PARAMETERIZATION AND OPTIMIZATION OF THE ERROR PROPAGATION IN STAR TRACKERS FOR LEO SPACECRAFT

A Thesis

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Chapter 1

MAJOR CONCEPTS IN STAR TRACKER ERROR ANALYSIS

- Different steps in the Star Tracker process
  - Centroiding
  - Identification
  - Attitude Determination
- Errors can exist on several levels
  - Hardware (permanent non-varying)
  - Hardware noise (varies, function of hardware)
    - CCD noise
    - Dark Current
  - Environment (varies, can affect hardware which)
    - Thermal Environment on Lens
    - Radiation Environment on focal plane
    - Atomic Oxygen on lens
  - Dynamics (varies, can affect centroiding process)
  - Different identification algorithms can yield different results
  - Wahba's problem which will affect attitude determination from identification algorithm
- "The most important factors that affect star tracker accuracy include **thermal drift, optical aberration, detector noise, and systematic error of star image centroid estimation algorithm**"[4]

- Most of the literature describes the centroiding/calibration being the prime mover of error; on-orbit effects are rarely discussed although do pose additional error in attitude determination

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Chapter 2

CALIBRATION TECHNIQUES AND NOTES

2.1 Key Concepts[6]

- Paper strictly talks about camera calibration and not environmental considerations
- Measurement precision is determined by
  - Focal length, f
  - Principal Point offsets, (x_0, y_0)
  - Focal Plane distortions
- ... these are usually calibrated out prior launch
- We can model the defocused image by imagining a new focal plane of focal length, f'
  - we can determine the ideal centroid of the star given

$$u = f' tan\theta$$

$$v = f' tan\phi$$

and the direction vector of the star given by

$$\hat{v} = \begin{bmatrix} \hat{v}_x \\ \hat{v}_y \\ \hat{v}_z \end{bmatrix} = \frac{1}{u^2 + v^2 + f'^2} \begin{bmatrix} u \\ v \\ f' \end{bmatrix}$$

- 4 types of errors described in the paper
  - Distortion Model Error
  - Principal Point Error
  - Measurement Error
  - Calibration Data Error

2.2 Distortion Model[6]

- Ideal cameras do not exist; a new camera model was developed considering radial, decentering, and thin prism distortions

$$\begin{bmatrix} \delta_u(u',v') \\ \delta_v(u',v') \end{bmatrix} = \begin{bmatrix} u' \\ v' \end{bmatrix} - \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} (g_1+g_3)u'^2 + g_4u'v' + g_1v'^2 + \kappa u'(u'^2+v'^2) \\ g_2u'^2 + g_3u'v' + (g_2+g_4)v'^2 + \kappa v'(u'^2+v'^2) \end{bmatrix}$$

where

(u, v) are the ideal, nonobservable image coordinates

(u', v') are the observable image coordinates with image distortion

$$\begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix} = \begin{bmatrix} s_1+p_1 \\ s_2+p_2 \\ 2p_1 \\ 2p_2 \end{bmatrix}$$

where s_1, s_2 represent thin prism distortion

p_1, p_2 represent decentering distortion

and κ represents radial distortion

Chapter 3

ERROR SOURCES

List of models grouped by error source

3.1 CCD Noise

3.1.1 Key Concepts

- high resolution, high quantum efficiency (QE), wide spectral response, low noise, linearity, geometric fidelity, fast response, small size, low power consumption, durability [2]
- discrete "buckets" where photons are trapped/measured (called photoelectrons)
- Uniformly illuminating a focal plane will lead to non-uniformity in collected charge - called the fixed pattern noise
- Normally collection site values are independent of each other; in reality a surplus of charge can "spill over" adjacent buckets and creates codependence; this is called blooming
- Thermal energy creates free electrons that can be stored in buckets and become indistinguishable from real photoelectrons; called **Dark Current**; Dark Current $\propto T$, Integration Time
- Shot noise is a result of quantum nature of light and characterizes uncertainty in the number of electrons stored in a bucket

- Amplifier noise dominates shot noise at low signal levels; determines real noise floor signal; high-frequency noise is filtered out via Low-Pass Filter
- Spatial variation in illumination and reflectance leads to spatial variance in the collected charge across the focal plane
  - Number of electrons follows a Poisson's Ratio such that variance equals its mean

3.1.2 Models

1. Typical noise floor size[5]

$$NF = \bar{B} + 5 * \sigma_B$$

where $\bar{B} \equiv$ the average brightness value and $\sigma_B \equiv$ the standard deviation in brightness

2. Ideal number of electrons collected in a collection site (bucket)[2]

$$I = T \int_\lambda \int_y \int_x B(x, y, \lambda) S_r(x, y) q(\lambda)\, dx\, dy\, d\lambda$$

where

$$T \equiv \text{integration time [sec]}$$

$$(x, y) \equiv \text{continuous coordinates on the focal plane}$$

$$B(x, y, \lambda) \equiv \text{incident spectral irradiance [W/unit area]}$$

$$q(\lambda) \equiv \text{ratio of electrons collected per incident light energy as a function of wavelength [electrons/Joule]}$$

$$S_r \equiv \text{spatial response of the collection site}$$

3. Model of electrons collected by site[2]

$$N = KI$$

where $K \equiv$ a constant associated with a collection site that accounts for product of $q(\lambda)$ and $S_r(x, y)$ and $\bar{K} = 1$ and variance of σ_K^2

4. Number of electrons integrated at a collection site[2]

$$N = KI + N_{DC} + N_S$$

where $N_{DC} \equiv$ electrons from Dark Current, $N_S \equiv$ zero mean Poisson shot noise with variance a function of KI and N_{DC}

5. Magnitude of Video Signal, V[2]

$$V = (KI + N_{DC} + N_S + N_R)A$$

where $\bar{N_R} = 0$ and $N_R \equiv$ amplifier read noise, $A \equiv$ combined gain of output amplifier and camera circuitry

6. Quantization of Video Signal[2]

$$D = (KI + N_{DC} + N_S + N_R)A + N_Q$$

where $\bar{N_Q} = 0$ and $N_Q \equiv$ random variable independent of V and has a uniform probability over range $[-\frac{1}{2}a, \frac{1}{2}q]$ with variance $\frac{q^2}{12}$ where q is a quantization step such that

$$(n - \frac{1}{2})q < V \leq (n + \frac{1}{2})q)$$

where n is an integer satisfying $0 \le n \le 2^b - 1$ where b is the number of bits to represent D

7. Spatially varying spectral reflectance, R

$$R(x, y, \lambda) = \bar{R}(\lambda) + r(x, y, \lambda)$$

where

(x, y, z) is a coordinate system where z is the boresight axis and x, y span the focal/sensor plane

$\bar{R}(\lambda)$ is the mean reflectance

$r(x, y, \lambda)$ represents random spatial variance of reflectance

thus

$$E[R(x, y, \lambda)] = \bar{R}(\lambda) \quad \lambda \in [\lambda_1, \lambda_2]$$

and

$$E[r(x, y, \lambda)] = 0 \quad \lambda \in [\lambda_1, \lambda_2]$$

where $[\lambda_1, \lambda_2]$ is the range over which $q(\lambda)$ is nonzero

and E is the expected value operator over (x, y)

8. Spectral irradiance [2]

$$L(x, y, \lambda) = \bar{L}(\lambda) + l(x, y, \lambda)$$

where

$$E[L(x, y, \lambda)] = \bar{L}(\lambda) \quad \lambda \in [\lambda_1, \lambda_2]$$

and

$$E[l(x,y,\lambda)] = 0 \quad \lambda \in [\lambda_1, \lambda_2]$$

9. (Many additional derivations to transform this in terms of camera pixels; too specific to include here) ...

10. Generalizing (6) to the camera model

$$D(a,b) = (K(a,b)I(a,b) + N_{DC}(a,b) + N_S(a,b) + N_R(a,b))A + N_Q(a,b)$$

11. Expected Value of $D(a,b)$

$$D(a,b) = \mu(a,b) + N(a,b)$$

where

$$\mu(a,b) = K(a,b)I(a,b)A + E_{DC}(a,b)A$$

where E_{DC} is the expected value of $N_{DC}(a,b)$ and $\bar{N}_{AB} = 0$ and

$$N(a,b) = N_I(a,b) + N_C(a,b)$$

12. Variance of $N_I(a,b)$

$$\sigma_I^2(a,b) = A^2(K(a,b)I(a,b) + E_{DC}(a,b))$$

13. Variance of $N_C(a,b)$

$$\sigma_C^2 = A^2\sigma_R^2 + \frac{q^2}{12}$$

3.2 Thermal Distortion

3.2.1 Models

1. Refractive Index of Air at a given temperature [3]

$$n_T - 1 = (n_{15} - 1)(\frac{1.0549}{1 + 0.00366T})$$

where n_T is the refractive index at Temperature, T given in celsius and n_{15} is the refractive index of air at 15C

2. n_{15} as a function of wavelength, λ [3]

$$(n_{15} - 1) * 10^8 = 8342.1 + \frac{2406030}{130 - \nu^2} + \frac{15996}{38.9 - \nu^2}$$

where $\nu = \frac{1}{\lambda}$ and $[\lambda] \equiv$ microns

3. Relation between absolute and relative refractive indices [3]

$$n_{abs} = n_{rel} n_{air}$$

4. Relation between thermal expansion and focal Length [3]

$$x_f = \frac{1}{f}\frac{df}{dt} = x_g - \frac{1}{n - n_{air}}(\frac{dn}{dt} - n\frac{dn_{air}}{dt})$$

3.3 Radiation

3.3.1 Models

1. Radiation from a black body at a given wavelength and temperature (on lens)[5]

$$I(\lambda, T) = \frac{2 * \pi * h * c^2}{\lambda^5 * (e^{(h*c)/(\lambda*k_B*T)} - 1)}$$

   where $h = 6.626 * 10^-34 J * s$; $c = 2.997 * 10^8 m/s$; $k_B = 1.38 * 10^{-23} J/K$; $[\lambda] = m; [T] = K$

2. Photon Energy [5]

$$E = \frac{hc}{\lambda}$$

   where $[\lambda] = m$; $h = 6.626 * 10^{-34} J * s$; $c = 2.997 * 10^8 m/s$

3. Photoelectrons per exposure [5]

$$19100 \frac{photoelectrons}{s * mm^2} * \frac{1}{2.5^{M_V - 0}} * t \frac{sec}{exposure} * \pi * A$$

   where $M_V \equiv$ Apparent Magnitude; $t \equiv$ exposure time $[sec]$; $A \equiv$ Aperture Area $[m^2]$

3.4 Hardware

3.4.1 Models

1. Limit of Pixel Accuracy w/o intentional defocusing[5]

$$\int_{-0.5}^{0.5}\int_{-0.5}^{0.5}\sqrt{x^2+y^2}\,dx\,dy = 0.38$$

   where (x, y) is the number of pixels in the x and y direction respectively on the focal plane

2. Detection Limit[5]

$$A_{pixel} + 5 * \sigma_{pixel} * \frac{1}{\int_0^1\int_0^1 \frac{1}{2*\pi*\sigma_{PSF}} e^{\frac{-x^2+y^2}{2*\sigma_{PSF}}}\,dx\,dy}$$

   where $A_{pixel} \equiv$ mean value of pixels; $\sigma_{pixel} \equiv$ standard deviation of pixel values; $\sigma_{PSF} \equiv$ Point Spread Function (assumed Gaussian)

3. Estimated Noise Exclusion Angle in Cross-Boresight Axis

$$E_{cross-boresight} = \frac{A * E_{centroid}}{N_{pixel} * \sqrt{N_{star}}}$$

   where $A \equiv$ FOV, $E_{Centroid} \equiv$ average centroid accuracy [0.01 - 0.5], $N_{Pixels} \equiv$ number of pixels across plane; $N_{Stars} \equiv$ number of detected stars in image

4. Estimated Roll Accuracy

$$E_{roll} = atan(\frac{E_{centroid}}{0.3825 * N_{pixel}}) * \frac{1}{\sqrt{N_{stars}}}$$

3.5 Misc

3.5.1 Key Concepts

"The performance of the star tracker depends on the sensitivity to starlight, FOV, the accuracy of the star centroiding, the star detection threshold, the number of stars in the FOV, the internal star catalog, and the calibration" (Carl Christian Liebe)[5]

"Factors such as misalignment, aberration, instrument aging, and temperature effects could cause a departure of the star trackers from the ideal pinhole image model"[8] "...the factors that directly affect the results... include extraction error of star point position, principal point error, error of focal length, direction vectors of the navigation stars, and attitude solution algorithm error" [8]

- 2 Types of Error: Line of Sight Uncertainty and Relative Error
  - Line of Sight Uncertainty
    - Can't be calibrated out
    - Includes errors i.e., thermal expansion, launch effects, etc.
  - Relative Error
    - Ability to measure angles and characteristics between stars
    - Contains 4 categories: Calibration, S-Curve, NEA, Algorithmic
  - Calibration Error
    - errors in calibration

    * i.e, inaccurate focal length, intersection between boresight and focal plane, hardware flaws
    * optical distortion, chroma/astigmatism
  - S-Curve Error
    * pixel periodic errors
    * centroiding errors, homogeneity of pixel response, noise, dark current, PSF, brightness, etc.
    * Radiation has an effect on focal sensor; errors tend to get worse as sensitivity and dark-current gets more non-uniform
    * Can be calibrated by looking at a grid of evenly spaced pixels; transformation can be applied to correct errors; called an S-Curve Correction
  - NEA Error
    * Ability to get same attitude given same input
    * Exclusively reflects hardware
    * Photon noise, dark-current noise, read/amplifier noise, A/D resolution; can be estimated
    * Typical Roll Accuracy is 6-16x less accurate than cross-boresight accuracy
  - Algorithmic Error
    * Errors in algorithm i.e.,False stars, star catalog inaccuracies
- Extraction errors include...
  - background radiation
  - optical systems

- photoelectric detectors
- signal processing

- Subpixel processing is used for star extraction off the focal plane

3.5.2 Models

1. Right Ascension (α) and Declination (δ) to Direction Vector [8]

$$\overrightarrow{v} = \begin{bmatrix} cos\alpha * cos\delta \\ sin\alpha * cos\delta \\ sin\delta \end{bmatrix}$$

2. Position vector of star on focal plane from optical Lens [8]

$$w_i = \frac{1}{\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + f^2}} * \begin{bmatrix} -(x_i - x_0) \\ -(y_i - y_0) \\ f \end{bmatrix}$$

where $f \equiv$ the focal length of the camera and (x_0, y_0) is where the boresight meets the focal plane and (x_i, y_i) is the center pixel of the star on the focal plane

3. Fraction of Sky covered by FOV[5]

$$\frac{1 - cos(\frac{A}{2})}{2}$$

where $[A] \equiv$ deg

4. Number of stars brighter than a given magnitude, M; experimentally consistent[5]

$$N = 6.57 * e^{1.08*M}$$

5. Average number of stars in the FOV[5]

$$N_{FOV} = 6.57 * e^{1.08*M} * \frac{1 - cos(\frac{A}{2})}{2}$$

6. Pinhole Model to Reference Frame Transformation[5]

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} cos(atan2(x - x_0, y - y_0)) * cos(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \\ sin(atan2(x - x_0, y - y_0)) * cos(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \\ sin(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \end{bmatrix}$$

where $x, y \equiv$ focal plane coordinate; $(x_0, y_0) \equiv$ intersection of boresight and focal plane; $F \equiv$ Focal Length

7. Typical Form of Star Tracker Accuracy[5]

$$CrossBoresight_{RMS} = \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}}$$

8. Average distance from a star to the center of the focal plane[5]

$$\int_{-N/2}^{N/2} \int_{-N/2}^{N/2} \sqrt{x^2 + y^2}\, dx\, dy = 0.3825N$$

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