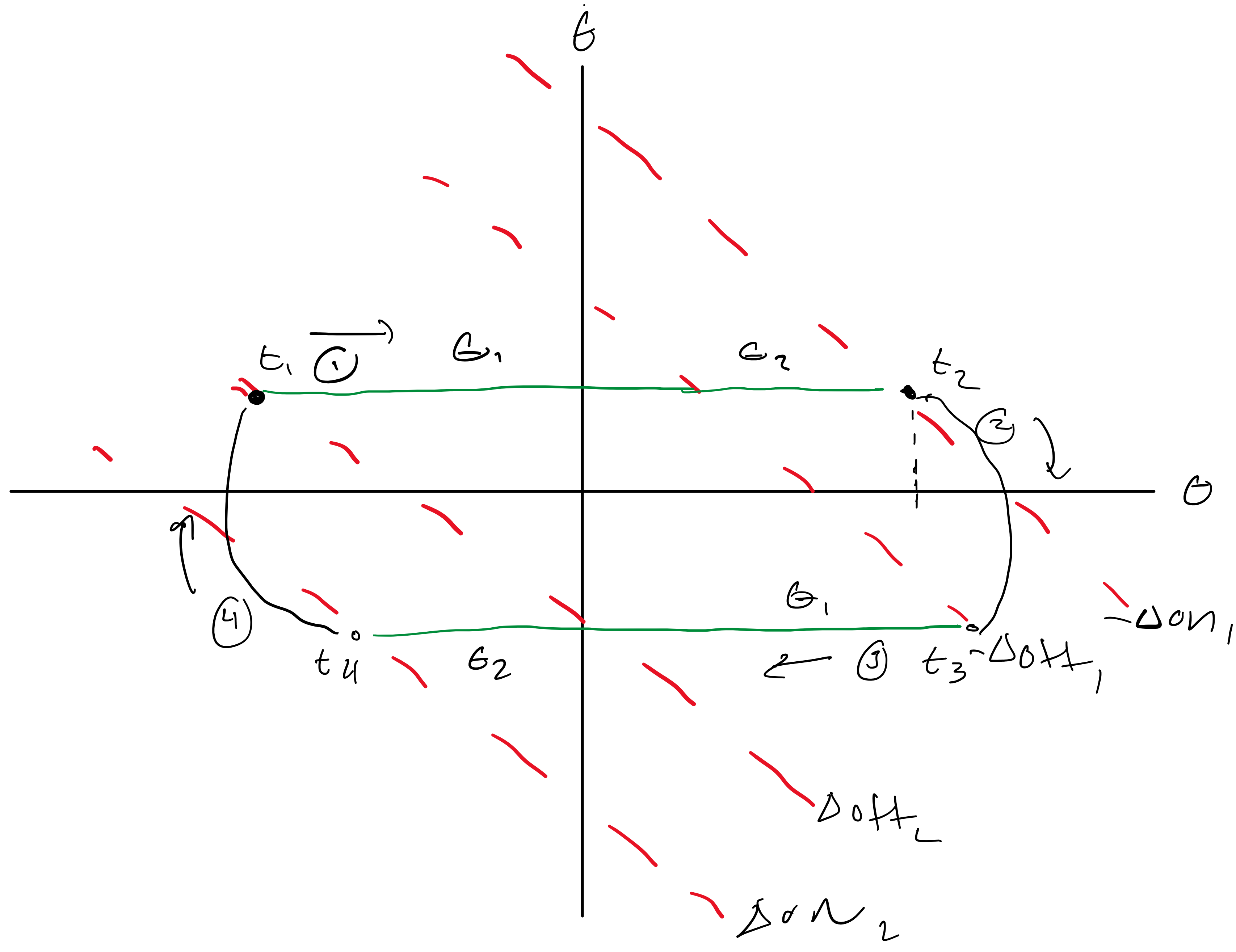
Homework 5

Sunday, November 13, 2022 8:34 PM

Gagandeep Thapar

1) a) Duty Cycle of Switch = 100% (never turns off

b) Schmitt Trigger



$\Delta on_1 \sim e + \tau\dot e = \theta + \tau\dot\theta$

$\Delta off_1 \sim e + \tau\dot e = \theta - \tau\dot\theta$

$\Delta on_2 \sim e + \tau\dot e = -\theta - \tau\dot\theta = -\Delta on$

$\Delta off_2 \sim e + \tau\dot e = -\theta + \tau\dot\theta = -\Delta off$

recall: $T = J\ddot\theta \quad \therefore \ddot\theta = \frac{T}{J}$ where $T = m$ $\quad \boxed{\ddot\theta = \frac{m}{J}}$

notice: $\Delta on + \Delta off = \theta + \tau\dot\theta + \theta - \tau\dot\theta$

$2\theta = \Delta on + \Delta off$

$$\boxed{\theta = \frac{\Delta on + \Delta off}{2}}$$

$\Delta on - \Delta off = \theta + \tau\dot\theta - \theta + \tau\dot\theta$

$2\tau\dot\theta = \Delta on - \Delta off$

$$\boxed{\dot\theta = \frac{\Delta on - \Delta off}{2\tau}}$$

notice: $t_{rest} = t_1 + t_3$ and $t_1 = t_3$

$t_{thrust} = t_2 + t_4$ and $t_2 = t_4$

$\therefore t_{rest} = 2t_1$

$t_{thrust} = 2t_2$

$$DC = \frac{t_{thrust}}{t_{rest} + t_{thrust}} = \frac{\cancel{2}t_2}{2t_1 + 2t_2} = \frac{t_2}{t_1 + t_2}$$

notice: $\theta(t) = \theta_0 + \dot\theta(t)$

$\theta(t_1) = -\theta + \dot\theta t_1 + \cancelto{0}{\tfrac{1}{2}\ddot\theta t_1^2}$

$\dot\theta t_1 = 2\theta$

$$t_1 = \frac{-2\theta}{\dot\theta}$$

$$= -\frac{2\left(\frac{\Delta on + \Delta off}{2}\right)}{\frac{\Delta on - \Delta off}{2\tau}} = \boxed{-\frac{2\tau(\Delta on + \Delta off)}{\Delta on - \Delta off} = t_1}$$

notice: $\dot\theta(t) = \dot\theta_0 + \ddot\theta t$

$\dot\theta(t_2) = \dot\theta + \ddot\theta t_2 = -\dot\theta$

$\ddot\theta t_2 = -2\dot\theta$

$$t_2 = \frac{-2\dot\theta}{\ddot\theta}$$

$$= -\frac{2\left(\frac{\Delta on - \Delta off}{2\tau}\right)}{\frac{m}{J}} = \boxed{-\frac{(\Delta on - \Delta off)}{\tau\frac{m}{J}} = t_2}$$

$$DC = \frac{t_2}{t_1 + t_2}$$

$$= \frac{\Delta on - \Delta off}{\tau\frac{m}{J}} \cdot \left(\frac{\Delta on - \Delta off}{\tau\frac{m}{J}} + \frac{2\tau(\Delta on + \Delta off)}{\Delta on - \Delta off}\right)^{-1}$$

$$= \frac{\Delta on - \Delta off}{\tau\frac{m}{J}} \cdot \left(\frac{(\Delta on - \Delta off)^2 + 2\tau^2\frac{m}{J}(\Delta on + \Delta off)}{(\Delta on - \Delta off)(\tau\frac{m}{J})}\right)^{-1}$$

$$= \frac{(\Delta on - \Delta off)\cdot(\Delta on - \Delta off)\cancel{(\tau\frac{m}{J})}}{\cancel{(\tau\frac{m}{J})}(\Delta on - \Delta off)^2 + 2\tau^2\frac{m}{J}(\Delta on + \Delta off)}$$

$$\boxed{= \frac{(\Delta on - \Delta off)^2}{(\Delta on - \Delta off)^2 + 2\tau^2\frac{m}{J}(\Delta on + \Delta off)}}$$