Assignment 5: The Resistor Problem

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Introduction

This assignment is about solving the laplace equation with boundary conditions in order to find the potential across a resistive sheet along with the current flow. The important learning outcome in this assignment is using vectorization in python.

1 Voltage across the sheet

The values taken into account in this report are:

`Nx = 25; Ny = 25; radius = 8; Niter = 1500`

The bottom edge of the sheet is grounded, and other three edges are hanging i.e., no current flow at those edges. The potential ϕ is initialized as a zero array with Nx columns and Ny rows. Now, a wire at the center of the sheet with radius 8 is held at a voltage of 1V. This is set by finding the indices of ϕ where the wire is present, and setting the value of the voltage as 1.0. The initial spread of ϕ over the sheet is plotted in Figure 1. The red dots indicate the region at 1V voltage.

2 Updating the potential

The following code is used to update the potential every iteration.

```
for k in range(Niter):
    oldphi = pl.copy(phi)
    phi[1:-1, 1:-1] = 0.25*(phi[:-2, 1:-1] + phi[2:, 1:-1]
                            + phi[1:-1, :-2] + phi[1:-1, 2:])
```

Vectorising the code makes it much faster than using nested for-loops to perform the operation.
Now, the boundary conditions are enforced. The voltage of region occupied by the wire is 1V and other three edges are hanging i.e., there is no gradient of voltage in the normal direction.

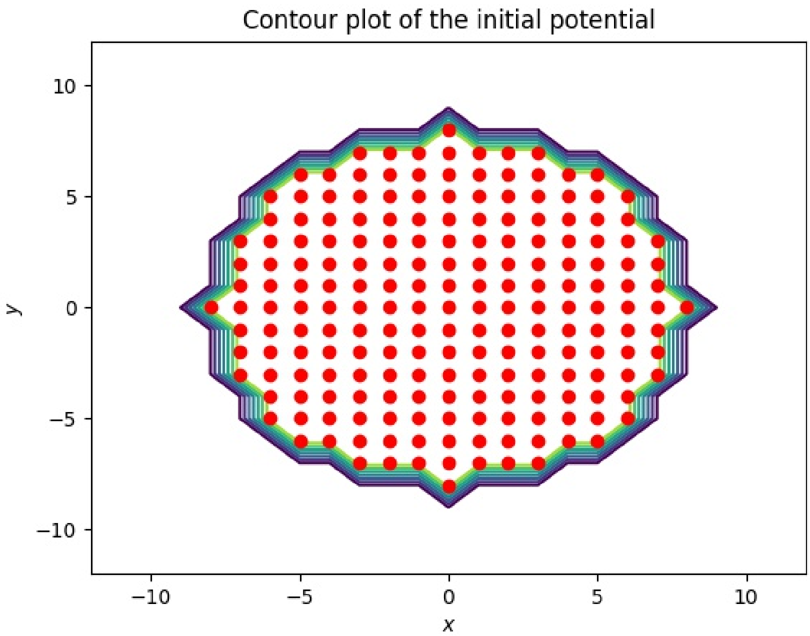

Figure 1: Initial potential on the surface of conductor

```
phi[:, 0] = phi[:, 1]
phi[:, -1] = phi[:, -2]
phi[0, :] = phi[1, :]
phi[ii] = 1.0
```

The above code is used to enforce the mentioned boundary conditions.

3 Change in voltage profile

The error is defined as the maximum difference between the voltage profiles before and after the iteration. These values are stored in a vector of size 1500 (Number of iterations). We see that the semilog plot of these errors vs number of iterations is linear after a particular number of iterations. Since the semilog plot is linear, we can model the error as

$$error_k = Ae^{Bk}$$

$$log(error_k) = log(A) + Bk$$

The least squares fit is found for $log(A)$ and B. One case (fit1) uses the entire error vector and the other case (fit2) uses errors after 500th iteration.

Figure 2: Plot of errors and least squares fit

4 Final voltage profile

We have seen that the change in voltage profile is very less after 1500 iterations. Now, we will see how the voltage varies across the surface of the conductor. The surface plot of the voltage is given in Figure 3. The contour plot of the voltage is given in Figure 4. We see that the voltage at the centre of the sheet is fixed at 1V (first boundary condition) and the voltages do not vary in the normal direction at the top 3 edges since they are hanging (second boundary condition)

5 Current flow in the sheet

The quiver plot for current flow is given in Figure 5. We see that most of the current flows in the bottom part of the conductor. This is because the voltage above the region held at 1V is also nearly 1V, whereas the voltage below that region gradually decreases to 0V since the bottom edge is grounded.

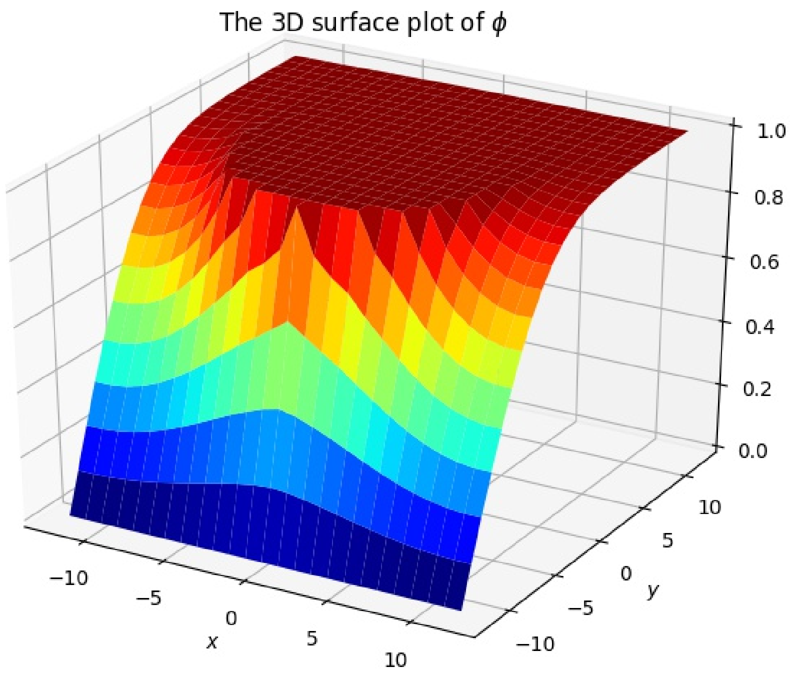

Figure 3: Surface plot of the potential

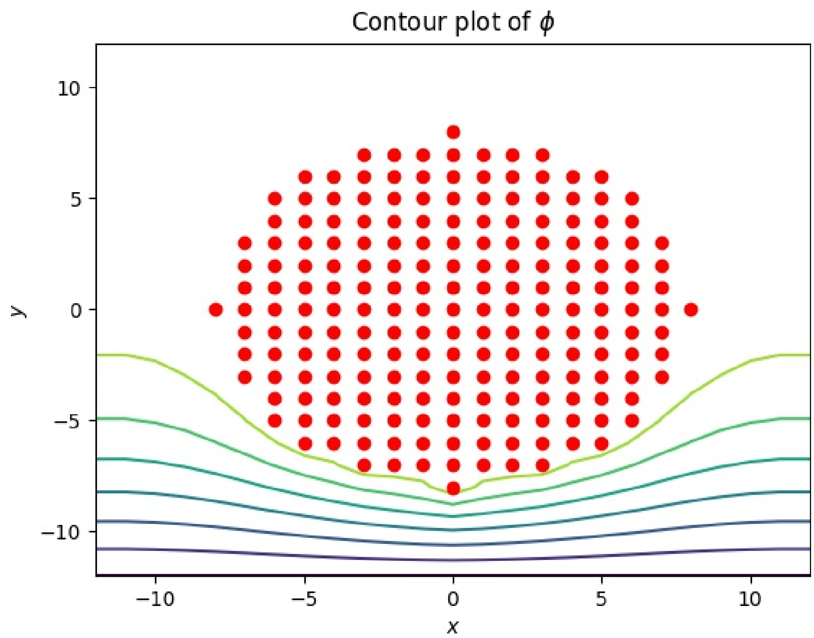

Figure 4: Contour plot of the potential

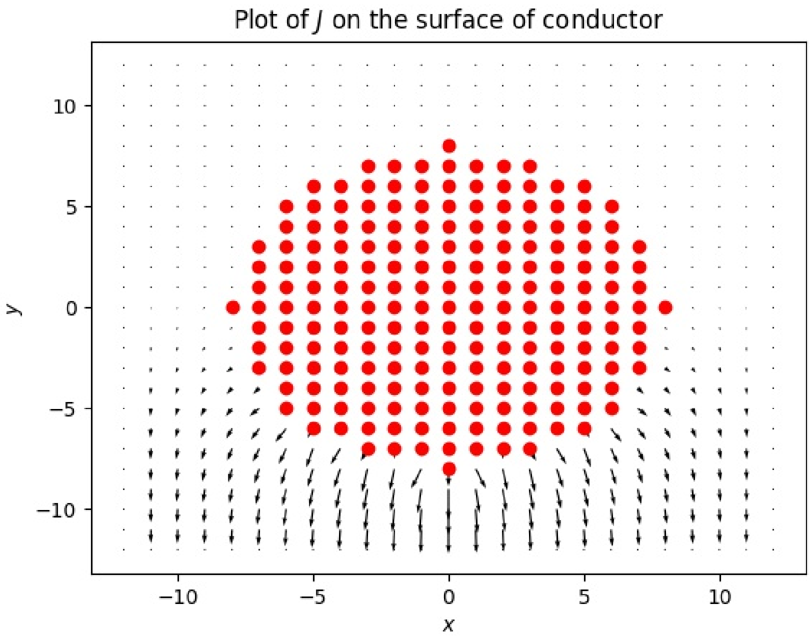

Figure 5: Flow of current in the conductor

Conclusion

In this assignment, we learnt the better method of handling arrays, **Vectorization**. Vectorized code is much faster than using nested for-loops which consumes a lot of time.