Assignment 7 : The Laplace Transform

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Introduction

In this assignment, we make use of the signal toolbox in python to simulate circuits

1 Question 1

The given equation is

$$\ddot{x} + 2.25x = f(t)$$

with initial conditions as $\dot{x}(0) = 0, x(0) = 0$.
Taking the Laplace transform of the equation, we get

$$X(s) = \frac{F(s)}{s^2 + 2.25}$$

$$\Rightarrow X(s) = \frac{s + 0.5}{(s^2 + 2.25)((s + 0.5)^2 + 2.25)}$$

$$\Rightarrow X(s) = \frac{s + 0.5}{s^4 + s^3 + 4.75s^2 + 2.25s + 5.625}$$

Using the following piece of code, $x(t)$ is computed

```
X = sp.lti([1, 0.5], [1, 1, 4.75, 2.25, 5.625])
t, x = sp.impulse(X, None, pl.linspace(0, 50, 501))
```

The plot of $x(t)$ is shown in Figure 1.

2 Question 2

In this question, the decay in $f(t)$ is reduced to 0.05. This results in

$$X(s) = \frac{s + 0.05}{s^4 + 0.1s^3 + 4.5025s^2 + 0.225s + 5.068125}$$

Again $x(t)$ is computed and plotted. The plot is shown in Figure 2. We see that since the decay is a lot less, it settles slowly.

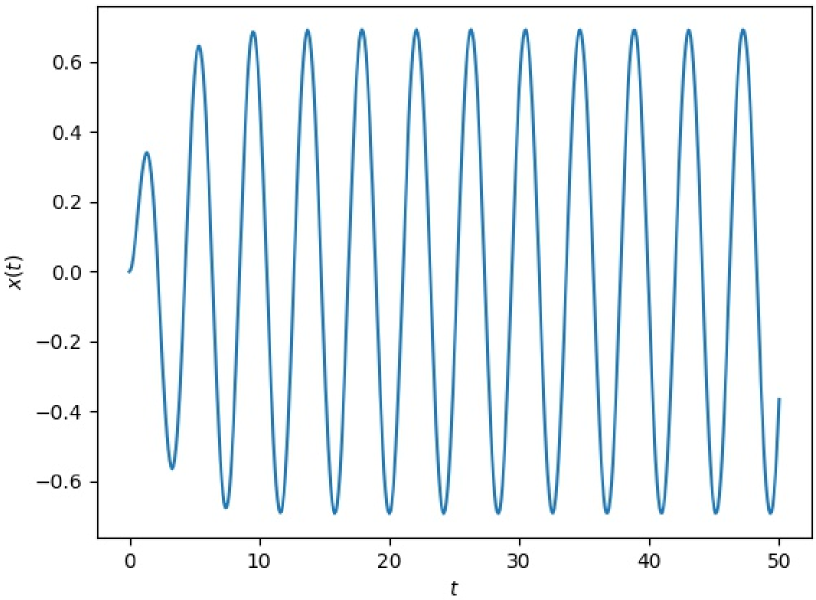

Figure 1: Plot of $x(t)$

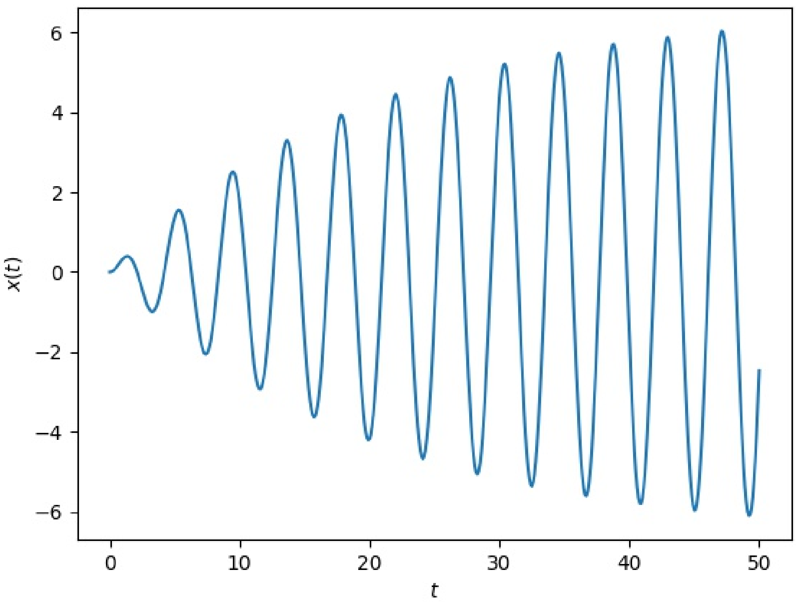

Figure 2: Plot of $x(t)$ with 0.05 decay in $f(t)$

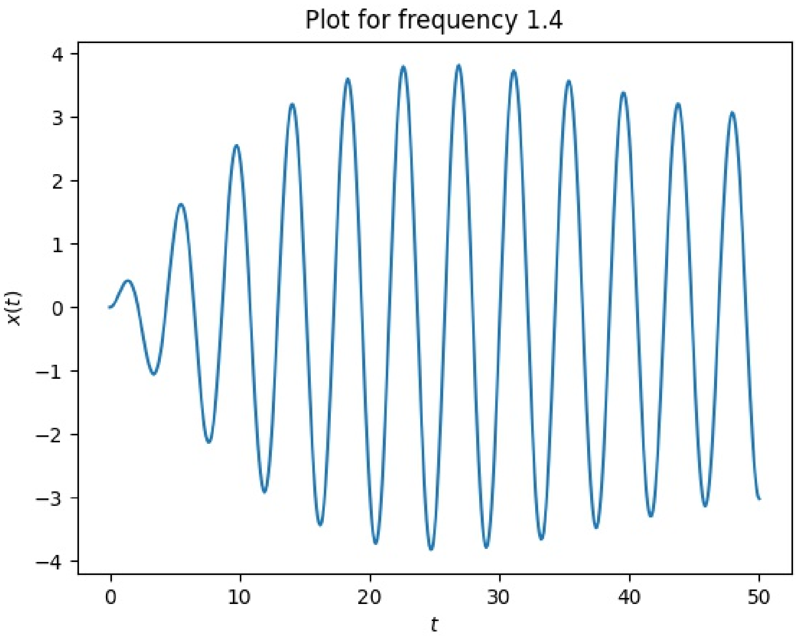

Figure 3: $x(t)$ for frequency 1.4

3 Question 3

In this question, we find the output $x(t)$ for different values of the frequency in $f(t)$. From the plots, we find that the amplitude is maximum for frequency of 1.5 because of resonance (1.5 is the natural frequency of the system).

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 2.25}$$

4 Question 4

The coupled equations are

$$\ddot{x} + (x - y) = 0$$

$$\ddot{y} + 2(y - x) = 0$$

Taking Laplace Transform with initial conditions $x(0) = 0, \dot{x}(0) = \dot{y}(0) = y(0) = 0$, we get the following relations

$$X(s) = \frac{s^2 + 2}{s^3 + 3s}$$

$$Y(s) = \frac{2}{s^3 + 3s}$$

The curves $x(t)$ and $y(t)$ are plotted in Figure 8. We see that they are out of phase.

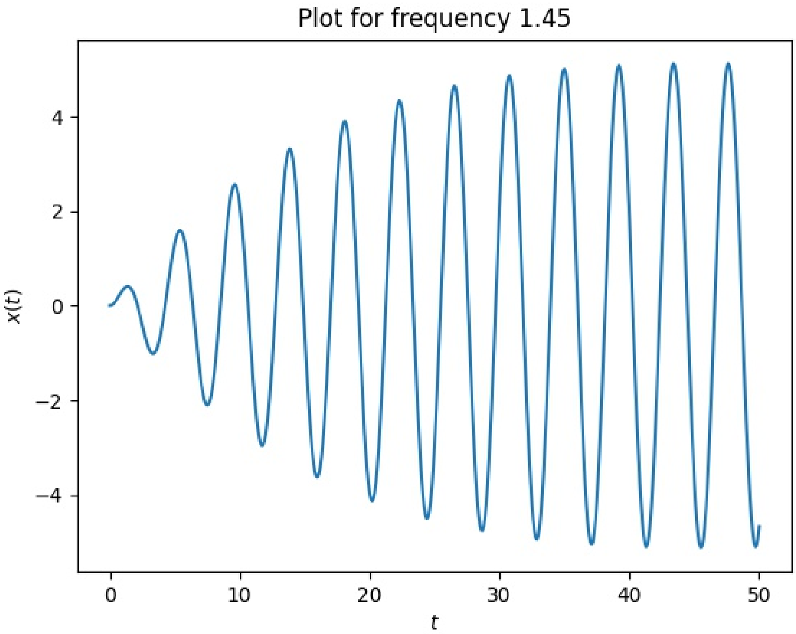

Figure 4: $x(t)$ for frequency 1.45

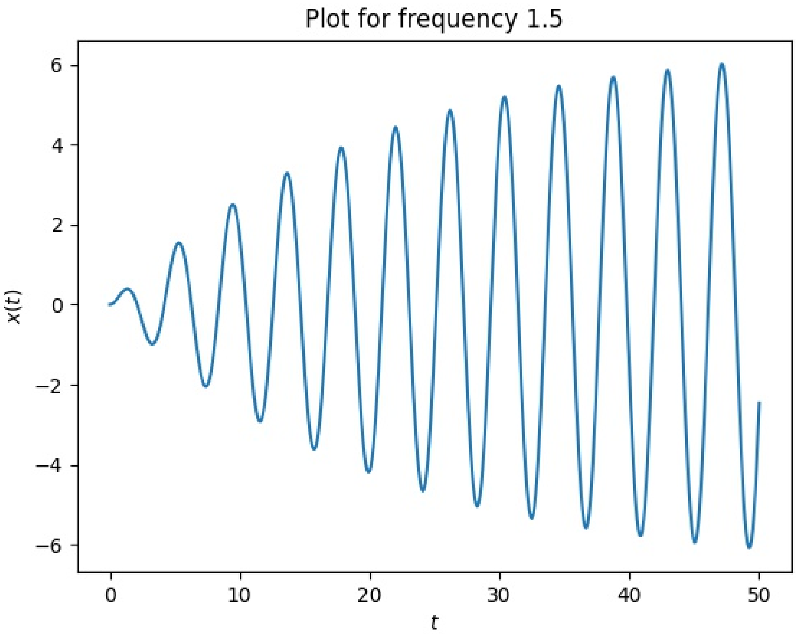

Figure 5: $x(t)$ for frequency 1.5

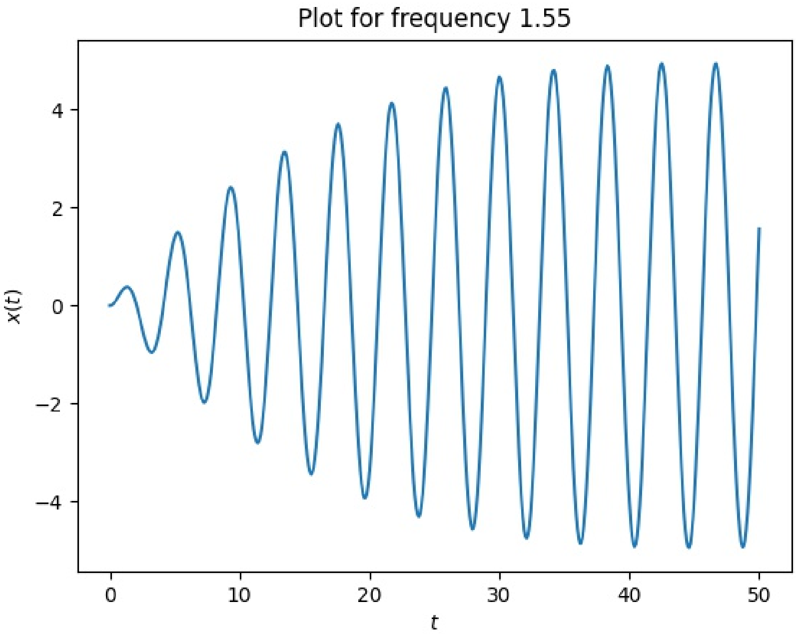

Figure 6: $x(t)$ for frequency 1.55

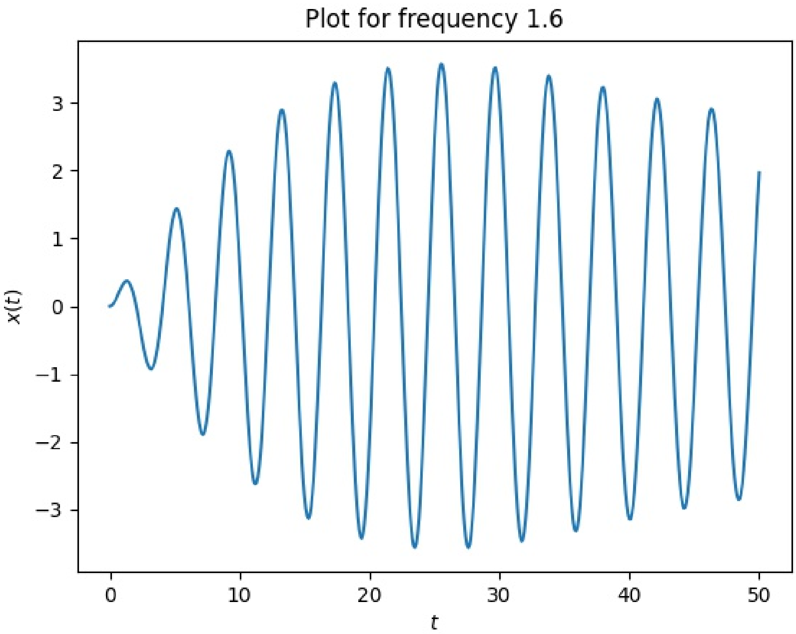

Figure 7: $x(t)$ for frequency 1.6

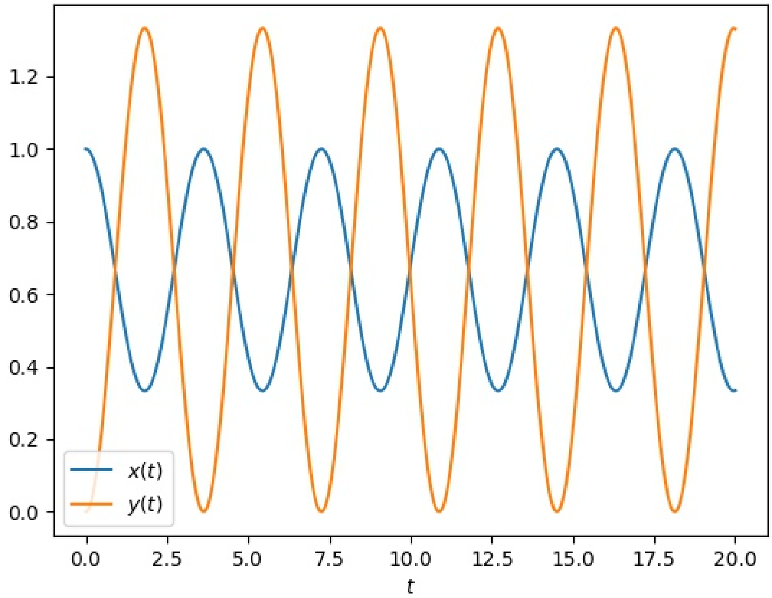

Figure 8: Solution for the coupled equations

5 Question 5

The transfer function of the given circuit is

$$H(s) = \frac{1}{s^2LC + sCR + 1}$$

$$\Rightarrow H(s) = \frac{1}{10^{-12}s^2 + 10^{-4}s + 1}$$

The magnitude and phase response of the system is given in Figure 9.

6 Question 6

The given input signal is

$$v_i(t) = cos(10^3 t)u(t) - cos(10^6 t)u(t)$$

The output is computed and plotted as shown in Figures 10 and 11. The plot in Figure 10 is for $0 < t < 10ms$ whereas the plot in Figure 11 is for $0 < t < 30\mu$s. Since the system is a lowpass filter, we see that the output is dominated by the low freqency component. Since the system is not an ideal lowpass filter, there is some component of the higher frequency visible in the 30μs plot. But from Figure 10, we see that the higher frequency is largely attenuated.

Conclusion

In this assignment, we have used the signal toolbox in Python to simulate various systems.

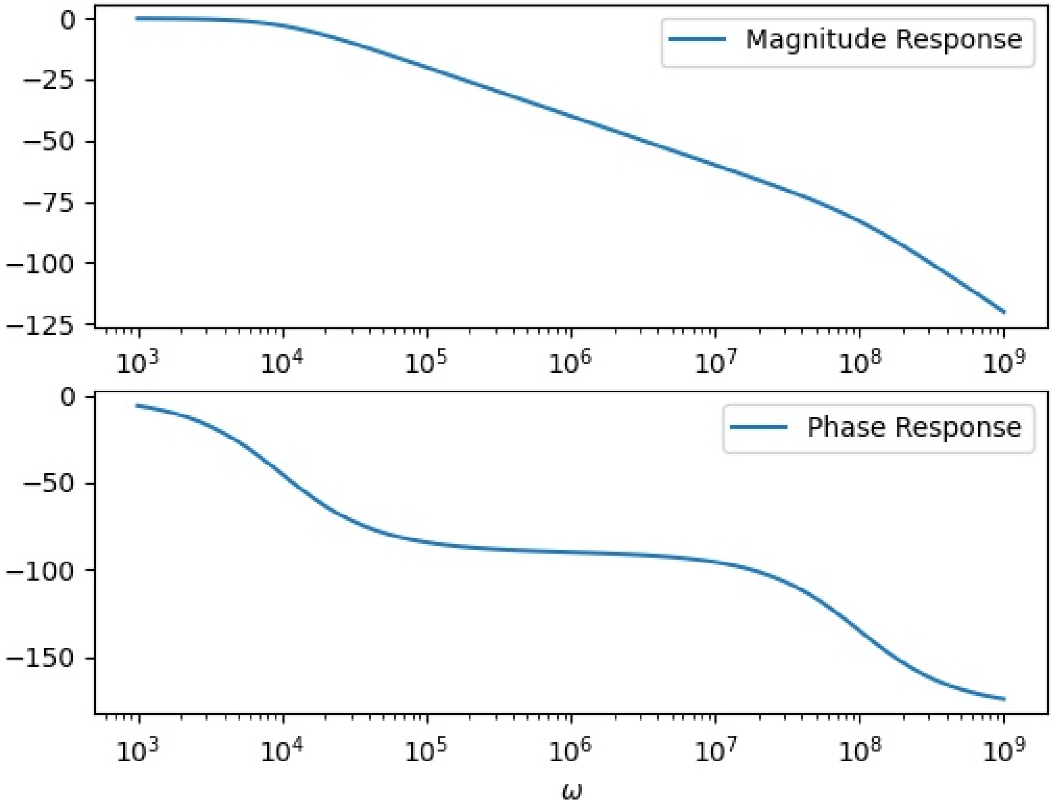

Figure 9: Magnitude and Phase Response of the circuit

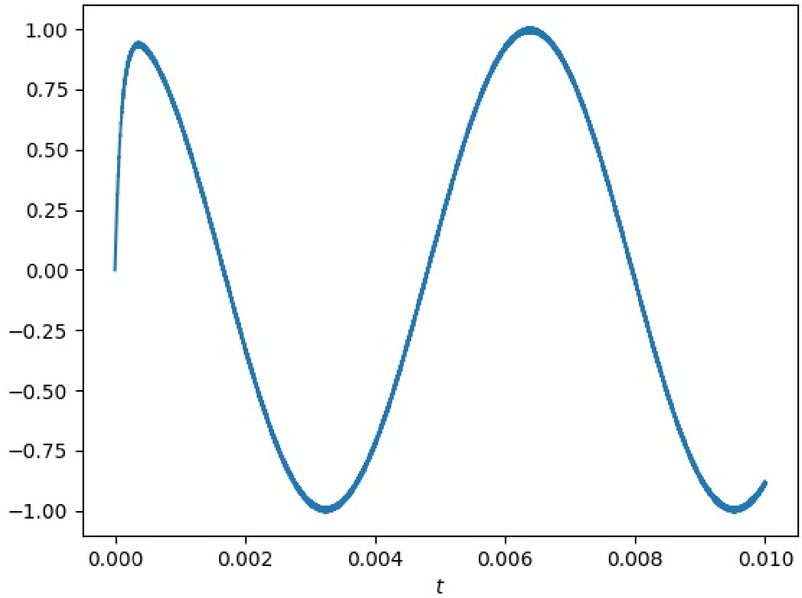

Figure 10: Output for $0 < t < 10ms$

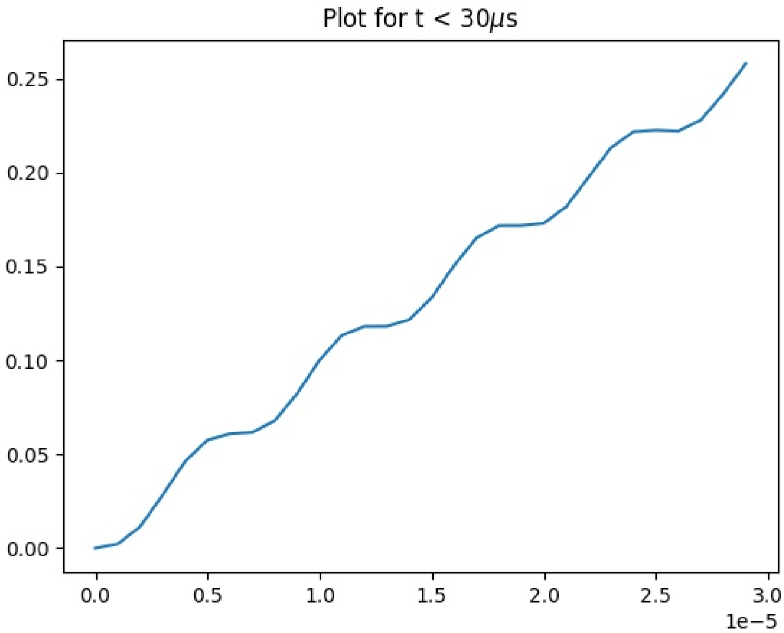

Figure 11: Output for $0 < t < 30\mu$s