPTW. They show that policies based on a certain amount of waiting, that is, a certain tolerance for changes in the current planned routes leads to better overall results.

Other novel research includes that of Montemanni *et al.* [42] solved a DVRP problem using the Ant Colony System paradigm. The algorithm was tested on simulated environments and also applied to a real-world case study in the city of Lugano, Switzerland. Hvattum *et al.* [30] addressed a DVRP problem that also had stochastic elements. Based on an actual application, the authors used historical data to generate probability distributions for the attributes of the dynamic customers. They formulate the problem as a multi-stage stochastic programming problem with recourse and propose a heuristic that gradually builds the routes by exploiting the information gathered on future customer demand. The computational results illustrate the superiority of the this method over a more standard pure dynamic heuristic that resolves the problem at the beginning of a number of consecutive time intervals. Chen and Xu [17] used an optimization-based dynamic approach to minimize the total distance traveled for a dynamic vehicle routing problem with hard time windows. At each decision epoch, the authors solve by column-generation a static Vehicle Routing Problem with Time Windows (VRPTW) consisting of all the known orders that have not been satisfied up to that point. Columns are generated dynamically over time by alternately solving a linear program and applying a fast local-search-based heuristic. The authors report that their approach outperformed an insertion-based heuristic on most test problems generalized from Solomon's problems.

5.2 The Dynamic Pick-up and Delivery Problem

Another stream of research has focused on the dynamic pick-up and delivery problem. Attanasio *et al.* [4] consider the dynamic DARP where as many as possible of the requests received throughout the planning horizon must be satisfied. They present parallel implementations of a tabu search method developed earlier by Cordeau and Laporte for the DARP. The authors report that the proposed algorithms are capable of a high service level as measured by the percentage of satisfied requests. Thomas and White [54] considered the single vehicle pickup and delivery problem where the origin and destination are known. Immediate request customers may demand service while the vehicle is in transit with a known probability. The authors seek a strategy to construct routes that minimize the expected travel time and lateness penalty incurred at the destination if the cutoff time is exceeded. Using a finite-horizon Markov decision process model they propose a policy that optimally build routes that consider anticipated immediate request customers. This policy is computationally compared to an industry standard reactive strategy where there is no information about an immediate request until this actually occurs for a very small number of anticipated requests. It seems that anticipatory routing outperformed the reactive strategy, especially when the immediate requests are likely to occur late during the day of operations.

Coslovich *et al.* [19] considered a dynamic DARPTW where customers can ask the vehicle driver for a trip at a vehicle stop. They proposed a two-phase insertion heuristic based on route perturbations consisting of 2-opt arc swaps. One phase is off-line and produces a feasible neighborhood of the current route and the other phase is on-line where an attempt is made to insert the unexpected customer in the planned route. The objective considered was to minimize the overall inconvenience of the advance customers. The authors report computational results that indicate that their solution was quite close to the static one and the number of unexpected requests that were not accepted was negligible. Gendreau *et al.* [22] examined a different dynamic DARPTW faced by courier services for the same-day local pick-up and delivery of small sized packages. The authors developed a tabu search heuristic where the neighborhood structure was based on ejection chains. Their experiments conducted in a parallel computing environment indicated that their method was superior to insertion based methods, even in highly dynamic scenarios.

Recently, Mitrovic-Minic and Laporte [41] have examined the benefits of waiting strategies in the context of a pickup and delivery problem with time windows. While Branke *et al.* [15] focused on maximizing the probability of being able to service an additional customer, Mitrovic-Minic and Laporte [41] sought waiting policies that can decrease the total detour and the number of required vehicles. When a new request arrives, if the vehicle assigned to it waits as long as possible before moving the total insertion cost can be reduced at the expense of more vehicles. If, on the other hand, the vehicle travels as soon as possible, the total number of vehicles is decreased but longer detours are experienced. The authors propose an intermediate strategy, called advanced dynamic waiting, that involves partitioning the overall tour into service zones and allocating the total time available for waiting proportionally to the time necessary to service each service zone. This method was able to simultaneously decrease the total detour and the required number of vehicles. Mitrovic *et al.* [40] further this line of research by proposing a double-horizon heuristic for the dynamic PDPTW. The short term horizon, the next two hours, accounts for immediate increase in routing cost created by the insertion of an immediate request. The long-term horizon, the rest of the day of operations, deals with the decrease in vehicle slack time. The authors propose a constructive heuristic, which is used for every immediate request, in conjunction with a tabu search heuristic which is called periodically.

Fleischmann *et al.* [21] propose a planning framework for the dynamic PDPTW. The authors then suggest three event-based dispatching policies which differ in the length of the planning horizon per event. Using dynamic travel time information, the procedures are compared on real-world data from an urban traffic management center and a logistics service provider. Finally, Yang *et al.* [56] address a real-time multivehicle truckload pickup and delivery problem. They present a mixed-integer programming formulation for the offline version of the problem and propose a new rolling horizon reoptimization strategy for the real-time version. This policy is compared with another previously introduced reoptimization policy and three other known heuristic rules. The results of a simulation study that considered varying traffic intensities, degrees of advance information, and degrees of flexibility for job-rejection decisions indicate that the new reoptimization policy systematically outperformed the others.

5.3 Emergency Service Systems

Yet another stream of research has dealt with emergency service systems. Due to its importance, this application has received substantial attention from the scientific community since the early 1970s. These systems are truly strongly dynamic since no requests are known in advance of the day of operation. The quality of an emergency system is generally measured by its response time. The emergency service providers and the public administration agree on a certain level of service which for instance defines that 90% of the calls should be served within 5 minutes whereas the remaining 10% of the calls should be served within 8 minutes.

Often, the quality of a-priori information such as the potential location of the next request is often quite poor. If, on the other hand, a-priori information on future requests is available it could potentially improve the solution quality. For example, this could involve moving an idle vehicle currently situated in a low demand area to a central location. This idea was explored by Gendreau *et al.* [25] who proposed a model for real-time relocation of ambulances. For a survey of emergency vehicle location and relocation problems, see Brotcorne *et al.* [16]. Recently, Gendreau *et al.* [26] proposed a dynamic relocation strategy for emergency vehicle waiting sites that maximizes the expected covered demand and controls the number of waiting site relocations. They formulate the problem as an integer linear program and solve it within reasonable computing time when the number of vehicles is relatively small. Simulations conducted with real-world emergency medical services data from the Montreal area confirm the feasibility of the proposed approach.

5.4 Competitive Analysis

A different stream of research has examined the DVRP using competitive analysis. Following Bertsimas and Van Ryzin [12], [13] and [14] who were the first to use it in this context, Ausiello *et al.* [5] have studied the on-line version of the classical Traveling Salesman Problem (TSP) using this type of analysis. The authors examine two versions of the problem and provide lower bounds for the competitive ratio. Recently, Jaillet and Wagner [34] have examined online versions of the TSP and TRP where each request has a disclosure date. This is the time when the location and release date of a request become known. This measure is similar to the reaction time introduced by Larsen et al. [37] except it utilizes release times instead of due dates. The authors propose online algorithms for a variety of scenarios and show that this advanced information leads to better competitive ratios. They also provide a general result on polynomial-time online algorithms for the online TSP.

Complexity results and competitive analysis for vehicle routing problems are the subject of the PhD-thesis by Paepe [43]. Paepe gives a thorough analysis of the on-line version of the Dial-a-ride problem in which a single capacitated vehicle serves a set of customers that requests to be picked-up at some geographical location and to be transported to another location. The requests appear in real-time and Paepe derives the competitive ratios of a number of routing policies. Angelelli *et al.* [2], [3] studied a dynamic multi-period routing problem. Here, the orders arriving during a period have to be completed either in that period or the next. This means that the system will hold customers that are to be served right away as well as customers that will have to wait to be served. The authors introduce simple routing policies and analyze these by examining their competitive ratios.

Other streams of research have been directed at problems related to the DVRP. These include the Dynamic Assignment Problem (see Spivey and Powell [51]), the Dynamic Vehicle Scheduling Problem (see Huisman *et al.* [29]) and the VRP and its variants with time dependent travel times (see Taniguchi and Shimamoto [53], and Haghania and Jung [28]). Their discussion is beyond the scope of this chapter.

6 Conclusions and Directions for Future Research

This chapter has highlighted the evolution of research on dynamic vehicle routing problems. We introduced the DVRP and illustrated how it is different from the generic VRP. We then examined the technological environment required. Next, we delineated the salient problem dimensions, including the degree of dynamism, the potential components to be considered in the system objective, and theoretical and empirical evaluation methods for the performance of algorithms. We then looked back at the research conducted prior to 2000 and discussed the many developments from 2000 to present. Several themes for future research have emerged from this discussion.

The level of dynamism of a system has a strong impact on the type of algorithm to be used. Therefore, one direction for future work should consider the design of more encompassing measures to determine the level of dynamism of a given system. We discussed natural extensions of the basic *dod* measure, *edod* and $edod_{tw}$, which capture information on when the immediate requests are received by the dispatcher. Other than the initial research done by Jaillet and Wagner [34] the *edod*, $edod_{tw}$ or similar measures have yet to be considered in system classification or algorithmic design. While the *edod* and $edod_{tw}$ are improvements over the definition of the *dod* there are still many subtle interactions in the way requests arrive that will elude analysis for the time being, thus making it hard to characterize the difficulty of dynamic routing problems.

The above ideas are part of a broader theme which seeks to exploit knowledge about future request arrivals and/or consider vehicle relocation and waiting. The research of Larsen *et al.* [37], Ichoua *et al.* [32], Jaillet and Wagner [34], Bent and Van Hentenryck [8], and van Hemert and La Poutre [55], among others, has paved the way for future approaches. In particular, consider the issue of choosing where vehicles should idle in anticipation of future requests.

When determining the attractiveness of each idle point, an algorithm may take the arrival intensities of not only a given subregion but also neighboring ones into consideration. More sophisticated methods, such as location analysis, should be investigated.

Other interesting research directions should consider robustness considerations with regards to unforeseen events (e.g., vehicle failures, traffic congestion and others) and the integration of industry practices (e.g., the use of bicycles in highly congested areas, load consolidation, and others) in the models used.

We believe a new generation of DVRP algorithms will blend the effectiveness of advanced methods, tailored to take advantage of special problem structures and advanced knowledge, with the efficiency of parallel implementations, and the ever growing computing power of workstations to solve increasingly larger and more realistic problems. The overnight courier mail service provider environment represents a good model for the use of such new hybrid approaches. The morning subproblem is often weakly dynamic while the afternoon one is moderately dynamic. Therefore, a reoptimization algorithm could first plan a set of morning delivery routes. In case of urgent call-in requests, the algorithm could insert the new requests into the predetermined delivery routes. In turn, the afternoon pickup problem, would use fast algorithms for online routing that would take advantage of a priori information on future requests.

As companies are continuing the computer integration of their operations through ERP systems, much more information is being transferred between logistics and other functional areas. Implementations of advanced distribution planning systems based on DVRPs is starting to be seen in medium sized companies; small enterprises are next in line. This will likely be accelerated during the coming years by the ever growing number of just-in-time global supply chains. An example of this could be the transportation of the elderly and handicapped. Hence, the interface between DVRP models and algorithms and other parts of the supply chain such as warehousing and manufacturing is an important research direction that has yet to be tapped.

We hope that this chapter has offered an insightful perspective on this rapidly moving field. While the DVRP has reached a certain level of maturity, many important problems remain open. We can only hope that this chapter has steered sufficient interest that many of its readers will embark on or continue their research in this field.

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