Constraint Programming

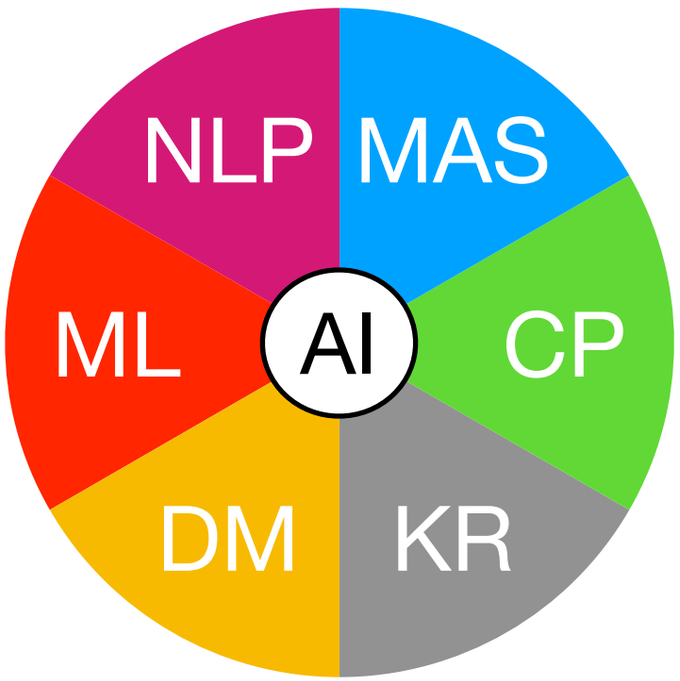
The user states the problem,
the computer solve it!

Nadjib Lazaar
Ing - Phd - Assistant Professor - University of Montpellier - COCONUT Team
http://www.lirmm.fr/~lazaar/

12/09/2022

Artificial Intelligence

AI Topics

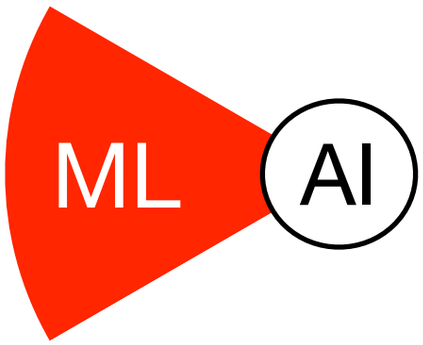


- NLP: Natural Language Processing
- MAS: Multi-Agent Systems
- KR: Knowledge Representation
- DM: Data Mining
- ML: Machine Learning
- CP: Constraint Programming

COCONUT

Artificial Intelligence

ML Topics



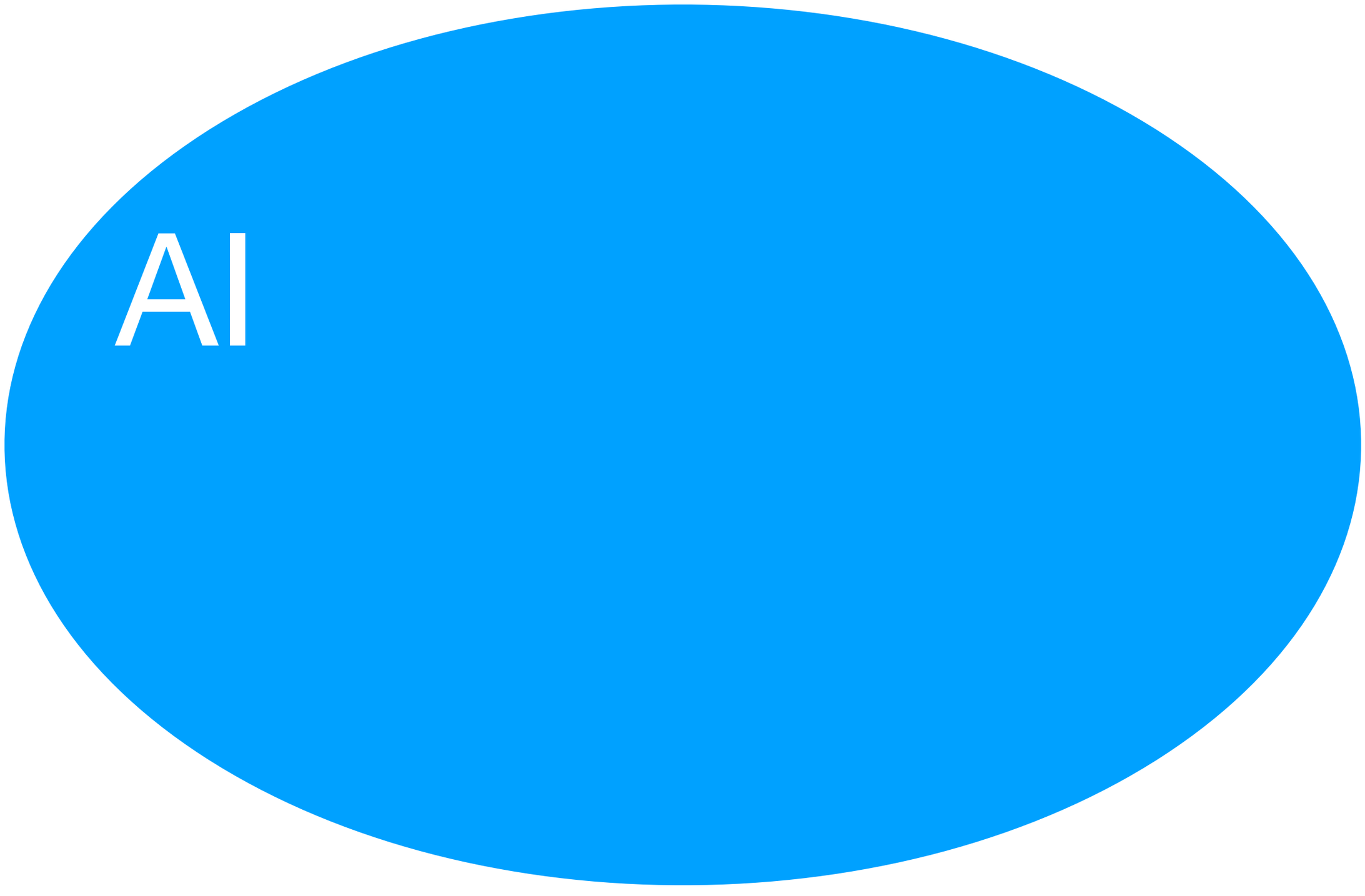
1. Active Learning
2. Adversarial Machine Learning
3. Bayesian Optimization
4. Classification
5. Clustering
6. Cost-Sensitive Learning
7. Deep Generative Models
8. Deep Learning
9. Developmental Learning
10. Dimensionality Reduction and Manifold Learning
11. Ensemble Methods
12. Explainable Machine Learning
13. Feature Selection
14. Learning Sparse Models
15. Federated Learning
16. Interpretability
17. Kernel Methods
18. Knowledge-based Learning
19. Learning Generative Models
20. Learning Graphical Models
21. Learning Preferences or Rankings
22. Learning Theory
23. Multi-instance;Multi-label;Multi-view learning
24. Neuro-Symbolic Methods
25. Online Learning
26. Probabilistic Machine Learning
27. Recommender Systems
28. Reinforcement Learning
29. Relational Learning
30. Semi-Supervised Learning
31. Structured Prediction
32. Tensor and Matrix Methods
33. Time-series;Data Streams
34. Transfer, Adaptation, Multi-task Learning
35. Trusted Machine Learning
36. Unsupervised Learning

Artificial Intelligence

ML Topics

Artificial Intelligence

ML Topics



Artificial Intelligence

ML Topics

Artificial Intelligence

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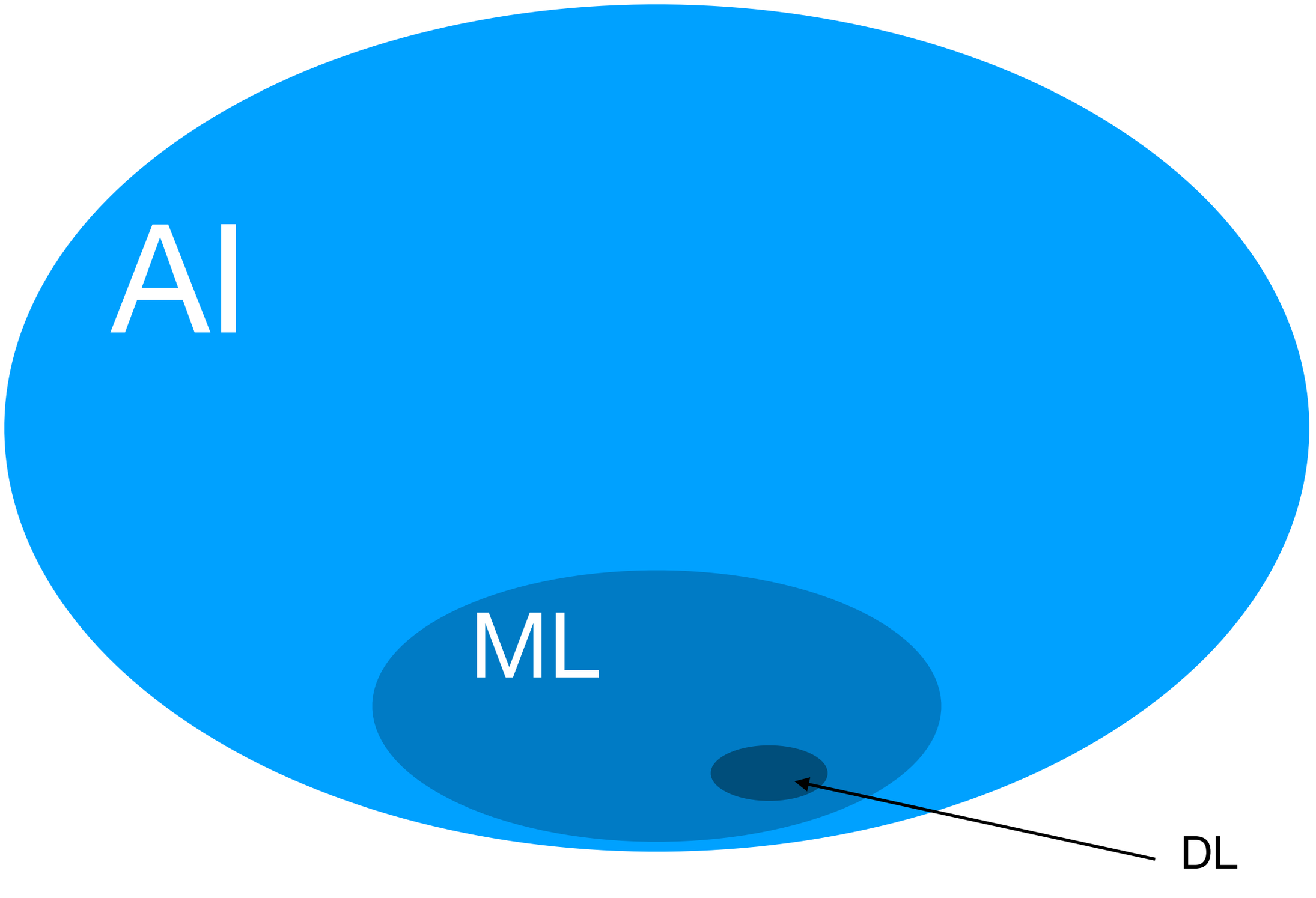

Constraint Programming

Definition

Constraint Programming

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A formalism to model (**constraint network**) and to solve (**constraint solver**) combinatorial problems (scheduling, planning,…).

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Examples of constraints :

- $X_1+X_2+X_3=5$
- allDifferent(X_1,..X_n)
- c(X_i,X_j)= {(1,1), (2,1),(2,3), (4,4)}

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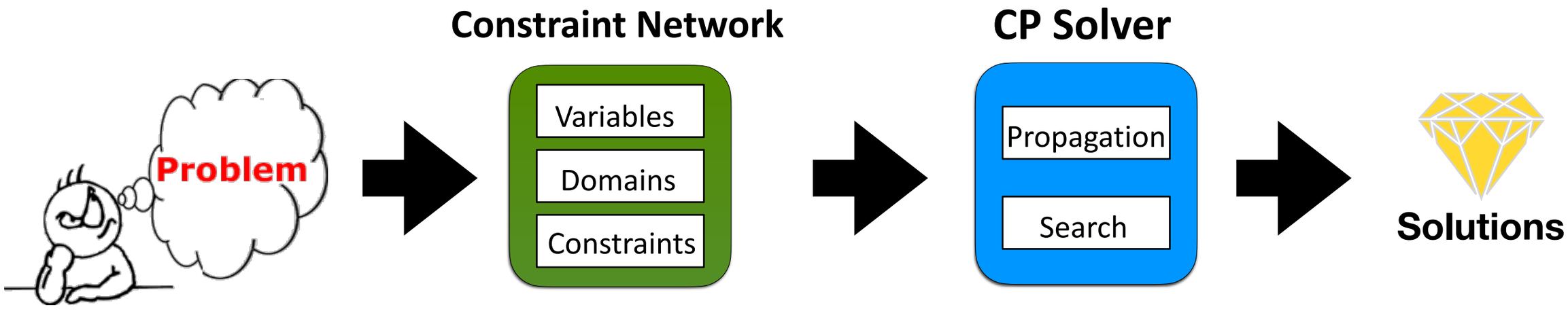
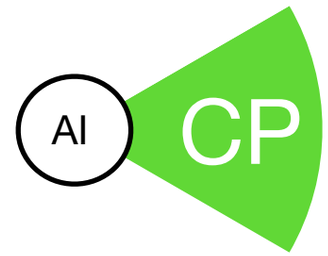
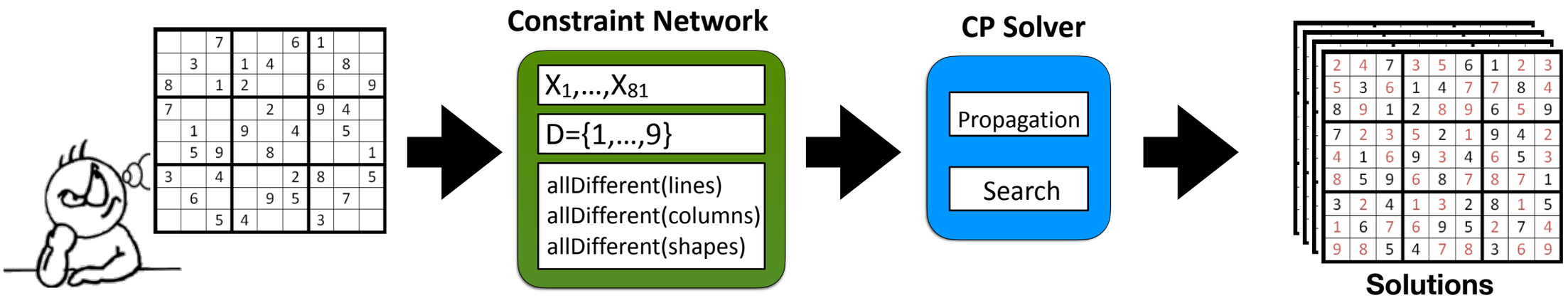


Constraint Programming

Example

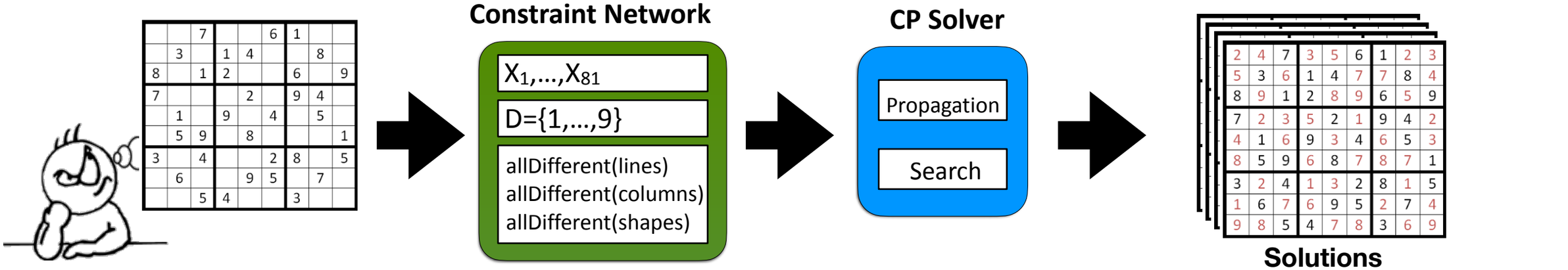




Switch

Constraint Programming

Example



Propagation:

X1={~~1~~,2,~~3~~,4,5,~~6~~,~~7~~,~~8~~,9}

…

Search:

X2 <— 2

		7			6	1		
	3		1	4			8	
8		1	2			6		9
7				2		9	4	
	1		9		4		5	
	5	9		8				1
3		4			2	8		5
	6			9	5		7	
		5	4			3		

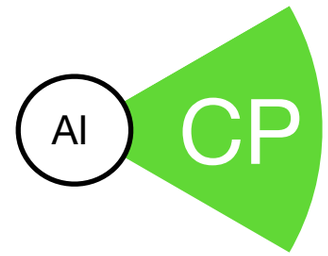
Solution:

X1 = 2
x2 = 4
X3 = 7
…
X81 = 9

Switch

Constraint Programming

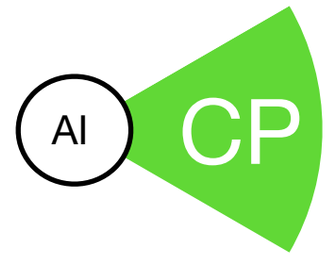
Why CP?



- Combinatorial problems can be solved by Integer Linear Programming (ILP) or by propositional satisfiability (SAT)
- Advantages of CP :
  - Compactness
  - Expressiveness
  - And (often) efficiency

Constraint Programming

Why CP?



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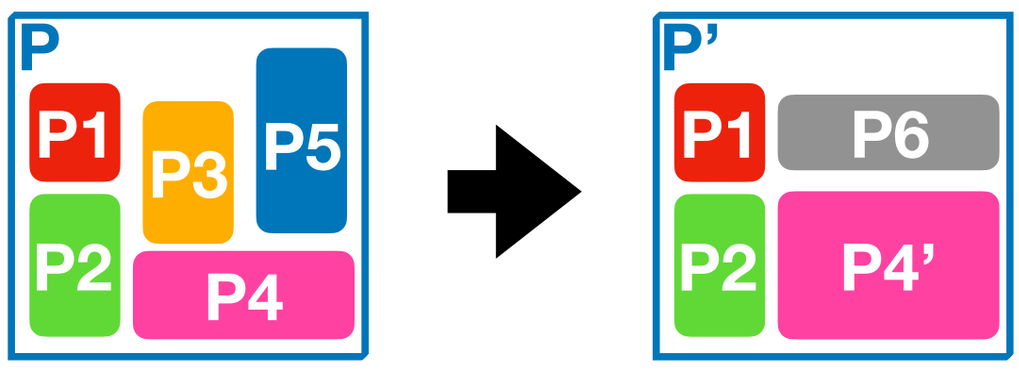
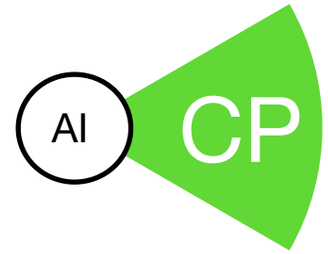


Constraint Programming

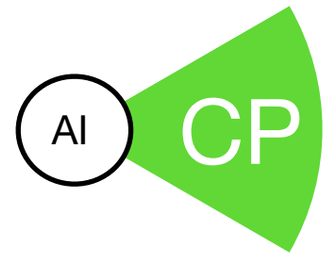
Why CP?



- Sudoku 9x9:
  - SAT = 6 500 clauses
  - CP: `sudoku.mod` with n=9

Constraint Programming

Why CP?



- Sudoku 9x9:
  - SAT = 6 500 clauses
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```
Variables X= {X1,1, …Xn,n}

Domaine des variables : {1,…,n}

Contraintes :
allDifferent(Xi,1,..,Xi,n), ∀i (rows)
allDifferent(Xi,1,..,Xi,n), ∀i (columns)
allDifferent(Xi,j), ∀i,j (squares)
```

sudoku.mod

Sudoku 36x36

17		5				16		24										2		19				27		13								
	22			18					32		9	4		21	3		12		10			28					8		15	6		23	25		31
2		19				27		13										17		5				16		24								
						4		21				16	32	24		9								6		23				27	8	13		15	
	10			28					8		15	6		23	25		31		22			18					32		9	4		21	3		12
						6		23				27	8	13		15								4		21				16	32	24		9	
32		9							16		24				17		5	8		15							27		13				2		19
			17		5							3		12							2		19							25		31			
8		15							27		13				2		19	32		9							16		24				17		5
7		20				1		11							16		24	26		34				29		35							27		13
			2		19							25		31							17		5							3		12			
26		34				29		35							27		13	7		20				1		11							16		24
	17		22	5	18				36		14								2		10	19	28				30		33						
1		11	16		24	32		9					36			14		29		35	27		13	8		15					30			33	
	2		10	19	28				30		33								17		22	5	18				36		14						
									4		21				1		11										6		23				29		35
29		35	27		13	8		15					30			33		1		11	16		24	32		9					36			14	
									6		23				29		35										4		21				1		11
5		17				24		16										19		2				13		27								
	18			22					9		32	21		4	12		3		28			10					15		8	23		6	31		25
19		2				13		27										5		17				24		16								
						21		4				24	9	16		32								23		6				13	15	27		8	
	28			10					15		8	23		6	31		25		18			22					9		32	21		4	12		3
						23		6				13	15	27		8								21		4				24	9	16		32	
9		32							24		16				5		17	15		8							13		27				19		2
			5		17							12		3							19		2							31		25			
15		8							13		27				19		2	9		32							24		16				5		17
20		7				11		1							24		16	34		26				35		29							13		27
			19		2							31		25							5		17							12		3			
34		26				35		29							13		27	20		7				11		1							24		16
	5		18	17	22				14		36								19		28	2	10				33		30						
11		1	24		16	9		32					14			36		35		29	13		27	15		8					33			30	
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									21		4				11		1										23		6				35		29
35		29	13		27	15		8					33			30		11		1	24		16	9		32					14			36	
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Constraint Programming

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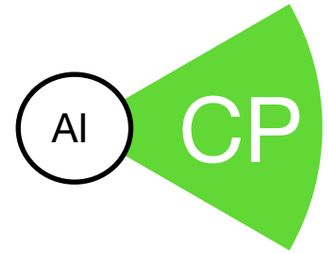

- Sudoku 36x36:
  - SAT ~ 2 millions of clauses : cnf file of 160Gb
  - CP: `sudoku.mod` with n=36

Switch

Constraint Programming

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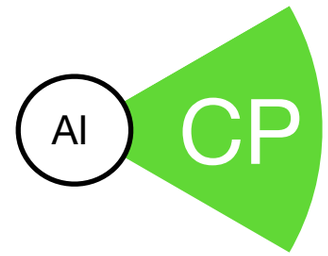

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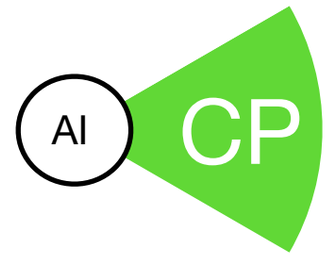
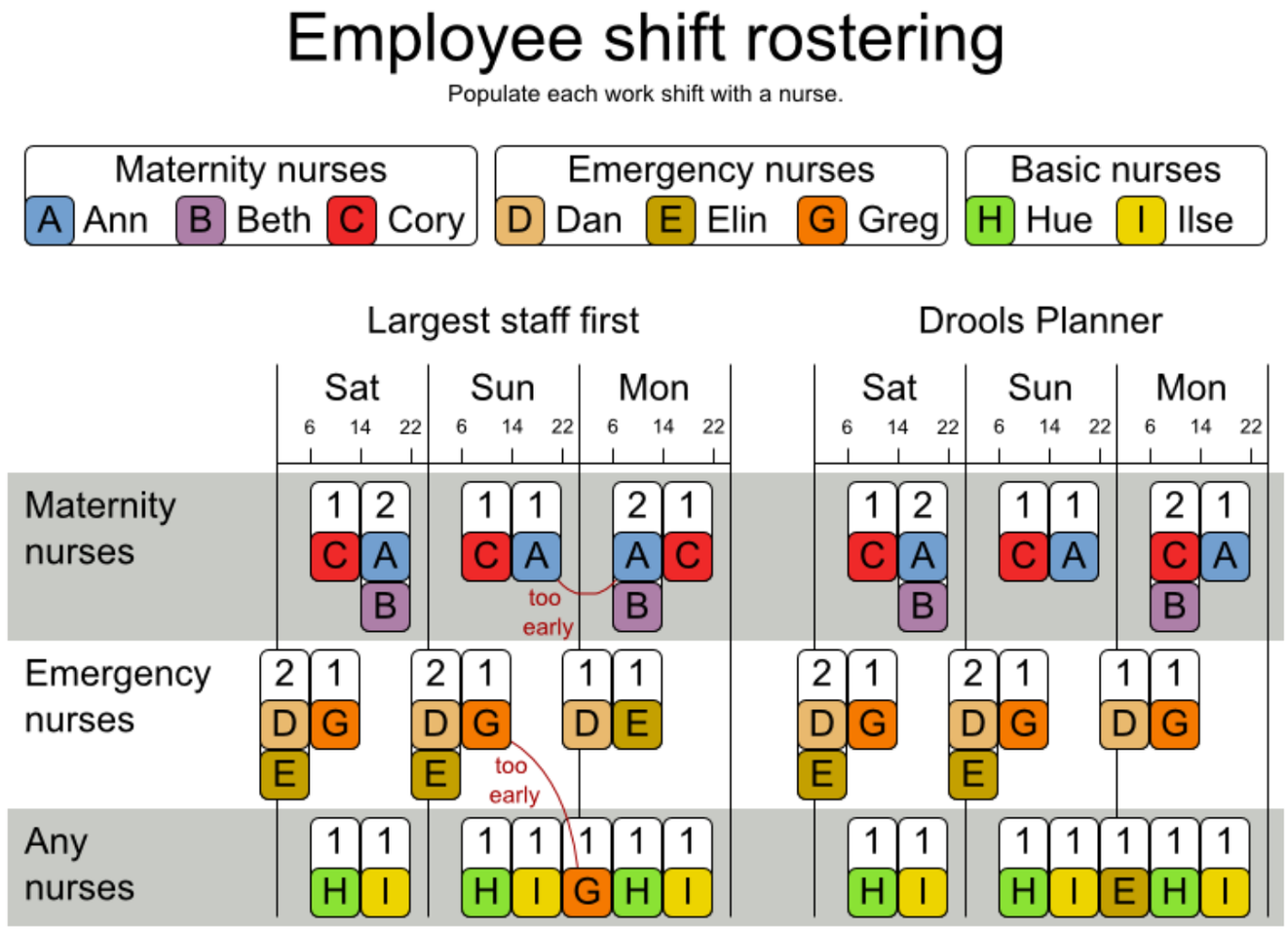


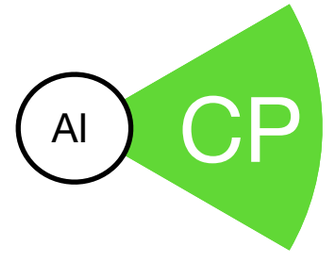
- Nurse Rostering Problem
  - Best Linear Program: more than 10 000 lines



Switch

Constraint Programming

Solvers and CP platforms



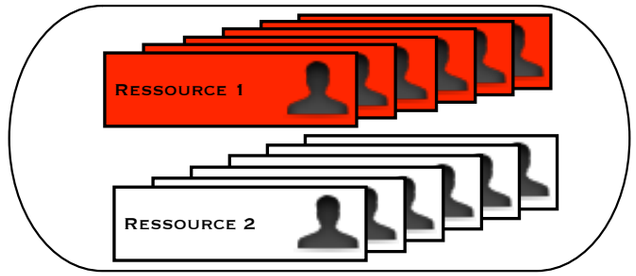
IBM ILOG CP Optimizer (Java, C++, Python, .NET)	IBM
Google OR-Tools (C++, Java, C#, Python)	Google OR-Tools
Artelys Kalis (Java, C++, Python)	ARTELYS KALIS™ Constraint Programming Library
SICStus Prolog (CLPFD bib in prolog)	SICStus 4
Gecode (C++)	G
Choco (Java)	CHOCO
Minizinc (high-level, solver-independent)	Mini Zn

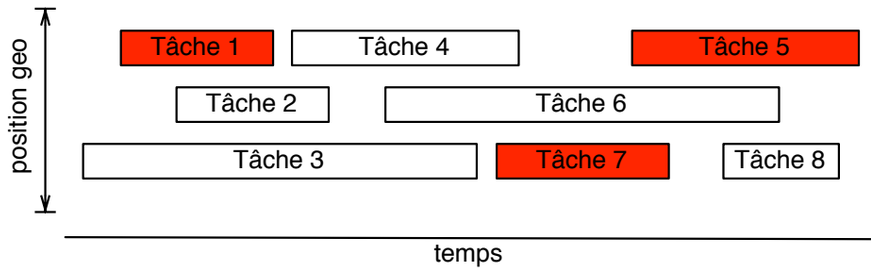
Constraint Programming

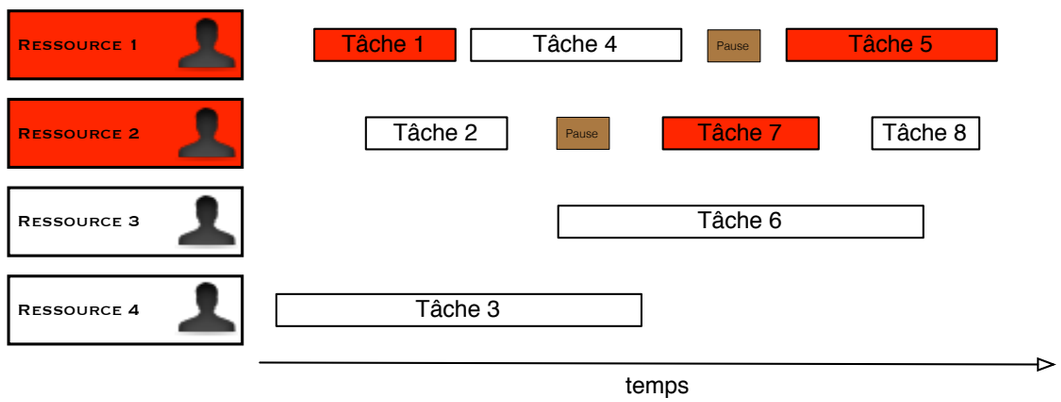
SNCF train driver planning using CP

AI CP

- Legal rules constraints:
  - daily working time
  - daily rest periods
  - …
- Preferences:
  - Type of lines
  - Day of rest
  - Place of rest
  - …
- Solution:
  - Best in terms of ressources
  - Robust one
  - Cheapest one
  - …



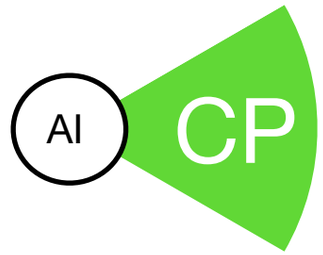




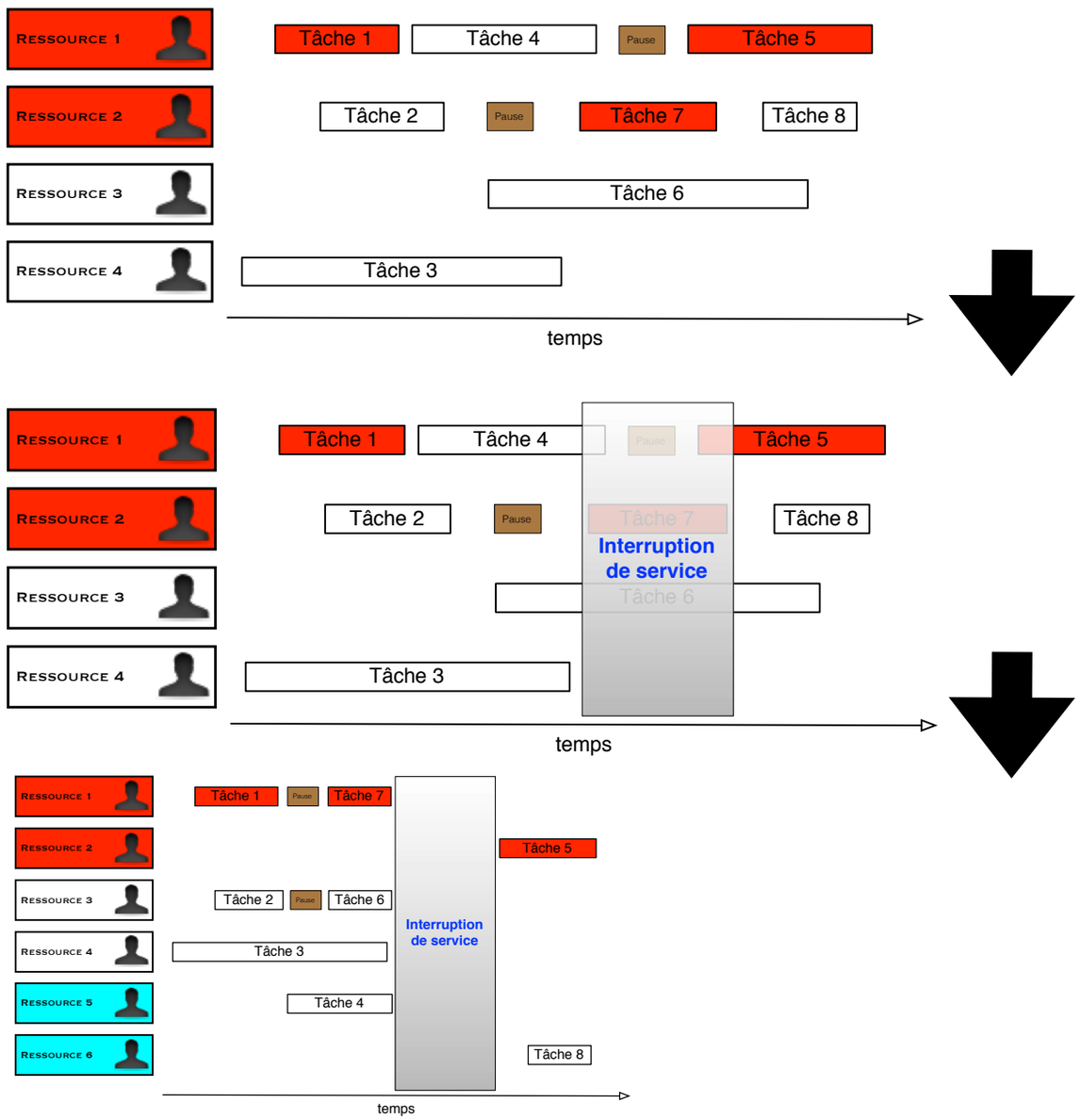
SNCF

Constraint Programming

SNCF train driver planning using CP



- Maintaining an existing planning



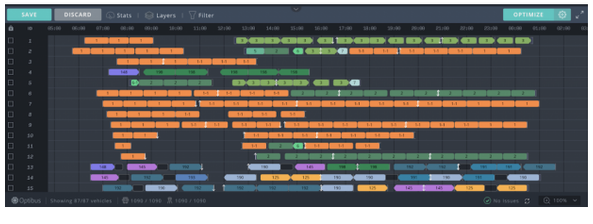
Constraint Programming

ABB Robotics partner projects

AI CP

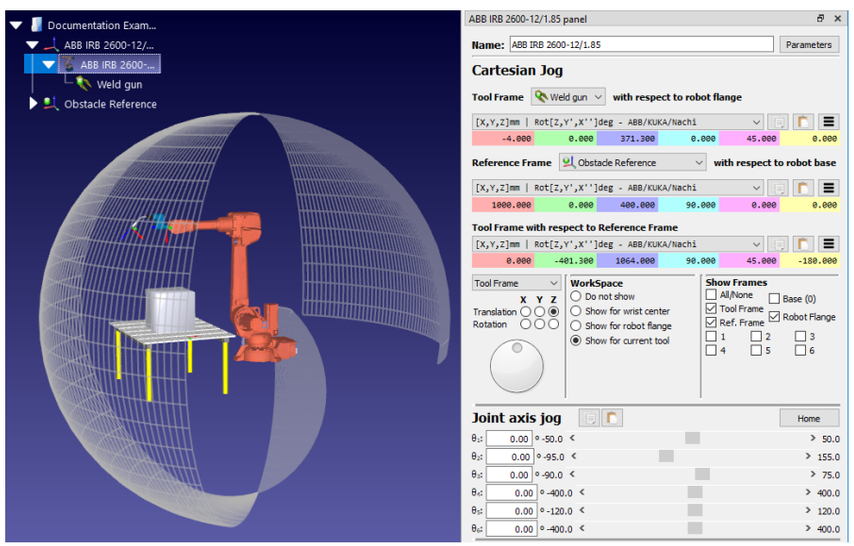
- SWMOD: Test Case Execution Scheduling with CP

"SWMOD deployed at ABB Robotics and used every day to schedule tests throughout several ABB centers in the world (Norway, Sweden, India, China)"



https://github.com/Makouno44/Robtest

- Robtest: Optimal Stress Test Trajectories for Robots with CP

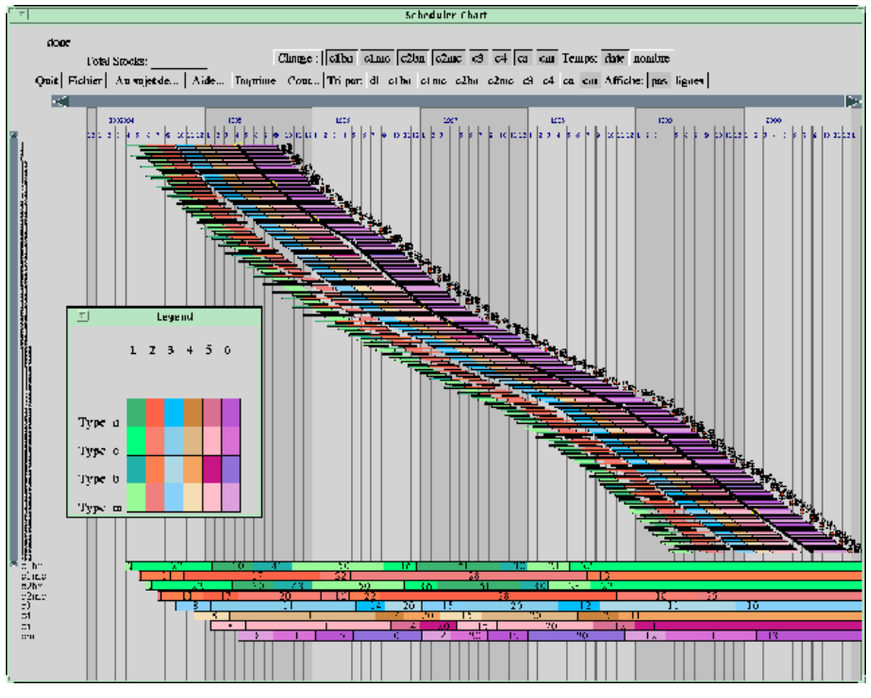


ABB

Constraint Programming

Aircraft Industry - Dassault Aviation

Assembly of Mirage aircraft

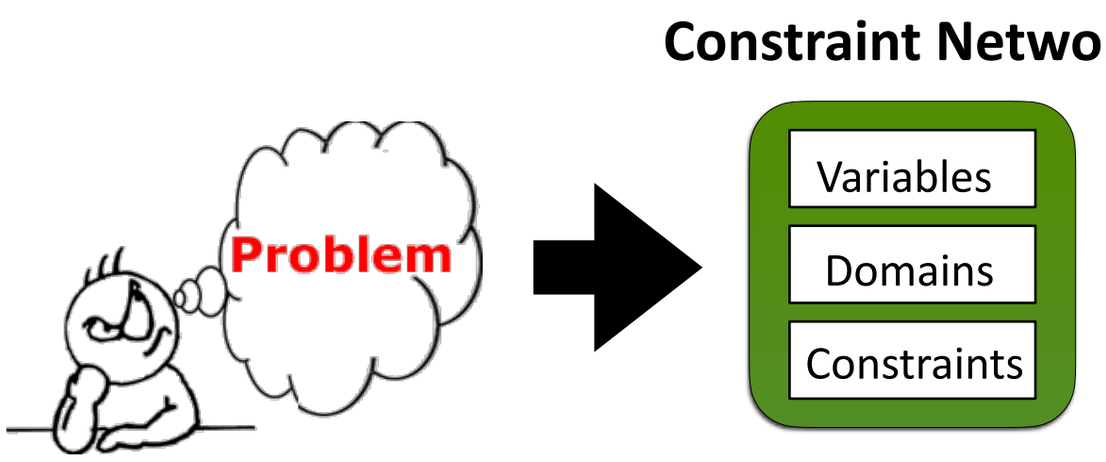


Constraint Programming

Modeling

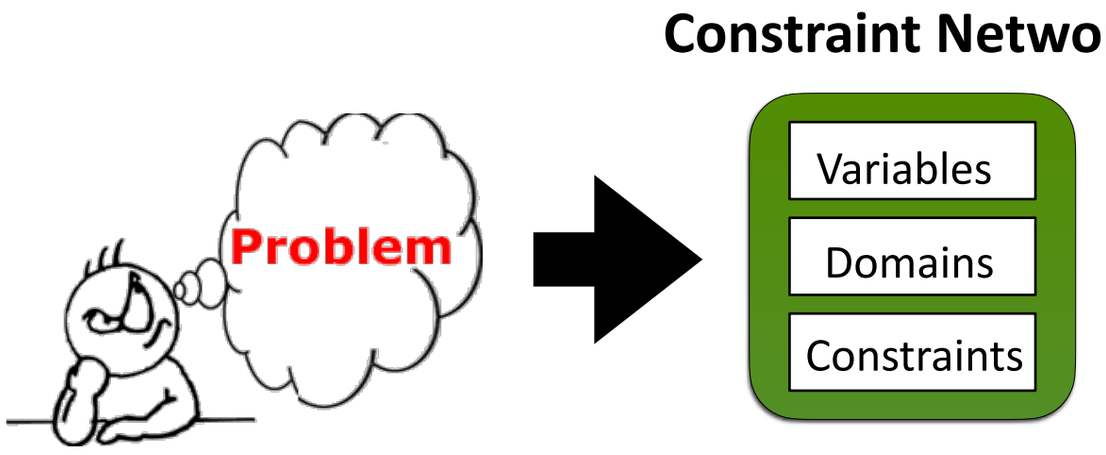
Constraint Programming

Modeling



Constraint Programming

Modeling

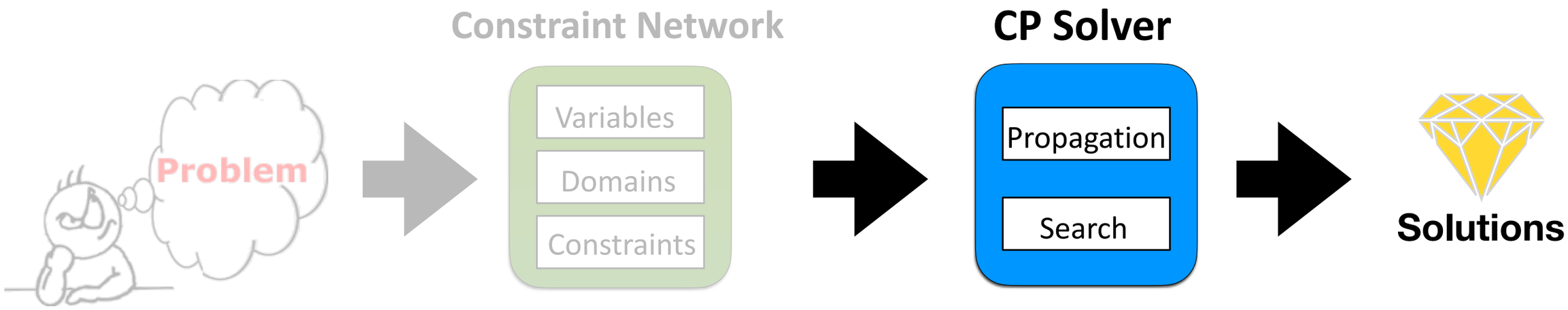


Constraint Network N=(X,D,C)

X: finite set of variables
D: domain on X, where D(Xi) is a finite set of values for Xi
C: a set of constraints

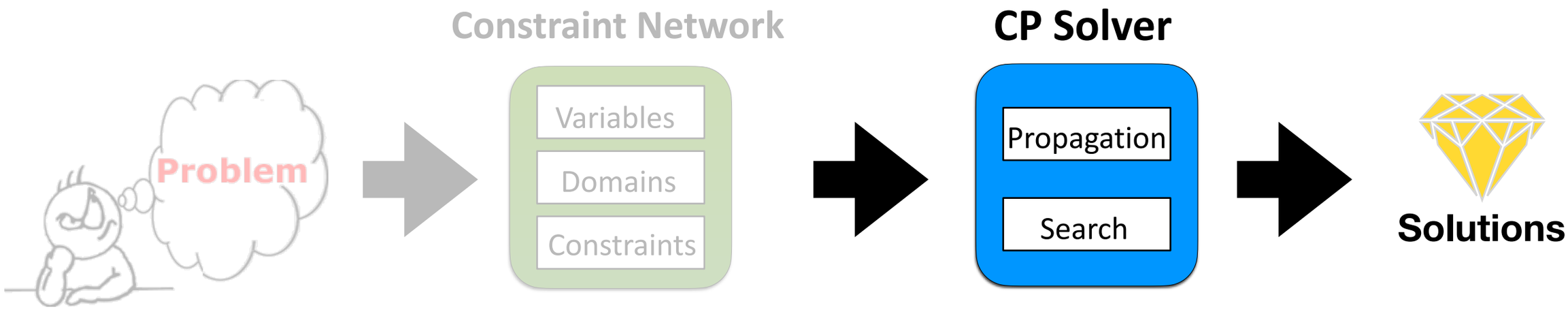
Constraint Programming

Solving



Constraint Programming

Solving



Instantiation I

- Complete: assignment on X
- Partial: assignment on Y subset X
- Valid: values in D
- Violating c: assignment on var(c) is not in c
- Locally consistent: assignment not violating any constraint on Y
- Solution: locally consistent on X

Constraint Programming

Solving - search

Constraint Programming

Solving - search

```
Backtrack(<X,D,C>, I):

If I is complete then return true

Select a variable Xi not in I

ForEach v in D(Xi) do

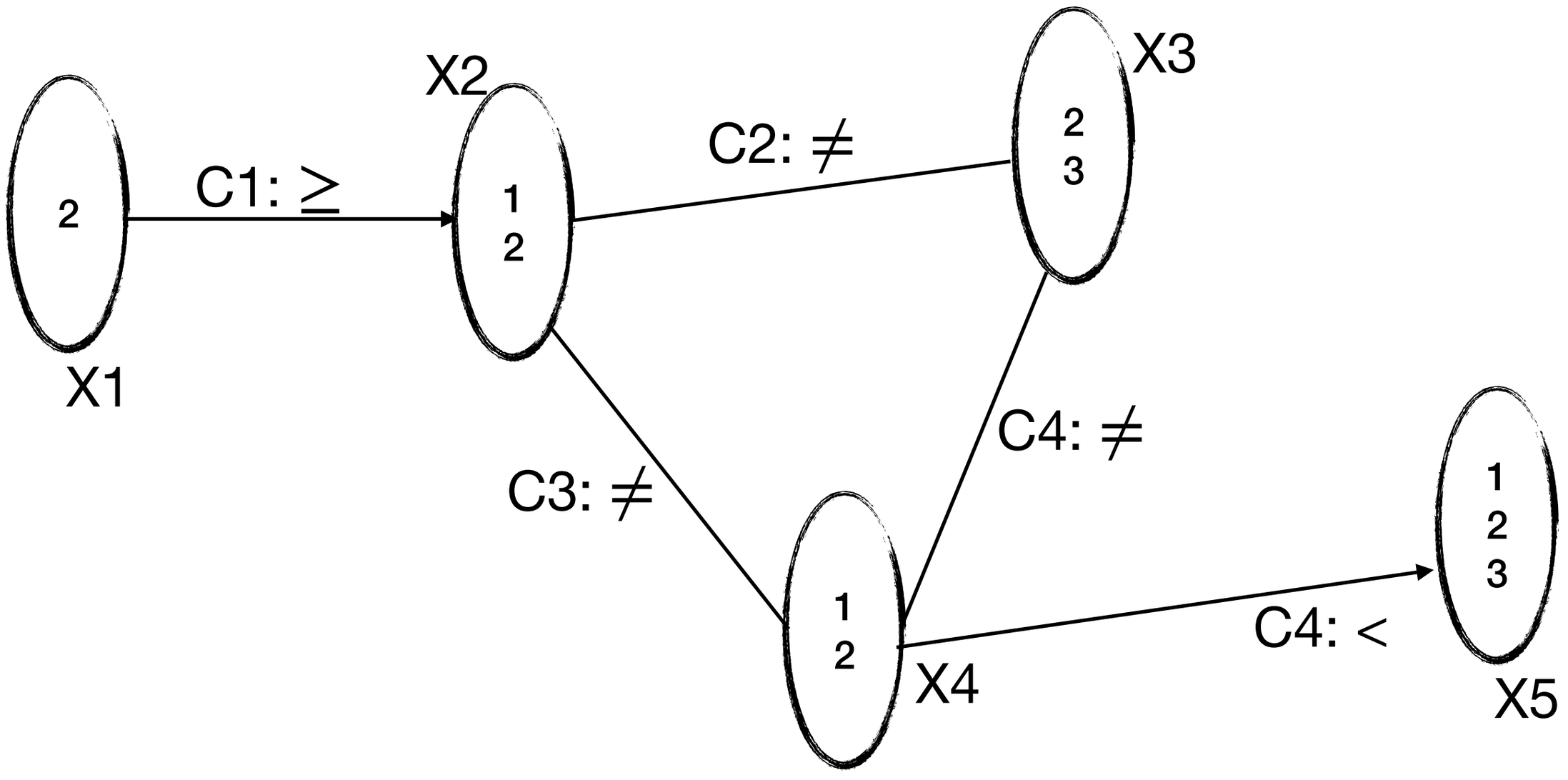
   If I union <Xi, v> is locally consistent then

      If Backtrack(<X,D,C>, I union <Xi, v>) then
          return true

return false
```

Constraint Programming

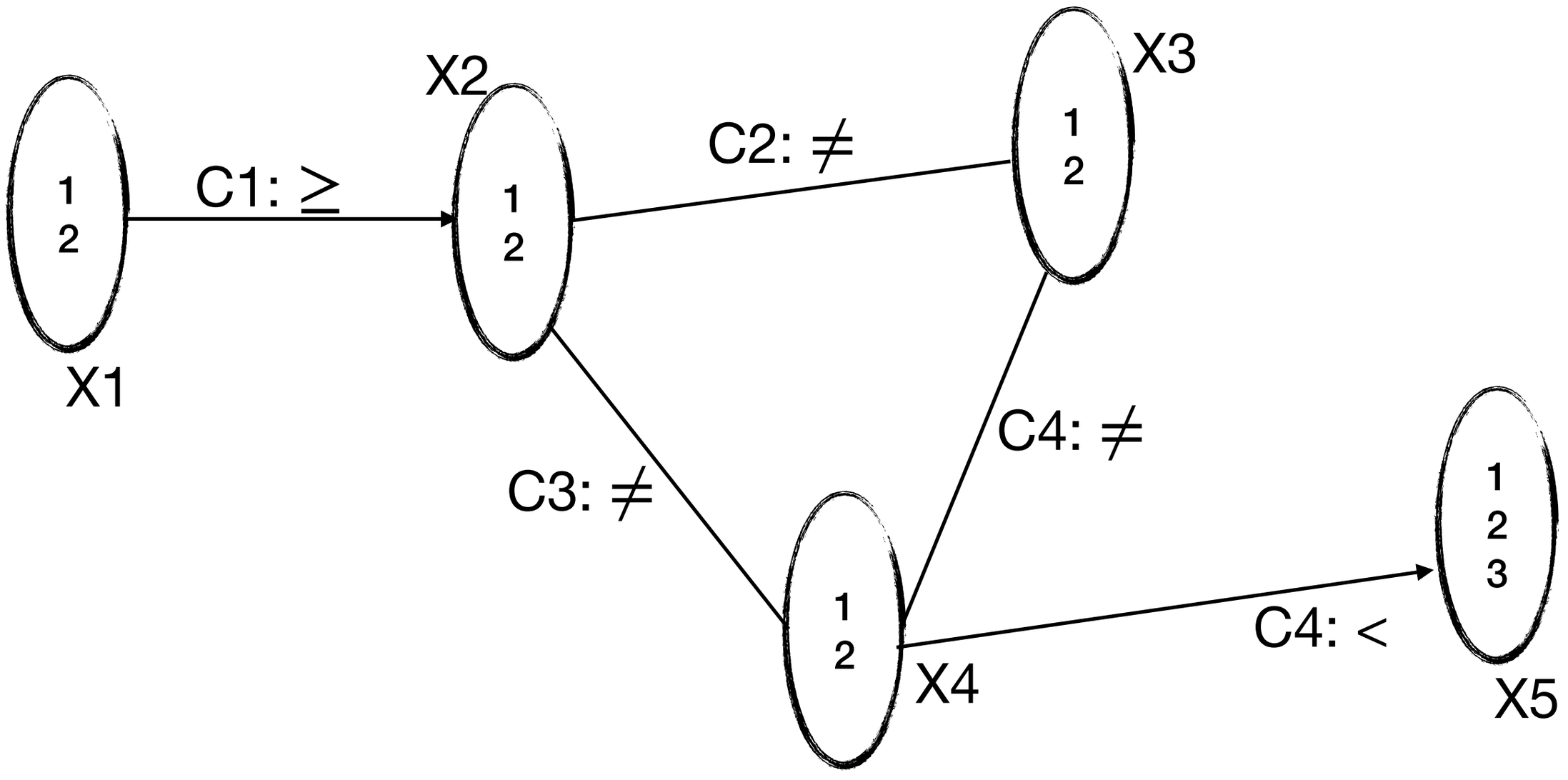
Solving - propagation (local consistency)



Situation 1

Constraint Programming

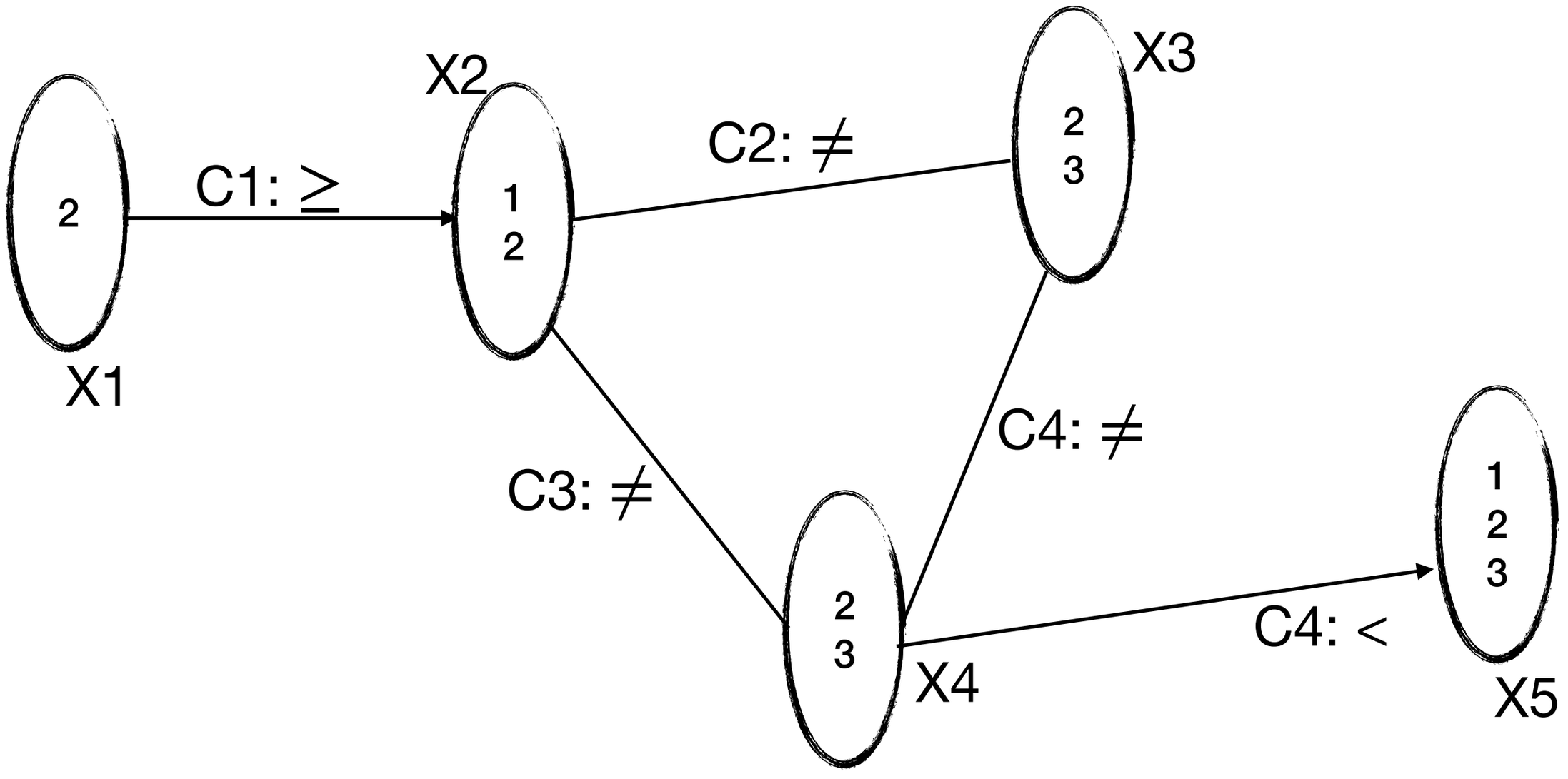
Solving - propagation (local consistency)



Situation 2

Constraint Programming

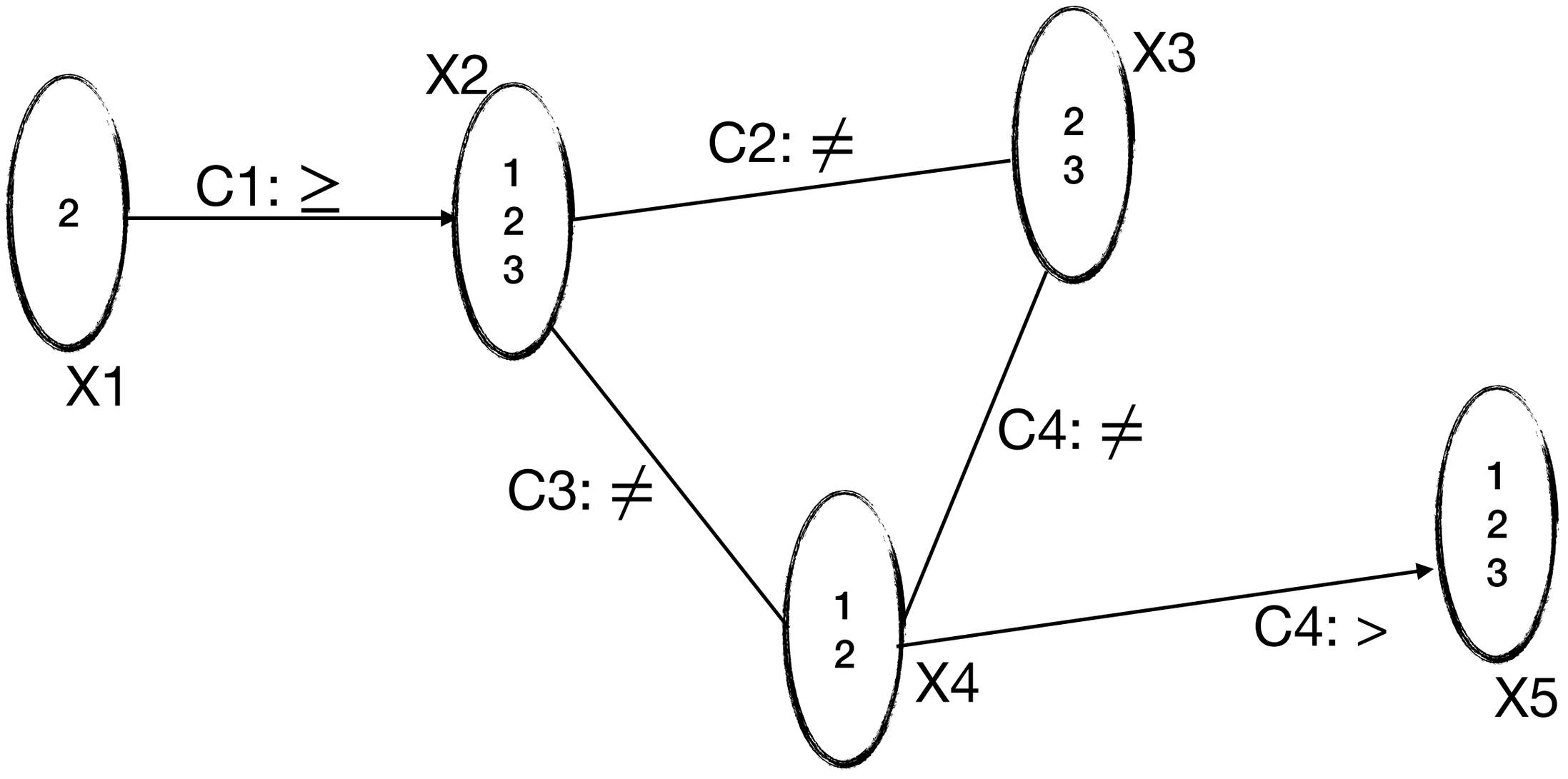
Solving - propagation (local consistency)



Situation 3

Constraint Programming

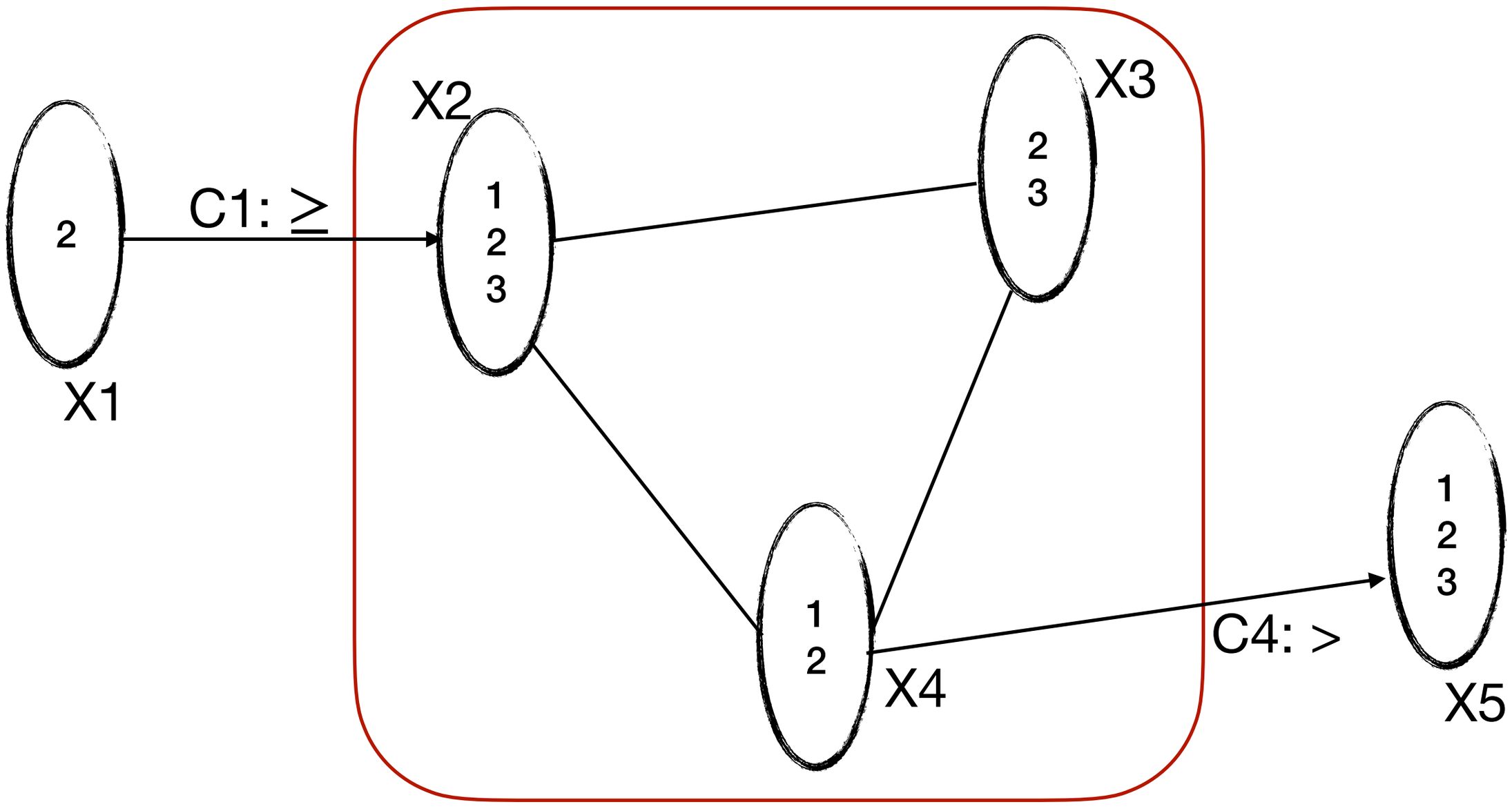
Solving - propagation (local consistency)



Situation 4

Constraint Programming

Solving - propagation (local consistency)



Situation 5

Constraint Programming

Arc Consistency (AC)

- AC closure implemented by several AC algorithms
- AC3 [Mackworth 1977]:

Constraint Programming

Arc Consistency (AC)

- AC closure implemented by several AC algorithms
- AC3 [Mackworth 1977]:

```
AC3(<X,D,C>):

Q = {(Xi, Cij) | Cij in C}

While Q is not empty

  Pick (Xi, Cij) in Q

    If revise(Xi, Cij) then

       If D(Xi) = emptyset then return false
       Else Q = Q union {(Xk, Cki)| k ≠ j}

return true
```

Constraint Programming

Arc Consistency (AC)

- AC closure implemented by several AC algorithms
- AC3 [Mackworth 1977]:

Constraint Programming

Arc Consistency (AC)

- AC closure implemented by several AC algorithms
- AC3 [Mackworth 1977]:

```
revise(Xi, Cij):

flag <– false

foreach vi in D(Xi) is not empty
 vj <– first(D(Xj))
   while (vj ≠ nil) and ((vi,vj) not in Cij) do
       vj <– next(vj, D(Xj))

   if vj = nil then
       remove vi from D(Xi)
       flag <– true

return flag
```

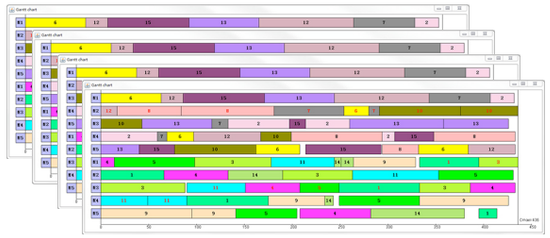
Constraint Programming

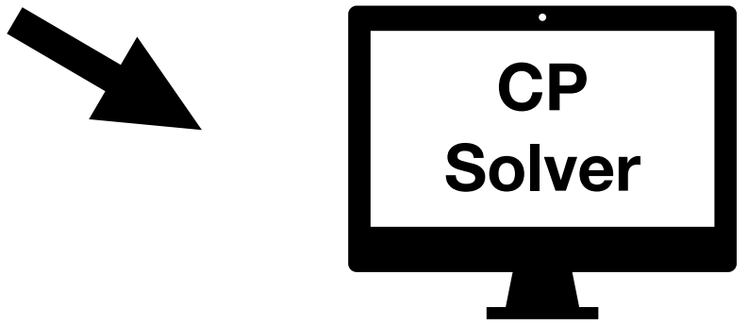
Global constraints

Constraint Programming

Global constraints

Scheduling instances

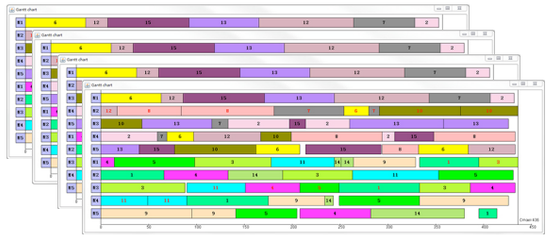


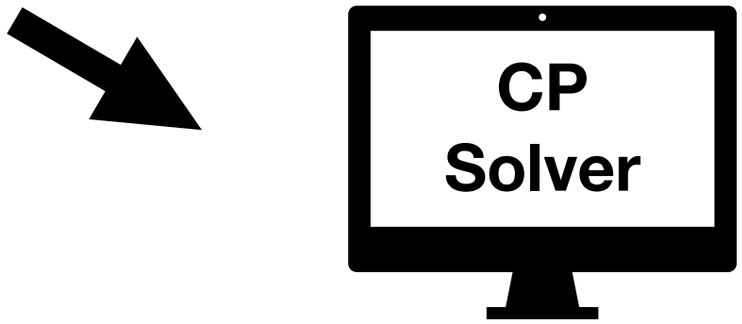


Constraint Programming

Global constraints

Scheduling instances



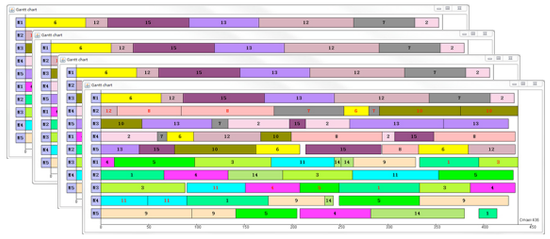


`Cumulative`

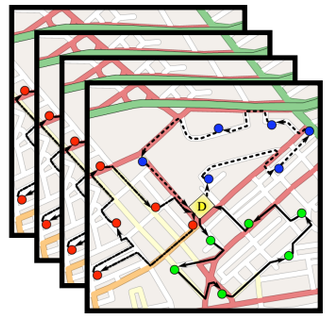
Constraint Programming

Global constraints

Scheduling instances



Vehicule routing instances

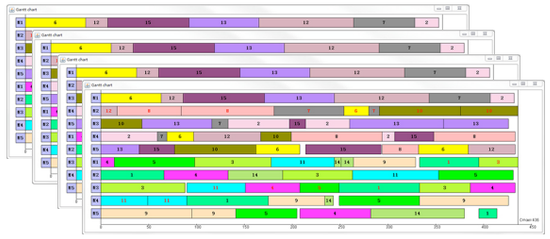


`Cumulative`

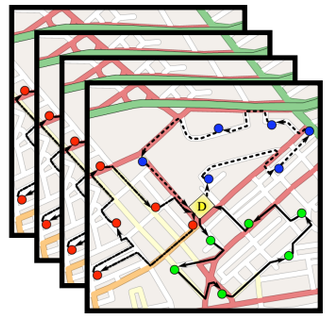
Constraint Programming

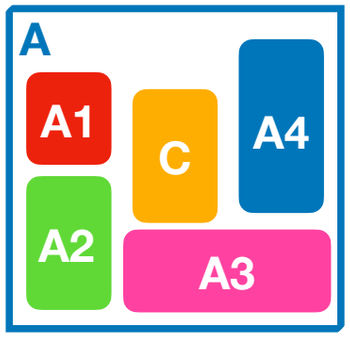
Global constraints

Scheduling instances



Vehicule routing instances



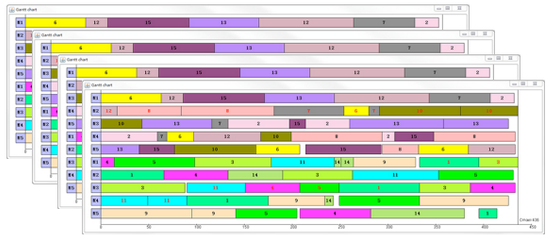


`Cumulative`

Constraint Programming

Global constraints

Scheduling instances



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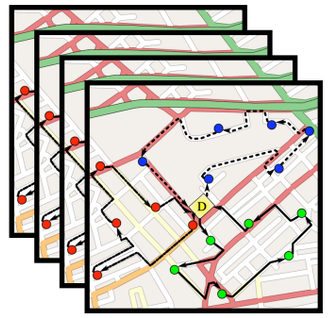
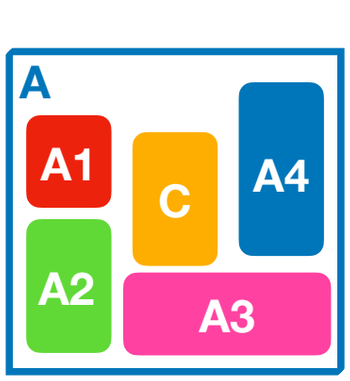
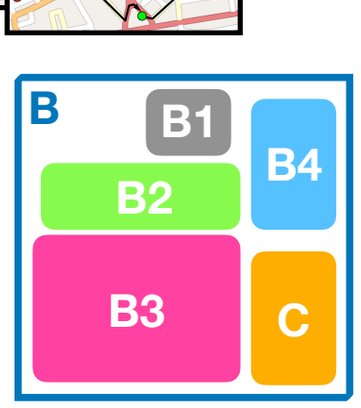




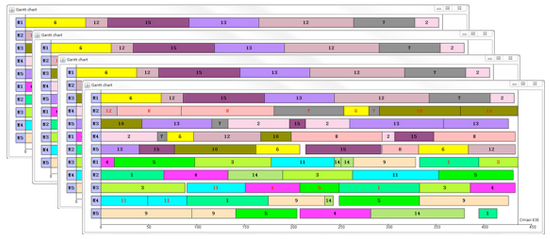


`Cumulative`

Constraint Programming

Global constraints

Scheduling instances



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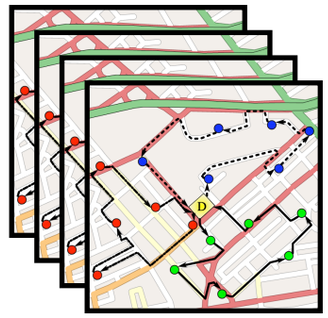

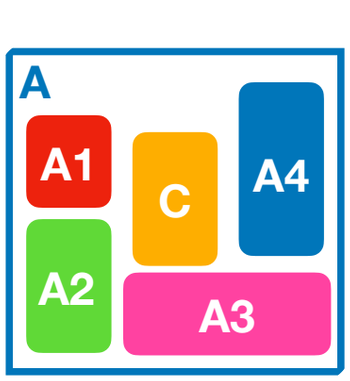

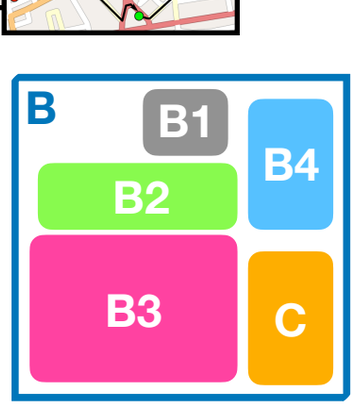


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Cumulative
Alldifferent
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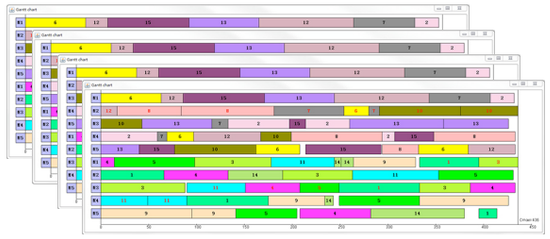
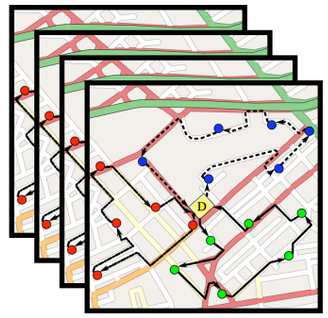
Constraint Programming

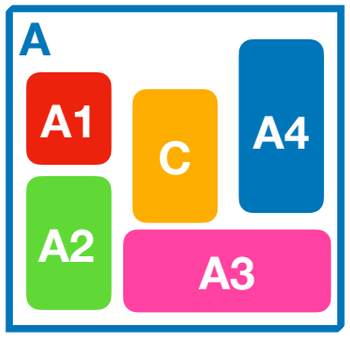
Global constraints

Scheduling instances

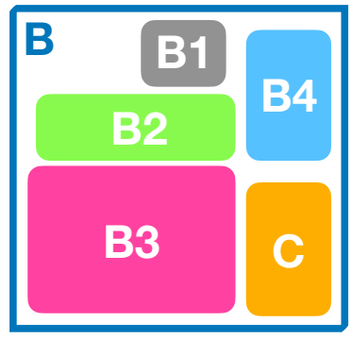


Vehicule routing instances





CP Solver

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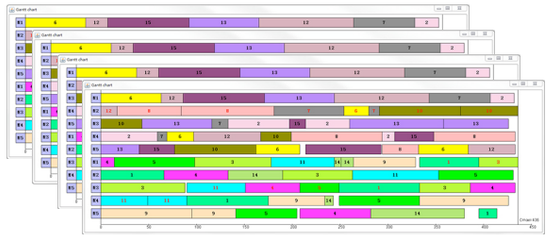
Constraint Programming

Global constraints

Scheduling instances



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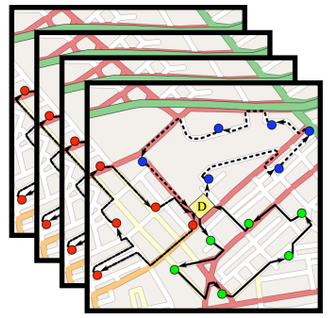
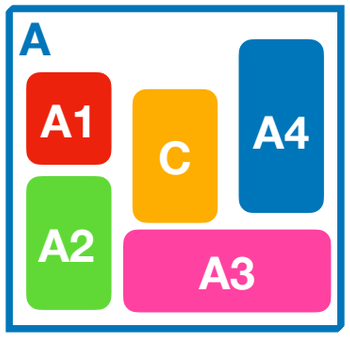
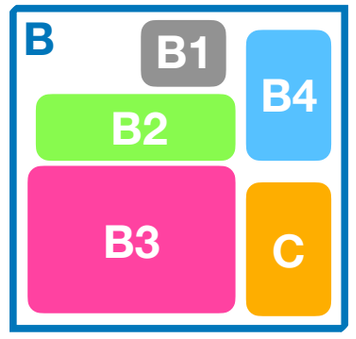




CP Solver

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Cumulative
Alldifferent
Sum
knapsack
Element
GlobalCardinality
Regular
…
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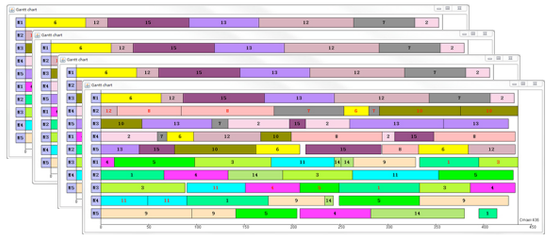
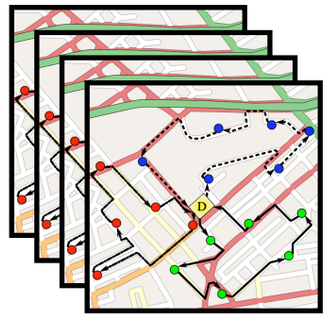
Constraint Programming

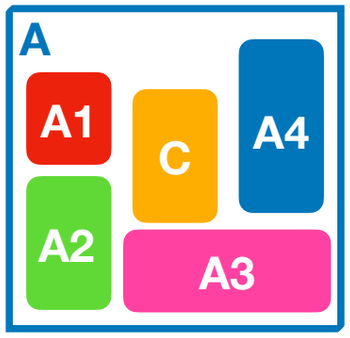
Global constraints

Scheduling instances



Vehicule routing instances





CP Solver

toolbox

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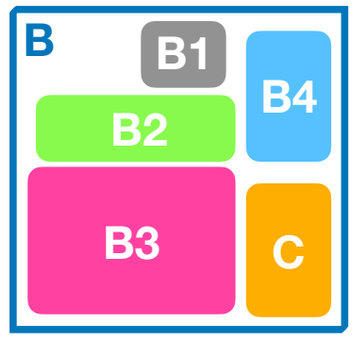
Cumulative
Alldifferent
Sum
knapsack
Element
GlobalCardinality
Regular
…
```



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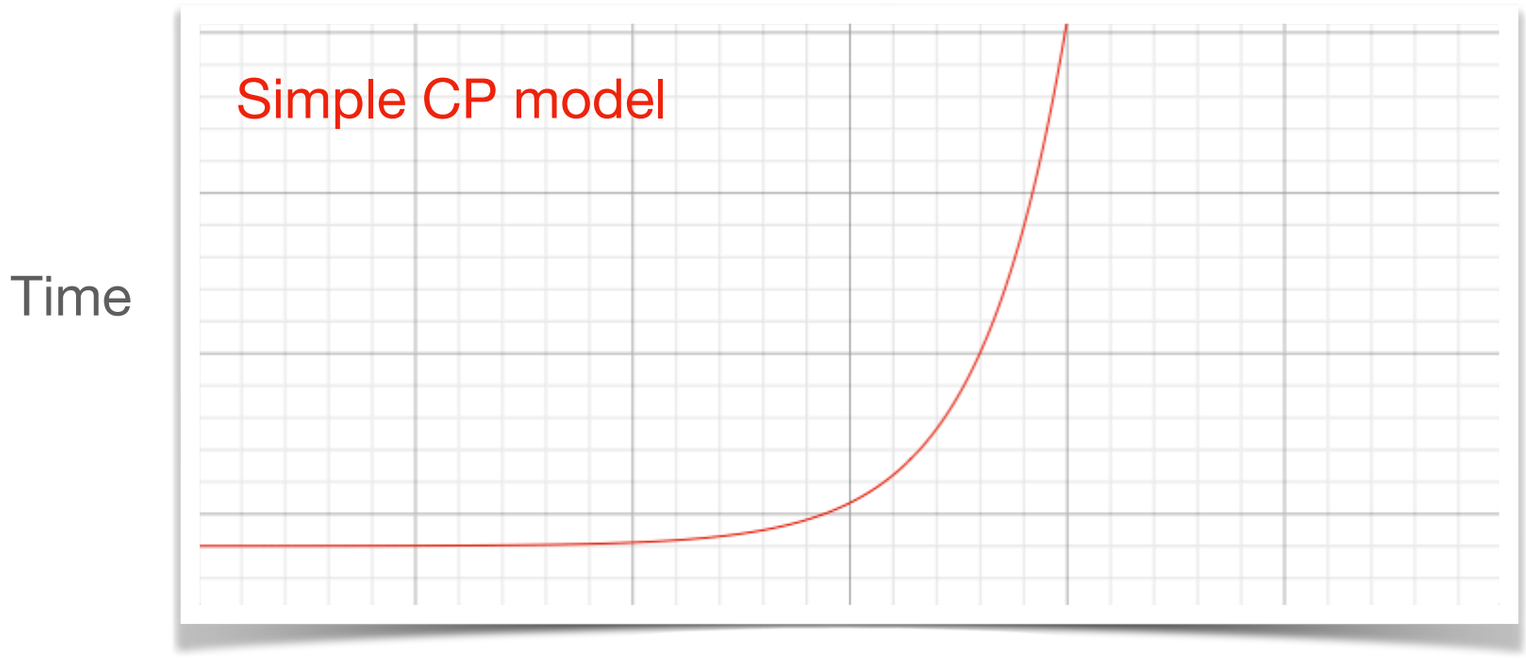
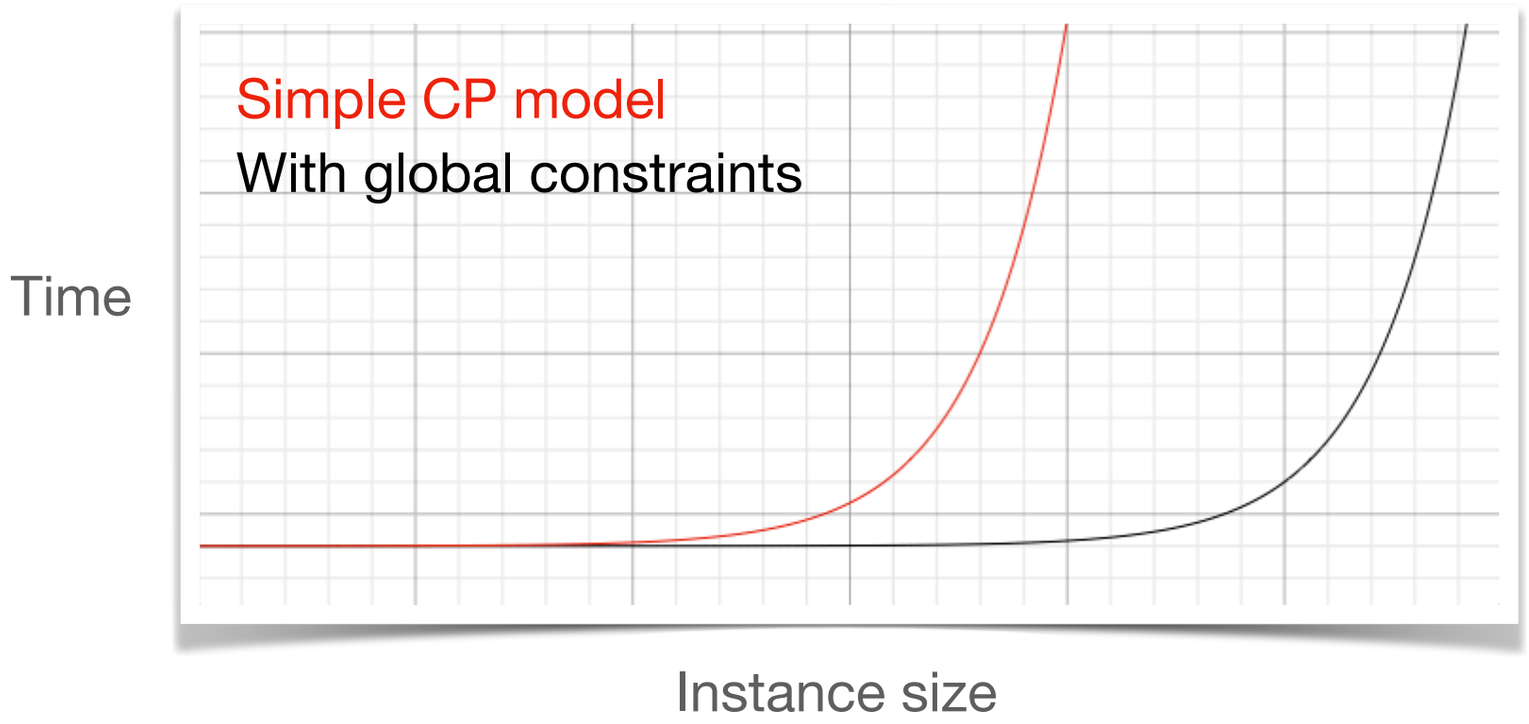
Constraint Programming

Global constraints



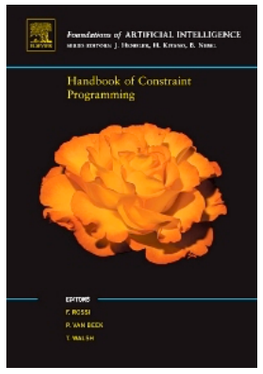
Constraint Programming

Global constraints



Constraint Programming

References & links



- **Handbook of Constraint Programming.** Francesca Rossi Peter van Beek Toby Wals. Elsevier Science 2006

- ACP: Association for Constraint Programming

- Guide to Constraint Programming

- CP conference Series

- Global Constraint Catalog

- COCONUT Team, LIRMM