PARAMETERIZATION AND OPTIMIZATION OF THE ERROR PROPAGATION IN STAR TRACKERS IN THE CONTEXT OF LEO SPACECRAFT

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by

Gagandeep Thapar

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COMMITTEE MEMBERSHIP

TITLE:	Parameterization and Optimization of the Error Propagation in Star Trackers in the context of LEO Spacecraft
AUTHOR:	Gagandeep Thapar
DATE SUBMITTED:	June 2023
COMMITTEE CHAIR:	Leo Torres, Ph.D. Professor of Aerospace Engineering
COMMITTEE MEMBER:	John Bellardo, Ph.D. Professor of Computer Science
COMMITTEE MEMBER:	Kira Abercromby, Ph.D. Professor of Aerospace Engineering
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ABSTRACT

Parameterization and Optimization of the Error Propagation in Star Trackers in the context of LEO Spacecraft

Gagandeep Thapar

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ACKNOWLEDGMENTS

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Chapter 1

RELEVANT MODELS AND SOURCES

"The performance of the star tracker depends on the sensitivity to starlight, FOV, the accuracy of the star centroiding, the star detection threshold, the number of stars in the FOV, the internal star catalog, and the calibration" (Carl Christian Liebe)[2]

Accuracy performance of star trackers - a tutorial[2]

1. Limit of Pixel Accuracy w/o intentional defocusing

$$\int_{-0.5}^{0.5}\int_{-0.5}^{0.5}\sqrt{x^2+y^2}\,dx\,dy = 0.38$$

2. Radiation from a black body at a given wavelength and temperature (on Lens)

$$I(\lambda,T) = \frac{2*\pi*h*c^2}{\lambda^5*(e^{\frac{h*c}{\lambda*k_B*T}}-1)}$$

where

$$h = 6.626{*}10^-34J{*}s; c = 2.997{*}10^8m/s; k_B = 1.38{*}10^{-23}J/K; [\lambda] = m; [T] = K$$

3. Photon Energy

$$E = \frac{hc}{\lambda}$$

where

$$[\lambda] = m; h = 6.626*10^{-34}J*s; c = 2.997*10^8m/s$$

4. photoelectrons per exposure

$$19100\frac{photoelectrons}{s * mm^2} * \frac{1}{2.5^{M_V - 0}} * t\frac{sec}{exposure} * \pi * A$$

where

$$M_V \equiv Apparent\ Magnitude; [t] \equiv exposure\ time; A \equiv Aperture\ Area$$

5. Typical noise floor size

$$NF = AvgBrightness + 5 * \sigma_{brightness}$$

6. Detection Limit

$$A_{pixel} + 5 * \sigma_{pixel} * \frac{1}{\int_0^1 \int_0^1 \frac{1}{2*\pi*\sigma_{PSF}} e^{\frac{-x^2+y^2}{2*\sigma_{PSF}}}\, dx\, dy}$$

where $A_{pixel} \equiv meanvalueofpixels; \sigma_{pixel} \equiv standarddeviationofpixelvalues; \sigma_{PSF} \equiv$ $Point\ Spread\ Function\ assuming\ Gaussian$

7. Fraction of Sky covered by FOV

$$\frac{1 - cos(\frac{A}{2})}{2}$$

where

$$[A] = deg$$

8. Number of stars brighter than a given magnitude, M; experimentally consistent

$$N = 6.57 * e^{1.08*M}$$

9. Average number of stars in the FOV

$$N_{FOV} = 6.57 * e^{1.08*M} * \frac{1 - cos(\frac{A}{2})}{2}$$

10. Pinhole Model to Reference Frame Transformation

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} cos(atan2(x - x_0, y - y_0)) * cos(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \\ sin(atan2(x - x_0, y - y_0)) * cos(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \\ sin(\frac{\pi}{2} - atan(\sqrt{(\frac{x-x_0}{F})^2 + (\frac{y-y_0}{F})^2})) \end{bmatrix}$$

where $x, y \equiv focalplanecoordinate; x_0, y_0 \equiv intersectionofboresightandfocalplane; F \equiv Focal\ Length$

11. Typical Form of Star Tracker Accuracy

$$CrossBoresight_{RMS} = \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}}$$

12. Estimated Noise Exclusion Angle in Cross-Boresight Axis

$$E_{cross-boresight} = \frac{A * E_{centroid}}{N_{pixel} * \sqrt{N_{star}}}$$

$whereA == FOV, E_{Centroid} == averagecentroidaccuracy[0.01{-}0.5], N_{Pixels} == numberofpixelsacrossplane; N_{Stars} == numberofdetectedstarsinimage$

13. Average distance from a star to the center of the focal plane

$$\int_{-N/2}^{N/2} \int_{-N/2}^{N/2} \sqrt{x^2 + y^2}\, dx\, dy = 0.3825N$$

14. Estimated Roll Accuracy

$$E_{roll} = atan(\frac{E_{centroid}}{0.3825 * N_{pixel}}) * \frac{1}{\sqrt{N_{stars}}}$$

KEY CONCEPTS:

- 2 Types of Error: Line of Sight Uncertainty and Relative Error
  - Line of Sight Uncertainty
    * Can't be calibrated out
    * Includes errors i.e., thermal expansion, launch effects, etc.
  - Relative Error
    * Ability to measure angles and characteristics between stars
    * Contains 4 categories: Calibration, S-Curve, NEA, Algorithmic
  - Calibration Error
    * errors in calibration
    * i.e, inaccurate focal length, intersection between boresight and focal plane, hardware flaws
    * optical distortion, chroma/astigmatism
  - S-Curve Error
    * pixel periodic errors
    * centroiding errors, homogeneity of pixel response, noise, dark current, PSF, brightness, etc.
    * Radiation has an effect on focal sensor; errors tend to get worse as sensitivity and dark-current gets more non-uniform

    * Can be calibrated by looking at a grid of evenly spaced pixels; transformation can be applied to correct errors; called an S-Curve Correction

- NEA Error
    * Ability to get same attitude given same input
    * Exclusively reflects hardware
    * Photon noise, dark-current noise, read/amplifier noise, A/D resolution; can be estimated
    * Typical Roll Accuracy is 6-16x less accurate than cross-boresight accuracy

- Algorithmic Error
    * Errors in algorithm i.e.,False stars, star catalog inaccuracies

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[1] Cal Poly Github. `http://www.github.com/CalPoly`.

[2] C. Liebe. Accuracy performance of star trackers - a tutorial. *IEEE Transactions on Aerospace and Electronic Systems*, 38(2):587–599, 2002.