Assignment5

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Question 2: General VAR

A. We often use VARs in macro time series estimation instead of IVs and single-equation estimations methods because of endogeniety issues. The VARs is a good a way to work with these endogeniety issues. To illustrate this, let's look at the following example. Let:

$$y_{1t} = \frac{M}{P} = \text{real money balance} \tag{1}$$

$$y_{2t} = \text{Real GNP} \tag{2}$$

The money demand function is:

$$y_{1t} = \gamma_{10} + \beta_{12} y_{2t} + \gamma_{11} y_{1,t-1} + \gamma_{12} y_{2,t-1} + \epsilon_{1t} \tag{3}$$

In this function, ϵ_{1t} encompossases all other factors, β_{12} is the elasticity of real money balances with respect to real income and the lagged terms allow for a different long-run elasticity. As we can see in the function, money demand,y_{1t}, is a function of the money supply, y_{2t}. Similarly, the function of money supply is dependent on the money demand:

$$y_{2t} = \gamma_{20} + \beta_{21} y_{1t} + \gamma_{21} y_{1,t-1} + \gamma_{22} y_{2,t-1} + \epsilon_{2t} \tag{4}$$

What we are interested in these equations are the β_{12} and β_{21} coefficients but the problem is there is an inverse realtionship between the two equations. y_{2t} is affected by y_{1t} and vice versa. This leads us to have inconsistent β coefficients if we were to estimate them via OLS as they are endegenous. Since there is a correlation between income and money, as evident from the equations, we are unable to infer any causality. This is where the VAR comes in.

If we estimate both equations together, ϵ_{1t} and ϵ_{2t} may include omitted variables. In a fully specified strucutral model ϵ_{1t} and ϵ_{2t} represnt exogenous shocks, in this case money demand shock and productivity shock respectively. What this means is that if ϵ_{1t} is indeed an exogenous shock, a change in it will affect y_{1t} and also y_{2t} via β_{21}. This means that we can theoreticaly identify β_{21} but there is the identification problem due to exogeneity. To run them together, we simply the VAR.

B. Using the exampl above:

$$\begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{22} & 1 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \gamma_{10} \\ \gamma_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \tag{5}$$