In [2]:
```
# importing necessary packages
import numpy as np
import scipy as sci
import scipy.sparse
import matplotlib.pyplot as plt
import time
```

In [ ]:

In [3]:
```
# defining dense function with boundaries and N as input
def Finite_Difference_Dense(a,b,N):
    # defining h
    h = (b-a)/(N+1)
    # creating plain matrix for A
    A = np.zeros([N,N])
    # filling in new values of A with double for loop
    for i in range(N):
        for j in range(N):
            #conditions of Aij as in (5)
            if i == j+1:
                A[i][j] = -1/(h**2)
            elif i == j-1:
                A[i][j] = -1/(h**2)
            elif i == j:
                A[i][j] = 2/(h**2)
            else:
                A[i][j] = 0
    # returning the matrix as output
    return A
```

In [ ]:

In [4]:
```
# defining the sparse function
def Finite_Difference_Sparse(a, b, N):
    # creating plain array for e values
    h = np.ones(N)
    # filling h with the h equation
    for i in range(N):
        h[i] = (b-a)/(N+1)
    # classifying my B as the h conditions given in (5)
    B = [-1/(h**2), 2/(h**2), -1/(h**2)]
    # using spdiags to get matrix A
    A = sci.sparse.spdiags(B, [-1, 0, 1], N,N)
    # returning the resulting matrix
    return A
```

In [ ]:

In [5]:
```python
def Thomas_Solver(a, b, c, d):

    # number of equations
    numEq = len(d)
    # copying the diagonals into more readable names
    low, mid, upp, sol = a, b, c, d
    # looping through the number of equations
    for i in range(1, numEq):
        # following instructions in (6) to classify values
        subAB = low[i-1]/mid[i-1]
        mid[i] = mid[i] - subAB*upp[i-1]
        sol[i] = sol[i] - subAB*sol[i-1]

    # creating new variable copy to change last value
    xi = mid
    xi[-1] = sol[-1]/mid[-1]

    # looping to find the solution xi
    for j in range(numEq-2, -1, -1):
        xi[j] = (sol[j]-upp[j]*xi[j+1])/mid[j]

    return xi
```

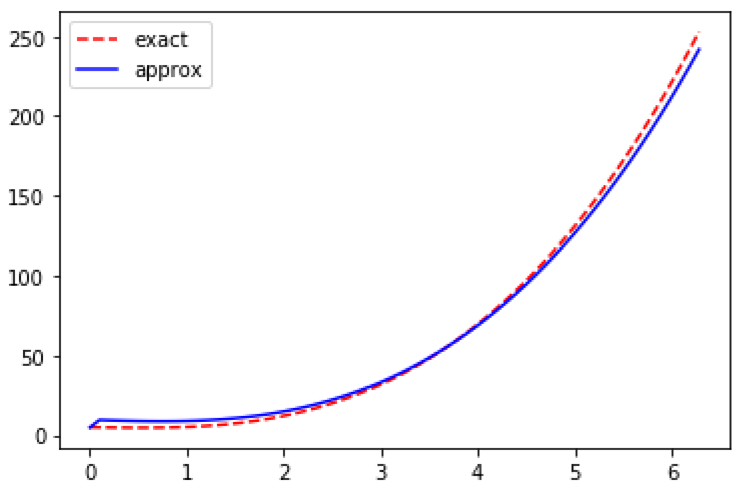
In [ ]:

In [6]:
```
def Poisson_Solver(string, f, a, b, ua, ub, N):
    # setting up conditions for different strings
    if string == 'dense':
        # initializing A using the dense equation
        A = Finite_Difference_Dense(a,b,N)
        # initializing timesteps list
        steps = np.linspace(a, b, N+1)
        # initializing structure for list x to be filled later
        x = np.ones(N)
        # calculating h
        h = (b-a)/(N+1)

        # looping through to find timesteps evaluated in function
        for i in range(1, N):
            x[i] = f(steps[i+1])

        # the second condition of fi changes here
        x[1] = x[1] + 1/(h**2)*ua
        x[-1] = x[-1] + 1/(h**2)*ub

        # using equivalent of backslash operator
        U = np.linalg.solve(A, x)

    # in the case the string is 'sparse'
    elif string == 'sparse':
        # using sparse function defined above
        A = Finite_Difference_Sparse(a,b,N).A
        # initializing list of timesteps
        steps = np.linspace(a, b, N+1)
        # structure of set to be filled later
        x = np.ones(N)
        # calculating h
        h = (b-a)/(N+1)

        # looping through to find timesteps evaluated in function
        for i in range(1, N):
            x[i] = f(steps[i+1])

        # the second condition of fi changes here
        # adding the 1/h^2*ua and 1/h^2*ub
        x[1] = x[1] + 1/(h**2)*ua
        x[-1] = x[-1] + 1/(h**2)*ub

        # using equivalent of backslash operator
        U = np.linalg.solve(A, x)

    # in the case the string is 'thomas'
    elif string == 'thomas':
        # using the sparse function to get A
        A = Finite_Difference_Sparse(a,b,N).A
        # initialize steps list
        steps = np.linspace(a, b, N+1)
        # calculating h
        h = (b-a)/(N+1)

        # grab the diagonals necessary (a=low b = mid, c = upp, x = d)
```

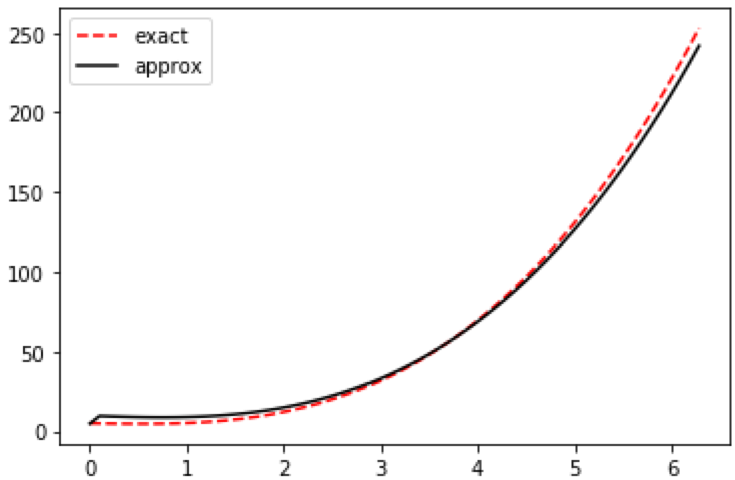
```
        low =  list(A.diagonal(-1))
        mid = list(A.diagonal(0))
        upp = list(A.diagonal(1))

        # initializing list to be filled later
        x = np.ones(N)

        # looping through to find timesteps evaluated in function
        for i in range(1, N):
            x[i] = f(steps[i+1])

        # the second condition of fi changes here
        # adding the 1/h^2*ua and 1/h^2*ub
        x[1] = x[1] + 1/(h**2)*ua
        x[-1] = x[-1] + 1/(h**2)*ub

        # solution using the thomas solver defined above
        U = Thomas_Solver(low, mid, upp, x)

    return U, x
```

In [ ]:

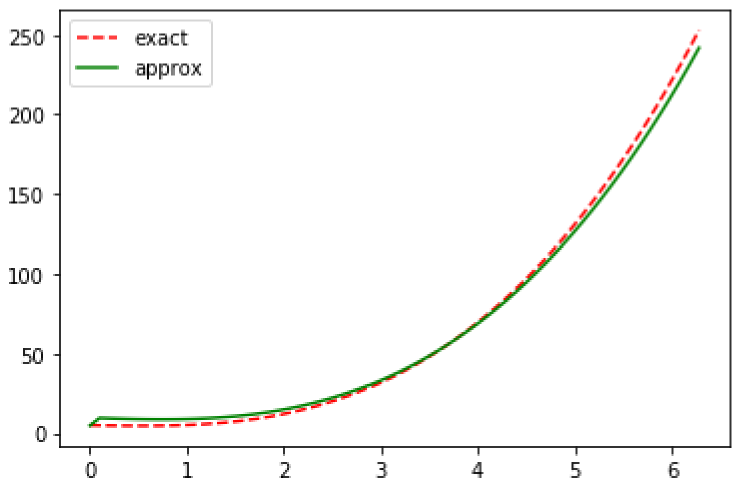
In [7]:
```
# setting up input variables
string = 'dense'
f = lambda x: -6*x - np.sin(x)
a = 0
b = 2*np.pi
ua = 5
ub = 8*(np.pi**3)+5
N = 64
# calling function
U, x = Poisson_Solver(string, f, a, b, ua, ub, N)

# steps as x for graphing U as y
steps = np.linspace(a, b, N)

# setting up values for exact equation
Xvals = np.linspace(0, 2*np.pi, 100)
u = lambda x: x**3 - np.sin(x) + 5
Yvals = u(Xvals)

# plotting both on the same graph
plt.plot(Xvals, Yvals, 'r--')
plt.plot(steps, U, 'b')
plt.legend(['exact', 'approx'])
plt.show()
```



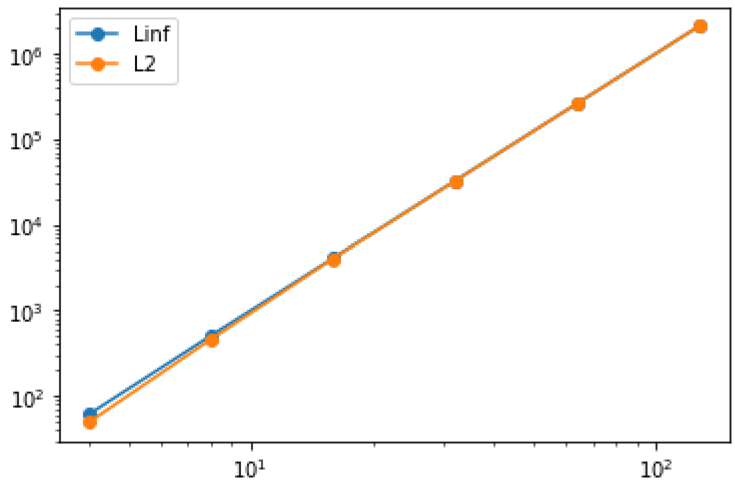
In [8]:
```
# setting up input variables
string = 'sparse'
f = lambda x: -6*x - np.sin(x)
a = 0
b = 2*np.pi
ua = 5
ub = 8*(np.pi**3)+5
N = 64
# calling function
U, x = Poisson_Solver(string, f, a, b, ua, ub, N)

# steps as x for graphing U as y
steps = np.linspace(a, b, N)

# setting up values for exact equation
Xvals = np.linspace(0, 2*np.pi, 100)
u = lambda x: x**3 - np.sin(x) + 5
Yvals = u(Xvals)

# plotting both on the same graph
plt.plot(Xvals, Yvals, 'r--')
plt.plot(steps, U, 'k')
plt.legend(['exact', 'approx'])
plt.show()
```



In [9]:
```
# setting up input variables
string = 'thomas'
f = lambda x: -6*x - np.sin(x)
a = 0
b = 2*np.pi
ua = 5
ub = 8*(np.pi**3)+5
N = 64

# calling function
U, x = Poisson_Solver(string, f, a, b, ua, ub, N)

# steps as x for graphing U as y
steps = np.linspace(a, b, N)

# setting up values for exact equation
Xvals = np.linspace(0, 2*np.pi, 100)
u = lambda x: x**3 - np.sin(x) + 5
Yvals = u(Xvals)

# plotting both on the same graph
plt.plot(Xvals, Yvals, 'r--')
plt.plot(steps, U, 'g')
plt.legend(['exact', 'approx'])
plt.show()
```



In [20]:
```python
# setting up input variables
string = 'sparse'
f = lambda x: -6*x - np.sin(x)
a = 0
b = 2*np.pi
ua = 5
ub = 8*(np.pi**3)+5
N = np.array([4, 8, 16, 32, 64, 128])

# exact equation xs and y
u = lambda x: x**3 - np.sin(x) + 5
Yexact = u(N)

# empty to be filled later
Ys = []
# appending the result for each N
for i in N:
    U, x = Poisson_Solver(string, f, a, b, ua, ub, i)
    Ys.append(U)

# empty to be filled later
L2norm = []
# appending l2 norm
for i in range(len(N)):
    L2norm.append(abs((1/np.sqrt(N[i]))*(np.sqrt(sum((Yexact[i]-Ys[i])**2)))))

# empty to be filled later
LInfnorm = []
# appending L infinity norm
for i in range(len(N)):
    LInfnorm.append(max(abs(Yexact[i]-Ys[i])))

# plotting on loglog scale both on the same graph
plt.loglog(N, LInfnorm, 'o-')
plt.loglog(N, L2norm, 'o-')

# legend labels
plt.legend(['Linf', 'L2'])
plt.show()

# slopes of loglog norms
print("L Inf Norm", np.log(LInfnorm[-1]-LInfnorm[0])/np.log(N[-1]-N[0]))
print("L2 Norm", np.log(L2norm[-1]-L2norm[0])/np.log(N[-1]-N[0]))
```




```
L Inf Norm 3.0197532635529023
L2 Norm 3.0197481487279703
```

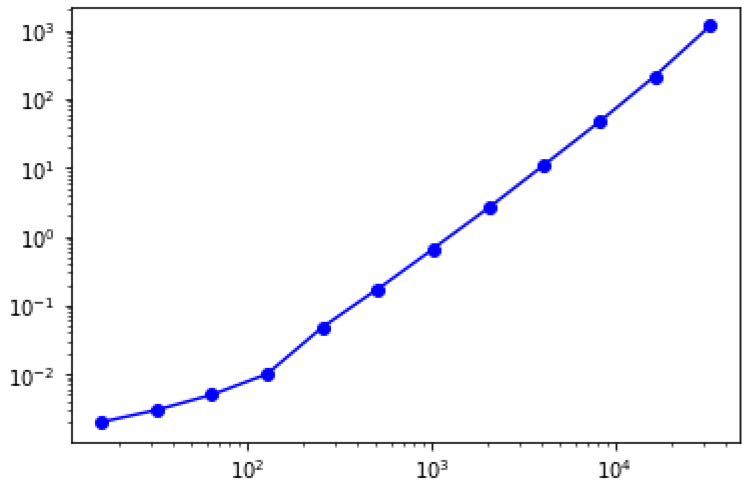
In [ ]:

In [16]:
```
# setting up variables to run function
string = 'dense'
f = lambda x: -6*x - np.sin(x)
a = 0
b = 2*np.pi
ua = 5
ub = 8*(np.pi**3)+5
# N values given in the question
N = [2**4, 2**5, 2**6, 2**7, 2**8, 2**9, 2**10, 2**11, 2**12, 2**13, 2**14, 2*
*15]

# empty times to be filled later
times = []

# looping through all of the N values
for i in range(len(N)):
    # initial time
    tic = time.time()
    # function running at N[i]
    U, x = Poisson_Solver(string, f, a, b, ua, ub, N[i])
    # ending time
    toc = time.time()
    # elapsed time by subtracting initial from final
    elapsed = toc-tic
    # appending this value to array of times
    times.append(elapsed)
print(times)
```

```
[0.0019998550415039062, 0.0030014514923095703, 0.004998922348022461, 0.010002
613067626953, 0.04800224304199219, 0.17401337623596191, 0.6680655479431152,
2.6122071743011475, 10.962820529937744, 46.535985231399536, 215.006127595901
5, 1156.9006271362305]
```

In [32]:
```
# plotting N on x and times on y for loglog plot
N = [2**4, 2**5, 2**6, 2**7, 2**8, 2**9, 2**10, 2**11, 2**12, 2**13, 2**14, 2*
*15]
timesDense = [0.0019998550415039062, 0.0030014514923095703, 0.0049989223480224
61, 0.010002613067626953, 0.04800224304199219, 0.17401337623596191, 0.66806554
79431152, 2.6122071743011475, 10.962820529937744, 46.535985231399536, 215.0061
275959015, 1156.9006271362305]
plt.loglog(N, timesDense, 'bo-')
```

Out[32]: [<matplotlib.lines.Line2D at 0x228eb01adc8>]

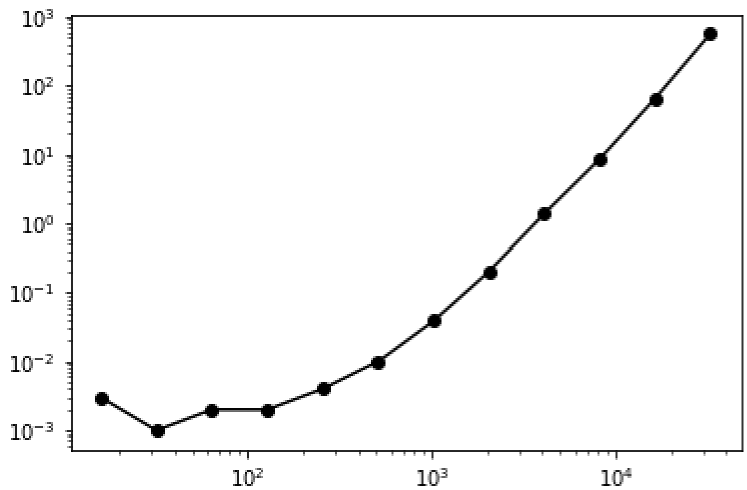


In [14]:
```
# setting up variables to run function
string = 'sparse'
f = lambda x: -6*x - np.sin(x)
a = 0
b = 2*np.pi
ua = 5
ub = 8*(np.pi**3)+5
# N values given in the question
N = [2**4, 2**5, 2**6, 2**7, 2**8, 2**9, 2**10, 2**11, 2**12, 2**13, 2**14, 2*
*15]

# empty times to be filled later
times = []

# looping through all of the N values
for i in range(len(N)):
    # initial time
    tic = time.time()
    # function running at N[i]
    U, x = Poisson_Solver(string, f, a, b, ua, ub, N[i])
    # ending time
    toc = time.time()
    # elapsed time by subtracting initial from final
    elapsed = toc-tic
    # appending this to times list
    times.append(elapsed)
print(times)
```

```
[0.0030002593994140625, 0.0010020732879638672, 0.002001047134399414, 0.001997
9476928710938, 0.004000425338745117, 0.009999990463256836, 0.039001703262329
1, 0.20301437377929688, 1.375103235244751, 8.535635471343994, 64.430854320526
12, 564.1538195610046]
```

In [36]:
```
# x and y values for our loglog plot
N = [2**4, 2**5, 2**6, 2**7, 2**8, 2**9, 2**10, 2**11, 2**12, 2**13, 2**14, 2*
*15]
timesSparse = [0.0030002593994140625, 0.0010020732879638672, 0.002001047134399
414, 0.0019979476928710938, 0.004000425338745117, 0.009999990463256836, 0.0390
017032623291, 0.20301437377929688, 1.375103235244751, 8.535635471343994, 64.43
085432052612, 564.1538195610046]


# plotting on loglog scale
plt.loglog(N, timesSparse, 'ko-')
```

Out[36]: [<matplotlib.lines.Line2D at 0x228ec74e448>]

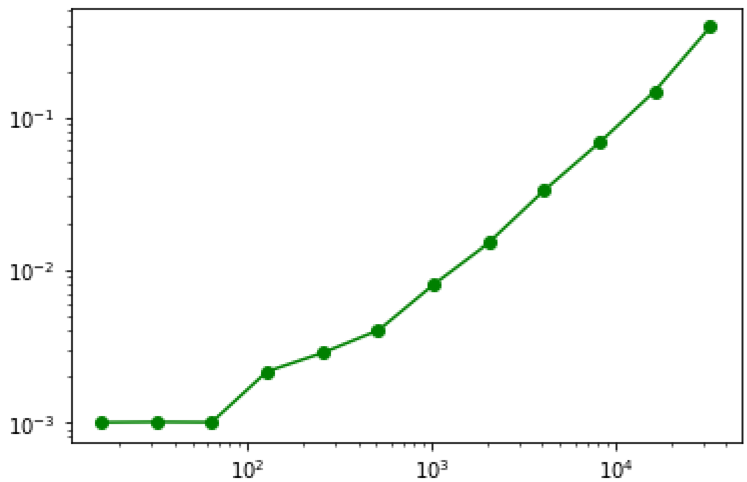


In [11]:
```
# setting up variables to run function
string = 'thomas'
f = lambda x: -6*x - np.sin(x)
a = 0
b = 2*np.pi
ua = 5
ub = 8*(np.pi**3)+5

# N values given in the question
N = [2**4, 2**5, 2**6, 2**7, 2**8, 2**9, 2**10, 2**11, 2**12, 2**13, 2**14, 2*
*15]

# empty times to be filled later
times = []

for i in range(len(N)):
    # initial time
    tic = time.time()
    # running function at N[i]
    U, x = Poisson_Solver(string, f, a, b, ua, ub, N[i])
    # ending time
    toc = time.time()
    # elapsed time by subtracting initial from final
    elapsed = toc-tic
    # appending this to times list
    times.append(elapsed)
print(times)
```

```
[0.0009980201721191406, 0.001003265380859375, 0.00099945068359375, 0.00215220
45135498047, 0.002847909927368164, 0.003998756408691406, 0.00800251960754394
5, 0.015004158020019531, 0.03299713134765625, 0.06800389289855957, 0.14801239
967346191, 0.3900301456451416]
```

In [34]:
```
N = [2**4, 2**5, 2**6, 2**7, 2**8, 2**9, 2**10, 2**11, 2**12, 2**13, 2**14, 2**15]
timesThomas = [0.0009980201721191406, 0.0010032653808593750, 0.0009994506835937
5, 0.0021522045135498047, 0.0028479099273681640, 0.0039987564086914060, 0.008002
519607543945, 0.015004158020019531, 0.03299713134765625, 0.06800389289855957,
0.14801239967346191, 0.3900301456451416]
plt.loglog(N, timesThomas, 'go-')
```

Out[34]: [<matplotlib.lines.Line2D at 0x228eb08fe08>]



In [ ]: