XIth HAND WRITTEN MATERIALS

CHAPTER – 2

BASIC ALGEBRA

MANIKANDAN S

P.G Assistant Mathematics

9655536357

Join our Telegram Channel: https://t.me/mathsums

CHAPTER : 2 ①

TOPIC : BASIC ALGEBRA

EX:2.1

1) classify each element of ($\sqrt{7}$, $-1/4$; 0, 3.14, 4, 22/7) as a member of N, Q, R − Q or Z.

Solution:

i) $\sqrt{7} \in R - Q$

ii) $-1/4 \in Q$

iii) $0 \in Z, Q$

iv) $3.14 \in Q$

v) $4 \in N, Z, Q$

vi) $22/7 \in Q$

---

2) Prove that $\sqrt{3}$ is an irrational number.

solution:

Suppose that $\sqrt{3}$ is a rational number.

Let $\sqrt{3} = \frac{m}{n}$ {where m and n are positive integers with no common factors greater than 1. $(n \neq 0)$

$$\sqrt{3} = \frac{m}{n}$$

$$\sqrt{3}n = m$$

$$3n^2 = m^2 \rightarrow ①$$

Since $3n^2$ is divisible by 3, it shows that m^2 is multiple of 3.

Also m is multiple of 3. So,

that $m = 3K$ (K is any constant)

$$① \Rightarrow 3n^2 = (3K)^2$$

$$3n^2 = 9K^2$$ ②

$$n^2 = 3K^2$$

since $3K^2$ is divisible by 3, it shows that n^2 is multiple of 3.

n also multiple of 3.

So, that $n = 3l$ (l is any constant)

It follows, that m and n are having a common factor 3.

Thus, we arrived at a contradiction.

Hence, $\sqrt{3}$ is an irrational number.

---

3) Are there two distinct irrational numbers such that their difference is a rational number? Justify.

Solution:

Let $a+\sqrt{b}$ & $c+\sqrt{b} \in R-Q$

$$\text{Difference} = (a+\sqrt{b}) - (c+\sqrt{b})$$

$$= a+\sqrt{b}-c-\sqrt{b} = a-c \in Q.$$

So that difference between two distinct irrational numbers may be a rational number.

---

4) Find two irrational numbers such that their sum is a rational number. Can you find two irrational numbers whose product is a rational number.

Solution:

i) Let $4+\sqrt{2}$ & $4-\sqrt{2} \in R-Q$

Sum $= (4+\sqrt{2}) + (4-\sqrt{2})$

$= 4+\sqrt{2}+4-\sqrt{2} = 8 \in Q$

So that sum of two distinct irrational numbers may be a rational numbers.

ii) Let $4+\sqrt{2}$ & $4-\sqrt{2} \in R-Q$

Product $= (4+\sqrt{2})(4-\sqrt{2})$

$= (4)^2 - (\sqrt{2})^2 = 16-4 = 12 \in Q$

So that product of two distinct irrational numbers may be a rational numbers.

5) Find a positive number smaller than $\frac{1}{2^{1000}}$. is $\frac{1}{2^{1001}}$ Justify.

Solution:

$1000 < 1001$

$2^{1000} < 2^{1001}$

$\frac{1}{2^{1000}} > \frac{1}{2^{1001}}$

Hence a positive number smaller than $\frac{1}{2^{1000}}$ is $\frac{1}{2^{1001}}$

Ex: 2.2

4

1) Solve for x

i) $|3-x| < 7$

Solution:

$$|3-x| < 7$$

$$-7 < 3-x < 7$$

$$-7-3 < -x < 7-3$$

$$-10 < -x < 4$$

$$10 > x > -4 \qquad [\therefore \text{Multiple by } -1]$$

$$\Rightarrow -4 < x < 10$$

ii) $|4x-5| \geq -2$

Solution:

$$|4x-5| \geq -2$$

For all the values of x, $|4x-5|$ is positive.

i.e., $\forall\, x \in R$ $|4x-5| \geq -2$

$x \in R$ will satisfy the above equation.

iii) $\left|3-\frac{3}{4}x\right| \leq \frac{1}{4}$

Solution:

$$\left|3-3/4\, x\right| \leq 1/4$$

$$-1/4 \leq 3 - 3/4\, x \leq 1/4$$

$$-1/4 - 3 \leq -3/4\, x \leq 1/4 - 3$$

$$-13/4 \leq -3/4\, x \leq -11/4$$

$-13/3 \le -x \le -11/3$ [∵ Multiply by 4/3]

$13/3 \ge x \ge \frac{11}{3}$ [∵ Multiply by −1]

$11/3 \le x \le 13/3$

iv) $|x| - 10 < -3$

Solution:

$|x| - 10 < -3$

$|x| < -3 + 10$

$|x| < 7$

$-7 < x < 7$

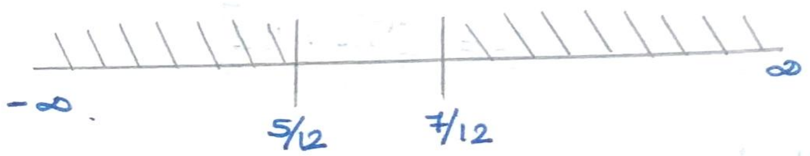
2) Solve $\frac{1}{|2x-1|} < 6$ and express the solution using the interval notation.

Solution:

$\frac{1}{|2x-1|} < 6$

$|2x-1| > 1/6$

$2x - 1 > 1/6$	$2x - 1 < -1/6$
$2x > 1/6 + 1$	$2x < -1/6 + 1$
$2x > 7/6$	$2x < 5/6$
$x > 7/12$	$x < 5/12$



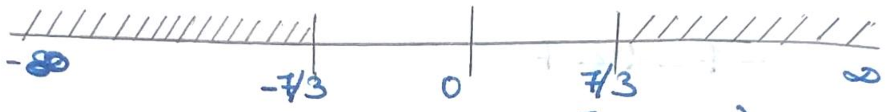
$x \in (-\infty, 5/2) \cup (7/12, \infty)$

⑥

3) Solve $-3|x|+5 \le -2$ and graph the solution set in a number line

Solution:

$$-3|x|+5 \le -2$$
$$-3|x| \le -2-5$$
$$-3|x| \le -7$$
$$3|x| \ge 7 \quad [\because \text{Multiply by } -1]$$
$$|x| \ge 7/3$$
$$x \ge 7/3 \text{ (or) } x \le -7/3$$

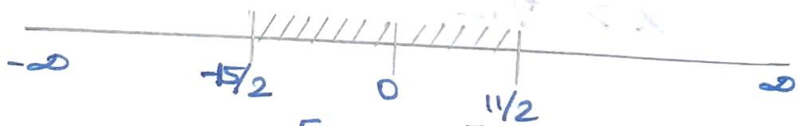


$$x \in (-\infty, -7/3] \cup [7/3, \infty)$$

4) Solve $2|x+1|-6 \le 7$ and graph the solution set in a number line.

Solution:

$$2|x+1|-6 \le 7$$
$$2|x+1| \le 7+6$$
$$2|x+1| \le 13$$
$$|x+1| \le 13/2$$
$$-13/2 \le x+1 \le 13/2$$
$$-13/2-1 \le x \le 13/2-1$$
$$-15/2 \le x \le 11/2$$



$$x \in [-15/2, 11/2]$$

5) Solve $1/5\,|10x-2| < 1$

Solution:

$$1/5\,|10x-2| < 1$$

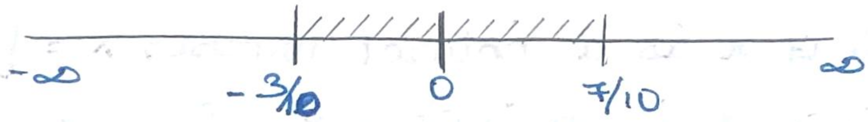
$|10x-2| < 5$

$-5 < 10x - 2 < 5$

$-5+2 < 10x < 5+2$

$-3 < 10x < 7$

$3/10 < x < 7/10$



6) Solve $|5x-12| < -2$

Solution:

$|5x-12| < -2$

Absolute value cannot be in negative. So x does not have any suitable solution to satisfy the above inequation.

x has no solution.

EX : 2.3

1) Represent the following inequalities in the interval notation:

QUESTION	INTERVAL NOTATION
a) $x \geq -1$ and $x < 4$	$x \in [-1, 4)$
b) $x \leq 5$ and $x \geq -3$	$x \in [-3, 5]$
c) $x < -1$ or $x < 3$	$x \in (-\infty, 3)$
d) $-2x > 0$ or $3x - 4 < 11$ $x < 0$ or $3x < 15$ $x < 5$	$x \in (-\infty, 5)$

2) Solve $23x < 100$ when (i) x is a natural number, ii) x is an integer.

Solution:

$$23x < 100$$
$$x < 100/23$$

i) If x is a natural number $x = \{1, 2, 3, 4\}$

ii) If x is an integer $x = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

---

3) Solve $-2x \geq 9$ when i) x is a real number, ii) x is an integer, iii) x is a natural number.

Solution:

$$-2x \geq 9$$
$$2x \leq -9$$
$$x \leq -9/2$$

i) If x is real number $x \in (-\infty, -9/2]$

ii) If x is an integer $x = \{\ldots, -7, -6, -5\}$

iii) If x is a natural number x has no solution.

---

4) Solve:

i) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

Solution:

$$\Rightarrow 9(x-2) \leq 25(2-x)$$
$$\Rightarrow 9x - 18 \leq 50 - 25x$$
$$\Rightarrow 34x \leq 68$$

9

$\Rightarrow 34x \leq 68$

$x \leq 2$

$x \in (-\infty, 2]$

ii) $\frac{5-x}{3} < \frac{x}{2} - 4$

Solution:

$\Rightarrow 2(5-x) < 3x - 24$

$\Rightarrow 10 - 2x < 3x - 24$

$\Rightarrow 34 < 5x$

$\Rightarrow 5x > 34$

$\Rightarrow x > 34/5$

$x \in (34/5, \infty)$

5) To secure A grade one must obtain an average of 90 marks or more in 5 subjects each of maximum 100 marks. If one scored 84, 87, 95, 91 in first four subjects, what is the minimum mark one scored in the fifth subject to get A Grade in the course?

Solution:

Marks in the first four subjects are 84, 87, 95, 91.

Let marks in fifth subject be "x".

$$\text{Average mark} = \frac{84+87+95+91}{5} = \frac{357+x}{5}$$

From given data required inequation to score A grade:

(10)

$$\Rightarrow \frac{357+x}{5} \geq 90$$

$$357+x \geq 450$$

$$x \geq 450-357$$

$$x \geq 93$$

Minimum marks to score in fifth subject for getting A Grade is 93.

---

b) A manufacture has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?

Solution:

Initial amount of solution = 600 liters,

$\left.\begin{array}{l}\text{% of acid present in} \\ \text{initial solution}\end{array}\right\} = 12\%$

$\left.\begin{array}{l}\text{Amount of acid precent in} \\ \text{initial solution}\end{array}\right\} = \frac{12}{100} \times 600$

$= 72$ litres

Let amount of mixing solution = x Liters

$\left.\begin{array}{l}\text{% acid present in mixing} \\ \text{solution}\end{array}\right\} = 30\%$

$\left.\begin{array}{l}\text{Amount of acid present in} \\ \text{mixing solution}\end{array}\right\} = \frac{30}{100} \times x$

$= \frac{30x}{100}$

Total amount of solution $= 600 + x$ (ii)

Total amount of acid present $= 72 + \dfrac{30x}{100}$

Total % acid present in mixture solution:

$$= \frac{\text{Total amount of acid present}}{\text{Total amount of solution}} \times 100$$

$$= \frac{72 + 30x/100}{600 + x} \times 100$$

$$= \frac{7200 + 30x}{600 + x}$$

From given data

$$15 < \frac{7200 + 30x}{600 + x} < 18$$

case (i) :	case (ii)
$15 < \dfrac{7200+30x}{600+x}$	$\dfrac{7200+30x}{600+x} < 18$
$15(600+x) < 7200+30x$	$7200+30x < 18(600+x)$
$9000+15x < 7200+30x$	$7200+30x < 10800+18x$
$9000-7200 < 30x-15x$	$30x-18x < 10800-7200$
$1800 < 15x$	$12x < 3600$
$120 < x$	$x < 300$

Hence x is must be $120 < x < 300$

7) Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40.

Solution:

Let x and $x+2$ are the consecutive odd numbers.

It is given that both natural numbers are greater than 10.

$x > 10 \longrightarrow (1)$

also given that their sum is less than 40.

i.e., $x + (x+2) < 40$

$2x + 2 < 40$

$2x < 38$

$x < 19 \rightarrow (2)$

From (1) and (2)

$10 < x < 19$

Possible values of x is 11, 13, 15, 17

Respective $x+1$ values are 13, 15, 17, 19

The required pairs of consecutive odd numbers are (11, 13), (13, 15), (15, 17) and (17, 19).

8) A model rocket is launched from the ground. The height h reached by the rocket after t secounds from lift off is given by $h(t) = -5t^2 + 100t$; $0 \le t \le 20$. At what time the rocket is 495 feet above the ground?

Solution:

From given data,

13

$$h(t) > 495$$
$$-5t^2+100t > 495$$
$$-5t^2+100t-495 > 0$$
$$5t^2-100t+495 < 0$$
$$t^2-20t+99 < 0$$
$$(t-9)(t-11) < 0$$

	$(t-9)$	$(t-11)$	$5t^2-100t+495$
$t<9$	$-$	$-$	$+$
$9<t<11$	$+$	$-$	$-$
$t>11$	$+$	$+$	$+$

At $9<t<11$ the rocket is 495 feet above the ground.

9) A plumber can be paid according to the following schemes: In the first scheme he will be paid rupees 500 plus rupees 70 per hour, and in the secound scheme he will paid rupees 120 per hour. If he works x hours, then for what value of x does the first scheme given better wages?

Solution:

Let no of working hours be "x"

From given data,

First scheme : $500+70x$

secound scheme : $120x$

Required linear inequation is

$$500+70x > 120x$$
$$500 > 120x-70x$$
$$500 > 50x$$
$$10 > x$$

For less than 10 working hours first scheme gives better wages. (14)

10) A and B are working on similar jobs but their annual salaries differ by more than Rs 6000. If B earns rupees 27000 per month, then what are the possibilities of A's salary per month?

Solution:

B's monthly salary $= 2700$

Let A's monthly salary $= x$

Difference in annual income $= 6000$ Rs

Difference in monthly salary $= \frac{6000}{12} = 500$

Required linear inequation is

$$|A\text{'s salary} - B\text{'s salary}| > 500$$

$$|x - 27000| > 500$$

$x - 27000 > 500$ $\qquad$ $x - 27000 < -500$

$x > 27500$ $\qquad$ $x < 26500$

Possible salary of A is either less than 26500 or more than 27500. which gives more than 6000 difference in annual income.

EX: 2.4

1) construct a quadratic equation with roots 7 and -3.

Solution:

Method 1 :

Factors of quadratic equation is $(x-7)$ & $(x+3)$

Required quadratic equation is ~~(x-7)~~

$$(x-7)\ (x+3)=0$$

$$x^2-7x+3x-21=0$$

$$x^2-4x-21=0$$

Method 2:

Sum of factors $= 7+(-3)=4$

Product of factors $= 7\times(-3) = -21$

Required quadratic equation is

$$x^2-(S.R)x+P.R=0$$

$$x^2-4x-21=0$$

2) A quadratic polynomial has one of its zeros $1+\sqrt{5}$ and it satisfies $P(1)=2$. Find the quadratic polynomial.

Solution:

Always irrational roots occur in conjugate pairs

If $1+\sqrt{5}$ is root then $1-\sqrt{5}$ also a root to the equation.

Sum of roots $= 1+\sqrt{5}+1-\sqrt{5} = 2$

Product of roots $= (1+\sqrt{5})(1-\sqrt{5}) = 1-5 = 4$

Required quadratic polynomial

$$P(x) = K(x^2-(S.R)x+P.R)$$

$$P(x) = K(x^2-2x-4) \rightarrow (1)$$

16

Also given $P(1) = 2$

$$P(1) = K\left((1)^2 - 2(1) - 4\right) \Rightarrow K = -2/5$$

$$① \Rightarrow P(x) = -2/5\,(x^2 - 2x - 4)$$

---

3) If α and β are the roots of the quadratic equation $x^2 + \sqrt{2}x + 3 = 0$, form a quadratic polynomial with zeroes $\frac{1}{\alpha}, \frac{1}{\beta}$

Solution:

α and β are the roots of the quadratic equation.

$$x^2 + \sqrt{2}x + 3 = 0$$

$$\text{Now } \alpha + \beta = -\sqrt{2}$$

$$\alpha\beta = 3$$

Required quadratic polynomial is:

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(1/\alpha \cdot 1/\beta\right) = 0$$

$$x^2 - \left(\frac{\alpha + \beta}{\alpha\beta}\right)x + \left(\frac{1}{\alpha\beta}\right) = 0$$

$$x^2 - \left(\frac{-\sqrt{2}}{3}\right)x + \left(\frac{1}{3}\right) = 0$$

$$x^2 + \left(\frac{\sqrt{2}}{3}\right)x + \left(\frac{1}{3}\right) = 0$$

$$3x^2 + \sqrt{2}x + 1 = 0$$

---

4) If one root of $k(x-1)^2 = 5x - 7$ is double the other root, show that $k = 2$ or -25.

Solution:

17

$$K(x-1)^2 = 5x-7$$

$$K(x^2-2x+1)-5x+7=0$$

$$Kx^2-2Kx+K-5x+7=0$$

$$Kx^2-(2K+5)x+(K+7)=0$$

from given data if a is a root then 2a is another root of

$$Kx^2-(2K+5)x+(K+7)=0$$

where sum of roots $a+2a = 2K+5$

$$3a = 2K+5$$

$$a = \frac{2K+5}{3K} \rightarrow (1)$$

Also product of the roots $a.(2a) = \frac{K+7}{K}$

$$2a^2 = \frac{K+7}{K}$$

$$2\left(\frac{2K+5}{3K}\right)^2 = \frac{K+7}{K} \quad [\because \text{From (1)}$$

$$2\left(\frac{4K^2+20K+25}{9K^2}\right) = \frac{K+7}{K}$$

$$2(4K^2+20K+25) = 9K^2\left(\frac{K+7}{K}\right)$$

$$8K^2+40K+50 = 9K^2+63K$$

$$-K^2-23K+50 = 0$$

$$K^2+23K-50 = 0$$

$$(K-2)(K+25) = 0$$

$K=2$ or $K=-25$

Hence proved.

5) If the difference of the roots of the equation $2x^2-(a+1)x+a-1=0$ is equal to their product, then prove that $a=2$ (18)

Solution:

α and β are the roots of the quadratic equation.

$$2x^2-(a+1)x+a-1=0$$

Here $\alpha+\beta = \frac{a+1}{2}$

$$\alpha\beta = \frac{a-1}{2}$$

From given data

Difference of the roots = product of the roots

i.e., $\alpha-\beta = \alpha\beta$

Now $(\alpha-\beta)^2 = (\alpha\beta)^2$

$$(\alpha+\beta)^2 - 4\alpha\beta = (\alpha\beta)^2$$

$$\left(\frac{a+1}{2}\right)^2 - 4\left(\frac{a-1}{2}\right) = \left(\frac{a-1}{2}\right)^2$$

$$\frac{a^2+2a+1}{4} - 2(a-1) = \frac{a^2-2a+1}{4}$$

Multiply by 4

$$a^2+2a+1-8(a-1) = a^2-2a+1$$

$$a^2+2a+1-8a+8 = a^2-2a+1$$

$$-4a+8=0$$

$$\Rightarrow a=2$$

Hence proved.

6) Find the condition that one of the roots of ax^2+bx+c may be i) negative of the other, ii) Thrice the other, iii) Reciprocal of the other. (19)

Solution:

Let $ax^2+bx+c=0$

i) Let the roots negative of other.

i.e., the roots α and $-\alpha$

Now sum $= \alpha + (-\alpha) = \frac{-b}{a}$

$$0 = \frac{-b}{a} \Rightarrow b=0$$

ii) Let the roots is thrice the other.

i.e., The roots α and 3α.

Now Sum $= \alpha + 3\alpha = -b/a$

$$4\alpha = -b/a$$

$$\alpha = -b/4a$$

Product $= \alpha(3\alpha) = c/a$

$$3\alpha^2 = c/a$$

$$3\left(\frac{-b}{4a}\right)^2 = c/a$$

$$\frac{3b^2}{16a^2} = c/a \Rightarrow \frac{3b^2}{16a} = c$$

$$\Rightarrow 3b^2 = 16ac$$

iii) Let the roots is reciprocal of the other

i.e., The roots α and $1/\alpha$

Product $= \alpha \times 1/\alpha = c/a \Rightarrow 1 = c/a$

$$\Rightarrow c = a$$

7) If the equation $x^2-ax+b=0$ and $x^2-ex+f=0$ have one roots in common (20) and if the second equation has equal roots, the prove that $ae=2(b+f)$

Solution:

$x^2-ax+b=0 \rightarrow (1)$	$x^2-ex+f=0 \rightarrow (2)$
α & β are the roots of (I) where α is the common root of (I) & (II)	α and α are the roots of (II) where roots of (II) are equal
Sum $=\alpha+\beta=a \rightarrow (1)$	Sum $\alpha+\alpha=e$ $\Rightarrow 2\alpha=e \rightarrow (2)$
Product $=\alpha\beta=b \rightarrow (3)$	Product $\alpha.\alpha=f$ $\alpha^2=f \rightarrow (4)$

To prove : $ae=2(b+f)$

$$LHS = ae = (\alpha+\beta)2\alpha \quad [\because \text{from (1) and (2)}]$$

$$=2(\alpha^2+\alpha\beta)$$

$$=2(b+f) \quad [\because \text{from (3) and (4)}]$$

$$=R.H.S$$

Hence proved.

8) Discuss the nature of roots of

i) $-x^2+3x+1=0$, ii) $4x^2-x-2=0$,

iii) $9x^2+5x=0$

Solution:

	VALUES OF a, b, c	b^2-4ac & SIGN	NATURE OF ROOTS
i) $-x^2+3x+1=0$	$a=-1$ $b=3$ $c=1$	$(3)^2-4(-1)(1)$ $=9+4=13$ Positive	Real & Distinct
ii) $4x^2-x-2=0$	$a=4$ $b=-1$ $c=-2$	$(-1)^2-4(4)(-2)$ $=1+32=33$ Positive	Real & Distinct
iii) $9x^2+5x=0$	$a=9$ $b=5$ $c=0$	$(5)^2-4(9)(0)$ $=25$ Positive	Real & Distinct

9) Without sketching the graphs, find whether the graphs of the following functions will intersect the x-axis and if so in how many points

i) $y=x^2+x+2$, ii) $y=x^2-3x-7$, iii) $y=x^2+6x+9$.

	VALUES OF a, b, c	b^2-4ac & SIGN	RESULTS ON INTERSECTION OF X-AXIS
i) $y=x^2+x+2$	$a=1$ $b=1$ $c=2$	$(1)^2-4(1)(2)$ $=1-8=-7$ Negative	Does not meet x-axis
ii) $y=x^2-3x-7$	$a=1$ $b=-3$ $c=-7$	$(-3)^2-4(1)(-7)$ $=9+28=37$ Positive	Intersects x-axis at two points
iii) $y=x^2+6x+9$	$a=1$ $b=6$ $c=9$	$(6)^2-4(1)(9)$ $=36-36=0$ zero	Touches x-axis at one point

MANIKANDAN.S, P.G ASST MATHEMATICS -9655536357

10) Write $f(x) = x^2+5x+4$ in complete square form. (22)

Solution:

$$f(x) = x^2+5x+4$$
$$= x^2+5x+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+4$$
$$= \left(x+\frac{5}{2}\right)^2-\frac{25}{4}+4$$
$$= (x+5/2)^2 - 9/4$$
$$= (x+5/2)^2-(3/2)^2$$

Ex: 2.5

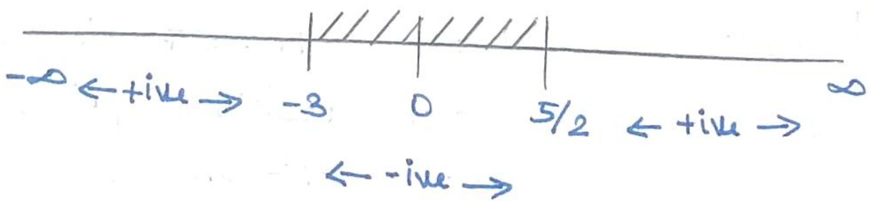
1) Solve $2x^2+x-15 \leq 0$

Solution:

$$2x^2+x-15 \leq 0$$
$$(x+3)(2x-5) \leq 0$$

The critical points are -3 and $5/2$

INTERVAL	SIGN OF $(x+3)$	SIGN OF $(2x-5)$	SIGN OF $2x^2+x-15$
$(-\infty, -3)$	$-$	$-$	+ive
$(-3, 5/2)$	$+$	$-$	-ive
$(5/2, \infty)$	$+$	$+$	+ive



$x \in [-3, 5/2]$ is satisfies $2x^2+x-15 \leq 0$

2) solve $-x^2+3x-2 \geq 0$

solution:

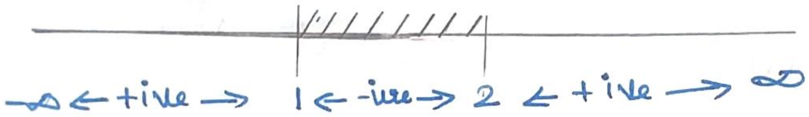
$$-x^2+3x-2 \geq 0$$

$$x^2-3x+2 \leq 0$$

$$(x-1)(x-2) \leq 0$$

critical numbers are 1, 2

INTERVAL	SIGN OF $(x-1)$	SIGN OF $(x-2)$	SIGN OF x^2-3x+2
$(-\infty,1)$	$-$	$-$	+ive
$(1,2)$	$+$	$-$	-ive
$(2,\infty)$	$+$	$+$	+ive



$x \in [1,2]$ is satisfies $-x^2+3x-2 \geq 0$

EX: 2.6

1) Find the zeros of the polynomial function $f(x) = 4x^2-25$.

solution:

$$f(x) = 4x^2-25 = 0$$

$$(2x)^2-5^2 = 0$$

$$(2x-5)(2x+5) = 0$$

zeros of the polynomials are $x = 5/2$ and $x = -5/2$.

2) If $x=-2$ is one root of $x^3-x^2-17x=22$, then find the other roots of equation.

(24)

Solution:

By the synthetic division

1	−1	−17	−22
0	−2	6	22
1	−3	−11	0

$Q = x^2 - 3x - 11 = 0$

$$x = \frac{3 \pm \sqrt{9+44}}{2} = \frac{3 \pm \sqrt{53}}{2}$$

Other two roots are $\frac{3+\sqrt{53}}{2}$ and $\frac{3-\sqrt{53}}{2}$

3) Find the real roots of $x^4 = 16$

Solution:

$x^4 - 16 = 0$

$(x^2)^2 - 4^2 = 0$

$(x^2-4)(x^2+4) = 0$

$(x+2)(x-2)(x^2+4) = 0$

$x = -2$, $x = 2$ other two roots are complex roots.

4) Solve $(2x+1)^2 - (3x+2)^2 = 0$

Solution:

$(2x+1)^2 - (3x+2)^2 = 0$

$|(2x+1) + (3x+2)|\,|(2x+1) - (3x+2)| = 0$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$$(5x+3)(x+1)=0$$

25

$x=-3/5$ and $x=-1$ are the required roots.

Ex: 2.7

1) Factorize : x^4+1. (Hint : try completing the square)

Solution:

$$x^4+1=0$$

$$(x^2)^2+2x^2+1-2x^2=0$$

$$(x^2+1)^2-(\sqrt{2}\,x)^2=0$$

$$(x^2+1+\sqrt{2}x)(x^2+1-\sqrt{2}x)=0$$

$$(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)=0$$

---

2) If x^2+x+1 is a factor of the polynomial $3x^3+8x^2+8x+a$ then find the value of a.

Solution:

$$\begin{array}{r|l} & 3x+5 \\ \hline x^2+x+1 & 3x^3+8x^2+8x+a \\ & \underline{\cancel{3x^3}+3x^2+3x} \\ & \quad 5x^2+5x+a \\ & \quad \underline{5x^2+5x+5} \\ & \qquad 0 \end{array}$$

$$\therefore a-5=0$$

$$a=5$$

Ex: 2.8

26

1) Find all values of x for which $\frac{x^3(x-1)}{x-2} > 0$

solution:

$$\frac{x^3(x-1)}{x-2} > 0$$

$$x^3(x-1) = 0$$

$$x = 0 \text{ or } x = 1 \text{ and } x \neq 2$$

INTERVAL	x^3	$(x-1)$	$x^3(x-1)$	$(x-2)$	$\frac{x^3(x-1)}{x-2}$
$(-\infty, 0)$	$-$	$-$	$+$	$-$	$-$
$(0, 1)$	$+$	$-$	$-$	$-$	$+$
$(1, 2)$	$+$	$+$	$+$	$-$	$-$
$(2, \infty)$	$+$	$+$	$+$	$+$	$+$

$$\frac{x^3(x-1)}{x-2} > 0 \text{ in } (0,1) \cup (2, \infty)$$

2) Find all values of x that satisfies the inequality $\frac{(2x-3)}{(x-2)(x-4)} < 0$

solution:

$$\frac{(2x-3)}{(x-2)(x-4)} < 0$$

$$(2x-3) < 0$$

$$\Rightarrow x = 3/2 \text{ and } x \neq 2 \text{ and } x \neq 4$$

INTERVAL	$(2x-3)$	$(x-2)(x-4)$	$\frac{(2x-3)}{(x-2)(x-4)}$
$(-\infty, 3/2)$	$-$	$+$	$-$
$(3/2, 2)$	$+$	$+$	$+$
$(2, 4)$	$+$	$-$	$+$
$(4, \infty)$	$+$	$+$	$+$

$$\frac{(2x-3)}{(x-2)(x-4)} < 0 \text{ in } \left(-\infty, \tfrac{3}{2}\right) \cup (2,4)$$

27

3) Solve $\dfrac{x^2-4}{x^2-2x-15} \leq 0$

Solution:

$$\frac{x^2-4}{x^2-2x-15} \leq 0$$

$$\frac{(x-2)(x+2)}{(x-5)(x+3)} \leq 0$$

$x = 2$ or $x = -2$ and $x \neq -3$, $x \neq 5$

INTERVALS	$(x-2)(x+2)$	$(x-5)(x+3)$	$\frac{x^2-4}{x^2-2x-15}$
$(-\infty,-3)$	+	+	+
$(-3,2]$	+	−	−
$[-2,2)$	−	−	+
$[2,5)$	+	−	−
$(5,\infty)$	+	+	+

$$\frac{x^2-4}{x^2-2x-15} \leq 0 \text{ in } (-3,-2] \cup [2,5)$$

Ex: 2.9

Resolve the following rational expressions into partial fraction.

1) $\dfrac{1}{x^2-a^2}$

Solution:

$$\frac{1}{x^2-a^2} = \frac{1}{(x-a)(x+a)}$$

28

Let $\frac{1}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a}$

$1 = A(x+a) + B(x-a)$

put $x = a$; $1 = A(2a) \Rightarrow A = 1/2a$

$x = -a$; $1 = B(-2a) \Rightarrow B = -1/2a$

The Partial fraction are $\frac{1}{2a(x-a)} - \frac{1}{2a(x+a)}$

2) $\frac{3x+1}{(x-2)(x+1)}$

Solution:

$\frac{3x+1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

$3x+1 = A(x+1) + B(x-2)$

Put $x=2$; $7 = A(3) \Rightarrow A = 7/3$

Put $x=-1$; $-2 = B(-3) \Rightarrow B = 2/3$

Thus,

$\frac{3x+1}{(x-2)(x+1)} = \frac{7/3}{x-2} + \frac{2/3}{x+1}$

(Or)

$\frac{7}{3(x-2)} + \frac{2}{3(x+1)}$

3) $\frac{x}{(x^2+1)(x-1)(x+2)}$

Solution:

Let $\frac{x}{(x^2+1)(x-1)(x+2)} = \frac{A}{(x-1)} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1}$

$\therefore x = A(x+2)(x^2+1) + B(x-1)(x^2+1) +$ 29

$(Cx+D)(x-1)(x+2).$

put $x = 1$; $1 = A(3)(2) \Rightarrow A = 1/6$

$x = -2$; $-2 = B(-3)(5) \Rightarrow B = 2/15$

Equating the coefficient of x^3 on both sides

$$0 = A + B + C$$

$$0 = 1/6 + 2/15 + C$$

$$\therefore C = -1/6 - 2/15 = \frac{-5-4}{30} = \frac{-9}{30}$$

$$\therefore C = -3/10$$

$$0 = A(2)(1) + B(-1)(1) + D(-1)(2)$$

$$0 = 2A - B - 2D$$

$$2D = 2A - B$$

$$= \frac{2}{6} - \frac{2}{15} = \frac{10-4}{30}$$

$$= \frac{6}{30} = \frac{2}{10} = \frac{1}{10}$$

$$\therefore \frac{x}{(x^2+1)(x-1)(x+2)} = \frac{1/6}{x-1} + \frac{2/15}{x+2} + \frac{-3/10\,x + 1/10}{x^2+1}$$

$$= \frac{1}{6(x-1)} + \frac{2}{15(x+2)} + \frac{1-3x}{10(x^2+1)}$$

4) $\frac{x}{(x-1)^3}$

30

Solution:

Let $\frac{x}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$

So that $x = A(x-1)^2 + B(x-1) + C$

put $x=1 \Rightarrow 1 = C$

Equate the coefficient of x^2 on both sides

$0 = A$

put $x=0 \Rightarrow 0 = A(-1)^2 + B(-1) + C$

$\Rightarrow 0 = 0 - B + 1 \Rightarrow B = 1$

$\therefore \frac{x}{(x-1)^3} = \frac{1}{(x-1)^2} + \frac{1}{(x-1)^3}$

---

5) $\frac{1}{x^4-1}$

Solution:

$\frac{1}{x^4-1} = \frac{1}{(x-1)(x+1)(x^2+1)}$

Let $\frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$

$1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) +$
$(Cx+D)(x-1)(x+1)$

put $x=1$; $1 = A(2)(2) \Rightarrow A = 1/4$

$x=-1$; $1 = B(-2)(2) \Rightarrow B = -1/4$

Equate the coefficient of x^3 on both sides

$0 = A + B + C$

$\Rightarrow 0 = 1/4 - 1/4 + C$

$\Rightarrow C=0$ (31)

put $x=0 \Rightarrow 1 = A-B-D$

$1 = 1/4 + 1/4 - D$

$D = 1/4 + 1/4 - 1 = -1/2$

Thus $\frac{1}{x^4-1} = \frac{1/4}{x-1} + \frac{-1/4}{x+1} + \frac{-1/2}{x^2+1}$

$= \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x^2+1)}$

6) $\frac{(x-1)^2}{x^3+x}$

solution :

$\frac{(x-1)^2}{x^3+x} = \frac{(x-1)^2}{x(x^2+1)}$

Let $\frac{(x-1)^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$\therefore (x-1)^2 = A(x^2+1) + (Bx+C)x$

put $x=0$; $(-1)^2 = A \Rightarrow A=1$

Equate the coefficient of $x^2 \Rightarrow 1 = A+B \Rightarrow B=0$

put $x=1$; $0 = A(2) + (B+C)$

$0 = 2+0+C$

$C = -2$

$\frac{(x-1)^2}{x^3+x} = 1/x + \frac{-2}{x^2+1}$

32

7) $\dfrac{x^2+x+1}{x^2-5x+6}$

Solution:

$\dfrac{x^2+x+1}{x^2-5x+6}$ It is improper fractions

on division.

$$x^2-5x+6 \overline{)\, x^2+x+1 \,(}\, 1$$
$$\underline{x^2-5x+6}$$
$$\underline{-6x-5}$$

$$\frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{6x-5}{x^2-5x+6}$$

consider

$$\frac{6x-5}{x^2-5x+6} = \frac{6x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$6x-5 = A(x-3) + B(x-2)$$

put $x=2 \Rightarrow 12-5 = A(-1) \Rightarrow A = -7$

$x=3 \Rightarrow 18-5 = B(1) \Rightarrow B = 13$

$$\therefore \frac{6x-5}{x^2-5x+6} = \frac{-7}{x-2} + \frac{13}{x-3}$$

Thus,

$$\frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{13}{x-3} - \frac{7}{x-2}$$

8) $\dfrac{x^3+2x+1}{x^2+5x+6}$

Solution :

It is improper fractions

$$
\begin{array}{r}
x^2+5x+6 \,\big)\, x^3+0x^2+2x+1 \,\big(\, x-5 \\
x^3+5x^2+6x \\
\hline
-5x^2-4x+1 \\
-5x^2-25x-30 \\
\hline
21x+31 \\
\hline
\end{array}
$$

$$\therefore \frac{x^3+2x+1}{x^2+5x+6} = x-5+\frac{21x+31}{x^2+5x+6}$$

Consider $\dfrac{21x+31}{x^2+5x+6} = \dfrac{21x+31}{(x+2)(x+3)}$

$$= \frac{A}{x+2}+\frac{B}{x+3}$$

$$21x+31 = A(x+3)+B(x+2)$$

put $x=-2 \Rightarrow -11 = A(1) \Rightarrow A=-11$

$x=-3 \Rightarrow -32 = B(-1) \Rightarrow B=32$

$$\frac{21x+31}{x^2+5x+0} = \frac{-11}{x+2}+\frac{32}{x+3}$$

Thus,

$$\frac{x^3+2x+1}{x^2+5x+6} = x-5+\frac{-11}{x+2}+\frac{32}{x+3}$$

34

9) $\dfrac{x+12}{(x+1)^2(x-2)}$

solution:

Let $\dfrac{x+12}{(x+1)^2(x-2)} = \dfrac{A}{x-2} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$x+12 = A(x+1)^2 + B(x-2)(x+1) + C(x-2)$

put $x=2 \Rightarrow 14 = A(9) \Rightarrow A = 14/9$

$x=-1 \Rightarrow 11 = C(-3) \Rightarrow C = -11/3$

Equate the coefficient of $x^2 \Rightarrow 0 = A+B$

$\Rightarrow B = -14/9$

Thus,

$$\frac{x+12}{(x+1)^2(x-2)} = \frac{14/9}{x-2} + \frac{-14/9}{x+1} + \frac{-11/3}{(x+1)^2}$$

$$\Rightarrow \frac{14}{9(x-2)} - \frac{14}{9(x+1)} - \frac{11}{3(x+1)^2}$$

---

10) $\dfrac{6x^2-x+1}{x^3+x^2+x+1}$

solution:

$$\frac{6x^2-x+1}{x^3+x^2+x+1} = \frac{6x^2-x+1}{(x+1)(x^2+1)}$$

$$= \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$6x^2-x+1 = A(x^2+1) + (Bx+C)(x+1)$

put $x=-1 \Rightarrow 6+1+1 = A(2) \Rightarrow A=4$

Equate the coefficient of $x^2 \Rightarrow 6 = A+B$ 35

$\Rightarrow 6 = 4 + B = B = 2$

put $x=0 \Rightarrow 1 = A+C \Rightarrow 1 = 4 + C = C = -3$

Thus,

$$\frac{6x^2 - x + 1}{x^3 + x^2 + x + 1} = \frac{4}{x+1} + \frac{2x-3}{x^2+1}$$

11) $\dfrac{2x^2+5x-11}{x^2+2x-3}$

solution:

It is improper fraction

$x^2-2x-3)\ 2x^2+5x-11\ (2$

$2x^2+4x-6$

$x-5$

$$\frac{2x^2+5x-11}{x^2+2x-3} = 2 + \frac{x-5}{x^2+2x-3}$$

Consider $\dfrac{x-5}{x^2+2x-3} = \dfrac{x-5}{(x+3)(x-1)}$

$$= \frac{A}{x+3} + \frac{B}{x-1}$$

$x-5 = A(x-1) + B(x+3)$

put $x=1 \Rightarrow -4 = B(4) \Rightarrow B = -1$

$x=-3 \Rightarrow -8 = A(-4) \Rightarrow A = 2$

$$\therefore \frac{x-5}{x^2+2x-3} = \frac{2}{x+3} + \frac{-1}{x-2}$$

36

Thus,

$$\frac{x-5}{x^2+2x-3} = 2 + \frac{2}{x+3} + \frac{-1}{x-1}$$

12) $\dfrac{7+x}{(1+x)(1+x^2)}$

Solution:

Let $\dfrac{7+x}{(1+x)(1+x^2)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}$

$x+7 = A(x^2+1) + (Bx+C)(x+1)$

put $x=-1$; $6 = A(2) \Rightarrow A = 3$

Equate the coefficient of x^2, $0 = A+B$

$\Rightarrow B = -3$

$x=0$, put $\Rightarrow 7 = A + C \Rightarrow C = 4$

Thus, $\dfrac{7+x}{(1+x)(1-x^2)} = \dfrac{3}{x+1} + \dfrac{-3x+4}{x^2+1}$

Ex : 2.10

1) Determine the region in the plane determined by the inequalities

1. $x \le 3y$, $x \ge y$

Solution:

Let $x = 3y \Rightarrow y = x/3$

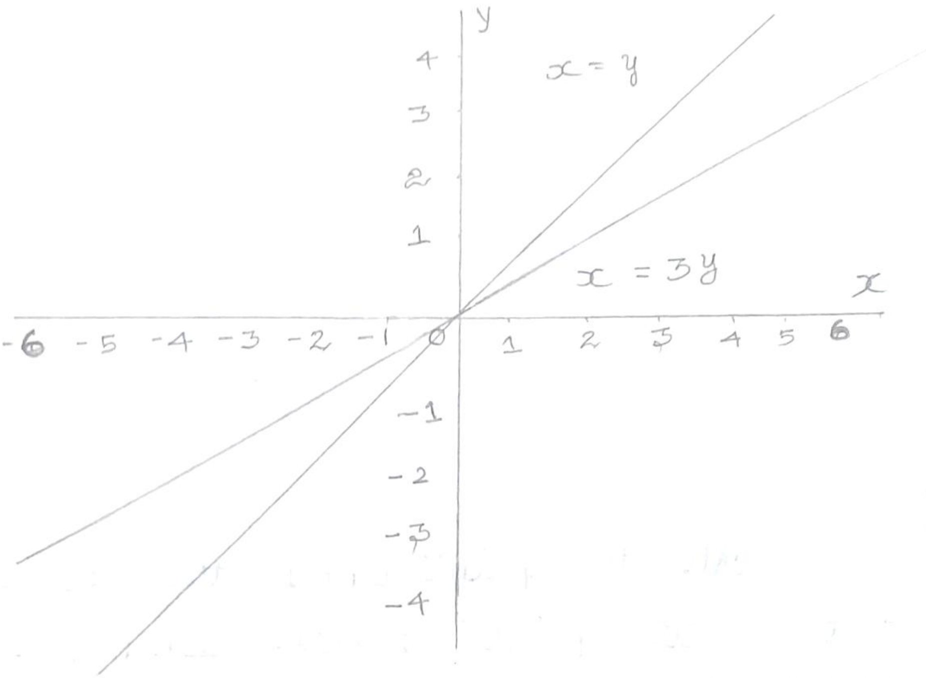
x	0	3
y	0	1

Joining the points (0,0) and (3,1) (37)
to get $x = 3y$.

Let $y = x$

x	0	1
y	0	1

Joining the points (0,0) and (1,1) to get $x = y$



All the point above the $x \leq 3y$ and all the points below $x \geq y$ is required region. Darkly shaded area will represents the solution set of the given linear inequalities

2) $y \geq 2x$, $-2x + 3y \leq 6$

Solution :

Let $y = 2x$

x	0	1
y	0	2

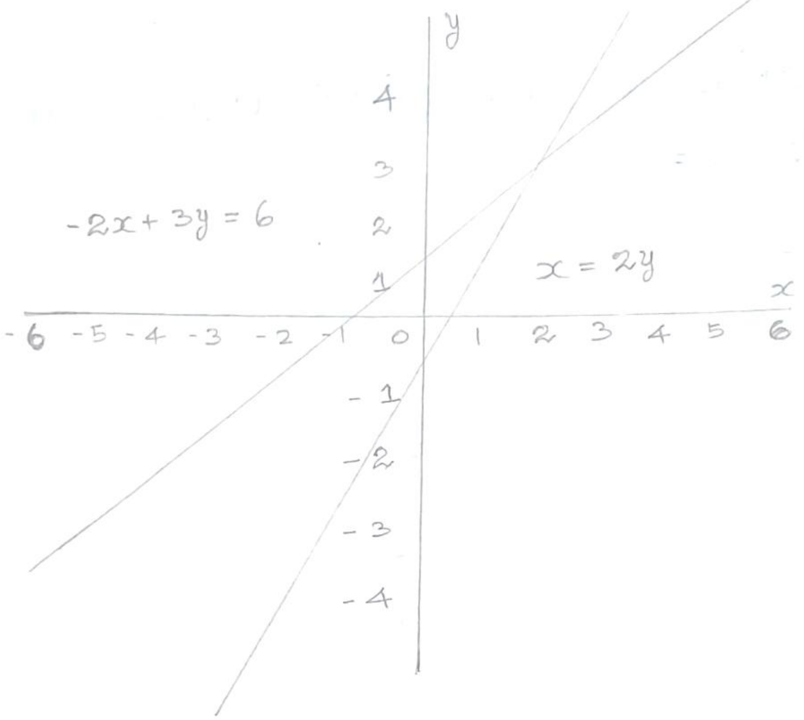
By joining (0, 0) and (1,2) we get $y = 2x$

Let $-2x+3y=6$

By joining (0,2) and (-3,0)

we get $-2x+3y=6$

x	0	-3
y	2	0



All the points above the $y \geq 2x$ and all the points below $-2x+3y \leq 6$ is required region. Darkly shaded area will represents the solution set of the given linear inequalities

3) $3x+5y \geq 45$, $x \geq 0$, $y \geq 0$

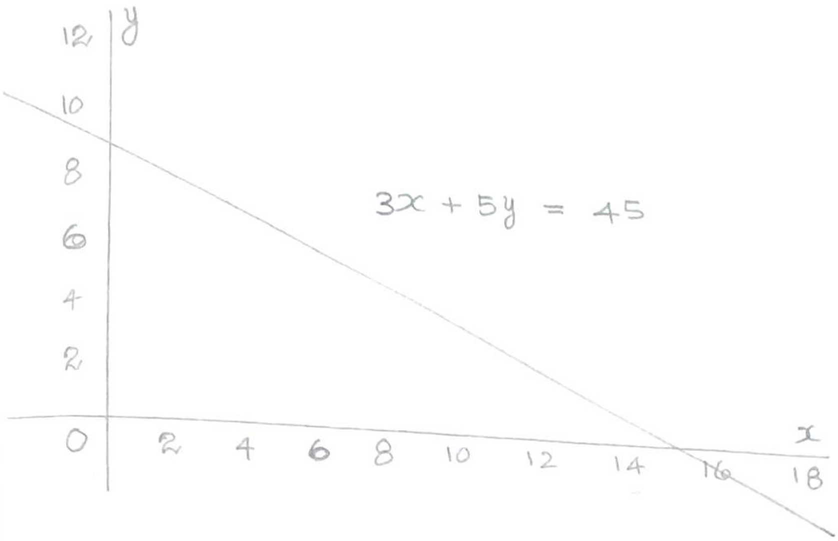
<u>Solution</u>

Let $3x+5y=45$ by joining (0,9) and (15,0) we get $3x+5y=45$.

x	0	15
y	9	0

$x=0$ represents y axis ; and $y=0$ represents x axis

39



All points bounded above $x=0$, $y=0$ and $3x+5y=45$ is required region. Darkly shaded area will represents the solution set of the given linear inequalities

---

4) $2x+3y \leq 35$, $x \geq 5$, $y \geq 2$

<u>solution</u>

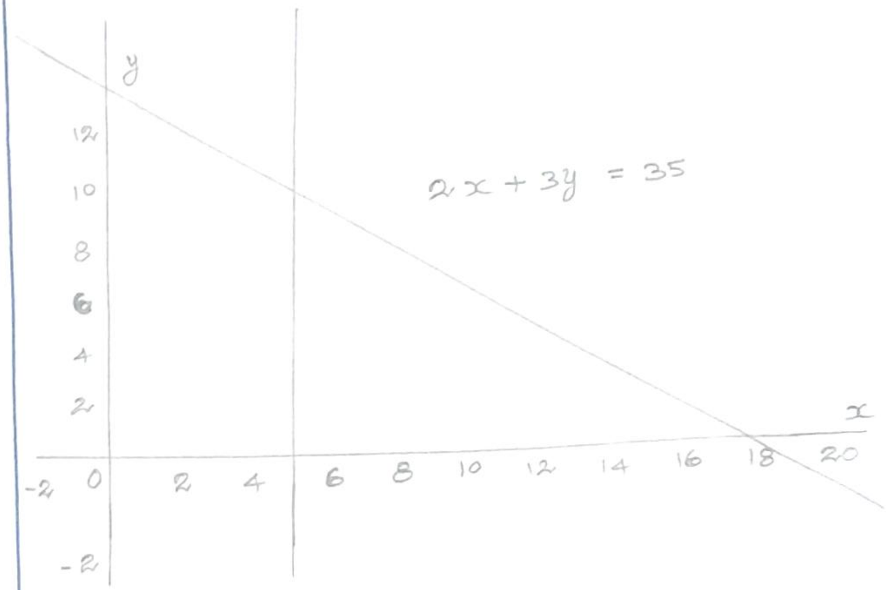
Let $2x + 3y = 35$

By joining (1,11) and (16, 1) we get $(2x + 3y = 35)$

x	1	16
y	11	1

Draw $x=5$ and $y=2$

40



All points bounded between $x=5$, $y=2$ and $2x+3y=35$ is required region. Darkly shaded area will represent the solution set of the given linear inequalities

5) $2x+3y \leq 6$, $x+4y \leq 4$, $x \geq 0$, $y \geq 0$

Solution:

Let $2x+3y=6$

By joining (0, 2) and (3, 0) we get

$2x+3y=6$.

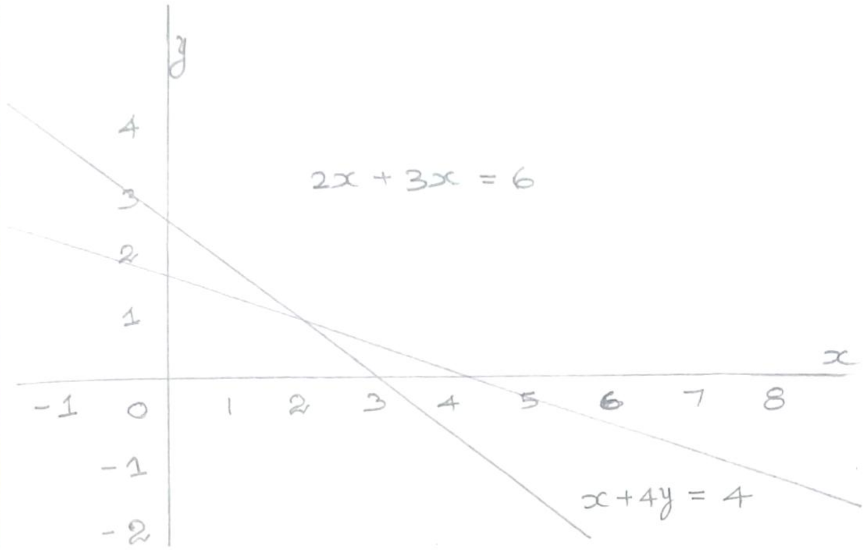
x	0	3
y	2	0

Let $x+4y=4$

By joining (0, 1) and (4, 0) we get

$x+4y=4$

x	0	4
y	1	0



All points bounded between $x=0$, $y=0$, $x+4y=4$ and $2x+3y=6$ is required region. Darkly shaded and area will represents the solution set of the given lines inequalities.

6) $x-2y\geq 0$, $2x-y\leq -2$, $x\geq 0$, $y\geq 0$

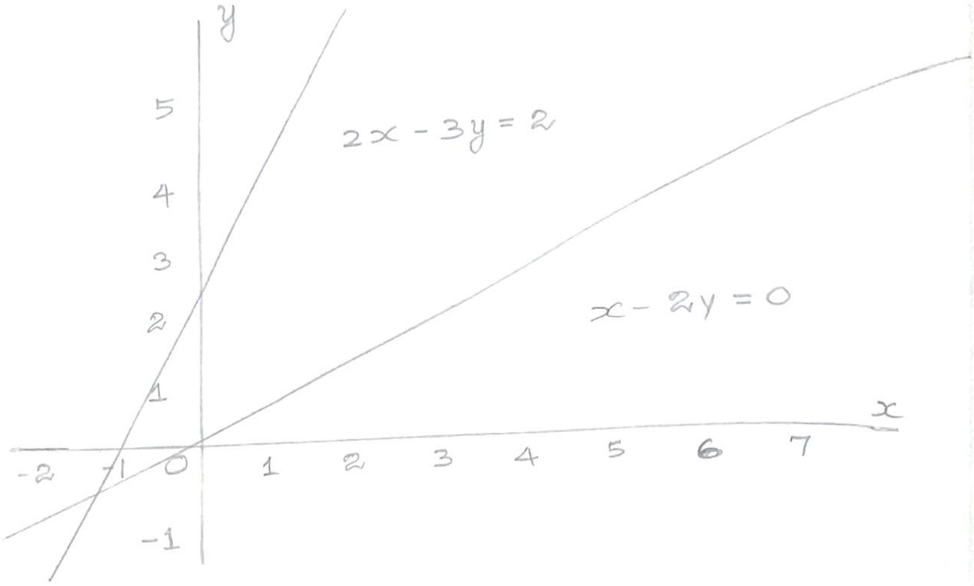
Solution:

$x-2y=0$; By joining $(0,0)$, $(2,1)$ and $(4,2)$ to get $x-2y=0$.

x	0	2	4
y	0	1	2

$2x-y=2$; by joining $(0,2)$, $(-1,0)$ we get $2x-y=2$

x	0	-1
y	2	0



No common solution region occur to the given inequalities $2x + 3y \leq 6$, $x + 4y \leq 4$, $x \geq 0$, $y \geq 0$.

---

7) $2x + 3y \geq 8$, $x + 2y \geq 8$, $x + y \leq 6$

Solution:

Let $2x + 3y = 8$

By joining (0,8) and (4,0) we get $2x + 3y = 8$

x	0	4
y	8	0

Let $x + 2y = 8$

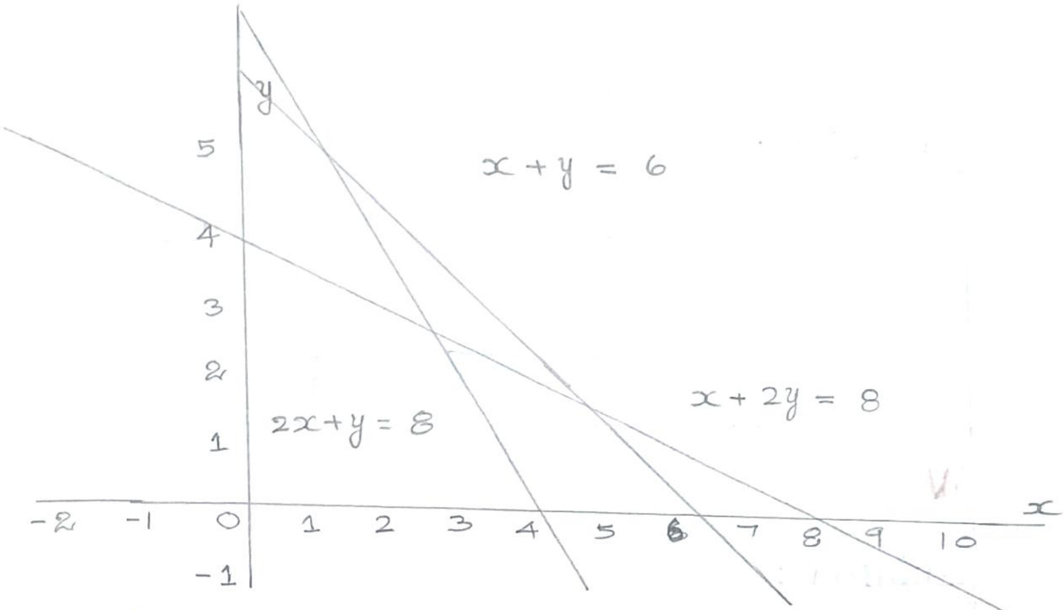
By joining (0,4) and (8,0) we get $x + 2y = 8$

x	0	8
y	4	0

Let $x+y=6$

By joining $(0,6)$ and $(6,0)$ we get $x+y=6$

x	0	6
y	6	0



All points bounded between $2x+3y=8$, $x+2y=8$, and $x+y=6$ is required region. Darkly shaded area will respresents the solution set of the given linear inequalities

---

Ex : 2.11

1) Simplify

i) $(125)^{2/3}$

Solution:

$$(125)^{2/3} = (5^3)^{2/3} = 5^2 = 25$$

44

2) ii) $16^{-3/4}$

Solution:

$$(16)^{-3/4} = (2^4)^{-3/4} = (2)^{-3} = \frac{1}{2^3}$$

$$= \frac{1}{8}$$

iii) $(-1000)^{-2/3}$

Solution:

$$(-1000)^{-2/3} = ((-10)^3)^{-2/3} = (-10)^{-2}$$

$$= \frac{1}{(-10)^2} = \frac{1}{100}$$

iv) $(3^{-6})^{1/3}$

Solution:

$$(3^{-6})^{1/3} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

v) $\frac{(27)^{-2/3}}{(27)^{-1/3}}$

Solution:

$$\frac{(27)^{-2/3}}{(27)^{-1/3}} = (27)^{-2/3+1/3} = (27)^{-1/3}$$

$$= \frac{1}{(27)^{1/3}} = \frac{1}{\sqrt[3]{27}}$$

$$= \frac{1}{3}$$

45

2) Evaluate : $\left(\left((256)^{-1/2}\right)^{-1/4}\right)^3$

Solution :

$$\left(\left((256)^{-1/2}\right)^{-1/4}\right)^3 = (256)^{-1/2 \times -1/4 \times 3}$$

$$= (2^8)^{3/8}$$

$$= 2^3 = 8$$

3) If $(x^{1/2} + x^{-1/2})^2 = 9/2$ then find the value of $(x^{1/2} - x^{-1/2})$ for $x > 1$

Solution :

We know that

$$(a-b)^2 = (a+b)^2 - 4ab$$

$$(x^{1/2} - x^{-1/2})^2 = (x^{1/2} + x^{-1/2})^2 - 4(x^{1/2} \cdot x^{-1/2})$$

$$= (x^{1/2} + x^{-1/2})^2 - 4\, x^{(1/2 + -1/2)}$$

$$= (x^{1/2} + x^{-1/2})^2 - 4$$

$$= \frac{9}{2} - 4 = \frac{1}{2}$$

$$x^{1/2} - x^{-1/2} = \frac{1}{\sqrt{2}} \text{ for } x > 1$$

4) Simplify and hence find the value of n :

$$\frac{3^{2n}\, 9^2\, 3^{-n}}{3^{3n}} = 27$$

Solution :

46

$$\frac{3^{2n}\, 9^{2}\, 3^{-n}}{3^{3n}} = 27$$

$$\Rightarrow \frac{3^{2n}\,(3^2)^2\, 3^{-n}}{3^{3n}} = 27$$

$$\Rightarrow \frac{3^{2n+4-n}}{3^{3n}} = 27$$

$$\Rightarrow 3^{n+4-3n} = 27$$

$$\Rightarrow 3^{4-2n} = 3^3$$

$$\Rightarrow 4-2n = 3$$

$$\Rightarrow -2n = -1 \Rightarrow n = 1/2$$

5) find the radius of the spherical tank whose volume is $32\pi/3$ units.

Solution:

volume of the sphere $\Rightarrow \frac{4}{3}\pi r^3 = \frac{32\pi}{3}$

$$r^3 = \frac{32\pi}{3} \times \frac{3}{4\pi}$$

$$r^3 = 8$$

$$r^3 = 2^3$$

$$r = 2$$

6) Simplify by rationalising the ~~demon~~ denominator $\frac{7+\sqrt{6}}{3-\sqrt{2}}$

solution:

$$\frac{7+\sqrt{6}}{3-\sqrt{2}} = \frac{7+\sqrt{6}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$= \frac{21+7\sqrt{2}+3\sqrt{6}+\sqrt{12}}{9-2}$$

$$= \frac{21+7\sqrt{2}+3\sqrt{6}+2\sqrt{3}}{7}$$

7) simplify $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

solution:

By rationalizing denominator of each terms we get

$$\frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{8-7} = \sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}+\sqrt{5}}{6-5} = \sqrt{6}+\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$$

48

$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

$$= (3+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+2)$$

$$= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$$= 3+2 = 5$$

8) If $x = \sqrt{2}+\sqrt{3}$ find $\frac{x^2+1}{x^2-2}$

Solution:

$$x = \sqrt{2}+\sqrt{3}$$

Now $x^2 = (\sqrt{2}+\sqrt{3})^2 = 2+2\sqrt{6}+3 = 5+2\sqrt{6}$

$$\frac{x^2+1}{x^2-2} = \frac{5+2\sqrt{6}+1}{5+2\sqrt{6}-2} = \frac{6+2\sqrt{6}}{3+2\sqrt{6}} \times \frac{3-2\sqrt{6}}{3-2\sqrt{6}}$$

$$= \frac{18-12\sqrt{6}+6\sqrt{6}-24}{9-24}$$

$$= \frac{-6-6\sqrt{6}}{-15} = \frac{-3(2+2\sqrt{6})}{-15}$$

$$= \frac{2+2\sqrt{6}}{5}$$

EX : 2.12

1) Let $b>0$ and $b \neq 1$. Express $y = b^x$ in logarithmic form. also state the domain and range of the logarithmic function

Solution:

49

Logarithmic form : $\log_b y = x$

Domain : $x \in R$

Range : $(0, \infty)$

2) Compute $\log_9 27 - \log_{27} 9$.

Solution:

$\log_9 27 = \log_9 3^3 \Rightarrow 3 \log_9 3$ (∵ power rule)

$= 3 \log_9 9^{1/2}$

$= \frac{3}{2} \log_9 9 = 3/2$

i.e., $\log_9 27 = \frac{3}{2}$

also $\log_{27} 9 = \frac{1}{\log_9 27}$ (∵ change of base rule)

$\log_{27} 9 = \frac{1}{3/2} = \frac{2}{3}$

i.e., $\log_{27} 9 = 2/3$

Now $\log_9 27 - \log_{27} 9 = 3/2 - 2/3 = \frac{9-4}{6}$

$= 5/6$

3) Solve $\log_8 x + \log_4 x + \log_2 x = 11$

Solution:

$\log_8 x + \log_4 x + \log_2 x = 11$

$$\frac{1}{\log_x 8} + \frac{1}{\log_x 4} + \frac{1}{\log_x 2} = 11$$

$$\frac{1}{\log_x 2^3} + \frac{1}{\log_x 2^2} + \frac{1}{\log_x 2} = 11$$

$$\frac{1}{3\log_x 2} + \frac{1}{2\log_x 2} + \frac{1}{\log_x 2} = 11$$

$$\frac{1}{\log_x 2}\left(\frac{1}{3} + \frac{1}{2} + 1\right) = 11$$

$$\frac{1}{\log_x 2}\left(\frac{11}{6}\right) = 11$$

$$\frac{1}{\log_x 2} = 11 \times \frac{6}{11}$$

$$\frac{1}{\log_x 2} = 6$$

$$\log_2 x = 6 \Rightarrow x = 2^6 = 64.$$

---

4) solve $\log_4 2^{8x} = 2\log_2 8$

solution:

$$\log_4 2^{8x} = 2\log_2 8$$

$$8x \log_4 2 = 2\log_2 2^3$$

$$8x \log_4 4^{1/2} = 2^3 \log_2 2$$

$$\frac{8x}{2} \log_4 4 = 2^3 \log_2 2$$

51

$$4x = 2^3$$

$$x = 8/4$$

$$x = 2$$

5) If $a^2 + b^2 = 7ab$. Show that $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$.

solution:

Given $a^2 + b^2 = 7ab$

Adding $2ab$ on both sides

$$a^2 + b^2 + 2ab = 7ab + 2ab$$

$$(a+b)^2 = 9ab$$

$$\frac{(a+b)^2}{9} = ab$$

$$\left(\frac{a+b}{3}\right)^2 = ab$$

$$\frac{a+b}{3} = (ab)^{1/2}$$

take log on both sides.

$$\log\left(\frac{a+b}{3}\right) = \log (ab)^{1/2}$$

$$\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log ab)$$

$$\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$$

Hence proved

6) Prove that $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab} = 0$

Solution:

$$\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab}$$

$$= \log \left(\frac{a^2}{bc} \cdot \frac{b^2}{ca} \cdot \frac{c^2}{ab} \right)$$

$$= \log \left(\frac{a^2 b^2 c^2}{a^2 b^2 c^2} \right) = \log 1 = 0$$

7) Prove that $\log 2 + 16 \log 16/15 + 12 \log 25/24 + 7 \log 81/80 = 1$

Solution:

$$L.H.S: \log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80}$$

$$= \log 2 + \log \left(\frac{16}{15}\right)^{16} + \log \left(\frac{25}{24}\right)^{12} + \log \left(\frac{81}{80}\right)^{7}$$

$$= \log 2 + \log \left[\left(\frac{16}{15}\right)^{16} \cdot \left(\frac{25}{24}\right)^{12} \cdot \left(\frac{81}{80}\right)^{7} \right]$$

$$= \log 2 + \log \left[\left(\frac{2^4}{5 \cdot 3}\right)^{16} \cdot \left(\frac{5^2}{3 \cdot 2^3}\right)^{12} \cdot \left(\frac{3^4}{5 \cdot 2^4}\right)^{7} \right]$$

$$= \log 2 + \log \left[\frac{2^{64} \cdot 5^{24} \cdot 3^{28}}{5^{16} \cdot 3^{16} \cdot 3^{16} \cdot 2^{36} \cdot 5^{7} \cdot 2^{28}} \right]$$

$$= \log 2 + \log 5 = \log (2 \times 5)$$

$= \log 10 = \log_{10} 10 = 1$. proved (53)

8) Prove $\log_{a^2} a \ \log_{b^2} b \ \log_{c^2} c = 1/8$

solution:

L.H.S $\Rightarrow \log_{a^2} a \ \log_{b^2} b \ \log_{c^2} c$

$$= \frac{1}{\log_a a^2} \cdot \frac{1}{\log_b b^2} \cdot \frac{1}{\log_c c^2}$$

$$= \frac{1}{2\log_a a} \cdot \frac{1}{2\log_b b} \cdot \frac{1}{2\log_c c}$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} \ (R.H.S)$$

Hence proved.

9) Prove $\log a + \log a^2 + \log a^3 + \cdots + \log a^n$
$= \frac{n(n+1)}{2} \log a$

solution:

L.H.S $\Rightarrow \log a + \log a^2 + \log a^3 + \cdots + \log a^n$

$\Rightarrow \log a + 2\log a + 3\log a + \cdots + n\log a$

$\Rightarrow \log a \, [1 + 2 + 3 + \cdots + n]$

$\Rightarrow \frac{n(n+1)}{2} \log a \ [R.H.S]$

Hence proved.

54

10) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ then prove that $xyz = 1$

Solution:

Let $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = K$

$\frac{\log x}{y-z} = K \Rightarrow \log x = K(y-z)$

$\Rightarrow \log x = Ky - KZ$

$\frac{\log y}{\log z-x} = K \Rightarrow \log y = K(Z-x)$

$\Rightarrow \log y = KZ - Kx$

$\frac{\log z}{x-y} = K \Rightarrow \log z = K(x-y)$

$\Rightarrow \log z = Kx - Ky$

Now,

$\log x + \log y + \log z = Ky - KZ + KZ - Kx + Kx - Ky$

$\log xyz = 0 \Rightarrow$ i.e., $xyz = 1$ $(\because \log 1 = 0)$

---

11) solve $\log_2 x - 3 \log_{1/2} x = 6$

Solution:

Let $\log_2 x = t$

Now $\log_{1/2} x = \frac{1}{\log_x 1/2}$

55

$$= \frac{1}{\log_x 1 - \log_x 2}$$

$$= \frac{1}{\log_x 1 - \frac{1}{\log_2 x}}$$

$$= \frac{1}{0 - 1/t} = -t$$

i.e., $\log_{1/2} x = -t$.

Given: $\log_2 x - 3\log_{1/2} x = 6$

$$t - 3(-t) = 6$$

$$4t = 6$$

$$t = 6/4$$

$$t = 3/2$$

$$\log_2 x = 3/2$$

$$x = 2^{3/2} = 8^{1/2} = \sqrt{8} = 2\sqrt{2}$$

$$x = 2\sqrt{2}$$

12) solve $\log_{5-x}(x^2 - 6x + 65) = 2$

Solution:

$$\log_{5-x}(x^2 - 6x + 65) = 2$$

$$x^2 - 6x + 65 = (5-x)^2$$

$$x^2 - 6x + 65 = 25 - 10x + x^2$$

$$4x = -40$$

$$x = -10$$