Assignment 4: Fourier Approximations

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Introduction

Assingnment 4 is about approximating a function using the well known Fourier Expansion. We have explored two ways of generating the fourier coefficients, the integral method and the least squares method. In this report, we will see how the two methods have fared against each other.

1 Functions to be approximated

The two given functions are $exp(x)$ and $cos(cos(x))$. The plots of these functions are given in Figure 1 and Figure 2 respectively. Along with the actual values, the expected plots from Fourier expansion are also plotted in the respective figures.

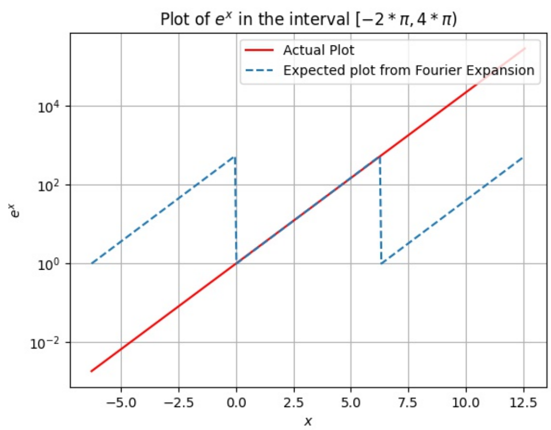

Figure 1: Actual and Expected plots of $exp(x)$

We know that the Fourier Expansion generates only periodic functions. So, we see that the actual plot of $exp(x)$ is different from what is expected from the Fourier approximation. On the other hand, since $cos(cos(x))$ is

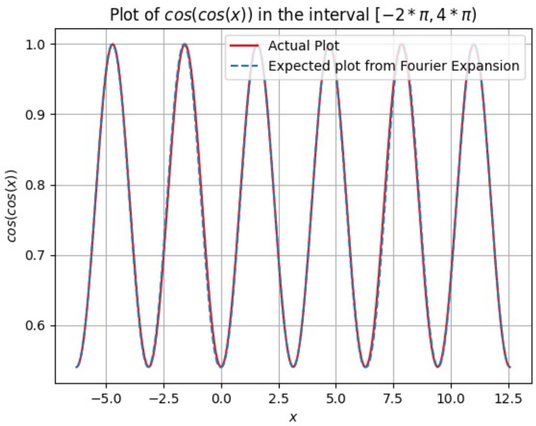

Figure 2: Actual and Expected plots of $cos(cos(x))$

inherently periodic, the Fourier approximation coincides with the actual value

2 Fourier Coefficients by Integration

The plots of Fourier Coefficients of $exp(x)$ are given in Figure 3 & Figure 4 and the plots of Fourier Coefficients of $cos(cos(x))$ are given in Figure 5 & Figure 6

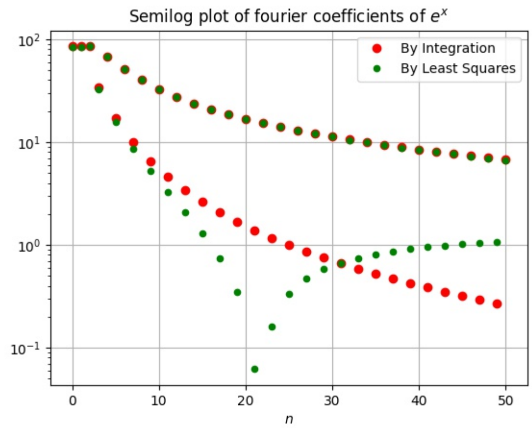

Figure 3: Semilog Plot of Fourier Coefficients of $exp(x)$

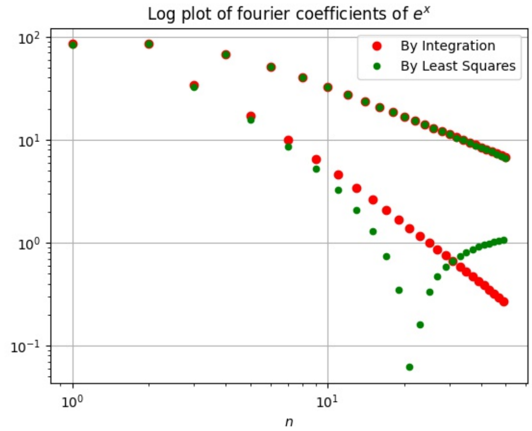

Figure 4: Log-Log Plot of Fourier Coefficients of $exp(x)$

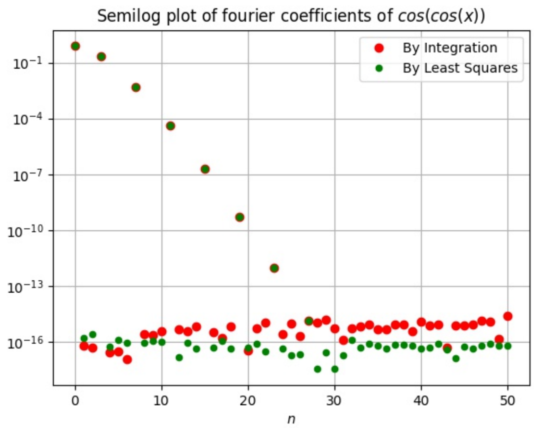

Figure 5: Semilog Plot of Fourier Coefficients of $cos(cos(x))$

2.1 Q3 (a)

In the second case i.e., $cos(cos(x))$, the function itself is an even function. Because of this, the b_n values will be zero, since they are the sin components, which is an odd function.

2.2 Q3 (b)

For $exp(x)$, the higher frequencies also have significant components whereas for $cos(cos(x))$ the frequency is $\frac{1}{\pi}$ and hence it doesn't have significant components from the higher frequencies. This is the reason the coefficients decay quickly in the second case.

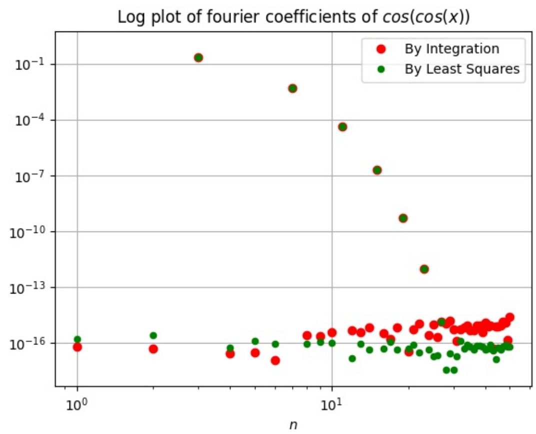

Figure 6: Log-Log Plot of Fourier Coefficients of $cos(cos(x))$

2.3 Q3 (c)

For the function $exp(x)$, the fourier coefficients

$$a_n \propto \frac{1}{n^2+1}$$

$$b_n \propto \frac{n}{n^2+1}$$

Now for large n, it can be approximated as

$$\frac{1}{n^2+1} \simeq \frac{1}{n^2}, \frac{n}{n^2+1} \simeq \frac{1}{n} \tag{1}$$

Taking logarithm gives

$$\log(\frac{1}{n^2+1}) \simeq -2\log(n), \log(\frac{n}{n^2+1}) \simeq -\log(n) \tag{2}$$

This is the reason for the linearity of the loglog plot in Figure 4.
Now, for $cos(cos(x))$, the fourier coefficients are vary exponentially with n, and hence the semilog plot in Figure 5 looks linear.

3 Fourier Coefficients by Least Squares Method

The fourier coefficients found by using the Least Squares Method are plotted in their respective figures. As we see, the plots partly argee with each other. The deviation of values calculated by least squares from those calculated by integration are

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Largest error for the function exp(x) is 1.3327308703353964
Largest error for the function cos(cos(x)) is 2.6566469738662125e-15
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The values computed by least squares method should agree with the ones computed by integration. We see this deviation because of the small sample size.

4 Comparing Fourier approximation with actual value

Using the fourier coefficients produced by Least Squares method, the function curve is generated and plotted, along with the actual function. $exp(x)$ is plotted in Figure 7 and $cos(cos(x))$ is plotted in Figure 8. As we see, the Fourier approximation of $cos(cos(x))$ nearly agrees with the actual value. On the other hand, the Fourier approximation of $exp(x)$ has a lot of deviation from the actual value. This is because we considered only the first 51 coefficients of the Fourier Series expansion. Since $exp(x)$ has significant components of the higher freqencies as well, we see a large difference.

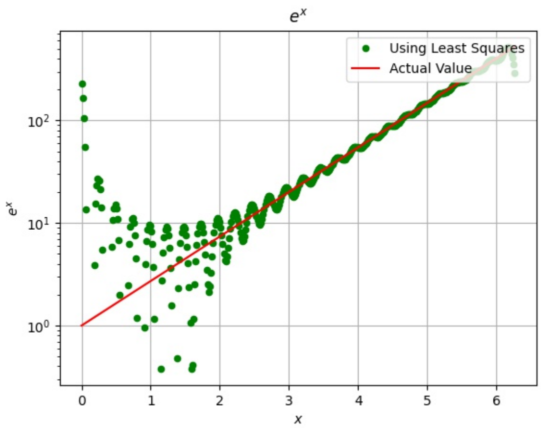

Figure 7: Fourier approximation of $exp(x)$

Conclusion

We have seen two ways of calculating the Fourier Coefficients, by integration and by Least Squares method. We have seen that the error in the coefficients when calculated by using least squares is not very large. Hence, it is safe to use the Least Squares method for these functions and reduce the computation complexity.

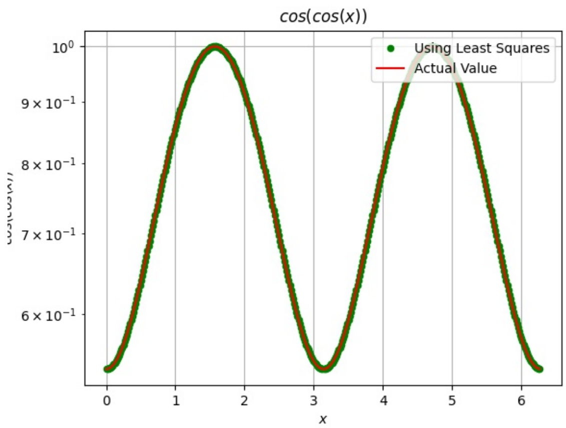

Figure 8: Fourier approximation of $cos(cos(x))$