

# Covariance Matrix Adaptation - Evolution Strategy: An summary

Gaëtan Serré

ENS Paris-Saclay - Centre Borelli

gaetan.serre@ens-paris-saclay.fr

## 1. Introduction

The *Covariance Matrix Adaptation - Evolution Strategy* (CMA-ES) is a global and black-box optimization algorithm. It is a randomized black-box search algorithm as it samples evaluation points from a distribution conditioned with the previous parameters:

For  $g$  in  $1 \dots k$ :

1. Let  $d_\theta$  a distribution on  $\mathcal{X}$  parametrized by  $\theta$ ;
2. Sample  $\lambda$  points:  $(x_i)_{1 \leq i \leq \lambda} \sim d_{\theta_i}$ ;
3. Evaluate the points:  $f((x_i)_{1 \leq i \leq \lambda})$ ;
4. Update the parameters  $\theta_{i+1} = F(\theta_i, (x_1, f(x_1)), \dots, (x_\lambda, f(x_\lambda)))$ .

Figure 1: Black-box search algorithm.

CMA-ES uses a multivariate Gaussian as the sampling distribution  $\mathcal{N}(m, C)$ . The authors made this choice as, given the variances and covariances between components, the normal distribution has the largest entropy in  $\mathbb{R}^d$ . To goal is to find how update the mean and covariance matrix of this distribution to minimize the trade-off between finding a good approximation of the optimum and evaluate the objective function as few times as possible. In this small paper, we will present the ideas behind CMA-ES. For more details, see [1].

## 2. Update the mean

Select the  $\mu$  best points and do a weighted average of their coordinates. The choice of  $\mu$ ,  $w$  and  $\sigma$  is crucial. The quantity  $\mu_{eff}$  is an indicator on the loss of variance due to the selection.

## 3. Update the covariance matrix

Unbiased estimator using empirical mean vs using real mean updated before and only the selected point. The second one redirects the search towards those points and turns out to be more efficient. Rank- $\mu$  update?

## Bibliography

- [1] N. Hansen, "The CMA Evolution Strategy: A Tutorial," arXiv, 2023. Accessed: Apr. 4, 2023. [Online]. Available: <http://arxiv.org/abs/1604.00772> (Comment: ArXiv e-prints, arXiv:1604.00772, 2016, pp.1-39)