Homework 1 Questions

1. Weight of a standard volume of "nitrogen" obtained from different sources.

Origin	NO	NO	NO	NO	Air	Air	Air	Air
Weight	2.30143	2.29816	2.30182	2.2989	2.31017	2.30986	2.31017	2.3101

Origin	N_2O	N_2O	NH_4NO_2	NH_4NO_2	Air	Air	Air
Weight	2.31001	2.29889	2.2994	2.29849	2.31024	2.3103	2.31028

Estimate the percentage of "Argon" in air by weight using the above dataset. Assume that the gas obtained is only a mixture of Nitrogen and Argon and no other gas is present.

2. Plot the box and whisker plot of standard normal distribution and exponential distribution with lambda=1.

3. If data is distributed as standard normal, what is the percentage of points would be considered outliers using the 1.5 IQR rule discussed in class.

4. Table 1 contains the expenditure made by a company from May 1,2017 to May 20, 2017. Prepare the 5-number summaries, box-and-whiskers plot and percentiles. Also prepare a histogram. Are there any outliers in this data?

Algorithm 1: Method to calculate percentiles

1. Arrange n number of data points in ascending order: $x_1, x_2, x_3, \dots, x_n$
2. Calculate the rank r for the percentile p you want to find: $r = \frac{p}{100}(n-1)+1$
3. If r is an integer, then the data value at location r, x_r, is the percentile p: $p = x_r$
4. If r is not an integer, p is interpolated using r_i, the integer part of r, and r_f, the fractional part of r: $p = x_{r_i} + r_f(x_{r_{i+1}} - x_{r_i})$

Date	Expense
5/1/2017	1199436
5/2/2017	1045515
5/3/2017	586111
5/4/2017	856601
5/5/2017	793775
5/6/2017	606535
5/7/2017	1112763
5/8/2017	1121218
5/9/2017	813844
5/10/2017	903343

5/11/2017	863465
5/12/2017	639224
5/13/2017	1030389
5/14/2017	1132645
5/15/2017	1018672
5/16/2017	1726870
5/17/2017	1378430
5/18/2017	532950
5/19/2017	828238
5/20/2017	823948

Table 1: Daily Expenses of a Company

5. Find the number of independent columns (rank) of the following matrices:

a. $\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 10 \\ 1 & 2 & 7 \\ 0 & 1 & 3 \end{bmatrix}$ b. $\boldsymbol{B} = \begin{bmatrix} 1 & 4 & -7 \\ 2 & 5 & -8 \\ 3 & 6 & -9 \end{bmatrix}$

$c.\boldsymbol{C} = \begin{bmatrix} 2 & 6 & 4 \\ 1 & 3 & 2 \\ 3 & 9 & 6 \\ 2 & 6 & -4 \end{bmatrix}$ d. **D**= $\begin{bmatrix} 1 & 2 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

6. Linear combination of two vectors **x**= (1, 2, 0) and **y**= (0, 1, 2) produce a plane in $\mathbb{R}^3$
   a. Find a vector **z** that is orthogonal to this plane.
   b. Find a vector **u** that is not in the plane. Show that $\boldsymbol{u}^T\boldsymbol{z} \neq 0$
7. **W**= $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix}$. Compute $\boldsymbol{W}^T\boldsymbol{W}$ and $\boldsymbol{W}^{-1}$
8. Find the eigenvalues and eigenvectors for $\boldsymbol{A} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$. As k increases, what will be the value of $\boldsymbol{A}^k$?
9. For what numbers e and f, are the matrices A and B positive definite?
   $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e & 1 \\ 1 & 1 & e \end{bmatrix}$ $\boldsymbol{B} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & d & 2 \\ 2 & 1 & 3 \end{bmatrix}$
10. Suppose S is a $n \times n$ symmetric matrix, i.e., $\boldsymbol{S} = \boldsymbol{S}^T$ with eigenvalues $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_n$ and orthonormal eigenvectors $v_1 \ldots v_n$.
    Then show that the maximum value of $R(x) = \frac{x^T S x}{x^T x} = \lambda_1$