PARAMETERIZATION AND OPTIMIZATION OF THE ERROR PROPAGATION IN STAR TRACKERS FOR LEO SPACECRAFT

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Chapter 1

THE LITERATURE

1.1 The Fundamental Question

Observing and measuring nature are the basic tools we can use to characterize the world around us. We can use increasingly accurate sensors to measure different properties of the universe, however, we must come to terms with a fundamental reality. True values of these measurements are non-observable, that is, we can never verify our measurements in nature and are doomed to contain some error in our work. We can attribute these errors to our measurement techniques and the idea that we only have discrete tools trying to measure a continuous reality.

In space, a key property to measure is the attitude, or orientation, of our spacecraft. The direction we point in is imperative for a variety of reasons such as pointing for power generation, pointing to observe interesting phenomena, and pointing to survive the environment. One such sensor to measure our attitude is the star tracker; a clever technology that utilizes the positions of the stars in the universe to ascertain its attitude. The star tracker, similar to any other sensor, is ladened with pitfalls in its measurement process and, due to a variety of factors, is prone to error. Before we move forward, we must determine why analyzing the error propagation in a star tracker is worth doing; what will benefit from studying how star trackers err.

CubeSats, developed by the Cal Poly CubeSat Lab in 1999 by Prof. Jordi Puig-Suari and Prof. Bob Twiggs, have long been used as a bus for university programs and commercial businesses to fly experiments in Low Earth Orbit, or, LEO. CubeSats typically use COTS, or commercial-off-the-shelf, components as they're characterized by their ease and low-cost of development. While CubeSats have typically been used for simpler observation missions, the technology has been maturing and, as a consequence, as have their mission requirements. Attitude determination and control have been an increasingly popular requirement imposed on CubeSats as a means to accomplish more complex missions i.e., optical-communication demonstrations (SOTA, OSIRISv2)[1]. With the attitude determination requirements becoming increasingly stringent, the sensor selection space converges to only a few select types considering the constraints CubeSat developers are forced to deal with such as lack of payload volume, limited power options, and especially total cost. Star trackers are a prime example of a sensor that fits the mission criteria but are overlooked due to their cost. Star trackers for CubeSats can start at $50,000 and only go up from there; a cost usually equal to if not greater than the cost of a typical CubeSat. However, with attitude determination requirements becoming stricter, a need for a low-cost solution emerges.

The fundamental question to ask is whether or not a star tracker, traded on performance for cost, is a viable solution for CubeSats in the future. This thesis aims to answer the question by analyzing where errors can be allowed to exist based on mission requirements to minimize the total cost for a developer.

The literature offers several suggestions in analyzing the error propagation within the sensor such as hardware considerations, "thermal drift, optical aberration, detector noise"[2], and systematic algorithmic errors. However, before we can analyze where error is prone to occur, we must first understand what a star tracker is and how they work.

1.2 What Is a Star Tracker and How Do They Work?

A star tracker is an optical-based sensor that uses the position of stars to determine its attitude; it relies on the notion that relative star movement is low and can be effectively mapped to each other. This mapping exists within catalogs formulated over several years from different organizations such as the Yale Bright Star Catalog 5 (BSC5). The catalog contains key information such as Star ID, right ascension and declination in the celestial sphere, brightness magnitude, right ascension and declination motion, and other features.

Image of example YBSC5 entry

The catalog can then be modified to include additional information, called features, such as inter-star angles, ratio of triangle legs created by 3 stars, or even pyramidal information; the specific feature generated is determined by the specific identification method employed. A quaternion is paired with each of these entries indicating the attitude should this specific feature set be seen.

Image of modified catalog entry with feature set

Onboard the star tracker, images are captured of the stars and processed by the star tracker. During the centroiding process, the star tracker will read the image and determine where the stars are located within the image plane. It will then create the aforementioned features and store it in memory. During the identification process, the features in memory are recalled and compared to against the modified catalog where potential matches are found. Once a suitable match, typically characterized by high likelihood of similarity in combination with a filter, the attitude is recalled from the entry in the modified catalog. It is not uncommon for the star tracker to fail in finding a sufficiently confident match due to the nigh-infinite number of variations a single feature set can have due to image positioning and centroiding errors. To combat this, star trackers typically take multiple images per second to try to find better matches. This process repeats indefinitely and will generate high-accuracy determinations of attitude.

Image of generic star tracker process

The star tracker, in addition to its 2 main process phases, has 2 operating phases. If the star tracker has some *a priori* knowledge of its attitude in recent time, i.e., based on previous star tracker readings, then the star tracker only has to search a small portion of the modified catalog in order to find a match as the catalog can be ordered by attitude. If, however, the star tracker has no *a priori* knowledge of its attitude, i.e., post-detumble, then the star tracker must search the entire modified catalog. Because the modified catalog is so large and can contain a great number of combinations for an even greater number of stars, the process to determine the attitude can take significantly longer. This operating phase is typically called the *Lost in Space* problem and is a unique metric in comparing star tracker performance.

Image of lost in space and normal star tracker process

1.3 Sources of Error

Now that we understand what a star tracker is and how they work, it's imperative to describe the different sources of error we can come across during operation. According to Jia et al. (2010), "[t]he most important factors that affect star tracker accuracy include **thermal drift, optical aberration, detector noise, and systematic error of star image centroid estimation algorithm**"[2]. We'll be analyzing these errors, in addition to others, in order to totally understand where star trackers err.

This thesis aims to identify major concerns and sources of error in star tracker attitude determination such as hardware limitations, sensor noise, thermal distortions, etc. to identify limitations and completely characterize larger order errors and how they propagate to attitude accuracy during star tracker operations.

1.3.1 Hardware Limitations

The physical hardware of the star tracker will always be the largest source of error. Inaccuracies in the optical axis, focal plane flatness, and lens distortion all contribute to errors in the centroiding process, leading to errors in attitude determination. Camera calibration is a common technique to reduce hardware inconsistencies propagating error through the attitude determination model. Sun et al (2013) proposes a model to handle the uncertainty of where the incident light rays from the navigation stars fall on the focal plane. [3]

$$\xi_A = \left(\frac{\left(\frac{f+\Delta f+\Delta s * tan(\theta)}{cos(\theta+\beta_{ri})} * sin\left(\beta_{ri}\right) + \frac{\Delta s}{cos(\theta)} + \Delta x + \Delta d \right)}{f} \right) - arctan\left(\frac{\Delta s}{f cos(\theta)} \right) - \beta_{ri} \tag{1.1}$$

where

ξ_A defines the star tracker measurement accuracy

f represents the focal length of the star tracker

Δs represents the deviation of the optical axis with respect to the boresight

Δx represents the star point-extraction error

Δd represents radial distortion in the lens

β_{ri} represents the angle between the optical axis and incident light ray

θ represents in the inclination of the focal plane

While Sun et al. (2013) devised this model to generate estimated attitude determination accuracy, it should be noted that only hardware limitations were considered. In addition to the inaccuracies of the focal system on the star tracker, additional considerations from Liu et al. (2010) address radial distortion, decentering, and thin prism distortions [4]

$$\begin{bmatrix} \delta_u(u', v') \\ \delta_v(u', v') \end{bmatrix} = \begin{bmatrix} u' \\ v' \end{bmatrix} - \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} (g_1 + g_3)u'^2 + g_4 u'v' + g_1 v'^2 + \kappa u'(u'^2 + v'^2) \\ g_2 u'^2 + g_3 u'v' + (g_2 + g_4)v'^2 + \kappa v'(u'^2 + v'^2) \end{bmatrix} \tag{1.2}$$

where

$[g_1, g_2, g_3, g_4]$ each represent a different distortion property.

1.3.2 CCD Noise

Sensor noise, specifically from the CCD, is another factor we must analyze when determining how our system and the environment affect attitude determination. CCD Noise often appears as partially illuminated pixels in the absence of a real incident light and is caused by the noise in the Analog to Digital Converter during the image capture process. G.E. Healey et al. (1994) proposes a model that relates the irradiance, dark current, and quantization to the expected noise level in a given pixel[5].

$$D(a, b) = \mu(a, b) + N(a, b) \tag{1.3}$$

$$N(a, b) = N_I(a, b) + N_C(a, b) \tag{1.4}$$

where

D represents the digital value of a pixel at location (a, b) on the focal plane

$\mu(a, b)$ represents the expected digital value based on standard camera principles

$N(a, b)$ represents the noise of a given pixel

$N_I(a, b)$ represents the noise part that varies with the image

$N_C(a, b)$ represents the noise part that is invariant of the electrons captured

CCD Noise is composed of several different sources, as hinted at in equation 1.4 such as unintentional light scattering from other bodies, analog-digital converter noise, and Dark Current. Dark Current is a form of CCD Noise that is a result of residual electric current in the pixel buckets while there is no incident light being projected on the focal plane. Dark Current can worsen with age of the sensor, among other factors.

G.E. Healey et al. (1994) proposes a model to account to account for different noises in CCD sensors.

$$D_C(a, b) = \left(I(a, b) + \frac{N_S(a, b)}{\hat{K}(a, b)} + \frac{N_R(a, b)}{\hat{K}(a, b)} \right) A + \frac{N_Q(a, b)}{\hat{K}(a, b)} \tag{1.5}$$

where

D_C represents the corrected digital value of a pixel at location (a, b)

I represents the irradiance on a given pixel

N_S represents shot noise, a type of noise that deals with the uncertainty of the number of electrons captured

N_R represents zero-mean amplifier noise

N_Q represents noise from the quantization process

$$\hat{K} \text{ represents the estimate of number of captured photoelectrons}$$

CCD Noise varies between cameras and can come in the form of several kinds of noises. To reduce noise (and improve centroiding), it's important that the noise be calculated to set a *Noise Floor*, or a filter that attempts to remove as much noise from the image without removing stars. A typical Noise Floor can be calculated using the average and standard deviation of the brightness of the pixels in the image [6]:

$$NF = \bar{B} + 5 * \sigma_B \tag{1.6}$$

where

$$\bar{B} \text{ represents the average brightness value}$$
$$\sigma_B \text{ represents the standard deviation in brightness}$$

The Noise Floor is a parameter that is typically calculated during operation and will change between image captures. This ensures the Noise Floor is not over-filtering clean photos or under-filtering noisy photos.

1.3.3 Thermal Effects

One of the larger effects from the environment is the rapidly changing thermal environment. In Low-Earth Orbit, or, LEO, spacecraft can experience a temperature swing between -65C and +125C[7] which can cause thermal stresses on all hardware. For star trackers in particular, the lens is of interest when analyzing thermal cycles. For one, the lens can change shape, lengthening or shortening the focal length in times of extreme heat or cold respectively. Jamieson et al. (1981) thoroughly describes the relation between optical systems and thermal effects, especially noting how the focal length changes with temperature [8]

$$x_f = \frac{1}{f}\frac{df}{dt} = x_g - \frac{1}{n - n_{air}}\left(\frac{dn}{dt} - n\frac{dn_{air}}{dt}\right) \tag{1.7}$$

$$n_t - 1 = (n_{15} - 1)\left(\frac{1.0549}{1 + 0.00366t}\right) \tag{1.8}$$

$$(n_{15} - 1) * 10^8 = 8342.1 + \frac{2406030}{130 - \nu^2} + \frac{15996}{38.9 - \nu^2} \tag{1.9}$$

$$n_{abs} = n_{rel} n_{air} \tag{1.10}$$

$$x_g = \frac{1}{R_1}\frac{dR_1}{dt} = \frac{1}{R_2}\frac{dR_2}{dt} \tag{1.11}$$

where

f is the focal length

x_g is the change of surface radii of the lens with respect to temperature

n is the refractive index at a given temperature

n_{air} is the refractive index of air $\equiv 1.0$

Thermal expansion will directly affect the hardware and, in turn, will influence centroiding processes. Understanding how best to approach minimal thermal deformation in the lens can help lead to greater consistency if not accuracy with star tracker measurements.

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