Module 1: Sets and Operations

1. Introduction to Sets (Week -1)

Basics of Sets

Definition of a Set:

A set is a fundamental mathematical concept that represents a collection of distinct objects or elements. These elements can be anything—numbers, letters, objects, or even other sets. The elements in a set are considered unordered, which means that the arrangement of elements within a set doesn't matter. Sets are commonly denoted by uppercase letters (e.g., A, B, C).

Elements of a Set:

The individual objects or items within a set are called its elements. Elements are often represented by lowercase letters (e.g., a, b, c). An element either belongs to a set or does not belong to it; there is no in-between. In set notation, we use braces {} to enclose the elements of a set, separated by commas. For example:

Set A containing elements 1, 2, and 3 is represented as: $A = \{1, 2, 3\}$

Set B containing elements "apple," "banana," and "cherry" is represented as: B = {"apple", "banana", "cherry"}

Set Notation:

Set notation is a standardized way of representing sets and set operations using mathematical symbols and conventions. Here are some common notations and symbols used with sets:

Element Membership ($\in$): This symbol, $\in$, is used to indicate that an element belongs to a particular set. For example, if "x" belongs to set A, we write it as: $x \in A$.

Element Notation ($\notin$): This symbol, $\notin$, is used to indicate that an element does not belong to a particular set. For example, if "y" does not belong to set B, we write it as: $y \notin B$.

Set Equality (=): Two sets are equal if they have exactly the same elements. For example, if $A = \{1, 2, 3\}$ and $B = \{1, 2, 3\}$, then $A = B$.

Empty Set ($\emptyset$): The empty set, denoted as $\emptyset$, is a set with no elements. It is also referred to as the null set. For example, $\emptyset = \{\}$.

Subset ($\subseteq$): A set A is said to be a subset of another set B (denoted as $A \subseteq B$) if every element of A is also an element of B. $A \subseteq B$ if, for every x, if $x \in A$, then $x \in B$.

Proper Subset (⊂): A set A is a proper subset of B (denoted as $A \subset B$) if A is a subset of B, but A is not equal to B.

Universal Set (U): The universal set, often denoted as U, represents the set that contains all elements under consideration in a particular context.

Examples:

Let's illustrate these concepts with some examples:

Set A representing the first three natural numbers: A = {1, 2, 3}

Set B representing vowels in the English alphabet: B = {"a", "e", "i", "o", "u"}

Set C representing even numbers: C = {2, 4, 6, 8, 10}

Set D representing prime numbers less than 20: D = {2, 3, 5, 7, 11, 13, 17, 19}

In these examples, you can see how sets are defined, how their elements are represented, and how set notation is used to describe relationships between sets and elements.

Exercise : 1.1

Practice Problems:

1. Define a set E containing the first five even numbers.
2. Create a set F containing the names of days of the week.
3. Determine if the set G = {2, 4, 6} is a subset of the set H = {1, 2, 3, 4, 5, 6}.
4. Define the empty set L using set notation.
5. Determine if the set M = {red, blue, green} is a proper subset of the set N = {red, blue, green, yellow}.

Set Operations

Set Operations

1. Union (U)

Definition:

- The union of two sets, A and B, denoted as $A \cup B$, is a new set that contains all the elements that belong to either A or B or both.
- In other words, an element x is in $A \cup B$ if it is in A, in B, or in both A and B.

Example:

- Let A = {1, 2, 3} and B = {3, 4, 5}.

- The union of sets A and B, $A \cup B$, is {1, 2, 3, 4, 5}.
- Notice that duplicate elements are not repeated in the union.

Set Notation:

- $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

2. Intersection (∩)

Definition:

- The intersection of two sets, A and B, denoted as $A \cap B$, is a new set that contains all the elements that belong to both A and B.
- In other words, an element x is in $A \cap B$ if it is in both A and B.

Example:

- Let A = {1, 2, 3} and B = {3, 4, 5}.
- The intersection of sets A and B, $A \cap B$, is {3}.
- Only the element 3 is common to both sets.

Set Notation:

- $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

3. Complement (')

Definition:

- The complement of a set A, denoted as A', is a new set that contains all the elements from the universal set U that do not belong to A.
- In other words, an element x is in A' if it is not in A.

Example:

- Let U be the universal set of natural numbers, and A = {1, 2, 3}.
- The complement of set A, A', is {4, 5, 6, ...} (all natural numbers except 1, 2, and 3).

Set Notation:

- $A' = \{x \mid x \in U \text{ and } x \notin A\}$

Properties of Set Operations:

Set operations, including union, intersection, and complement, follow certain fundamental properties that are useful for solving problems:

1. Commutative Property:
   - $A \cup B = B \cup A$ (Union is commutative)
   - $A \cap B = B \cap A$ (Intersection is commutative)
2. Associative Property:
   - $(A \cup B) \cup C = A \cup (B \cup C)$ (Union is associative)
   - $(A \cap B) \cap C = A \cap (B \cap C)$ (Intersection is associative)
3. Distributive Property:
   - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Intersection distributes over union)
   - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Union distributes over intersection)
4. Identity Element:
   - $A \cup \emptyset = A$ (Union with an empty set is the set itself)
   - $A \cap U = A$ (Intersection with the universal set is the set itself)

MATLAB Examples:

Let's use MATLAB to demonstrate these set operations:

Example 1: Union in MATLAB

```
% Define sets A and B
A = [1, 2, 3]; B = [3, 4, 5];
% Calculate the union of A and B
union_result = union(A, B);
disp(union_result);
```

Example 2: Intersection in MATLAB

```
% Define sets A and B
A = [1, 2, 3]; B = [3, 4, 5];
% Calculate the intersection of A and B
intersection_result = intersect(A, B); disp(intersection_result);
```

Example 3: Complement in MATLAB

```
% Define set A and the universal set U

A = [1, 2, 3];

U = 1:10;

% Universal set of natural numbers 1 to 10

% Calculate the complement of A

complement_result = setdiff(U, A);

disp(complement_result);
```

These MATLAB examples illustrate how to perform union, intersection, and complement operations on sets. You can adapt and expand upon these examples for more complex sets and operations as needed.

Exercise : 1.2

1. Given sets X = {1, 2, 3, 4} and Y = {3, 4, 5, 6}, calculate $X \cup Y$ and $X \cap Y$.
2. Let A = {a, b, c, d} and B = {c, d, e, f}. Find $A \cup B$ and $A \cap B$.
3. Define set C = {red, blue, green} and set D = {green, yellow, orange}. Calculate $C \cup D$ and $C \cap D$.
4. Consider the universal set U = {a, b, c, d, e, f, g, h, i, j}. If A = {a, c, e, g, i}, find the complement of A, A'.
5. Given sets P = {1, 2, 3, 4, 5} and Q = {4, 5, 6, 7}, determine the complement of Q, Q'.
6. Given A = {1, 2, 3, 4} and B = {3, 4, 5, 6}, find $A \cup B$ and $A \cap B$.
7. If A = {x, y, z}, B = {y, z, w}, and C = {w, x}, find $(A \cap B) \cup C$.
8. For A = {a, b, c, d, e} and B = {c, d, e, f}, find $A \cap B$.
9. Let U = {1, 2, 3, 4, 5} be the universal set. If A = {1, 2, 3} and B = {3, 4, 5}, find $A' \cup B$.

MATLAB Practice Problems:

1. Find $A \cup B$, $A \cap B$, and A' for A = {1, 2, 3} and B = {2, 3, 4}.
2. Given A = {a, b, c} and B = {b, c, d}, find $A \cup B$ and $A \cap B$.
3. If U = {1, 2, 3, 4, 5} is the universal set, and A = {1, 2, 3}, find A'.

4. Calculate (A ∩ B) ∪ C for A = {1, 2}, B = {2, 3}, and C = {3, 4}.

2. Advanced Sets (Week -2)

Advanced Sets

1.Finite Sets:

A finite set is a set that contains a specific, countable number of elements.

Example: The set {1, 2, 3, 4, 5} is a finite set because it contains 5 elements.

Finite sets are often denoted using curly braces {}.

Problem: Let A = {2, 4, 6, 8} and B = {3, 6, 9, 12}. Find the union and intersection of sets A and B.

Solution:

- Union (A ∪ B): {2, 3, 4, 6, 8, 9, 12}
- Intersection (A ∩ B): {6}

Problem: Given C = {red, green, blue} and D = {blue, orange, purple}, find the union and intersection of sets C and D.

Solution:

- Union (C ∪ D): {red, green, blue, orange, purple}
- Intersection (C ∩ D): {blue}

Practice Problems:

1. Calculate the union of sets X = {1, 2, 3, 4} and Y = {3, 4, 5, 6}.
2. Determine the intersection of sets P = {a, b, c} and Q = {b, c, d}.
3. Let E = {1, 2, 3, 4, 5} and F = {4, 5, 6, 7, 8}. Find E ∪ F and E ∩ F.
4. Given sets G = {apple, banana, cherry} and H = {cherry, date, fig}, compute G ∪ H and G ∩ H.
5. Calculate the union of sets M = {10, 20, 30} and N = {20, 30, 40}.
6. Determine the intersection of sets R = {x, y, z} and S = {y, z, w}.
7. Let U = {a, b, c, d, e} and V = {c, d, e, f, g}. Find U ∪ V and U ∩ V.
8. Given sets W = {red, green, blue, yellow} and X = {blue, orange, yellow, pink}, compute W ∪ X and W ∩ X.

2. Power Set:

The power set of a set A is the set of all possible subsets of A, including the empty set and A itself.

If set A has n elements, its power set has 2^n elements.

Example: If A = {1, 2}, then the power set of A is P(A) = {{}, {1}, {2}, {1, 2}}.

3. Cardinality of Finite Sets:

The cardinality of a set is the number of elements it contains.

For finite sets, the cardinality is simply the count of elements in the set.

Example: If A = {3, 7, 9}, then the cardinality of A is |A| = 3.

Problem: Calculate the cardinality of set A = {7, 14, 21, 28}.

Solution:

- Cardinality of A (|A|): 4

Problem: Find the cardinality of set B = {p, q, r, s, t, u}.

Solution:

- Cardinality of B (|B|): 6

Practice Problems:

1. Determine the cardinality of set C = {apple, banana, cherry, date}.
2. Calculate the cardinality of set D = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
3. Find the cardinality of set E = {x, y, z}.
4. Determine the cardinality of set F = {red, green, blue, orange, yellow}.

4. Cartesian Product:

The Cartesian product of two sets A and B, denoted as A × B, is a set of all possible ordered pairs (a, b) where a is in A and b is in B.

Example: If A = {1, 2} and B = {a, b}, then A × B = {(1, a), (1, b), (2, a), (2, b)}.

Problem: Compute the Cartesian product of sets A = {1, 2} and B = {a, b}.

Solution:

- Cartesian product (A × B): {(1, a), (1, b), (2, a), (2, b)}

Problem: Calculate the Cartesian product of sets X = {red, blue} and Y = {circle, square}.

Solution:

- Cartesian product (X × Y): {(red, circle), (red, square), (blue, circle), (blue, square)}

Practice Problems:

1. Find the Cartesian product of sets P = {1, 2, 3} and Q = {a, b}.
2. Calculate the Cartesian product of sets R = {x, y, z} and S = {0, 1, 2}.
3. Compute the Cartesian product of sets G = {red, green} and H = {circle, triangle}.

Properties of Sets:

Sets in mathematics have several properties and operations that are essential for understanding and working with them effectively. These properties help you manipulate sets and solve a wide range of mathematical problems. Here are some of the key properties of sets:

1. Commutative Property:

- Union (∪): The union of sets A and B is commutative, which means A ∪ B = B ∪ A. In other words, the order in which you combine sets in a union operation does not affect the result.
- Intersection (∩): The intersection of sets A and B is also commutative, meaning A ∩ B = B ∩ A.

2. Associative Property:

- Union (∪): The union of sets A, B, and C is associative, which means (A ∪ B) ∪ C = A ∪ (B ∪ C). You can group sets in any order when performing union operations.
- Intersection (∩): The intersection of sets A, B, and C is also associative, meaning (A ∩ B) ∩ C = A ∩ (B ∩ C).

3. Identity Property:

- Union (∪): The identity element for union is the empty set (∅). For any set A, A ∪ ∅ = A.
- Intersection (∩): The identity element for intersection is the universal set (U). For any set A, A ∩ U = A.

4. Complement Property:

- Complement ('): The complement of a set A (A') contains all elements not in A. For any set A, A ∪ A' = U (universal set), and A ∩ A' = ∅ (empty set).

5. Distributive Laws:

- Sets obey the distributive laws, just like arithmetic operations.
  - Union over Intersection: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
  - Intersection over Union: A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

6. Subset Property:

- Set A is a subset of set B (denoted as A ⊆ B) if every element of A is also in B. In other words, A is contained within B.

7. Superset Property:

- Set A is a superset of set B (denoted as A ⊇ B) if every element of B is also in A. In other words, A contains B.

8. Empty Set Property:

- The empty set (∅) is a subset of every set. For any set A, ∅ ⊆ A.

9. Universal Set Property:

- The universal set (U) is a superset of every set. For any set A, A ⊆ U.

Practice Problems:

1. Calculate the union of sets A = {1, 2, 3} and B = {3, 4, 5}.
2. Determine the intersection of sets C = {a, b, c, d} and D = {c, d, e, f}.
3. Let P = {10, 20, 30} and Q = {20, 30, 40}. Find P ∪ Q and P ∩ Q.
4. Given sets G = {apple, banana, cherry} and H = {cherry, date, fig}, compute G ∪ H and G ∩ H.
5. Calculate the union of sets M = {red, green, blue} and N = {blue, orange, yellow}.
6. Determine the intersection of sets R = {x, y, z} and S = {y, z, w}.
7. Let U = {a, b, c, d, e} and V = {c, d, e, f, g}. Find U ∪ V and U ∩ V.
8. Given sets W = {red, green, blue, yellow} and X = {blue, orange, yellow, pink}, compute W ∪ X and W ∩ X.

MATLAB

Finite Sets:

1. Given two sets A = [1, 2, 3] and B = [2, 3, 4, 5], write MATLAB code to find their union and display the result.
2. Create two sets C and D in MATLAB, and then write code to calculate their intersection. C = [10, 20, 30] and D = [20, 30, 40].

Power Set:

3. Define a set E = [1, 2, 3] in MATLAB and write code to compute its power set. Display the power set as a cell array.
4. Create a set F = ["apple", "banana", "cherry"] in MATLAB. Write code to find its power set and count the number of subsets it contains.

Cardinality of Finite Sets:

5. Define a set G = [5, 10, 15, 20, 25] in MATLAB. Write code to calculate its cardinality and display the result.
6. Given a set H = ["cat", "dog", "fish", "bird"] in MATLAB, write code to determine its cardinality and store it in a variable.

Cartesian Product:

7. Create two sets I = [1, 2] and J = ["a", "b"] in MATLAB. Write code to find their Cartesian product and display it as a cell array of tuples.
8. MATLAB Question 8: Define sets K = [x, y, z] and L = [1, 2, 3] in MATLAB. Write code to compute their Cartesian product and display the result.

Properties of Sets:

9. Given sets M = [2, 4, 6] and N = [4, 6, 8] in MATLAB, write code to find their union and intersection. Display both results.
10. Create two sets P = ["red", "green", "blue"] and Q = ["blue", "orange", "yellow"] in MATLAB. Write code to determine their union and intersection, and display the outcomes.

Exercise : 1.3

1. Find the power set of set A = {1, 2, 3}.
2. Calculate the cardinality of the set B = {x, y, z, w}.
3. Compute the Cartesian product A × B for A = {a, b} and B = {1, 2, 3}.
4. Prove that A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) for sets A, B, and C.

5. Find the power set of set $C = \{1, 2, 3, 4\}$.
6. If $|D| = 5$ and $|E| = 3$, find $|D \cap E|$.
7. Calculate the Cartesian product $X \times Y$ for $X = \{a, b, c\}$ and $Y = \{1, 2\}$.
8. Prove that $(A \cup B)' = A' \cap B'$ for sets A and B.

3. Visualizing Sets (Week -3)

Visualizing Sets: Venn Diagrams

A Venn diagram is an illustration that shows logical relationships between two or more sets (grouping items). Venn diagrams use circles (both overlapping and non-overlapping) or other shapes. Commonly, Venn diagrams show how given items are similar and different. Despite Venn diagrams with 2 or 3 circles being the most common type, there are also many diagrams with a larger number of circles (5,6,7,8,10…).

The general formula for a Venn diagram is

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$,

where n denotes the number of elements in a set.

This formula shows the relationship between two overlapping sets A and B.

The intersection of A and B is represented by $A \cap B$,

while the union of A and B is represented by $A \cup B$.

the formula for 3 circles Venn diagram:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

Here's an example problem:

Suppose that in a town, 800 people are selected by random types of sampling methods. 280 go to work by car only, 220 go to work by bicycle only and 140 use both ways – sometimes go with a car and sometimes with a bicycle. Here are some important questions we will find the answers:

How many people go to work by car only?

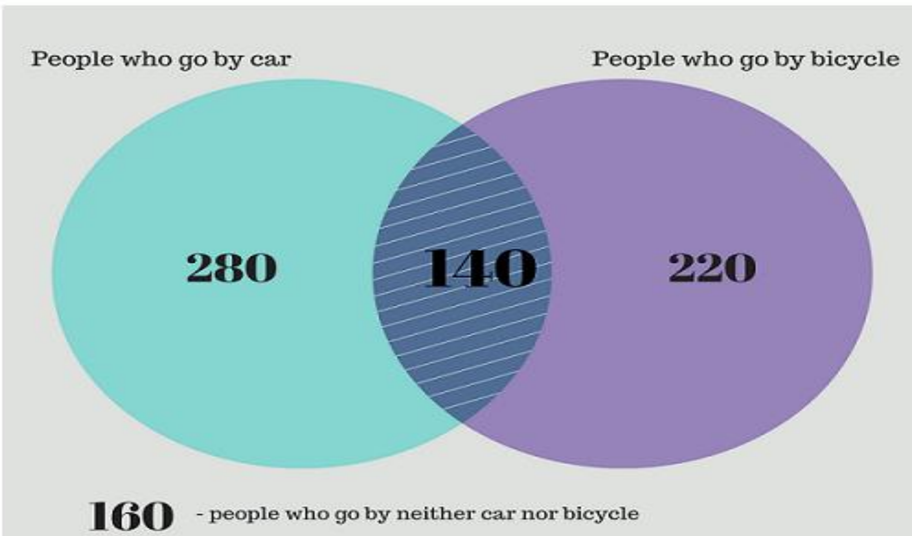
How many people go to work by bicycle only?

How many people go by neither car nor bicycle?

How many people use at least one of both transportation types?

How many people use only one of car or bicycle?

The following Venn diagram represents the data above:



- Number of people who go to work by car only = 280
- Number of people who go to work by bicycle only = 220
- Number of people who go by neither car nor bicycle = 160
- Number of people who use at least one of both transportation types = n(only car) + n(only bicycle) + n(both car and bicycle) = 280 + 220 + 140 = 640
- Number of people who use only one of car or bicycle = 280 + 220 = 500

Number of people who use at least one of both transportation types = n

(only car) + n(only bicycle) + n(both car and bicycle) = 280 + 220 + 140 = 640 [1].

Here are five solved problems on representing sets using Venn diagrams:

1. In a group of 100 students, 60 students play cricket, 40 students play football and 20 students play both games. Draw a Venn diagram and find how many students play neither game.

   P(cricket)=P(A)=60

   P(cricket & tennis)=P(A∩B)=20

$P(\text{total}) = P(A \cup B) = 100$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$100 = 60 + P(B) - 20$

$P(B) = 60$

P(tennis but not cricket)=P(tennis)-P(both tennis and cricket)

$= 60 - 20 = 40$

2. In a group of 200 employees, 120 employees have a college degree, while 80 employees have experience working in the industry. If there are 40 employees who have both a college degree and experience working in the industry, how many employees have neither?

Number of employees with neither = Total employees - (Employees with a degree + Employees with industry experience - Employees with both)

Number of employees with neither = 200 - (120 + 80 - 40)

Number of employees with neither = 200 - (200 - 40)

Number of employees with neither = 200 - 200 + 40

Number of employees with neither = 40

So, 40 employees in the group have neither a college degree nor experience working in the industry.

3. In a group of 50 people, there are 30 people who like coffee and 20 people who like tea. If there are 10 people who like both coffee and tea, how many people do not like either coffee or tea?

Let A be the set of people who like coffee, and B be the set of people who like tea.

Number of people who like coffee ($P(A)$) = 30
Number of people who like tea ($P(B)$) = 20
Number of people who like both coffee and tea ($P(A \cap B)$) = 10
Total number of people ($n(U)$) = 50
We want to find the number of people who do not like either coffee or tea, which is the complement of the union of A and B:

Number of people who do not like either coffee or tea = $n(U) - n(A \cup B)$

Using the formula for the union of two sets:

Number of people who do not like either coffee or tea =
$50 - (P(A) + P(B) - P(A \cap B))$

Number of people who do not like either coffee or tea = 50 - (30 + 20 - 10)

Number of people who do not like either coffee or tea = 50 - (50 - 10)

Number of people who do not like either coffee or tea = 50 - 50 + 10

Number of people who do not like either coffee or tea = 10

So, in this group of 50 people, there are 10 people who do not like either coffee or tea.
4. In a group of 80 students, there are 50 students who study math and 30 students who study science. If there are 20 students who study both math and science, how many students study either math or science?
Let A be the set of students who study math, and B be the set of students who study science.

Number of students who study math (P(A)) = 50
Number of students who study science (P(B)) = 30
Number of students who study both math and science ($P(A \cap B)$) = 20
Total number of students (n(U)) = 80
We want to find the number of students who study either math or science, which is the union of A and B:

Number of students who study either math or science = $P(A \cup B)$

Using the formula for the union of two sets:

Number of students who study either math or science = $P(A) + P(B) - P(A \cap B)$

Number of students who study either math or science = 50 + 30 - 20

Number of students who study either math or science = 80 - 20

Number of students who study either math or science = 60

So, in this group of 80 students, there are 60 students who study either math or science.
5. Consider three sets A = {1, 2, 3}, B = {2, 3, 4}, and C = {3, 4, 5}. Draw a Venn diagram to represent the sets A, B, and C and find:

a) $A \cap B$ b) $A \cup C$

a) $A \cap B$ (Intersection of A and B):
This represents the elements that are common to both sets A and B.
$A \cap B = \{2, 3\}$
b) $A \cup C$ (Union of A and C):
This represents the combination of all elements from sets A and C, including any duplicates (but not repeating elements).
$A \cup C = \{1, 2, 3, 4, 5\}$
So, the solutions are:
a) $A \cap B = \{2, 3\}$
b) $A \cup C = \{1, 2, 3, 4, 5\}$
6.Consider two sets X = {a, b, c, d} and Y = {c, d, e}. Draw a Venn diagram to represent the sets X and Y and find:

a) $X \cap Y$ b) $X - Y$ (Set difference)

$X = \{a, b, c, d\}$

$Y = \{c, d, e\}$

a) $X \cap Y$ (Intersection of X and Y):
This represents the elements that are common to both sets X and Y.
$X \cap Y = \{c, d\}$

b) $X - Y$ (Set difference of X and Y):
This represents the elements that are in set X but not in set Y.
$X - Y = \{a, b\}$
So, the solutions are:
a) $X \cap Y = \{c, d\}$
b) $X - Y = \{a, b\}$
7. Represent the universal set $U = \{1, 2, 3, 4, 5\}$ and its subset $V = \{2, 3, 4\}$ using a Venn diagram.

$U = \{1, 2, 3, 4, 5\}$

$V = \{2, 3, 4\}$

The set U, which is the universal set, contains all elements $\{1, 2, 3, 4, 5\}$.

The set V, which is a subset of U, contains elements $\{2, 3, 4\}$.

The Venn diagram visually represents how set V is a subset of the universal set U.

8. Consider two disjoint sets $P = \{a, b, c\}$ and $Q = \{x, y, z\}$. Draw a Venn diagram to illustrate their relationship.

Solution $P = \{a, b, c\}$

$Q = \{x, y, z\}$

Set P contains elements $\{a, b, c\}$.

Set Q contains elements $\{x, y, z\}$.

These two sets, P and Q, are disjoint, which means they have no elements in common. The Venn diagram visually illustrates that there is no overlap between the elements of P and Q.

9.Represent three sets X, Y, and Z using a Venn diagram in such a way that

$X \cap Y = \emptyset$, $Y \cap Z = \emptyset$, and $X \cap Z \neq \emptyset$.

Solution: Draw three circles for sets X, Y, and Z. Ensure that there is no overlap between X and Y and no overlap between Y and Z. However, there should be an overlap between X and Z to satisfy the conditions.

Exercise: 1.4

Practice Problems:

1. Draw a Venn diagram to represent two sets A and B where A is a proper subset of B ($A \subset B$).

2. Create a Venn diagram to show three sets P, Q, and R, where $P \cup Q = Q \cup R$, but $P \neq R$.

3. Draw a Venn diagram to illustrate the complement of a set A within a universal set U.

4. Represent sets M = {1, 2, 3, 4} and N = {3, 4, 5, 6} using a Venn diagram and find $M \cap N$.

5. Create a Venn diagram for four sets A, B, C, and D such that $A \cap B = \emptyset$ and $C \cup D = A$.

6. Draw a Venn diagram to represent the universal set U and a set S such that S is the empty set.

7. Represent two sets X = {a, b, c, d} and Y = {c, d, e} using a Venn diagram and find $Y \cap X'$.

8. Create a Venn diagram for sets P, Q, and R, where $P \cap Q = \emptyset$ and $P \cup Q = R$.

9.Draw a Venn diagram to represent two sets A and B such that $A \cap B = A$.

10. Illustrate the concept of the power set of a set A using a Venn diagram.

MATLAB Problems:

1. Write MATLAB code to create a Venn diagram for two sets A = [1, 2, 3] and B = [2, 3, 4]. Label the regions for $A \cap B$, A - B, and B - A.
2. Using MATLAB, generate a Venn diagram to represent three sets X, Y, and Z with appropriate labels and regions for their intersections.
3. Write MATLAB code to draw a Venn diagram for sets P = [a, b, c] and Q = [x, y, z], ensuring that the sets do not overlap.
4. Create a MATLAB script to generate a Venn diagram for four sets A, B, C, and D such that $A \cap B = \emptyset$ and $C \cup D = A$.

5. Develop a MATLAB program to draw a Venn diagram representing the concept of the power set of a set A, where A = {1, 2, 3}. Display all subsets within the diagram.

Solving Problems and Making Connections:

This topic focuses on problem-solving strategies, critical thinking, and making connections between different mathematical concepts. It encourages students to approach problems systematically, think creatively, and recognize how different mathematical concepts are related.

Solved Problems:

Problem: You are given a right triangle with a hypotenuse of length 10 units. The two legs of the triangle have integer lengths. Find all possible sets of integer side lengths for this triangle.

Solution: To find all possible sets of integer side lengths for the right triangle with a hypotenuse of 10 units, we can use the Pythagorean theorem, which states that in a right triangle, the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

Let the lengths of the two legs be 'a' and 'b'. According to the Pythagorean theorem, we have:

a^2 + b^2 = 10^2
a^2 + b^2 = 100

Now, we can systematically find pairs of integers (a, b) that satisfy this equation by considering integer values for 'a' and calculating the corresponding 'b' value:

Let a = 1:
a^2 + b^2 = 100
1^2 + b^2 = 100
b^2 = 100 - 1
b^2 = 99

However, there are no integer solutions for 'b' since the square root of 99 is not an integer.

Let a = 2:
a^2 + b^2 = 100
2^2 + b^2 = 100

4 + b^2 = 100
b^2 = 100 - 4
b^2 = 96

Similarly, there are no integer solutions for 'b' in this case.

Let a = 3:
a^2 + b^2 = 100
3^2 + b^2 = 100
9 + b^2 = 100
b^2 = 100 - 9
b^2 = 91

Again, there are no integer solutions for 'b'.

Let a = 4:
a^2 + b^2 = 100
4^2 + b^2 = 100
16 + b^2 = 100
b^2 = 100 - 16
b^2 = 84

No integer solutions for 'b' here either.

Let a = 5:
a^2 + b^2 = 100
5^2 + b^2 = 100
25 + b^2 = 100
b^2 = 100 - 25
b^2 = 75

Once again, there are no integer solutions for 'b'.

After systematically considering different integer values for 'a', we find that there are no integer solutions for 'a' and 'b' that satisfy the Pythagorean theorem and the condition of the hypotenuse being 10 units. Therefore, there are no sets of integer side lengths for this particular right triangle.

Problem: write the detailed soution to the problem " Problem: You are presented with three boxes: one contains only apples, one contains only oranges, and one contains a mix of both apples and oranges. Each box is labeled incorrectly. Using logical reasoning, how can you determine the contents of each box?

First, pick a box labeled “apples and oranges.” Since this box is labeled incorrectly, it cannot contain both apples and oranges. Therefore, this box must contain either only apples or only oranges.

Next, pick a box labeled “apples.” Since this box is also labeled incorrectly, it cannot contain only apples. Therefore, this box must contain either apples and oranges or only oranges.

Finally, the remaining box must be the one labeled “oranges,” since it is the only one left. This means that the box labeled “apples and oranges” must contain only apples, and the box labeled “apples” must contain both apples and oranges.

Therefore, the contents of each box are as follows:

- Box 1 (labeled “apples and oranges”): contains only apples.
- Box 2 (labeled “apples”): contains both apples and oranges.
- Box 3 (labeled “oranges”): contains only oranges.

Problem 1: In a town, there are three barbershops: one with a barber who gives perfect haircuts, one with a barber who shaves perfectly, and one with a barber who does both, but poorly. Each barber can only perform one task per customer. If you need both a haircut and a shave, where should you go?

You should go to the barber with the poor haircut skills but perfect shaving skills. If you go to the perfect haircut barber, you'll get a perfect haircut but a poor shave. If you go to the perfect shaving barber, you'll get a perfect shave but a poor haircut. Going to the barber with poor haircut skills but perfect shaving skills ensures you get a perfect shave and an acceptable haircut.

Problem: Find the next number in the sequence: 1, 3, 6, 10, 15, ...

Solution: The sequence represents triangular numbers, which are formed by adding consecutive natural numbers. To find the next number, add 6 and 21 to the last number in the sequence:

15 + 6 + 21 = 42

So, the next number in the sequence is 42.

Problem: Decode the following cryptic message: "Gsv zmbgsrmt ozmtrmt!"

The given message is a Caesar cipher encrypted message. This type of encryption is a substitution cipher in which each letter in the plaintext is shifted a certain number of places down the alphabet. For example, with a shift of 1, A would be replaced by B, B would become C, and so on.

To decode the given message, we need to shift each letter by 13 places down the alphabet. This is because the Caesar cipher is a symmetric encryption algorithm, meaning that the same key can be used for both encryption and decryption.

Using this method, we can decode the given message as follows:

Gsv zmbgsrmt ozmtrmt! => The ciphertext decrypted!

Therefore, the original message was "The ciphertext decrypted!".

Problem: Solve the following problem involving algebraic equations:

Problem: Solve for x: 3x + 7 = 16.

Solution: Start by subtracting 7 from both sides: 3x = 9. Then, divide both sides by 3: x = 3.

Problem 4: Solve a geometry problem involving connections between angles:

Problem: In a triangle, the measure of the first angle is twice that of the second angle, and the third angle is 30 degrees more than the second angle. Find the measures of all three angles.

Solution:

Let the second angle be x degrees.

Then, the first angle is 2x degrees,

the third angle is x + 30 degrees.

Using the fact that the angles in a triangle add up to 180 degrees,

we can solve for x:

x + 2x + (x + 30) = 180.

Solving for x,

we find x = 40 degrees, so the angles are 40 degrees, 80 degrees, and 70 degrees.

Problem 5: Solve a real-world problem that requires mathematical modeling:

Problem: You have $500 to spend on groceries for a month. On average, you spend $10 per day on groceries. How many days can you sustain this spending before running out of money?

Solution: You can sustain this spending for $500 / $10 = 50 days.

Problem: You have 12 identical-looking balls, and one of them is slightly lighter than the others. You also have a balance scale. Using the scale only three times, how can you identify the lighter ball?

Divide the balls into three groups of 4 each: A, B, and C.

Weigh group A against group B. If they balance, the lighter ball is in group C.

If one side is lighter in step 2, take the lighter group (either A or B) and weigh two of those balls against each other.

If they balance, the lighter ball is the one not weighed.

If one side is lighter in step 3, you've found the lighter ball.

You've used the scale three times to identify the lighter ball.

MATLAB Examples

Problem 1: Creating a Basic Venn Diagram Create a Venn diagram to represent the sets A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, and C = {4, 5, 6, 7}. Plot the Venn diagram to show the intersections and differences between these sets.

Problem 2: Customizing Venn Diagram Appearance Using MATLAB, create a Venn diagram with three sets: X, Y, and Z. Customize the appearance of the diagram by changing the colors and labels for each set. Make sure to include a legend.

Problem 3: Visualizing Set Operations Generate a Venn diagram to illustrate the result of set operations. Create three sets A, B, and C, and then visualize the union (A ∪ B), intersection (A ∩ B), and set difference (A - B) using a Venn diagram.

Problem 4: Probability with Venn Diagrams Simulate a probability problem using a Venn diagram. Suppose you have a set U containing 100 elements, and subsets A and B with 40 elements each. Create a Venn diagram to represent this scenario and visually calculate the probability of an element belonging to both A and B.

Problem 5: Dynamic Venn Diagram Create a MATLAB script that generates a dynamic Venn diagram. Allow the user to input the sizes and elements of two sets A and B. Update the Venn diagram to reflect the input data and display the intersections and differences between the sets.

Problem 6: Advanced Venn Diagram Visualization Design an advanced Venn diagram to visualize multiple sets. Create four sets A, B, C, and D, and use MATLAB to generate a

Venn diagram that clearly shows all intersections and differences between these sets. Customize the appearance, labels, and colors for better clarity.

Exercise: 1.5

1. A cube has six faces, each painted with a different color. If you randomly select three faces, what is the probability that all three faces have different colors?
2. In a school, there are three clubs: Math Club, Science Club, and Art Club. If every student is a member of at least one club, and there are 40 students in the Math Club, 30 students in the Science Club, and 25 students in the Art Club, how many students are members of all three clubs?
3. A farmer has a rectangular field that measures 30 meters in length and 20 meters in width. If the farmer wants to build a circular pond with the maximum possible area inside the field, what should be the radius of the pond?
4. A company manufactures three types of smartphones: Model A, Model B, and Model C. Each Model A phone costs $150 to produce, Model B costs $200, and Model C costs $250. The company wants to produce 1000 phones with a total cost of $200,000. How many phones of each model should they produce to meet this goal?
5. A baker has three different sizes of cakes: small, medium, and large. If they sell small cakes for $10 each, medium cakes for $15 each, and large cakes for $20 each, and they want to earn $500 in total, how many cakes of each size should they sell?
6. A group of friends goes on a road trip. The distance between their starting point and destination is 300 miles. If they drive at an average speed of 60 miles per hour for the first half of the trip and 40 miles per hour for the second half, what is their average speed for the entire journey?
7. In a deck of cards, there are 52 cards, including 4 suits (hearts, diamonds, clubs, spades). Each suit has 13 cards (Ace, 2, 3, ..., 10, Jack, Queen, King). What is the probability of drawing a red card or a face card (Jack, Queen, King) from the deck?
8. A company sells three products: Product A, Product B, and Product C. If the company wants to increase its revenue by 20% this year and expects to sell 1000 units of Product A, 500 units of Product B, and 300 units of Product C, how much should they increase the price of each product to meet their revenue target?
9. You have a rectangular garden with a fixed perimeter of 60 meters. What dimensions should you choose for the garden to maximize its area, and what will be the maximum area of the garden?
10. A teacher wants to distribute 30 identical books among her students in such a way that each student receives at least one book. How many different ways can she distribute the books among her 10 students?