

Extra assignment: Inference for continuous detection and discrimination

Theoretical Neuroscience II

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December 12, 2016

1 Maximum joint density/probability

I determined the peak density of the joint probability density function with three different computational methods.¹ The results are summed up in the following table. For the first method, I wrote a function that evaluates the pdf directly and used a library function to iteratively find the maximum. For the second method I sampled the same function at fixed points at linearly spaced values for θ_k and logarithmically spaced values for x_k and c_s , and selected the maximum from those results. Transforming the density function into a probability function, I also obtained an analytic result for the peak (discretized) probability. Lastly I generated stimuli and responses according to the probability distributions and calculated the empirical/numerical peak density and probability from the results. The chosen parameters are $c_0 = 1$ and $\theta_k = 0$.

	Method	x_k	c_s	θ_s
peak density	direct optimization	$\approx 1 \times 10^{-100}$	$\approx 1 \times 10^{-100}$	≈ 3.7
	analytic	1×10^{-8}	2×10^{-8}	2
	numeric	0	3×10^{-8}	3.7
peak probability	analytic	10	1	0
	numeric	7.3	0.65	0

The results for the location of the peak probability do not agree closely for the response and contrast, but are within an order of magnitude. Since they are on a logarithmic scale, we cannot expect much better.

The results for the peak density show x_k and c_s close to zero. However the density function with $x_k = 0$ is equal to zero (so not maximal), and it is not defined for $c_s = 0$. The response rate x_k is Rayleigh distributed with a mean determined by the stimulus contrast c_s and orientation θ_s . The mean tends towards zero with c_s . A Rayleigh distribution with the mean tending towards zero degenerates into a Dirac delta function $\delta(x - \epsilon)$. This means the density at that point tends to infinity.

¹I also tried to derive the maximum directly (analytically) from the probability density function, but gave up after still getting apparently wrong results after multiple attempts. One result included $c_s = -c_0$, another gave $x_k = \sqrt{\frac{1}{2\pi}}, \theta_s = \theta_k, c_s = \frac{c_0}{2} + \sqrt{\frac{c_0^2}{4} + \frac{c_0}{8e^2}}$

2 Marginal response distribution

Figure 1 shows the marginal distribution of the response rate, once derived analytically, once numerically, as described in Section 1. Qualitatively the analytic and numeric solutions agree well. The absolute values for the numeric solution are a bit smaller than for the analytic solution.

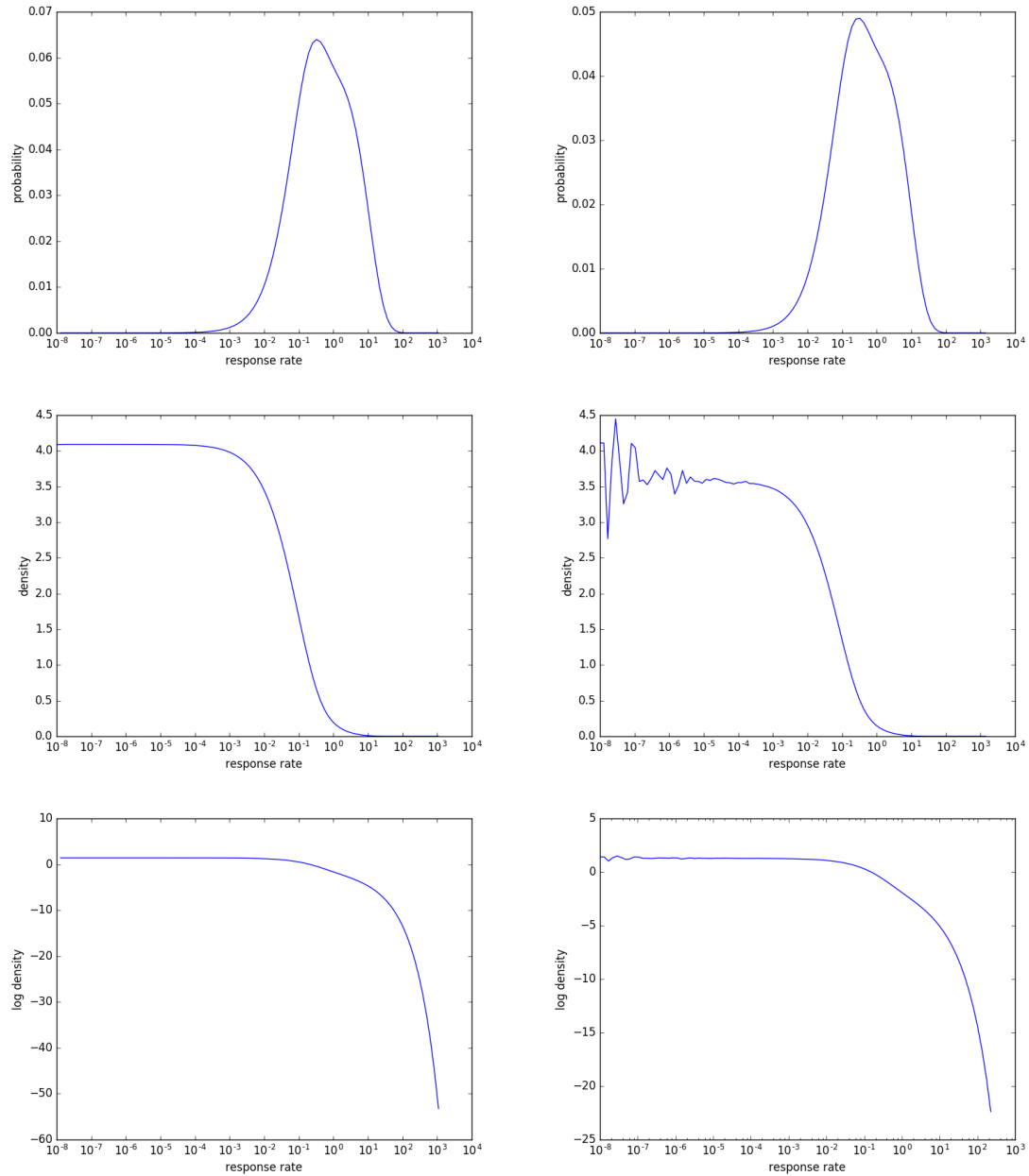


Figure 1: Left column: Analytic solution. Right column: Numeric solution. Rows, top to bottom: Marginal (discretized) probability with logarithmically spaced values of x , marginal probability density, log of marginal probability density.

3 Posterior of contrast and orientation, given response

Figure 2 shows the posterior probability of contrast and orientation, given a response rate. For low response rates the most likely stimulus is of non-preferred orientation, and has a higher contrast than the most likely stimulus with the preferred orientation. For higher response rates the posterior for the preferred stimulus orientation is highest.

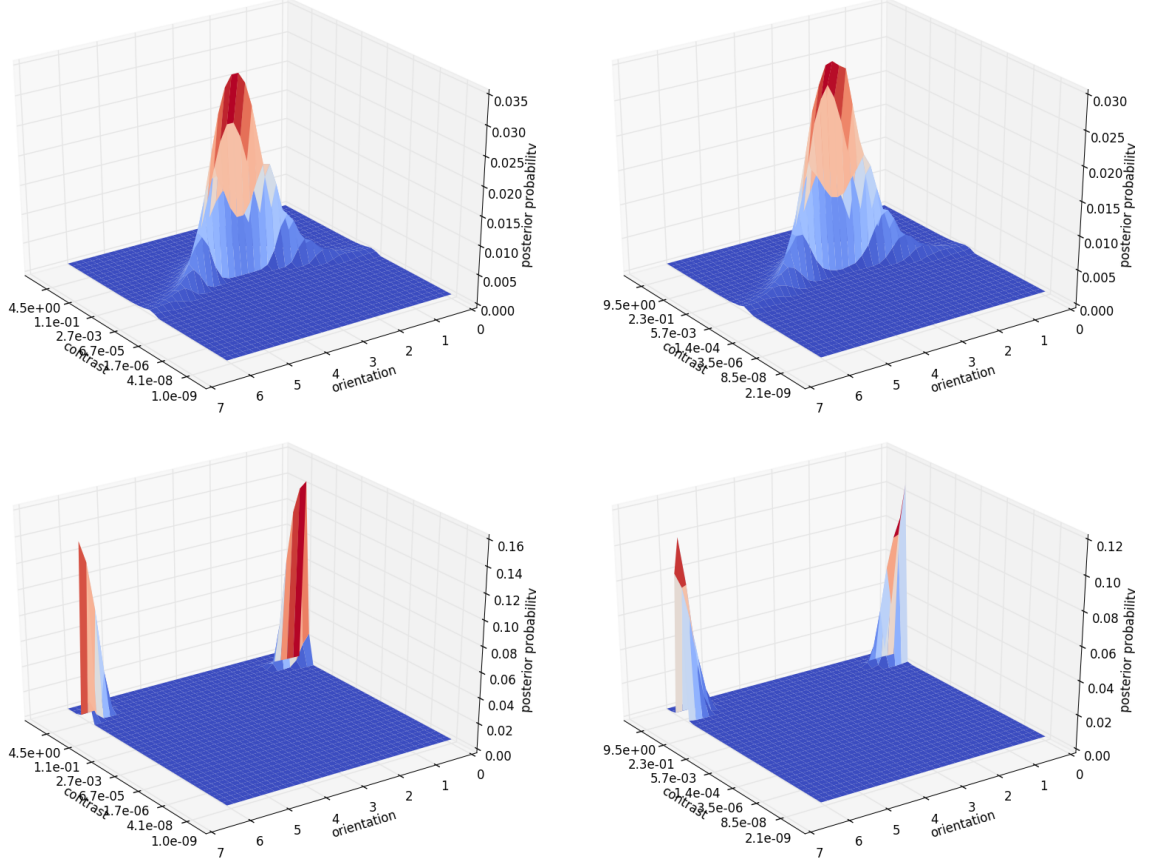


Figure 2: Left column: Analytic solution. Right column: Numeric solution. Posterior distribution for $x_k = 6.7 \times 10^{-4}$ (top) and for $x_k = 45.3$ (bottom). $c_0 = 1, \theta_k = 0$

4 Probability and latency of object detection

I estimated the probability and latency of correctly detecting an object as a function of object duration and contrast. For a given sequence of stimuli and their response rates and a fixed window size I determined the most likely stimulus for every time step. The differences between true and detected contrast and orientation were transformed to be in the range $[0, 1]$. For the orientation I used the function $f(\theta_s, \theta_d) = \frac{-\cos(\theta_s - \theta_d) + 1}{2}$ and for the contrast the function $f(c_s, c_d) = |\text{cdf}(c_s, c_0) - \text{cdf}(c_d, c_0)|$ where subscripts s and d refer to the stimulus and detected values respectively and cdf is the cumulative density function for the distribution of stimulus contrast. If the result of these functions was smaller than a given threshold for both contrast and orientation at at least one point in time the corresponding stimulus was said to be correctly detected and the latency was set to the difference between stimulus onset and first time of correct detection.

Some example results are shown in Figure 3. Overall trends appear to be as follows: Very low contrast and very high contrast objects are more difficult to detect than objects with average contrast. The latency is more or less independent of contrast, except for very high contrasts, where the latency is higher. Latency of detection is close to the worst possible value for short stimuli and otherwise independent of stimulus duration. Probability of detection is high for all stimuli longer than the window size, with perhaps a small positive correlation between object duration and detection probability. For objects with a shorter duration than the window size the probability of detection rises rapidly with a longer duration. See also Figure 4.

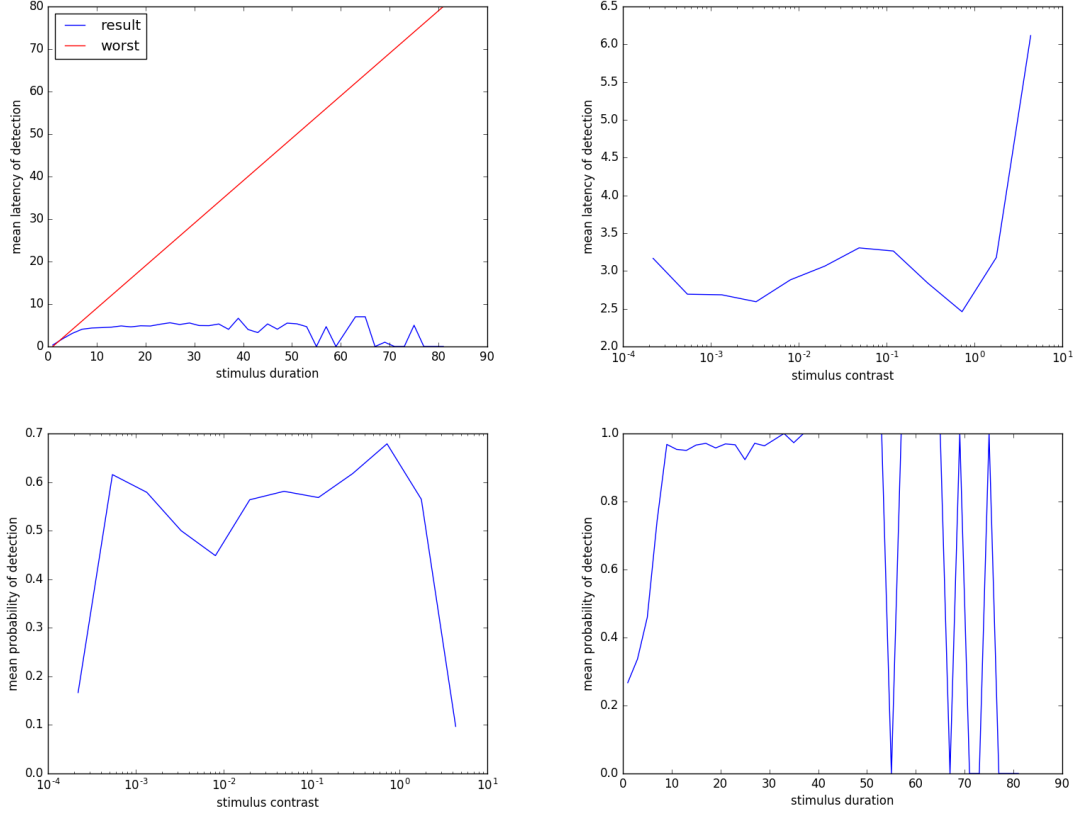


Figure 3: Probability and latency of stimulus detection as functions of stimulus duration and contrast. Chosen parameters: $c_0 = 1$, $\tau_0 = 8$, integration window size = 8, detection threshold = 0.1, number of channels = 4. The red line in the top left plot indicates the maximum possible latency. Points on the line mean that the stimulus was either not detected at all, or detected at the last point in time before the next stimulus began. Actual results can be slightly above the line because the results for multiple durations are put together. The mean of zero values was set to zero, which explains the zero points in the bottom right plot.

5 Standard deviation of error

I estimated the precision of the inferred stimuli as a function of object duration and contrast with the standard deviation of the error. I generated stimuli, responses and inferred stimuli as described in Section 4, but did not transform the difference into the $[0, 1]$ range. Some results are shown in Figure 5.

Comparing window sizes of 1 and 8, with the mean duration τ_0 also set to 1 or 8 show

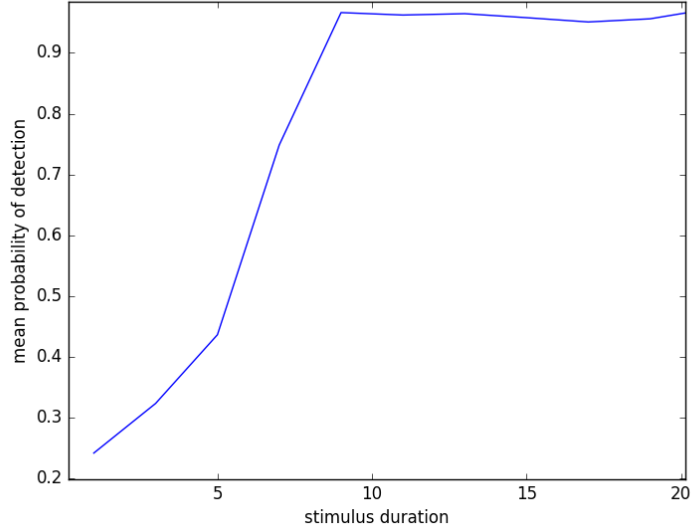


Figure 4: Chosen parameters: $c_0 = 1$, $\tau_0 = 8$, integration window size = 8, detection threshold = 0.1, number of channels = 12. Illustration of the initial rapid rise of detection probability for stimuli shorter than the window size.

that the inference is more precise with the smaller window size. The dependence of orientation and contrast precision on object duration are basically identical. For the small window size there is no clear dependence between object duration and precision. For the longer window size there is a clear trend of higher precision for longer duration objects. The absolute standard deviation values are much higher than for the short window though, approaching the short window performance only for very long duration objects.

The orientation precision seems to be independent of object contrast for small window sizes, and much larger for long windows, except for very low and very high contrast objects. The standard deviation of the contrast error grows with the object contrast for the short window, but is high for all object contrast values for the long window.

6 Optimal window size for parameters

Finally I estimated the optimal window size for a given set of environment parameters c_0 and τ_0 . For the given parameters I generated stimuli and responses, and from the responses the inferred stimulus for different window sizes. As in Section 4 the difference of true and inferred stimulus contrast and orientation was transformed to the range $[0, 1]$ so that errors in estimated contrast and orientation would be weighted roughly equally. An exemplary result is shown in Figure 6. We can see that with a higher τ_0 longer window sizes are preferred. The relationship between contrast and optimal window size is unclear.

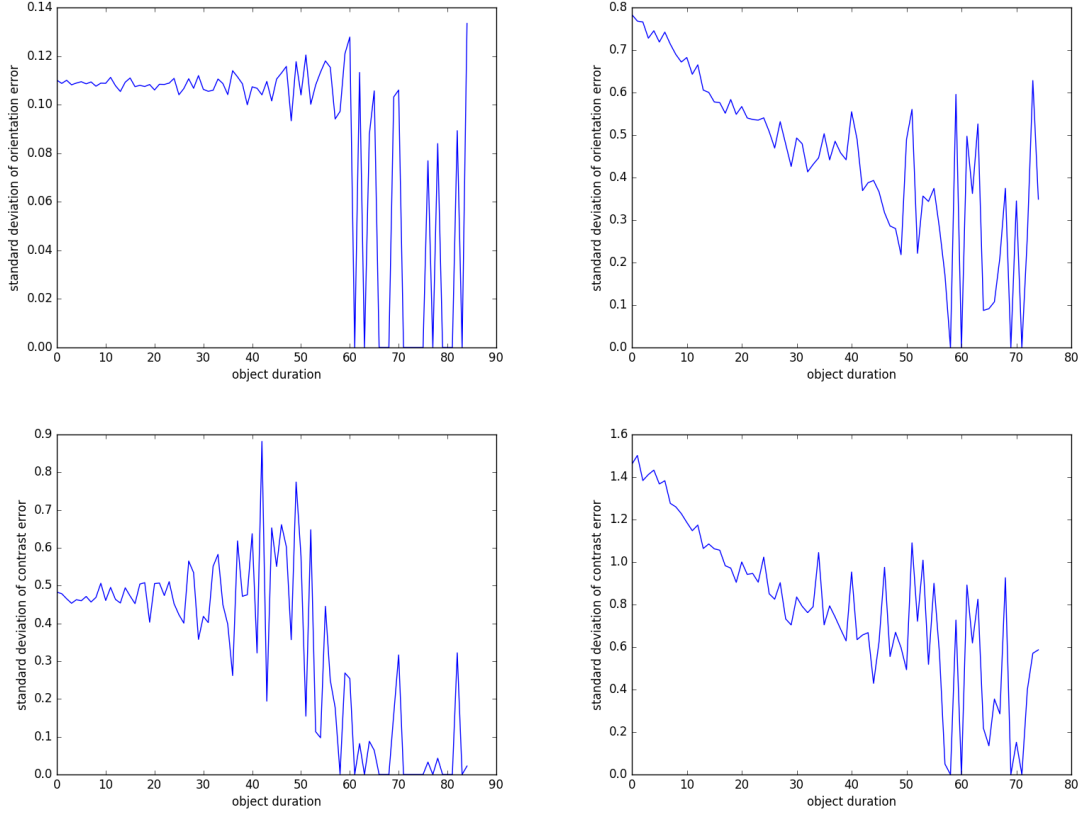


Figure 5: Standard deviation of orientation and contrast error as functions of object duration. Chosen parameters: $c_0 = 1, \tau_0 = 8$, number of channels = 8, left column integration window size = 1, right column integration window size = 8. Note the much larger scale of the y axis for the larger window size.

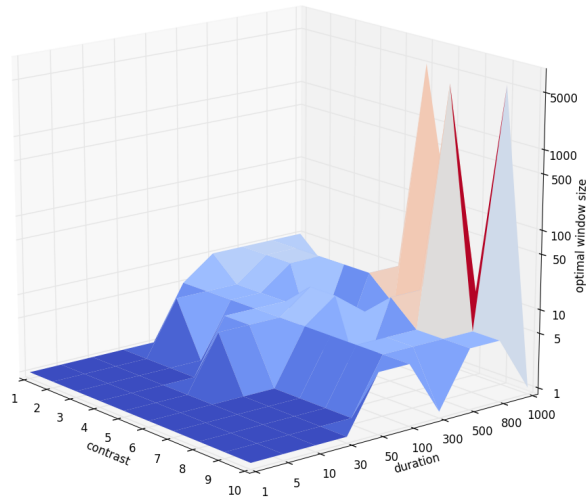


Figure 6: The “optimal” window size as a function of environment parameters c_0 (contrast) and τ_0 (duration), using 8 channels.