EE2703 : Applied Programming Lab
Final Exam 2021

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Introduction

A loop of wire is present on the XY plane, centered at origin and radius of 10 cm. The loop has a current

$$I = \frac{4\pi}{\mu_0} \cos(\phi) \exp(j\omega t)$$

where (r, ϕ) are polar co-ordinates. We need to find the magnetic field $(\vec{B})$ on the Z-Axis generated by this loop of wire.

1. Defining the space

The volume is divided into a $3 \times 3 \times 1000$ grid with the points separated by 1 cm. The grid is also placed off-axis by 1 cm so that the magnetic field doesn't become very small. The loop of wire is split into 100 small elements of current. The current elements are plotted as shown in Figure 1. The following code is used to find the vector $\vec{r}$ for every point in the grid

```
radius = 10; k = 0.1 #defining the parameters
phi = np.linspace(0, 2*np.pi, 101)[:-1]
rx = np.linspace(0, 2, 3)
ry = np.linspace(0, 2, 3)
rz = np.linspace(1, 1000, 1000)
rx, ry, rz = np.meshgrid(rx, ry, rz)
r = np.stack([rx, ry, rz], axis=-1)
```

The following code is used to find the vectors $\vec{r'}$ and $\vec{dl'}$ for every point on the loop

```
rprime = np.array([radius*np.cos(phi), radius*np.sin(phi),
                    np.zeros_like(phi)]).T
dlprime = np.array([-np.sin(phi)*2*np.pi*radius/100,
                    np.cos(phi)*2*np.pi*radius/100,
                    np.zeros_like(phi)]).T
```

2. Finding Vector Potential

In order to find $\vec{B}$, we first find the vector potential $\vec{A}$ using the formula

$$\vec{A}(r, \phi, z) = \frac{\mu_0}{4\pi} \int \frac{I(\phi)\hat{\phi}e^{-jkR}ad\phi}{R}$$

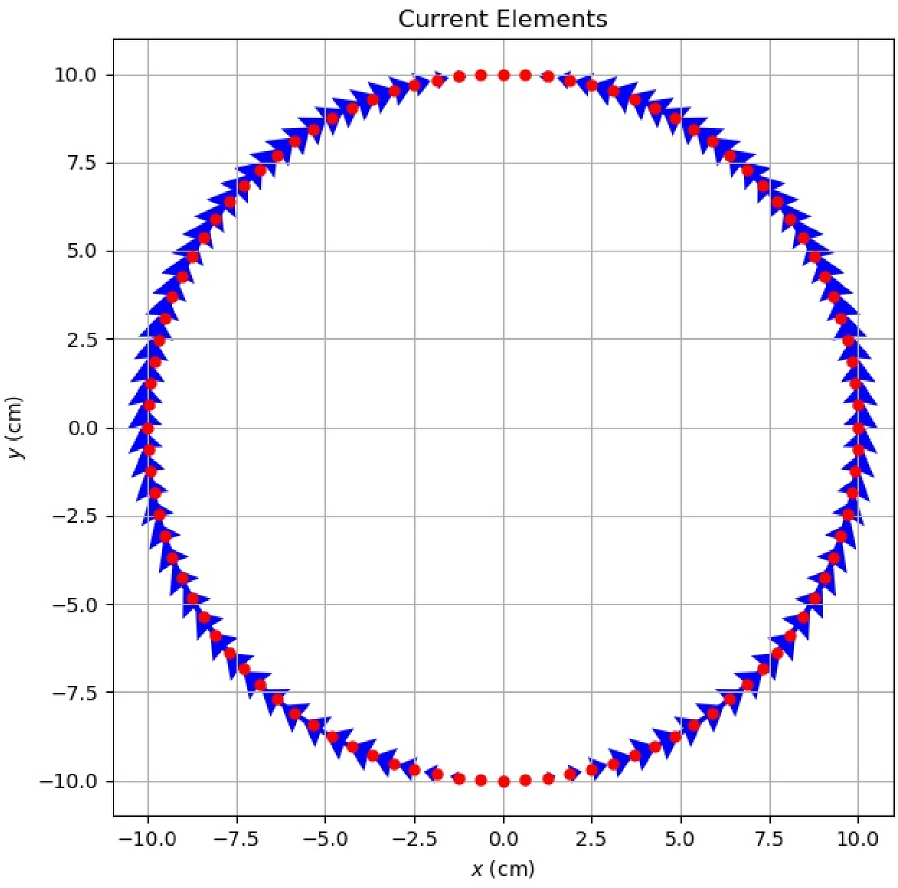

Figure 1: Loop of wire

where $\vec{R} = \vec{r} - \vec{r'}$ and wave vector $k = \frac{\omega}{c} = 0.1$. $\vec{r}$ is the point where we want to find the potential and $\vec{r'} = a\hat{r'}$ a point on the loop. This can be reduced to the sum:

$$\vec{A}_{ijk} = \sum_{l=0}^{N-1} \frac{\cos(\phi_l') \exp(-jkR_{ijkl}) d\vec{l'}}{R_{ijkl}}$$

Note that the index k is different from the wave vector defined above. The function `calc(l)` is used to find the term inside the summation

```
def calc(l):
    R = r - rprime[l]
    R = np.sqrt(np.sum(R**2, axis=-1)) #calculating R
    temp = np.cos(phi[l])*np.exp(-1j*k*R)/R
    temp = np.expand_dims(temp, axis=-1)
    term = temp*dlprime[l] #extension to find the entire term
    return term
```

The argument `l` is used to index the current elements. Now, $\vec{A}$ can be found by summing over `l`. This is done using a for loop because vectorization might become a little tricky.

3. Magnetic Field from Vector Potential

Once we finish finding $\vec{A}$, we can find the magnetic field $\vec{B}$ from it using (we are only interested in the z component)

$$\vec{B} = \nabla \times \vec{A}$$
$$\implies B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

This can be written as the difference

$$B_z = \frac{A_y(\Delta x, 0, z) - A_y(-\Delta x, 0, z)}{2\Delta x} - \frac{A_x(0, \Delta y, z) - A_x(0, -\Delta y, z)}{2\Delta y}$$

The following code is used to implement the above equation

```
B = 0.5*(A[1, 2, :, 1] - A[2, 1, :, 0]
            - A[1, 0, :, 1] + A[0, 1, :, 0])
```

Absolute value of Magnetic Field is plotted as shown in Figure 2.

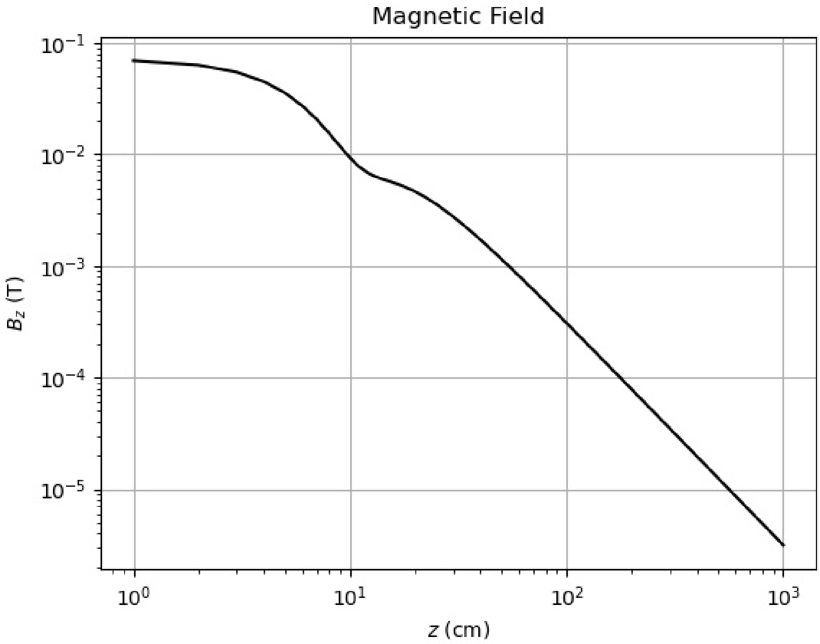

Figure 2: Magnetic Field

4. Fitting the data to cz^b

From the previous section we have found B_z as a function of z. The loglog plot is linear after some distance. Now we try to fit the data to the form cz^b. We use the points from $z = 50$ cm, so that only the linear region is covered.

$$B_z = cz^b$$
$$\implies \log(c) + b\log(z) = \log(B_z)$$
$$\implies \begin{bmatrix} 1 & \log(z_{50}) \\ 1 & \log(z_{51}) \\ ... & ... \\ 1 & \log(z_{1000}) \end{bmatrix} \begin{bmatrix} \log(c) \\ b \end{bmatrix} = \begin{bmatrix} \log(B_{z50}) \\ \log(B_{z51}) \\ ... \\ \log(B_{z1000}) \end{bmatrix}$$

The following code is used for fitting the data

```
M = np.c_[np.ones_like(z[50:]), np.log(z[50:])]
y = np.log(np.absolute(B[50:]))
x, res, rnk, s = lstsq(M, y)
c = np.exp(x[0]); b = x[1]
```

The values obtained are

```
b = -1.9894062771705439      c = 2.9340884765015973
```

The comparison between the calculated value and the fit is given in Figure 3. We see that the magnetic field falls off as $1/z^2$. However for the static case, the magnetic field on the Z-Axis is given as

$$\vec{B} = \frac{\mu_0 I r^2}{2(z^2 + r^2)^{3/2}}\hat{z}$$
$$z >> r$$
$$\implies B_z \approx \frac{\mu_0 I r^2}{2z^3}$$

where r is the radius of loop. We see that in the static case, the magnetic field falls off as $1/z^3$ but when the current is sinusoidally varying, magnetic field nearly falls off $1/z^2$. The difference comes because of the sinusoidal variation of current both in time and space.

Conclusion

We computed the Magnetic Field due to a loop of wire carrying sinusoidal current. This is done numerically by using numpy arrays. The arrays are efficiently manipulated in order to reduce the runtime. The Magnetic Field is then fit into a model which showed that the field varies as $1/z^2$. This is different from the static case, where the field varies as $1/z^3$.

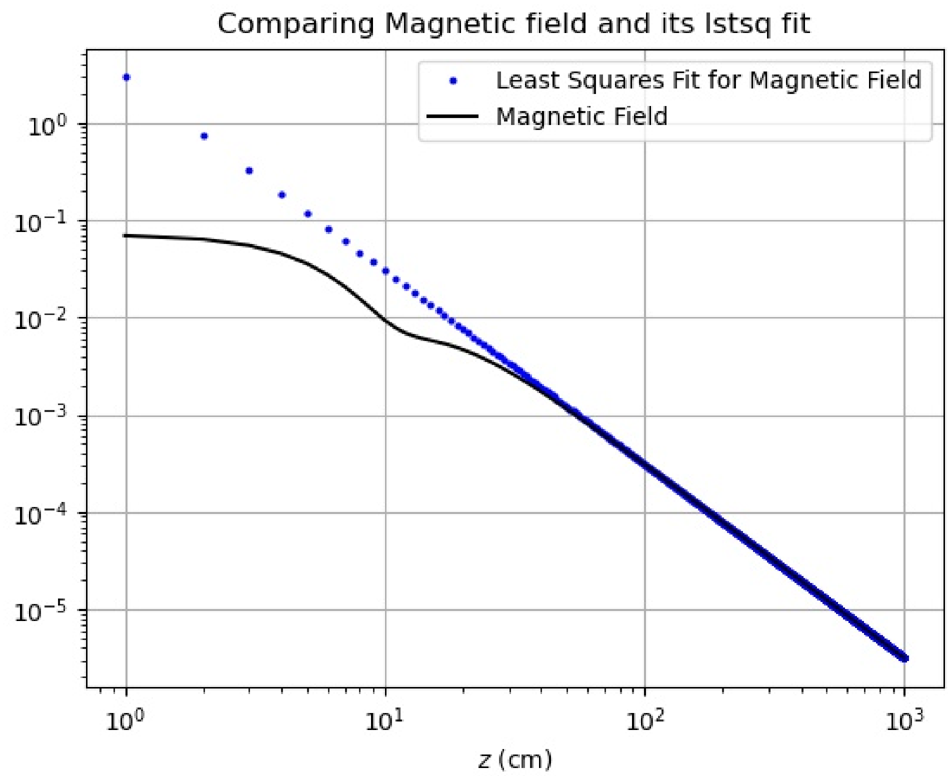

Figure 3: Comparing least square fit with original data