PARAMETERIZATION AND OPTIMIZATION OF THE ERROR PROPAGATION IN STAR TRACKERS IN THE CONTEXT OF LEO SPACECRAFT

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ABSTRACT

Parameterization and Optimization of the Error Propagation in Star Trackers in the context of LEO Spacecraft

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Chapter 1

RELEVANT MODELS AND SOURCES

1.1 Models

List of models group by error source

1.1.1 CCD Noise

1. Typical noise floor size[2]

$$NF = \bar{B} + 5 * \sigma_B$$

where $\bar{B} \equiv$ the average brightness value and $\sigma_B \equiv$ the standard deviation in brightness

1.1.2 Thermal Distortion

1.

1.1.3 Radiation

1. Radiation from a black body at a given wavelength and temperature (on lens)[2]

$$I(\lambda, T) = \frac{2 * \pi * h * c^2}{\lambda^5 * \left(e^{(h*c)/(\lambda * k_B * T)} - 1\right)}$$

where $h = 6.626 * 10^-34 J * s$; $c = 2.997 * 10^8 m/s$; $k_B = 1.38 * 10^{-23} J/K$; $[\lambda] = m; [T] = K$

2. Photon Energy [2]

$$E = \frac{hc}{\lambda}$$

where $[\lambda] = m$; $h = 6.626 * 10^{-34} J * s$; $c = 2.997 * 10^{8} m/s$

3. Photoelectrons per exposure [2]

$$19100 \frac{photoelectrons}{s * mm^2} * \frac{1}{2.5^{M_V - 0}} * t \frac{sec}{exposure} * \pi * A$$

where $M_V \equiv$ Apparent Magnitude; $t \equiv$ exposure time $[sec]$; $A \equiv$ Aperture Area $[m^2]$

1.1.4 Hardware

1. Limit of Pixel Accuracy w/o intentional defocusing[2]

$$\int_{-0.5}^{0.5} \int_{-0.5}^{0.5} \sqrt{x^2 + y^2}\, dx\, dy = 0.38$$

where (x, y) is the number of pixels in the x and y direction respectively on the focal plane

2. Detection Limit[2]

$$A_{pixel} + 5 * \sigma_{pixel} * \frac{1}{\int_0^1 \int_0^1 \frac{1}{2*\pi*\sigma_{PSF}} e^{\frac{-x^2+y^2}{2*\sigma_{PSF}}}\, dx\, dy}$$

where $A_{pixel} \equiv$ mean value of pixels; $\sigma_{pixel} \equiv$ standard deviation of pixel values; $\sigma_{PSF} \equiv$ Point Spread Function (assumed Gaussian)

3. Estimated Noise Exclusion Angle in Cross-Boresight Axis

$$E_{cross-boresight} = \frac{A * E_{centroid}}{N_{pixel} * \sqrt{N_{star}}}$$

where $A \equiv$ FOV, $E_{Centroid} \equiv$ average centroid accuracy [0.01 - 0.5], $N_{Pixels} \equiv$ number of pixels across plane; $N_{Stars} \equiv$ number of detected stars in image

4. Estimated Roll Accuracy

$$E_{roll} = atan(\frac{E_{centroid}}{0.3825 * N_{pixel}}) * \frac{1}{\sqrt{N_{stars}}}$$

1.1.5 Misc

1. Right Ascension (α) and Declination (δ) to Direction Vector [3]

$$\overrightarrow{v} = \begin{bmatrix} cos\alpha * cos\delta \\ sin\alpha * cos\delta \\ sin\delta \end{bmatrix}$$

2. Position vector of star on focal plane from optical Lens [3]

$$w_i = \frac{1}{\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + f^2}} * \begin{bmatrix} -(x_i - x_0) \\ -(y_i - y_0) \\ f \end{bmatrix}$$

where $f \equiv$ the focal length of the camera and (x_0, y_0) is where the boresight meets the focal plane and (x_i, y_i) is the center pixel of the star on the focal plane

3. Fraction of Sky covered by FOV[2]

$$\frac{1 - cos(\frac{A}{2})}{2}$$

where $[A] \equiv$ deg

4. Number of stars brighter than a given magnitude, M; experimentally consistent[2]

$$N = 6.57 * e^{1.08*M}$$

5. Average number of stars in the FOV[2]

$$N_{FOV} = 6.57 * e^{1.08*M} * \frac{1 - cos(\frac{A}{2})}{2}$$

6. Pinhole Model to Reference Frame Transformation[2]

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} cos(atan2(x - x_0, y - y_0)) * cos(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \\ sin(atan2(x - x_0, y - y_0)) * cos(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \\ sin(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \end{bmatrix}$$

where $x, y \equiv$ focal plane coordinate; $(x_0, y_0) \equiv$ intersection of boresight and focal plane; $F \equiv$ Focal Length

7. Typical Form of Star Tracker Accuracy[2]

$$CrossBoresight_{RMS} = \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}}$$

8. Average distance from a star to the center of the focal plane[2]

$$\int_{-N/2}^{N/2} \int_{-N/2}^{N/2} \sqrt{x^2 + y^2}\, dx\, dy = 0.3825N$$

1.2 KEY CONCEPTS

"The performance of the star tracker depends on the sensitivity to starlight, FOV, the accuracy of the star centroiding, the star detection threshold, the number of stars in the FOV, the internal star catalog, and the calibration" (Carl Christian Liebe)[2]

"Factors such as misalignment, aberration, instrument aging, and temperature effects could cause a departure of the star trackers from the ideal pinhole image model"[3] "...the factors that directly affect the results... include extraction error of star point position, principal point error, error of focal length, direction vectors of the navigation stars, and attitude solution algorithm error" [3]

- 2 Types of Error: Line of Sight Uncertainty and Relative Error
  - Line of Sight Uncertainty
    - Can't be calibrated out
    - Includes errors i.e., thermal expansion, launch effects, etc.
  - Relative Error
    - Ability to measure angles and characteristics between stars
    - Contains 4 categories: Calibration, S-Curve, NEA, Algorithmic
  - Calibration Error
    - errors in calibration
    - i.e, inaccurate focal length, intersection between boresight and focal plane, hardware flaws

    - optical distortion, chroma/astigmatism
  - S-Curve Error
    - pixel periodic errors
    - centroiding errors, homogeneity of pixel response, noise, dark current, PSF, brightness, etc.
    - Radiation has an effect on focal sensor; errors tend to get worse as sensitivity and dark-current gets more non-uniform
    - Can be calibrated by looking at a grid of evenly spaced pixels; transformation can be applied to correct errors; called an S-Curve Correction
  - NEA Error
    - Ability to get same attitude given same input
    - Exclusively reflects hardware
    - Photon noise, dark-current noise, read/amplifier noise, A/D resolution; can be estimated
    - Typical Roll Accuracy is 6-16x less accurate than cross-boresight accuracy
  - Algorithmic Error
    - Errors in algorithm i.e.,False stars, star catalog inaccuracies
- Extraction errors include…
  - background radiation
  - optical systems
  - photoelectric detectors
  - signal processing
- Subpixel processing is used for star extraction off the focal plane

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[3] T. Sun, F. Xing, and Z. You. Optical system error analysis and calibration method of high-accuracy star trackers. *Sensors (Basel, Switzerland)*, 13:4598–623, 04 2013.