Assignment 10 : Spectra of Non-periodic Signals

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Introduction

In this assignment, we explore ways of finding the Fourier Transform of non-periodic signals using a hamming window. The given signal is multiplied by the hamming window, and then the DFT is computed.

1 Given Examples

This section is all about working through the given examples in the document. We first start with the DFT of $\sin(\sqrt{2}t)$. The signal is not periodic with the period 2π. When the signal between $-\pi$ and π is repeated (Figure 1), we see that it is not $\sin(\sqrt{2}t)$. There are jumps at the ends of each period. We use the hamming window to reduce these jumps so that the DFT is cleaner. If we repeat the signal after multiplying the window, we see that the jump has reduced (Figure 2). The following code calculates the DFT of the signal with window. The plot is given in Figure 3.

```
t = np.linspace(-4*np.pi, 4*np.pi, 257)[:-1]
dt = t[1] - t[0]
fmax = 1/dt
n = np.arange(256)
wnd = np.fft.fftshift(0.54 + 0.46*np.cos(2*np.pi*n/255))
y = np.sin(np.sqrt(2)*t)*wnd
y[0] = 0
y = np.fft.fftshift(y)
Y = np.fft.fftshift(np.fft.fft(y))/256.0
w = np.linspace(-np.pi*fmax, np.pi*fmax, 257)[:-1]
```

2 Finding the spectrum of $\cos^3(\omega_0 t)$

We find the spectrum of the signal $\cos^3(\omega_0 t)$ for $\omega = 0.86$ with and without a hamming window. The spectrum for the signal without a hamming window is shown in Figure 4 and the spectrum with the hamming window is given in Figure 5. We see that the spectrum with hamming window only has significant magnitude around the frequencies present, whereas the one without the window has a slowly decaying magnitude.

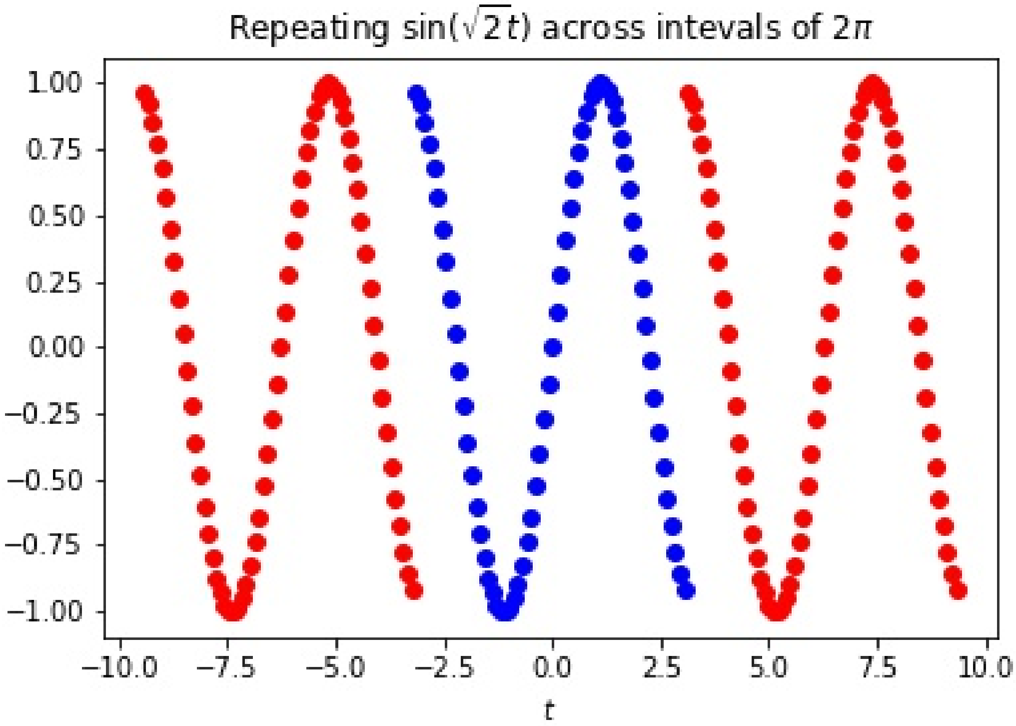

Figure 1: $\sin\left(\sqrt{2}t\right)$ repeated across intervals of 2π

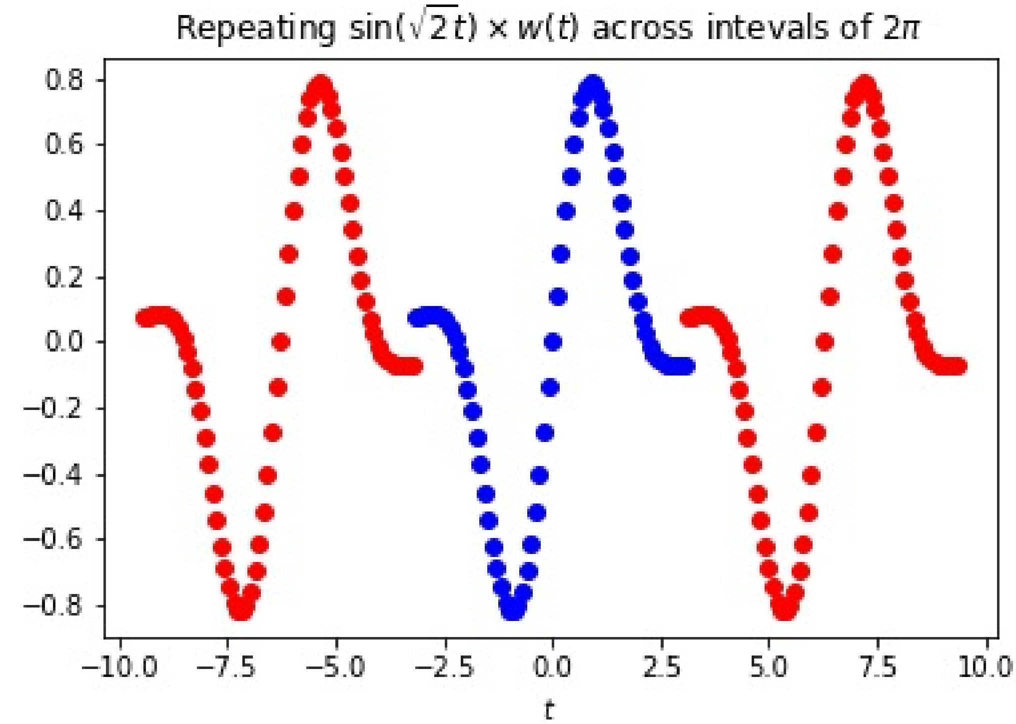

Figure 2: $\sin\left(\sqrt{2}t\right) \times w(t)$ repeated across intervals of 2π

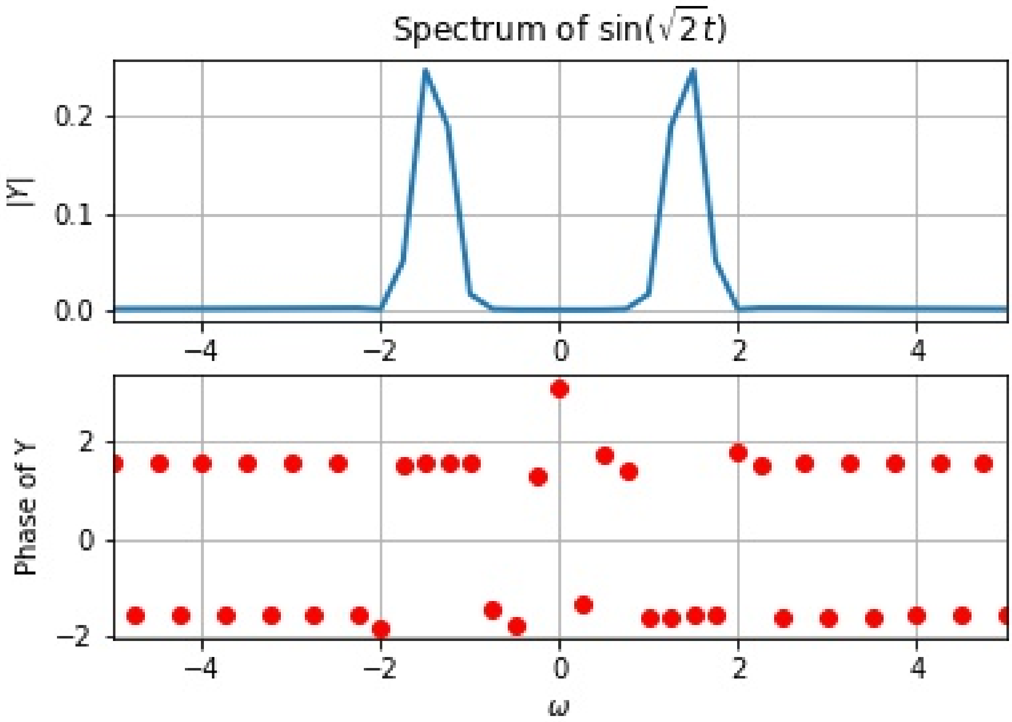

Figure 3: Spectrum of $\sin\left(\sqrt{2}t\right)$

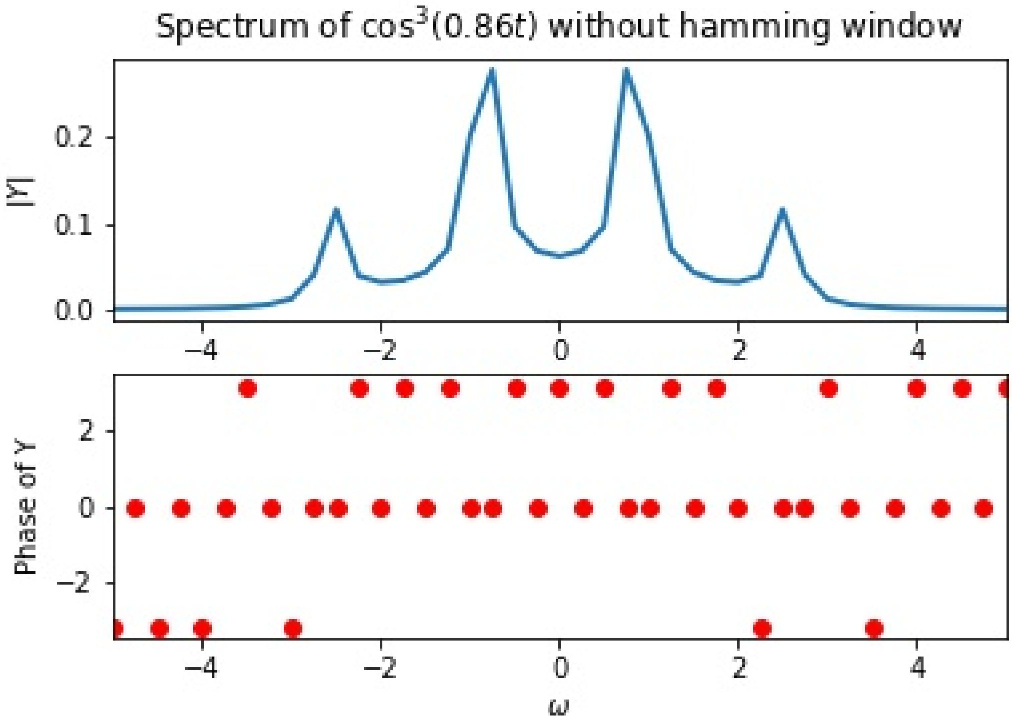

Figure 4: Spectrum of $\cos^3(0.86t)$ without hamming window

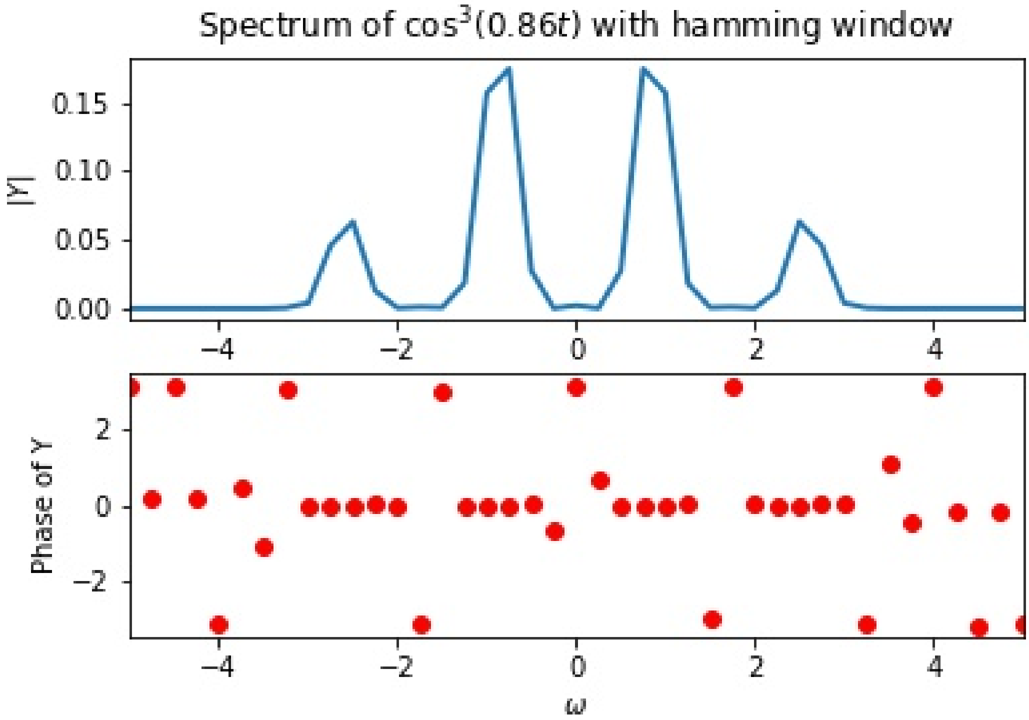

Figure 5: Spectrum of $\cos^3(0.86t)$ with hamming window

3 Estimating the frequency and phase of given sinusoid

In this section, we try to estimate the frequency and the phase of the sinusoid by finding its DFT. A 128 element vector is given with the t going from $-\pi$ to π. For the signal $\cos(1.5t + 0.7)$, the DFT plot is shown in Figure 6. The frequency is estimated by taking the weighted mean across the magnitude spectrum. Since $0.5 < \omega < 1.5$, it will be sufficient to take the magnitude of first few samples, so that the error is reduced. Once the frequency is found, the phase value at this frequency will be the phase shift of the sinusoid. This is true, since for a cosine function the phase at this frequency will be zero and any additional phase will just be added. To the same signal, white Gaussian noise is added and the estimate is found. The outputs for the signal $\cos(1.5t + 0.7)$ with and without noise are:

```
omega = 1.4625669131078334
delta = 0.6918203890706528
omega_noise = 1.485597255036664
delta_noise = 0.6918203890706528
```

4 DFT of the Chirped Signal

In this section, the DFT of the Chirped Signal is computed. The Chirped Signal is $\cos\big(16(1.5 + \frac{t}{2\pi})t\big)$. The frequency of this signal continuously varies from 16 to 32 rad/s. The plot of the spectrum without the hamming window is given in Figure 7 and the one with the hamming window is given in Figure 8.

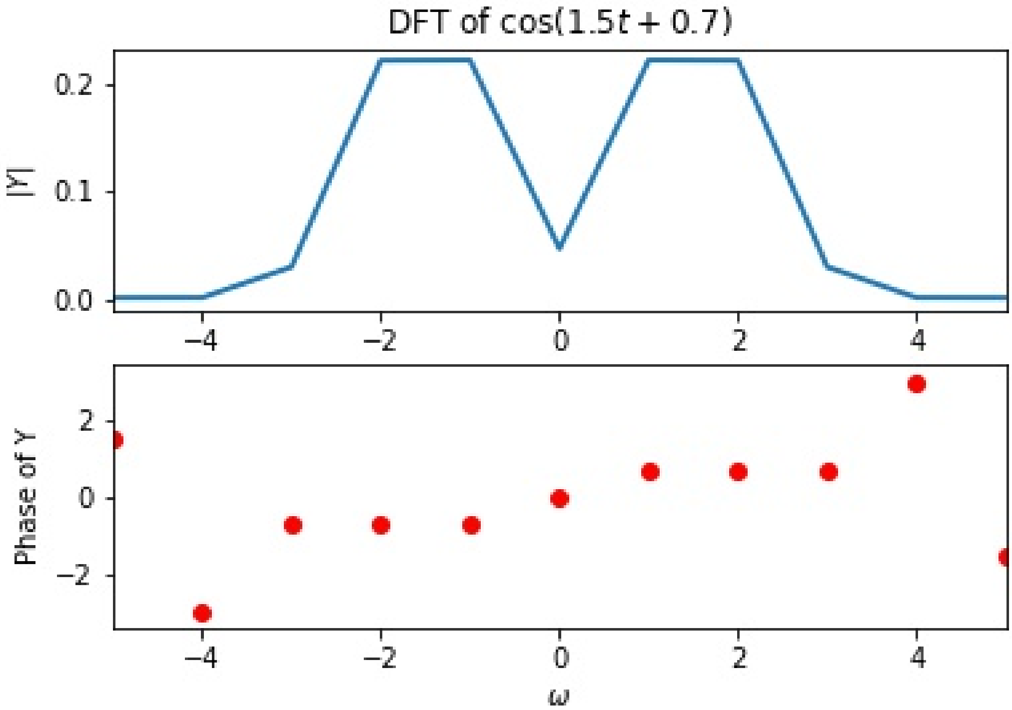

Figure 6: Spectrum of $\cos(1.5t + 0.7)$ with hamming window

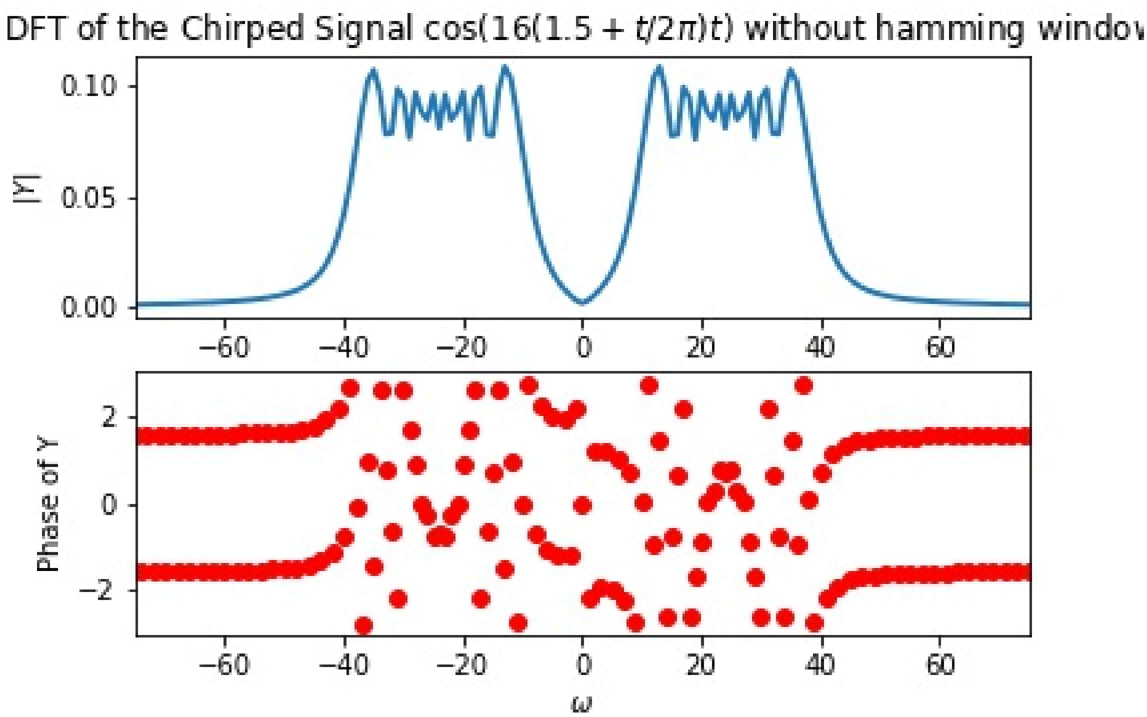

Figure 7: Spectrum of $\cos\left(16(1.5 + \frac{t}{2\pi})t\right)$ without hamming window

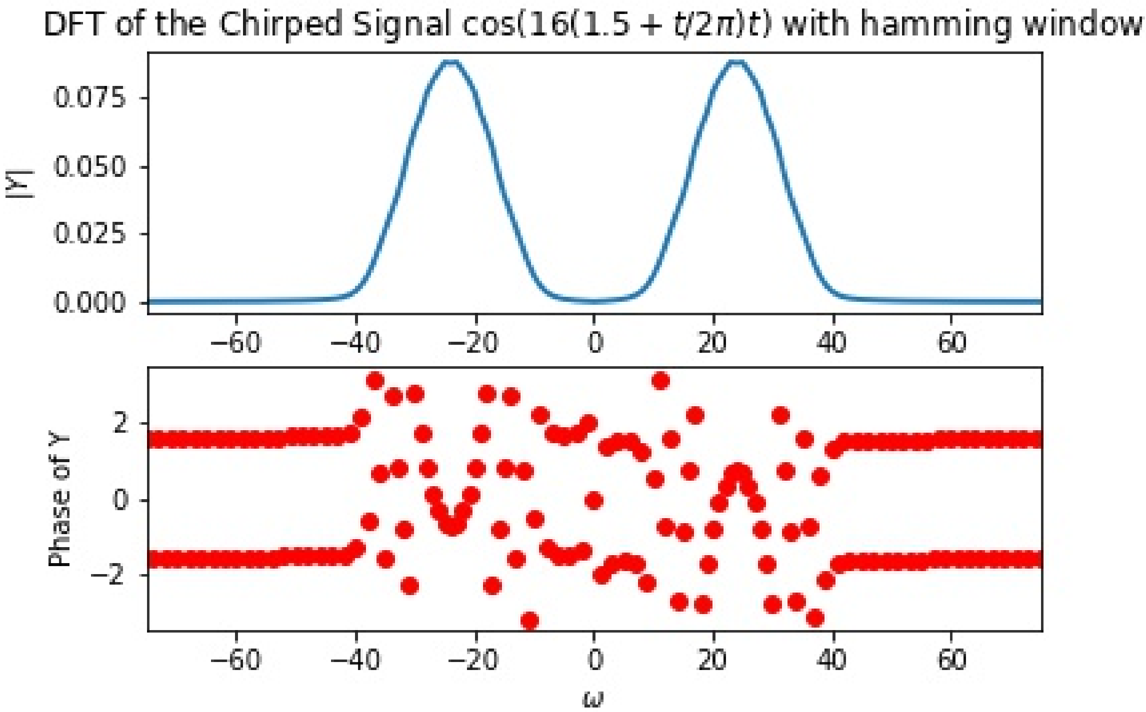

Figure 8: Spectrum of $\cos\left(16(1.5 + \frac{t}{2\pi})t\right)$ with hamming window

5 Time evolution of the DFT

We now split the time vector to 16 columns of 64 entries each. The DFT of the Chirped Signal is found for each of these vectors and the spectra are plot as a function of both frequency and time. This plot shows how the frequency varies with time. The plot of the spectrum without the hamming window is given in Figure 9 and the one with the hamming window is given in Figure 10.

Conclusion

We have seen how the frequencies can be estimated from the DFT of a non-periodic signal by making use of a hamming window.

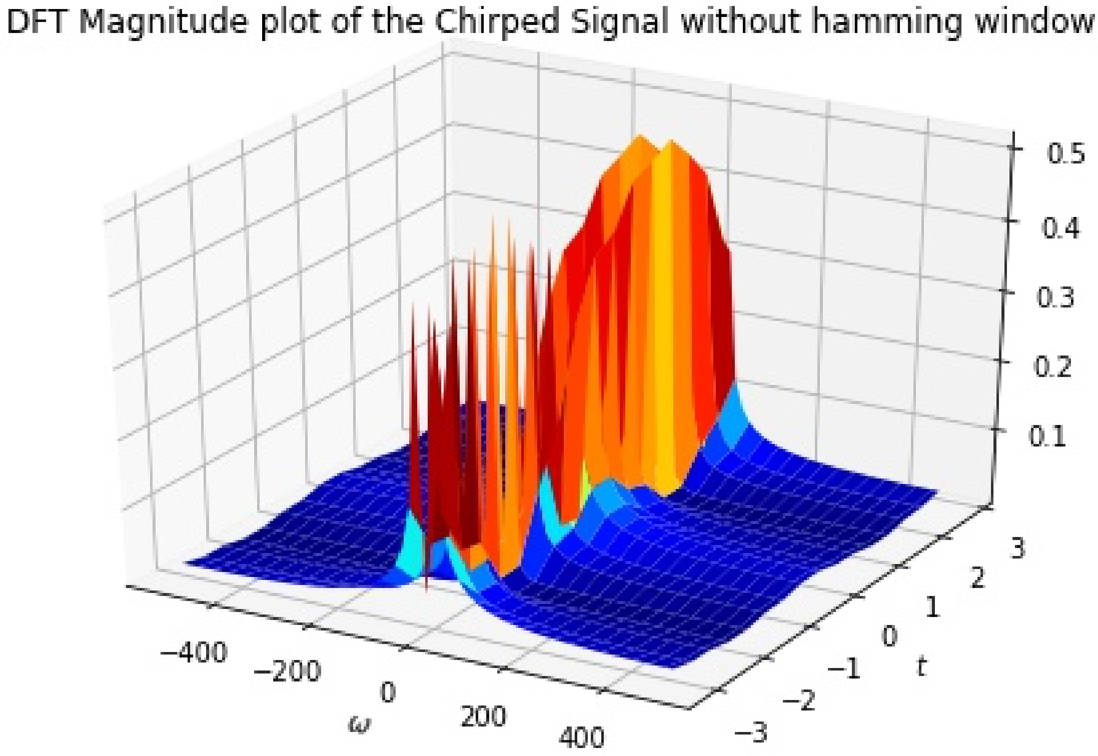

Figure 9: DFT Magnitude plot of Chirped Signal without hamming window

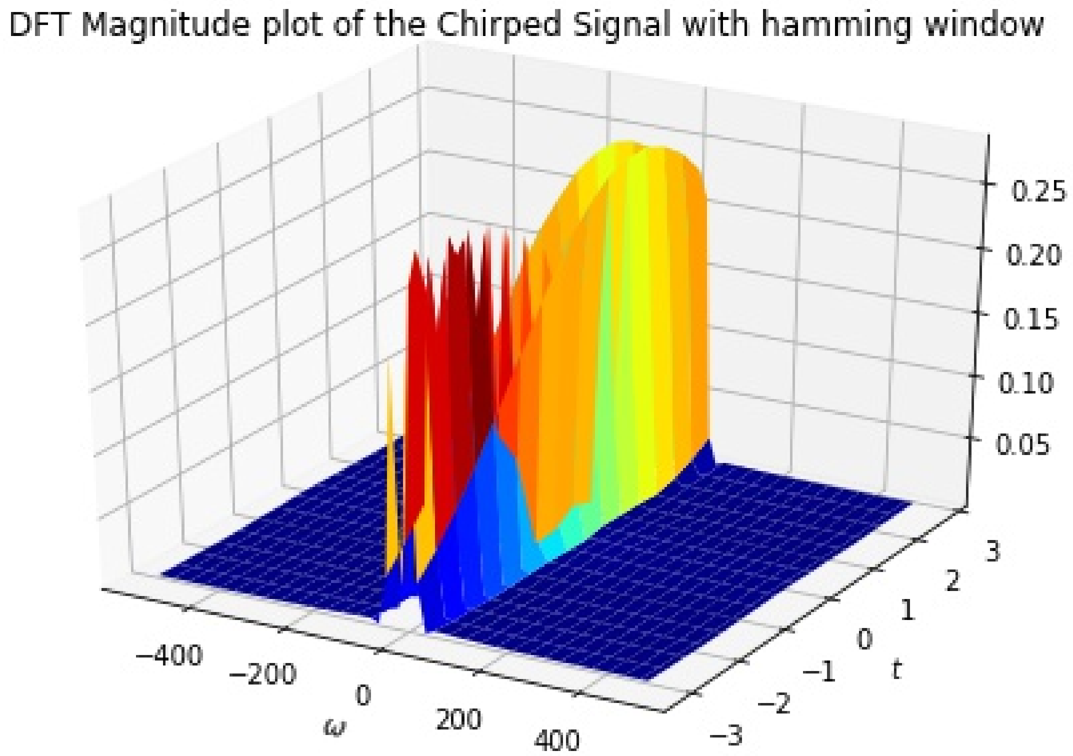

Figure 10: DFT Magnitude plot of Chirped Signal with hamming window