UNIVERSITÀ DI PISA

Logistics Project Report

Facility Location Problem

“Location of wifi-towers in Arizona”

Gaetano Antonicchio
Giulia Diamanti
Giulia Calvo
Davide Perra

Logistics

Academic year 2020/2021

General Problem

The aim of this project is to find optimal solution in terms of ILP to a facility location problem, by building two models able to satisfy the requirements defined by the text of exam.
The problem asked us to find the optimal number and location of wifi-towers for the Internet Agency (IA) company. The towers are to be built in the State of Arizona which was previously divided in 25 blocks. The model has to assure the realisation of constraints which will be listed in the next section.
For each tower built, the IA company has to pay a fixed cost of 180,000 $.
As per the expected revenue we were provided with the following matrix.

Revenue Matrix in (1,000 $)

$35	$44	$63	$43	$35
$65	$44	$72	$49	$66
$58	$58	$52	$62	$31
$33	$39	$71	$57	$41
$69	$74	$31	$57	$41

The exam text asks us the following:

1. Formulate an ILP model for the problem of IA.

2. Implement the model using the modelling language AMPL, and solve it by means of CPLEX.

3. Suppose IA is required to provide wireless service to at least the 92% of the blocks. Formulate the variant of the problem in terms of ILP.

4. Solve the variant of the problem using AMPL and CPLEX.

5. Compare the optimal solutions found in points 2 and 4: how much profit will IA make in the first year by considering the two alternative scenarios?

Notation

Let ***I*** be the set of blocks (demand nodes) and ***J*** the set of wifi-towers that can be potentially opened. Let R_i denote the revenue the company would get if node *i* is served by the wifi-tower *j,* and let f_j denote the fixed cost the company has to pay for opening wifi-tower *j.*
We used the binary variable x_i which assumes value 1 if the wifi-tower *i* is open, and 0 otherwise. The variable y_i is also binary, and takes value 1 if node *i* is served by a tower, and 0 otherwise.
The problem is presented as a coverage location problem, since we want to determine the optimal number and location of facilities (in our case towers) such that the company IA maximises its profit. Each block can be served by either a tower located at *i*, or by a nearby tower (provided the sides of the blocks are adjacent to the considered one).
Since we want to guarantee service to all the adjacent blocks, we defined the set N_i, which includes all the blocks adjacent to *i* , and *i* itself. This set indicates the blocks that would received the service (expressed as wifi-signal) if a tower is built at *i* or in the adjacent areas. Therefore, y_i takes value 1 if and only if at least one of the x_j for *j* in N_i is 1.
Furthermore, we indicate with f_i the fixed cost that IA company has to pay if it wants to open a tower at *i,* and with R_i the revenue expected if the service is provided to block *i.*

$$x_i = \begin{cases} 1 & \text{if wifi-tower is opened at } i \\ 0 & \text{otherwise} \end{cases} \quad , \ \forall i \in I$$

$$y_i = \begin{cases} 1 & \text{if block i is covered by the service} \\ 0 & \text{otherwise} \end{cases} \quad , \ \forall i \in I$$

N_i = *{set of blocks adjacent to i, including i }*

f_i = fixed cost for opening tower at *i*

R_i = Revenue if service is provided to block *i*

Methodology

This particular covering location problem has as *Objective Function* the maximisation of the profit. We were provided with data regarding the profits IA company would make if the service was provided to block i, along with information about the fixed costs needed to open one wifi tower at i (180,000 $). Since our goal is to maximise the profit, we have to maximise the difference between revenue and fixed costs.
Therefore the objective function is given by:

$$Max \sum_{i \in I} (R_i \times y_i) - \sum_{i \in I} (f_i \times x_i)$$

We can notice that given this maximisation function, the solver will try to set as many x_i to 0 (because it wants to minimise the fixed costs) and set as many y_i to 1 (so that the revenue is maximised).
Once we defined the objective function, we should provide a linking constraints that "*links*" the variable y_i to the variable x_i

$$\sum_{j \in N_i} x_j \geq y_i , \quad \forall i \in I$$

This constraint ensures that only opened facilities x_j belonging to N_i can provide service to y_i .
We then need only to set the variables to binary.

General Mathematical Formulation

Model 1#

$$Max \sum_{i \in I} (R_i \times y_i) - \sum_{i \in I} (f_i \times x_i)$$

$$\sum_{j \in N_i} x_j \geq y_i , \quad \forall i \in I$$

$$x_i \in (0,1) , \quad \forall i \in I$$

$$y_i \in (0,1), \quad \forall i \in I$$

Variation of the Model with minimal covering constraint

Model 2#

Given the constraints and objective function described above, we are asked to determine the location and number of wifi towers to be opened, so that the profit is maximised and the service is guaranteed to at least 92% of the total blocks.
Since the area of the state of Arizona was divided into 25 blocks, the problem now asks us to guarantee wifi service to at least 23 blocks (equivalent to 92% of the blocks).

This can be done without drastically modifying the original formulation. We need to add an additional constraint, which imposes that the sum of all blocks *i* should be greater or equal to 23 (in percentage 0.92).

Once we asserted this, the new constraint of the model is now:

$$\sum_{i \in I} y_i \geq 23, \quad \forall i \in I$$

This constraint will clearly have an impact on the profit expected by the company IA, because now by guaranteeing service to at least 92% of the blocks, we are forcing the company to open more wifi towers even if by doing so its profit will decrease (because the fixed cost will increase).

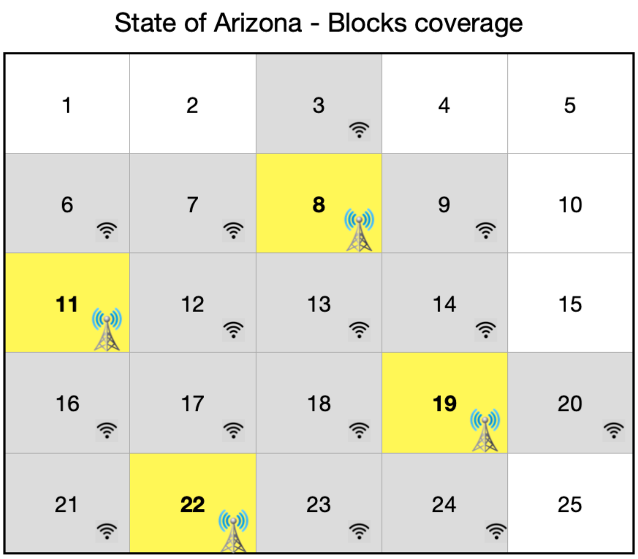
Solving the Problem

The problem was solved in AMPL and CPLEX. For each model described above, we calculated the expected profit and we visualised the location of the wifi-towers.
In order to compare the profit expected by applying one of the two models, we visualised the profits on a bar chart.

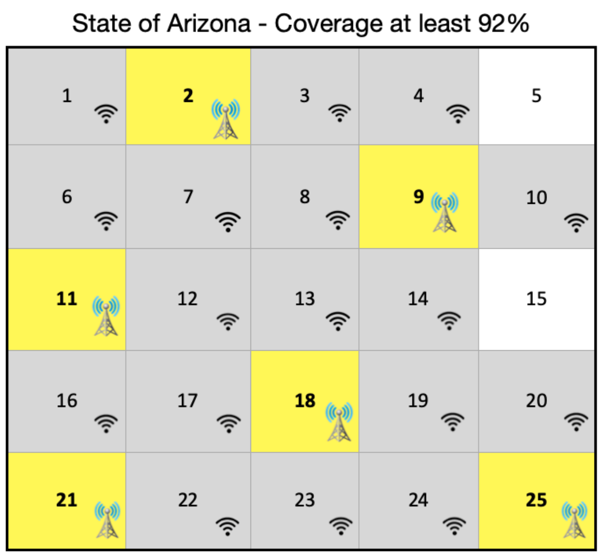
Results

The optimal solution for the first model (Model 1) is to open a wifi-tower at location { 8, 11, 19, 22}, which assures coverage to 18 blocks (included the ones in which the towers are built).
The corresponding expected profit is 275,000 $.

The optimal solution for the second model (Model 2) is to open a wifi-tower at location { 2, 9, 11, 18, 21, 25}. Since we instructed the solver to find a solution that would guarantee the service to at least 92% of the blocks, the optimal solution found was to provide service to 23 blocks in the Arizona State (only block 5 and 15 are not covered). This inevitably had repercussions on the profit, which is now expected to be 144,000 $.
Even though the second model was able to cover more blocks, the fix costs accounted for opening more towers have clearly a negative impact on the company's profit (the profit is reduced by half).
In conclusion, by comparing the two solutions, we can affirm that choosing model 1 is the best alternative for increasing the profit of IA company.



Model 1



Model 2