

Magnetism: Introduction, Bohr magneton, classification of dia, para and ferro magnetic materials on the basis of magnetic moment, Properties of anti-ferro and ferri magnetic materials, Hysteresis curve based on domain theory, Soft and hard magnetic materials.

UNIT – V:

(9 Hours)

LASERS & FIBER OPTICS

Characteristics of lasers, Absorption, Spontaneous and Stimulated emissions, Einstein's Coefficients, Population inversion, Meta stable state, types of pumping, lasing action, Construction and working of Ruby Laser, Helium-Neon Laser, Applications of lasers.

Introduction to optical fiber, Construction and working principle of an Optical Fiber, Acceptance angle and Numerical aperture, Types of Optical fibers - Mode and Propagation through step and graded index fibers, Attenuation, Optical Fiber in Communication System, Applications of optical fibers.

COURSE OUTCOMES: After completion of studying Applied Physics the student is able to,

1. Know the basic principles of quantum mechanics and the importance of behavior of a particle.
2. Realize the importance of band structure of solids and their applications in various electronic devices.
3. Learn concentration estimation of charge carriers in semiconductors and working principles of PN diode.
4. Learn dielectric, magnetic properties of the materials and apply them in material technology.
5. Learn the principles and production of LASER beams and transfer of information by optical fiber communication systems.

TEXT BOOKS:

1. Engineering Physics by Arumugam, Anuradha publications.
2. Engineering Physics- B.K.Pandey, S.Chaturvedi, Cengage Learning.

REFERENCES:

1. Engineering Physics – R.K. Gaur and S.L.Gupta, Dhanpat Rai Publishers.
2. Engineering Physics, S Mani Naidu- Pearson Publishers.
3. Engineering physics 2nd edition –H.K.Malik and A.K. Singh.
4. Engineering Physics – P.K. Palaniswamy, Scitech publications.
5. Physics by Resnick and Haliday.

CONTENTS

UNIT NO	NAME OF THE UNIT
I	QUANTUM MECHANICS
II	ELECTRONIC MATERIALS
III	SEMICONDUCTOR PHYSICS
IV	DIELECTRICS AND MAGNETIC PROPERTIES OF MATERIALS
V	LASERS & FIBER OPTICS

UNIT – I

QUANTUM MECHANICS

Classical Mechanics:-

It is the branch of physics, which explains the motion of macroscopic objects. It was introduced by sir Issac Newton.

Quantum Mechanics:-

It is the branch of physics which explains about the motion of microscopic particles like electrons, protons etc. It was introduced by Max Planck in 1990.

Waves & particles:

Wave:- A wave is nothing but spreading of disturbance in a medium. The characteristics/properties of waves are 1) Amplitude 2) Time period 3) Frequency 4) Wavelength 5) Phase 6) Intensity.

Particle:- A particle is a point in space which has mass & occupies space or region. The characteristics/properties of a particle are 1) Mass 2) velocity 3) Momentum 4) Energy etc.

Matter Waves or de-broglie-waves: The waves associated with a material particle are called as matter waves.

Difference between matter waves and electro-magnetic waves:-

Matter Wave	Electromagnetic waves(Light waves)
1. Matter wave is associated with moving particle or material particle.	1. Oscillating charged particle gives rise to EM wave.
2. Wavelength of matter wave is given as $\lambda = \frac{h}{mv}$	2. Wavelength of an electromagnetic wave is given by $\lambda = \frac{hc}{E}$
3. Wavelength of matter wave depends upon mass of the particle & velocity.	3. Wavelength of an electromagnetic wave depends upon the energy of the photon.
4. It can travel with a velocity greater than the velocity of light in vacuum.	4. It can travel with a velocity equal to the speed of light in vacuum i.e $c = 3 \times 10^8 \text{ m/s}$.
5. It is not a EM wave.	5. Electric field and Magnetic field oscillate perpendicular to each other & generate EM Waves.

de-broglie concept of dual nature of matter waves:-

In 1924, Louis de-broglie suggested that matter waves also exhibit dual nature like radiation(light). They are

- I. Wave nature
- II. Particle nature

Wave nature of matter waves is verified by Davisson & Germer experiment, G.P. Thomson experiment etc.

Particle nature of matter waves is verified by photo-electric effect, Compton effect etc.

de-broglie hypothesis:-

- 1) The universe consists of matter and radiation(light) only
- 2) Matter waves also exhibit dual nature like radiation.

- 3) The waves associated with the material particles are called as de-broglie-waves or matter waves & the wave length associated with matter waves are called as de-broglie wave-length or matter wave-length (λ).
- 4) de-broglie wave-length is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$

Expression for de-broglie wave-length(λ) in various form:-

According to the planck's theory of radiation, the energy of photon is given by

$$E = h\nu = \frac{hc}{\lambda} \text{----(1)}$$

h- planck's constant, ν -frequency of photon

According to Einstein mass energy relation

$$E = mc^2 \text{----(2)}$$

m-mass of a photon

c-velocity of light

From equation of (1) & (2)

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mv} = \frac{h}{p} \text{--- (3)}$$

Where p-momentum of photon = mc

m-mass of photon, c-speed of light

But according to de-broglie theory

Momentum of electron particle(p) = mv

m-mass of e's , ν -velocity of electron particle

$$\text{de-broglie wave-length}(\lambda) = \lambda = \frac{h}{mv} = \frac{h}{p} \text{----(4)}$$

Eq.(4) gives the expression for de-broglie wave-length.

Other forms of de-broglie wavelength(λ):-

(i) In terms of Energy(E):-

we know that the kinetic energy of particle i.e. $E = \frac{1}{2}mv^2$ --- (5)

Multiply Eq-(5) by 'm' on both sides, we get

$$mE = \frac{1}{2}mv^2$$

$$2Em = m^2v^2$$

$$mv = \sqrt{2mE}$$

$$\lambda = \frac{h}{\sqrt{2mE}} \text{---(6)}$$

(ii) de-broglie wavelength in terms of voltage(V):-

If a charged particle is accelerated through a potential difference(V), then the kinetic energy of the particle is given as

$$E = eV \text{---(7)}$$

But we have kinetic energy(E) of particle i.e. $E = \frac{1}{2}mv^2$

$$eV = \frac{1}{2}mv^2$$

$$2eV = mv^2$$

Multiply by 'm' on both sides we get

$$2meV = m^2v^2 \Rightarrow mv = \sqrt{2meV}$$

$$h = 6.6 \times 10^{-34} \text{ J-S}$$

$$\text{We have de-broglie wavelength } \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} \text{ or } \lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

$$h = 6.625 \times 10^{-34} \text{ Js}$$

$$m_e = 9.1 \times 10^{-31} \text{ Kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

(iii) de-broglie wavelength in terms of Temperature(T):-

According to the kinetic theory of gases, the average kinetic energy of a particle at temperature 'T' is given by

$$E = \frac{3}{2} k_B T$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m \times \frac{3}{2} k_B T}} = \frac{h}{\sqrt{3mk_B T}}$$

k_B is the Boltzmann constant.

Properties or characteristics of matter waves or de-broglie waves:-

- Lesser the mass of the particle, greater is the wavelength associated with it.
- Smaller the velocity of the particle, longer is the wavelength associated with the particle.
- When $V = 0$, $\lambda = \infty$ & $V = \infty$, $\lambda = 0$.
- Matter waves produced when the particles in motion are charged or uncharged.
- Matter waves are not electro-magnetic waves.
- Matter waves travel faster than the velocity of light.
- Wave nature of matter gives an uncertainty in the position of the particle.

Experimental Verification of matter waves:-

Here, we have two methods to verify the dual nature of matter waves. They are

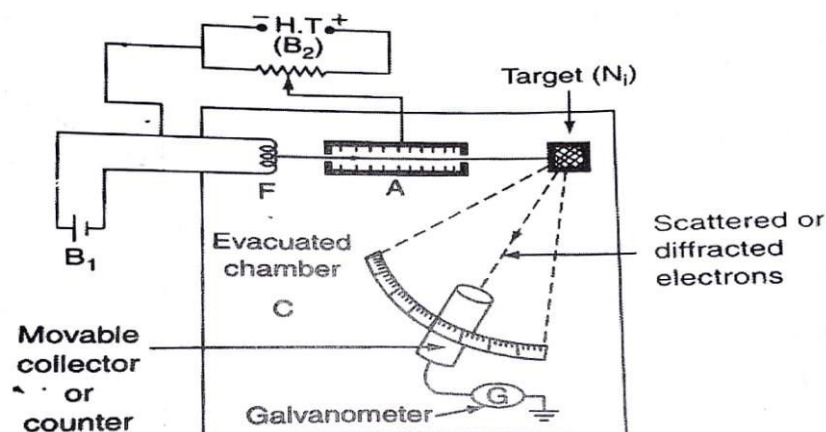
1. Davisson and Germer's experiment
2. G.P. Thomson's experiment

1. Davisson and Germer's experiment:-

First practical evidence for the wave nature of matter waves was given by C.J. Davisson and L.H. Germer in 1927. This was the first experimental support to de Broglie's hypothesis.

Principle: The e^- 's which are coming from the source are incident on the target and the e^- 's get diffracted. These diffracted e^- 's produce a diffraction pattern. It shows (explains) the wave nature of matter waves.

Experimental Arrangement:-



Construction:

The experimental setup is shown in above figure. It consists of mainly 3-parts

- (i) Filament
- (ii) Target
- (iii) Circular scale arrangement.

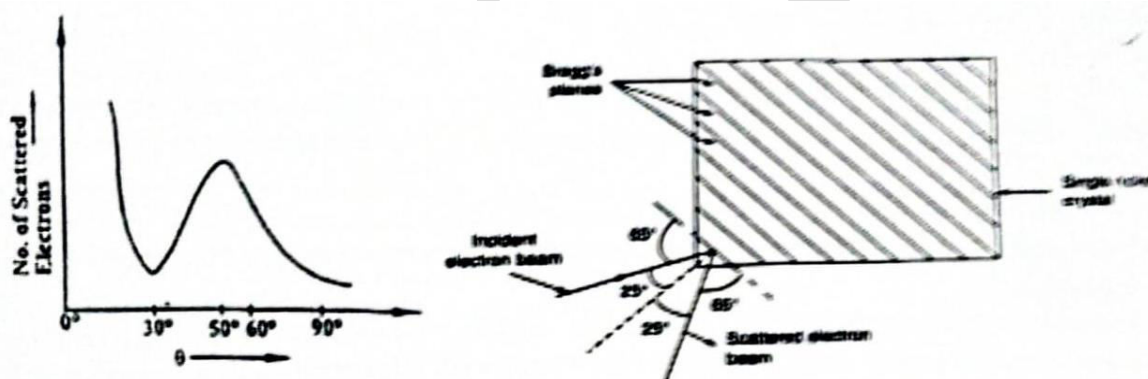
It also consists of a low tension battery(LTB), High tension battery(HTB) & a cylinder(A).

Working:

When tungsten filament 'F' is heated by a LTB then e's are produced. These e's are accelerated by High voltage(HTB). The accelerated e's are collimated into a fine beam of pencil by passing them through a system of pin-holes in the cylinder 'A'. This beam of electrons is allowed to incident on nickel crystal which acts as target. Then e's are scattered in all the directions. The intensity of scattered e's is measured by the circular scale arrangement. In this arrangement, an electron or movable collector(Double walled faraday cylinder) is fixed to circular scale which can collect the electrons and can move along the circular scale. The electron collector(Double walled faraday cylinder) is connected to a sensitive galvanometer to measure the intensity of electron beam entering the collector at different scattering angles(θ).

A graph is plotted between the scattering angle(θ) and the number of scattered electron's as shown in above figure.

The intensity of scattered e's is maximum at $\theta = 50^\circ$ & accelerating voltage =54V.

**Calculation of wave-length associated with e's:**

$$2d \sin \theta = n\lambda \text{---(1)}$$

For nickel crystal, $d = 0.909 \text{ \AA} = 0.909 \times 10^{-10} \text{ m}$

$n = 1$, First order

(i) Angle of diffraction(θ)(Practical value):

From the figure,

$$180^\circ = \theta + \theta + 50^\circ$$

$$180^\circ = 2\theta + 50^\circ$$

$$\therefore \text{Diffraction angle}(\theta) = 65^\circ \text{---(2)}$$

Substituting the above values in eq – 1 we get

$$2 \times 0.909 \times \sin 65^\circ = 1 \times \lambda$$

$$\lambda = 1.65 \text{ \AA} \text{---(3)}$$

(ii) From de-broglie wave length(λ):

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

$$\text{But } V = 54\text{v}$$

$$\lambda = \frac{12.27}{\sqrt{54}} \text{ \AA} = 1.67 \text{ \AA}$$

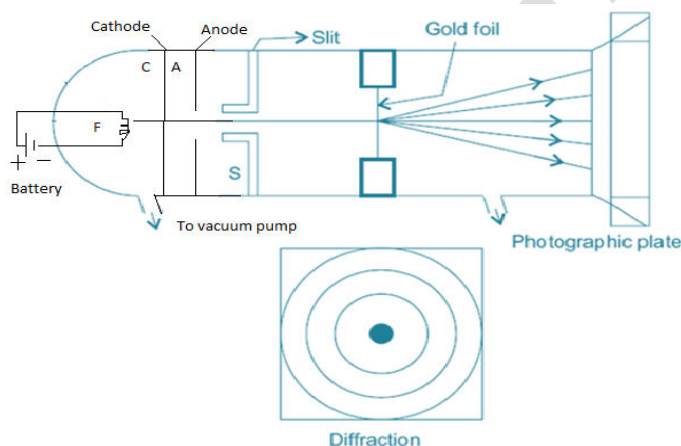
$$\lambda = 1.67 \text{ \AA} \text{ ----(4)}$$

From eq-(3) & eq-(4) it was been proved both the practical & theoretical wavelengths are almost equal. Hence the wave nature of particle is proved experimentally.

(2) G.P.Thomson experiment:

In 1928, G.P.Thomson, experimentally proved the dual nature of matter(particles).

Principle: The electrons which are coming from the source are incident on the thin metal foil e's are diffracted . These diffracted electrons produce a diffraction fringes on the photographic plate placed behind the foil. It explain the dual nature of matter.

Experimental Arrangement :

Construction: The experimental set up is shown in fig. It consists of

- (i) Filament(F)
- (ii) Anode(A)
- (iii) Cathode(C)
- (iv) Slit(S)
- (v) Gold foil(G)
- (vi) Photographic plate(P)
- (vii) Evacuated chamber.

Working:

Using a suitable battery the filament 'F' can be heated, so that electrons get emitted and pass through a high positive potential to the anode(A). Then 'e' beam passes through a slit incident on the gold foil(G) of thickness 10^{-8} cm.

The e's passing through the gold foil 'G' are recorded on a photographic plate(p). Since the gold foil consists of large no of micro crystals oriented at random ,the e's striking the gold foil diffracts according to bragg's law.

$$2d \sin \theta = n\lambda$$

After developing the plate, the diffraction pattern consists of concentric rings about a central spot is obtained. This pattern is similar as produced by x-rays.

To make sure this ,the pattern or fringes is deflected by application of magnetic field. Since the e's are deflected by magnetic field so that pattern is shifted. But x-rays are not deflected by electric & magnetic fields. So Thomson concluded that e's(matter) behave like waves.

Calculation's of wavelength of e's:

From fig, let us consider 'OA' is the radius(r) of the ring, 'O' is the center of the ring. Let 'L' be the distance between the gold foil(G) and the photographic plate(p).

Then from Δ^{le} QOA ,

$$\tan 2\theta = \frac{r}{L}$$

If 'θ' is small, then

$$\theta = \frac{r}{2L}$$

According to Bragg's law,

$$2d \sin \theta = n\lambda$$

If 'θ' is small, then $\sin \theta = \theta \Rightarrow 2d \cdot \theta = n\lambda$

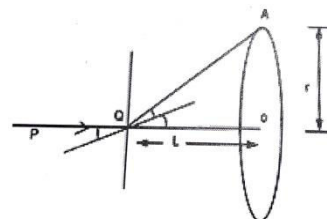
$$n\lambda = 2 \times d \times \frac{r}{2L}$$

$$n\lambda = \frac{dr}{L} \Rightarrow \lambda = \frac{dr}{nL} \text{---(1)}$$

de-Broglie wavelength of e's:

$$\therefore \lambda = \frac{12.27}{\sqrt{V}} \text{--- (2)}$$

eq-(1) & eq-(2) are almost equal. This confirms the existence of matter waves. The wave nature of matter(particles) is verified experimentally.

**Heisenberg uncertainty principle:-**

The discovery of dual nature of material particle imposes a serious hurdle in locating the exact position and momentum of a particle simultaneously.

This wax was removed by “**Werner. Heisenberg**” in 1927 by proposing a significant principle. Later it was called Heisenberg's uncertainty principle or Heisenberg's principle of indeterminacy.

“It is impossible to measure both the position and momentum of a particle simultaneously to any desired degree of accuracy.” If 'Δx' & 'Δp' are uncertainties in the measurement of position & momentum of the particle then mathematically this uncertainties of this physical variables is written as

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi} \text{---(1)}$$

Similarly the uncertainties in measuring energy and time interval we can write

$$(i) \quad \Delta E \cdot \Delta t \geq \frac{h}{4\pi} \text{ and the uncertainties in measuring angular momentum \& angular displacement as}$$

$$(ii) \quad \Delta J \cdot \Delta \theta \geq \frac{h}{4\pi}$$

Explanation:-

(i) If $\Delta x = 0$. i.e., the position of a particle is measured accurately, then from eq-(1).

$$\Delta p = \frac{h}{\Delta x \cdot 4\pi}$$

$$\Delta p = \frac{h}{0} = \infty$$

It means that, the momentum of the particle can't be measured.

(ii) If $\Delta p = 0$. i.e., the momentum of a particle is measured accurately, from eq. (1).

$$\Delta x = \frac{h}{\Delta p \cdot 4\pi} = \frac{h}{0} = \infty$$

From the above said observations made by Heisenberg, he clearly states that it is impossible to design an experiment to prove the wave & particle nature of matter at any given instant of time.

If one measures position or momentum accurately, then there will be an uncertainty in the other. Thus, the Heisenberg's uncertainty principle gives the probability of determining the particle at any given instant of time in place of certainty.

Applications:-

- (i). It explains the non-existence of e's in the nucleus.
- (ii). It gives the binding energy of an e's in atom.
- (iii). It calculates the radius of Bohr's first orbit.

Schrodinger Wave Equation:-

Schrodinger describes the wave nature of a particle in mathematical form and is known as Schrodinger wave equation (SWE). There are two types of SWE

- (i). Schrodinger Time independent wave equation (STIWE)
- (ii). Schrodinger Time dependent wave equation (STDWE)

(i) Schrodinger Time independent wave equation (STIWE):-

To derive an expression for "(STIWE)" let us consider an electron (particle) moving along a positive direction along the axes. Let x, y, z be the coordinates of the particle & ' ψ ' is the wave-displacement or wave-function of the matter wave at any time 't'. It is assumed that ' ψ ' is finite, single-valued, continuous and periodic function.

We can express the classical differential wave equation of the material particle in three-dimension axes is given as

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right)$$

$$\text{In Three dimension we write } \frac{\partial^2 \psi}{\partial t^2} = V^2 \nabla^2 \psi \text{ --- (1)}$$

The solution of eq-(1) is given by

$$\psi = \psi_0 \sin \omega t \text{ --- (2)}$$

$$\psi = \psi_0 \sin 2\pi \nu t \text{ --- (3)}$$

Where $\omega = 2\pi \nu$ is the angular frequency of the particle

Differentiating eq-(3) w.r.t. 't' twice we get

$$\frac{\partial \psi}{\partial t} = \psi_0 \times \cos 2\pi \nu t \times 2\pi \nu$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\psi_0 \times \sin 2\pi \nu t \times 2\pi \nu \times 2\pi \nu$$

$$\frac{\partial^2 \psi}{\partial t^2} = -4\pi^2 \nu^2 \psi \text{ --- (3)}$$

$$\nu = \frac{c}{\lambda} = \frac{v}{\lambda}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{4\pi^2 v^2 \psi}{\lambda^2} \text{ --- (4)}$$

Substituting eq-(4) in eq-(1), we get,

$$V^2 \nabla^2 \psi = -\frac{4\pi^2 v^2 \psi}{\lambda^2}$$

$$\boxed{\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \text{ --- (5)}}$$

According to de-broglie wave-length

$$\lambda = \frac{h}{p} = \frac{h}{mv} \text{ --- (6)}$$

Sub. eq(6) in eq(5), we get

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \text{ --- (7)}$$

We have, the total energy (E) is given by

$$E = \text{P.E.} + \text{K.E.}$$

$$E = V + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = E - V$$

$$mv^2 = 2(E - V)$$

$$m^2v^2 = 2m(E - V) \text{ --- (8)}$$

Sub. eq-(8) in eq-(7) we get,

$$\nabla^2 \psi + \frac{4\pi^2}{h^2} 2m(E - V)\psi = 0$$

$$\nabla^2 \psi + \frac{8\pi^2 m (E - V)\psi}{h^2} = 0 \text{ --- (9)}$$

$$\nabla^2 \psi + \frac{2m(E-V)\psi}{\hbar^2} = 0 \text{ --- (10)}$$

$$\text{Where } \hbar = \frac{h}{2\pi}$$

Eq-(9) & Eq-(10) is called as Schrodinger time independent wave equation in three dimension

For a free particle, $V=0$,

$$\nabla^2 \psi + \frac{2mE\psi}{\hbar^2} = 0 \text{ --- (11)}$$

(ii) Schrödinger Time dependent wave equation:-

$$\hat{H}\psi = \hat{E}\psi \text{ where } \hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V = \text{Hamiltonian operator}$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t} = \text{Energy operator}$$

Physical significance of wave-function(ψ)(Eigen – function): –

Wave-function(ψ) or Eigen-function(ψ):-

It is a variable or complex quantity that is associated with a moving particle at any position (x,y,z) and at any time 't'.

(i) ' ψ ' of a particle is represented by $\psi = \psi_0 e^{-i\omega t}$

(ii) ' ψ ' explains the motion of microscopic particles.

(iii) ' ψ ' is a complex quantity & it does not have any meaning.

(iv) $|\psi|^2 = \psi\psi^*$ is real and positive, it has physical meaning.

(v) $|\psi|^2$ represents the probability of finding the particle per unit volume.

(vi) For a given volume $d\tau$, the probability of finding the particle is given by

$$\text{probability density}(p) = \iiint |\psi|^2 d\tau \text{ where } d\tau = dx dy dz$$

(vii) ' ψ ' gives the information about the particle behavior.

(viii) ' p ' values are between 0 to 1.

(xi) wave-function ' ψ ' is a single valued, finite and periodic function.

(x) If $p = \iiint |\psi|^2 d\tau = 1$, then ' ψ ' is called normalized wave function.

Application of Schrödinger Time independent wave equation(STIWE):-**(1) Particle or electron in a one dimensional box or particle in an infinite square well potential:-**

Consider a particle or electron of mass 'm' moving along x-axis enclosed in a one dimensional potential box as shown in figure. Since the walls are of infinite potential the particle does not penetrate out from the box. i.e. potential energy of the particle $V=\infty$ at the walls.

The particle is free to move between the walls A & B at $x=0$ and $x=L$.

The potential energy of the particle between the walls is constant because no force is acting on the particle.

∴ The particle energy is taken as zero for simplicity

i.e. $V = 0$ between $x=0$ & $x=L$.

Boundary Conditions:-

(i) The potential energy for particle is given as

$$V(x) = 0, \text{ for } 0 \leq x \leq L \quad \text{----- (1)}$$

$$V(x) = \infty, \text{ when } 0 \geq x \geq L$$

The Schrödinger time independent wave equation for the particle is given by,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (E - V)\psi = 0$$

But $V=0$, between walls,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (E)\psi = 0 \text{---- (2)}$$

$$\text{Let } \frac{8\pi^2m}{h^2} (E) = k^2$$

Then eq-(2), becomes

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \text{---- (3)}$$

The general solution of eq-(3) is given by

$$\psi(x) = A\sin kx + B\cos kx \text{----- (4)}$$

Where A, B are two constants, 'k' is the wave-vector

Applying the boundary conditions eq-(4), we get

$$(i) \quad \psi = 0 \text{ at } x = 0$$

$$0 = A\sin(0) + B\cos(0)$$

$$0 = 0 + B$$

$$B=0$$

Then eq-(4) is written as

$$\psi(x) = A\sin kx \text{ or } \psi_n(x) = A\sin(kx) \text{---- (6)}$$

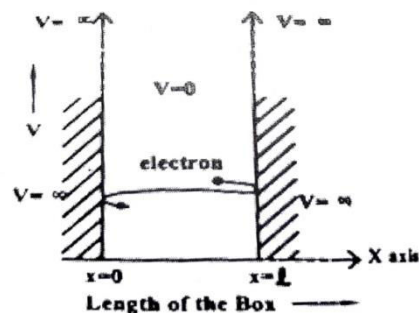
$$(ii) \quad \psi = 0 \text{ at } x = L$$

Eq-(6) can be written as

$$0 = A\sin(kL) \Rightarrow A \neq 0, \text{ and } \sin(kL) = 0$$

$$\sin(kL) = \sin(n\pi)$$

$$kL = n\pi \Rightarrow k = \frac{n\pi}{L} \text{----- (7)}$$



1) Energy of the particle(Electron):-

$$k = \frac{n\pi}{L} \Rightarrow k^2 = \frac{n^2 h^2}{L^2}$$

$$\text{But we have, } \frac{8\pi^2 m}{h^2} (E) = k^2$$

$$\frac{n^2 \pi^2}{h^2} = \frac{8\pi^2 m}{h^2} (E)$$

$$E = \frac{n^2 h^2}{8mL^2} \Rightarrow E_n = \frac{n^2 h^2}{8mL^2} \text{ --- (8)}$$

(ii) Energy levels of an particle(Electron):-

$$\text{when } n=1, \quad E_1 = \frac{h^2}{8mL^2}$$

$$\text{when } n=2, \quad E_2 = \frac{4h^2}{8mL^2} = 4E_1$$

$$\text{In general, } \boxed{E_n = n^2 E_1}$$

\therefore Energy levels of an electron are discrete.

(iii) Normalization of the wave-function of the particle to find 'A':-

To find the 'A' value, by applying the normalization conditions.

$$\int_0^L |\psi(x)|^2 dx = 1$$

$$\int_0^L A^2 \sin^2 \left(\frac{n\pi x}{L} \right) dx = 1$$

$$A^2 \int_0^L \sin^2 \left(\frac{n\pi x}{L} \right) dx = 1$$

By integrating and applying the appropriate limits, finally we obtain the value of 'A' as

$$A = \sqrt{\frac{2}{L}} \text{ --- (9)}$$

(iv) Electron wave functions ,probability density functions and energy functions:-

(i) Wave function of electron/particle

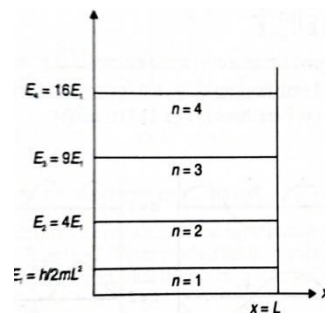
$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \text{ --- (10)}$$

(ii) Probability density function(P)

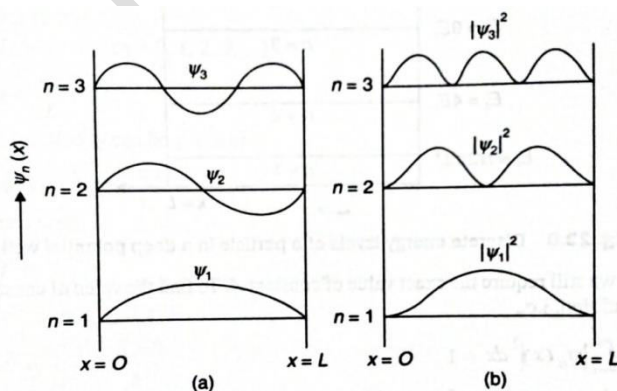
$$P = |\psi(x)|^2 = \frac{2}{L} \sin^2 \left(\frac{n\pi x}{L} \right) \text{ --- (11)}$$

(iii) Energy function of the particle

$$E_n = \frac{n^2 h^2}{8mL^2} \text{ --- (12)}$$



Discrete energy levels of a particle in a deep potential well



Wave function and probability density of a particle inside an infinite potential well

Unit II

ELECTRONIC MATERIALS

Free electron theory:

In solids, electrons in outer most orbits of atoms determine its electrical properties. Electron theory is applicable to all solids, both metals and non metals. In addition, it explains the electrical, thermal and magnetic properties of solids. The structure and properties of solids are explained employing their electronic structure by the electron theory of solids.

It has been developed in three main stages:

1. Classical free electron theory
 2. Quantum Free Electron Theory.
 3. Zone Theory.
- **Classical free electron theory:** The first theory was developed by Drude & Lorentz in 1900. According to this theory, metal contains free electrons which are responsible for the electrical conductivity and metals obey the laws of classical mechanics.
 - **Quantum Free Electron Theory:** In 1928 Sommerfield developed the quantum free electron theory. According to Sommerfield, the free electrons move with a constant potential. This theory obeys quantum laws.
 - **Zone Theory:** Bloch introduced the band theory in 1928. According to this theory, free electrons move in a periodic potential provided by the lattice. This theory is also called “Band Theory of Solids”. It gives complete informational study of electrons.

Classical free electron theory:

Even though the classical free electron theory is the first theory developed to explain the electrical conduction of metals, it has many practical applications. The advantages and disadvantages of the classical free electron theory are as follows:

Advantages:

1. It explains the electrical conductivity and thermal conductivity of metals.
2. It verifies ohm's law.
3. It is used to explain the optical properties of metals.
4. Metal composed of atoms in which electrons revolve around the nucleus are many states available for occupation. If the density of states is zero, no states can be occupied at that energy level.
5. The valence electrons are freely moving about the whole volume of the metals like the molecules of perfect gas in a container
6. The free electrons moves in random directions and collide with either positive ions or other free electrons. Collision is independent of charges and is elastic in nature
7. The movements of free electrons obey the laws of classical kinetic theory of gases
8. Potential field remains constant throughout the lattice.
9. In metals, there are large numbers of free electrons moving freely within the metal i.e. the free electrons or valence electrons are free to move in the metal like gaseous molecules, because nuclei occupy only 15% metal space and the remaining 85% space is available for the electrons to move.

Drawbacks:

1. It fails to explain the electric specific heat and the specific heat capacity of metals.
2. It fails to explain superconducting properties of metals.
3. It fails to explain new phenomena like photoelectric effect, Compton effect, black – body radiation, etc.
4. It fails to explain Electrical conductivity (perfectly) of semiconductors or insulators.
5. The classical free electron model predicts the incorrect temperature dependence of σ . According to the classical free electron theory, $\sigma \propto T^{-1}$.
6. It fails to give a correct mathematical expression for thermal conductivity.
7. Ferromagnetism couldn't be explained by this theory.
8. Susceptibility has greater theoretical value than the experimental value.

Quantum free electron theory of metals:**Advantages:**

1. All the electrons are not present in the ground state at 0 K, but the distribution obeys Pauli's exclusion principle. At 0 K, the highest energy level filled is called Fermi-level.
2. The potential remains constant throughout the lattice.
3. Collision of electrons with positive ion cores or other free electrons is independent of charges and is elastic in nature.
4. Energy levels are discrete.
5. It was successful to explain not only conductivity, but also thermionic emission paramagnetism, specific heat.

Drawbacks:

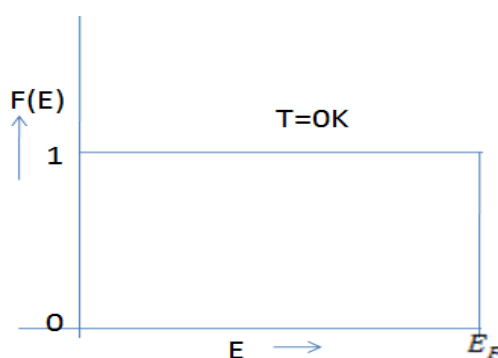
1. It fails to explain classification of solids as conductors, semiconductors and insulators.

Fermi level and Fermi energy:

The distribution of energy states in a semiconductor is explained by Fermi –Dirac statistics since it deals with the particles having half integral spin like electrons. Consider that the assembly of electrons as electron gas which behaves like a system of Fermi particles or fermions. The Fermions obeying Fermi –Dirac statistics i.e., Pauli's exclusion principle.

Fermi energy: It is the energy of state at which the probability of electron occupation is $\frac{1}{2}$ at any temperature above 0K. It separates filled energy states and unfilled energy states. The highest energy level that can be occupied by an electron at 0 K is called Fermi energy level

Fermi level: It is a level at which the electron probability is $\frac{1}{2}$ at any temp above 0K (or) always it is 1 or 0 at 0K.



Therefore, the probability function $F(E)$ of an electron occupying an energy level E is given by,

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)} \dots \dots \dots (1)$$

Where E_F known as Fermi energy and it is constant for a system,

K is the Boltzmann constant and T is the absolute temperature.

Case I : Probability of occupation at $T=0K$, and $E < E_F$

Therefore $F(E) = 1$, as per above, clearly indicates that at $T=0K$, the energy level below the Fermi energy level E_F is fully occupied by electrons leaving the upper level vacant.

Therefore, there is 100% probability that the electrons to occupy energy level below Fermi level.

Case II: Probability of occupation at $T=0K$, and $E > E_F$

Then

i.e., all levels below E_F are completely filled and all levels above E_F are completely empty. As the temperature rises $F(E)$.

Case III: Probability of occupation at $T=0K$, and $E = E_F$

$$F(E) = \frac{1}{1 + e^0} = \frac{1}{1 + 1} = \frac{1}{2} = 0.5$$

The above condition states that, $T=0K$, there is a 50% probability for the electrons to occupy Fermi energy.

The probability function $F(E)$ lies between 0 and 1.

Hence there are three possible probabilities namely

$F(E) = 1$ 100% probability to occupy the energy level by electrons.

$F(E) = 0$ No probability to occupy the energy levels by electrons and hence, it is empty.

$F(E) = 0.5$ 50% probability of finding the electron in the energy level.

Density of States (DOS):

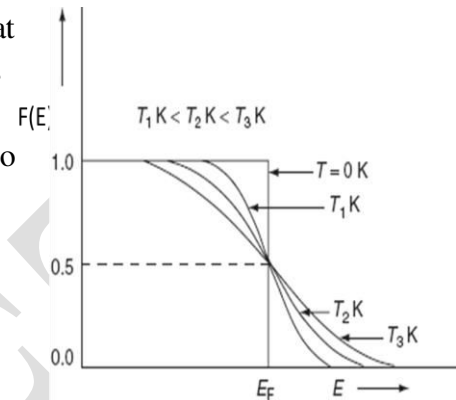
The number of electrons per unit volume in an energy level at a given temperature is equal to the product of density of states (number of energy levels per unit volume) and Fermi Dirac distribution function (the probability to find an electron).

$$n_c = \int g(E) \times f(E) dE \dots \dots \dots (1)$$

where n_c is the concentration of electrons, $g(E)$ is the density of states & $F(E)$ is the occupancy probability.

The number of energy states with a particular energy value E is depending on how many combinations of quantum numbers resulting in the same value n .

To calculate the number of energy states with all possible energies, we construct a sphere in 3D-space with ' n ' as radius and every point (n_x , n_y and n_z) in the sphere represents an energy state.



As every integer represents one energy state, unit volume of this space contains exactly one state .Hence; the number of states in any volume is equal to the volume expressed in units of cubes of lattice parameters). Also $n^2 = n_x^2 + n_y^2 + n_z^2$

Consider a sphere of radius n and another sphere of radius $n+dn$ with the energy values are E and $(E+dn)$ respectively.

Therefore, the number of energy states available in the sphere of radius ' n ' is by considering one octant of the sphere (Here, the number of states in a shell of thickness dn at a distance ' n ' in coordinate system formed by n_x, n_y and n_z and will take only positive values ,in that sphere $\frac{1}{8}$ of the volume will satisfy this condition).

The number of energy states within a sphere of radius $(n+dn)$ is

$$\frac{1}{8} \left(\frac{4\pi}{3} \right) (n + dn)^3$$

Thus the number of energy states having energy values between E and $E+dn$ is given by

$$\begin{aligned} g(E)dn &= \frac{1}{8} \left(\frac{4\pi}{3} \right) (n + dn)^3 - \frac{1}{8} \left(\frac{4\pi}{3} \right) n^3 \\ &= \frac{1}{8} \left(\frac{4\pi}{3} \right) [(n + dn)^3 - n^3] \\ &= \frac{\pi}{6} (3n^2 dn) = \frac{\pi}{2} n^2 dn \end{aligned}$$

compared to ' dn ', dn^2 and dn^3 are very small .

Neglecting higher powers of dn

$$g(E)dn = \frac{\pi}{2} n^2 dn \dots \dots (2)$$

The expression for n^{th} energy level can be written as ,

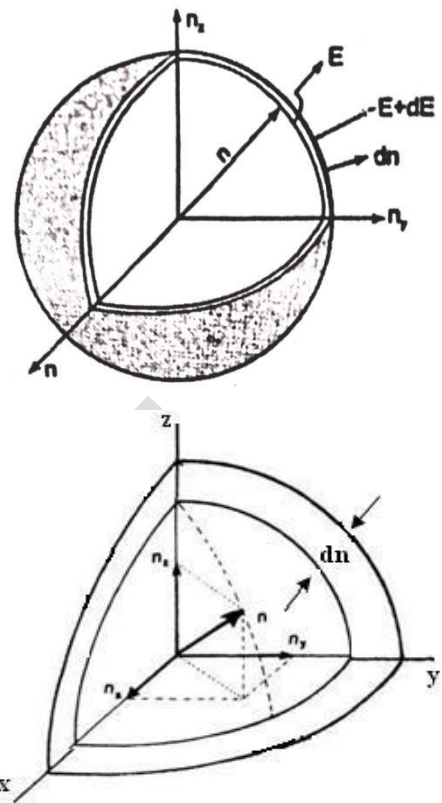
$$E = \frac{n^2 h^2}{8mL^2} \quad \text{or,} \quad n^2 = \frac{8mL^2 E}{h^2} \dots \dots (3)$$

$$\Rightarrow n = \left(\frac{8mL^2 E}{h^2} \right)^{\frac{1}{2}} \dots \dots (4)$$

Differentiating eq. (3):

$$2n dn = \frac{8mL^2}{h^2} dE \quad \Rightarrow \quad dn = \frac{1}{2n} \left(\frac{8mL^2}{h^2} \right) dE$$

\therefore by substituting $1/n$ value in dn ,



$$n = \frac{1}{2} \left(\frac{8mL^2}{h^2} \right)^{\frac{1}{2}} \frac{dE}{E^{\frac{1}{2}}} \dots \dots \dots (5)$$

Substitute n^2 and dn from eq. (3) and (5), we get

$$g(E)dE = \frac{\pi}{2} \left(\frac{8mL^2}{h^2} \right) E dE \times \frac{1}{2} \left(\frac{8mL^2}{h^2} \right)^{\frac{3}{2}} \frac{dE}{E^{\frac{1}{2}}}$$

$$g(E)dE = \frac{\pi}{4} \left(\frac{8mL^2}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE \dots \dots \dots (6)$$

According to Pauli's Exclusion Principle, two electrons of opposite spin can occupy each energy state

Equation (6) should be multiplied by 2

$$g(E)dE = 2 \times \frac{\pi}{4} \left(\frac{8mL^2}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

After mathematical simplification, we get g

$$g(E)dE = \frac{4\pi}{h^3} (2m)^{\frac{3}{2}} E^{\frac{1}{2}} L^3 dE$$

The density of energy states $g(E) dE$ per unit volume is given by,

$$g(E)dE = \frac{4\pi}{h^3} (2m)^{\frac{3}{2}} E^{\frac{1}{2}} dE \quad \because L^3 = 1$$

Bloch Theorem:

According to free electron model, a conduction electron in metal experiences constant potential. But in real crystal, there exists a periodic arrangement of positively charged ions through which the electrons move. As a consequence, the potential experienced by electrons is not constant but it varies with the periodicity of the lattice. In zone theory, as per Bloch, potential energy of electrons considered as varying potential with respect to lattice 'a'.

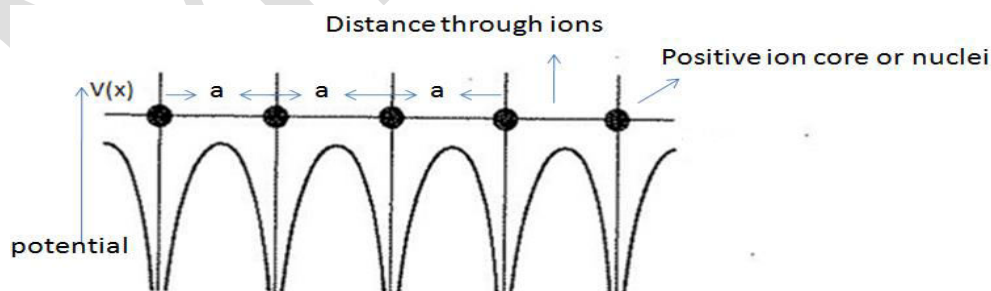


Fig: Variation of potential energy in a periodic lattice.

Let us examine one dimensional lattice as shown in figure. It consists of array of ionic cores along X-axis. A plot of potential V as a function of its position is shown in figure.

From graph:

At nuclei or positive ion cores, the potential energy of electron is minimum and in-between nuclei, the P.E. is considered as maximum w.r.to. Lattice constant 'a'.

This periodic potential $V(x)$ changes with the help of lattice constant a ,

$$V(x) = V(x + a) \quad ('a' \text{ is the periodicity of the lattice})$$

To solve, by considering Schrodinger's time independent wave equation in one dimension,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} [E - V(x)]\psi = 0 \dots \dots (1)$$

Bloch's 1D solution for Schrodinger wave equation (1) $\psi_k(x) = u_k(x)\exp(ikx) \dots \dots \dots (2)$

where $u_k(x) = u_k(x + a)$

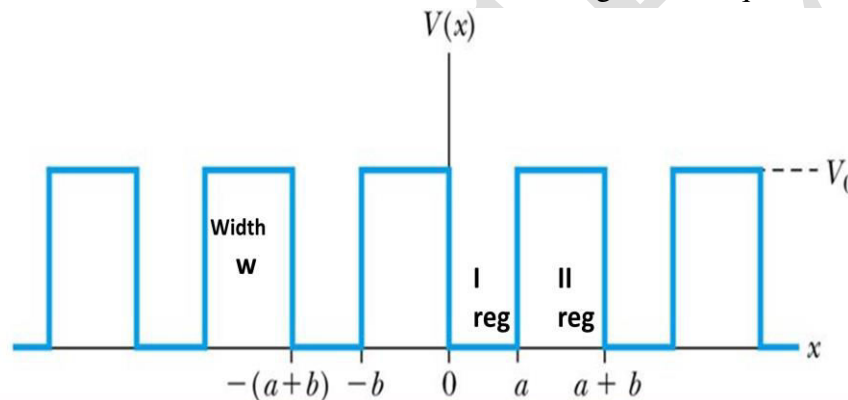
Here $u_k(x)$ -periodicity of crystal lattice, modulating function, k - propagation vector $= \frac{2\pi}{\lambda}$
 e^{ikx} is plane wave.

By applying eq.n (2) to eq.n (1), it is not easy to solve Schrodinger wave equation and Bloch cannot explain complete physical information about an e^- in periodic potential field. Then Kronig Penny model was adopted to explain the electrical properties of an e^- .

Kronig-Penney model:

Kronig-penny approximated the potentials of an e^- s inside the crystal in terms of the shapes of rectangular steps as shown, i.e. square wells is known as Kronig Penny model.

i.e. The periodic potential is taken in the form of rectangular one dimensional array of square well potentials and it is the best suited to solve Schrodinger wave equation.



It is assumed that the potential energy is zero when x lies between 0 and a , and is considered as I region. Potential energy is V_0 , when x lies between $-b$ and 0 . And considered as II region.

Boundary conditions:

$V(x) = 0$, where x lies between $0 < x < a$ - I region. $V(x) =$

V_0 , where x lies between $-b < x < 0$ - II region.

This model explains many of the characteristic features of the behavior of electrons in a periodic lattice.

The wave function related to this model may be obtained by solving Schrodinger equations for the two regions,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0, \text{ for } 0 < x < a \text{ with } V(x) = 0 \dots \dots \dots (1)$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V_0)\psi = 0, \text{ for } -b < x < 0 \text{ with } V(x) = V_0 \dots \dots \dots (2)$$

Again,

$$\frac{d^2\psi}{dx^2} + \alpha^2 \psi = 0 \dots\dots (3)$$

$$\text{where } \alpha^2 = \frac{2mE}{\hbar^2} \text{ and } \alpha = \frac{2\pi}{h} \sqrt{2mE}$$

$$\frac{d^2\psi}{dx^2} - \beta^2 \psi = 0 \dots\dots (4)$$

$$\text{where } \beta^2 = \frac{2m}{\hbar^2} (V_0 - E)$$

The solution of these equations from Bloch theorem, $\psi_k(x) = u_k(x)\exp(ikx)$. From figure, square well potentials, if V_0 increases, the width of barrier 'w' decreases, if V_0 decreases the width of barrier w increases. But the (product) barrier strength V_0w remains constant.

To get this, differentiating above Schrodinger wave equations 3 & 4 w.r.to x, and by applying boundary conditions of x (w.r.to their corresponding Ψ), to know the values of constants A, B of region -I, C,D-for reg-II, we get mathematical expression (by simplification)

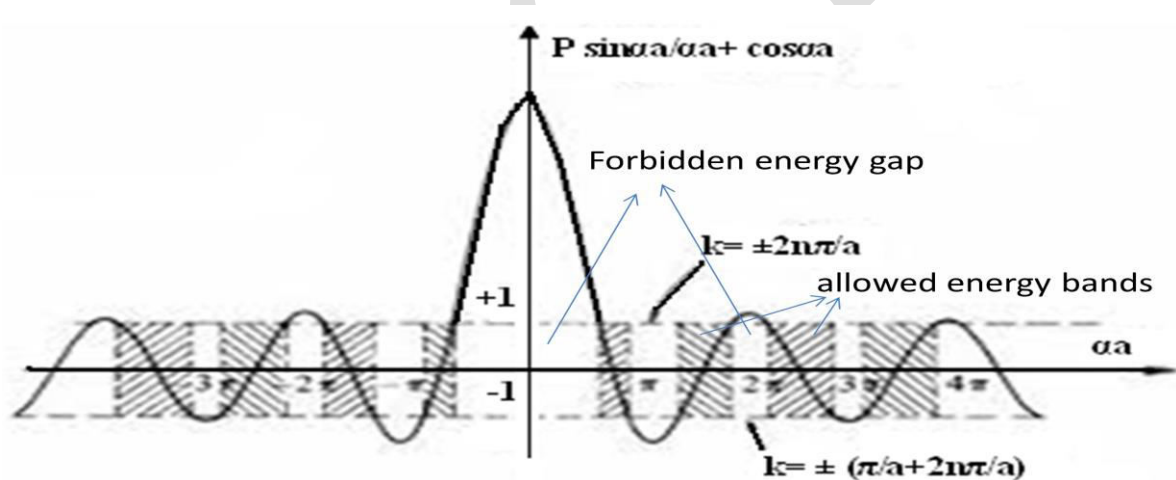
$$\cos ka = P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a$$

where,

$$P = \frac{4\pi^2 ma}{h^2} V_0 w \quad \text{and} \quad \alpha = \frac{2\pi}{h} \sqrt{2mE}$$

P-varying term, known as scattering power.

And ' $V_0 w$ ' is known as barrier strength.



Conclusions:

1. The L.H.S is a cosine term which varies between the limits -1 and +1, and hence the R.H.S also varies between these limits. It means energy is restricted within -1 to +1 only.
2. If the energy of e^- lies between -1 to +1, are called **allowed energy bands** and it is shown by shaded portion in energy spectrum. This means that ' αa ' can take only certain range of values belonging to allowed energy band.
3. As the value of αa increases, the width of the allowed energy bands also increases.
4. If energy of e^- s not lies between -1 to +1 are known as **forbidden energy bands** and it decreases w.r.to increment of αa .
5. Thus, motion of e^- s. in a periodic lattice is characterized by the bands of allowed & forbidden energy levels.

Case 1:

1. $P \rightarrow \infty$

If $P \rightarrow \infty$, the allowed band reduces to a single (line) energy level, gives us steeper lines.

We have

$$\cos ka \left(\frac{\alpha a}{p} \right) = \sin \alpha a + \cos \alpha a \left(\frac{\alpha a}{p} \right)$$

$$P \rightarrow \infty, \frac{1}{\infty} = 0 \quad \text{then} \quad \sin \alpha a = 0$$

$$\sin \alpha a = \sin n\pi$$

$$\alpha a = n\pi$$

$$\alpha^2 a^2 = n^2 \pi^2$$

$$\alpha^2 = \frac{n^2 \pi^2}{a^2}$$

$$\frac{2mE}{\hbar^2} = \frac{n^2 \pi^2}{a^2}$$

$$E = \frac{n^2 \pi^2 \hbar^2}{2ma^2} = \frac{n^2 \pi^2 \hbar^2}{2ma^2 4\pi^2}$$

$$E = \frac{n^2 \hbar^2}{8ma^2} \quad , \text{ here } a \text{ is lattice constant}$$

It means, it (zone theory) supports quantum free electron theory.

Case 2:

$P \rightarrow 0$, We have

$$\cos ka = P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a$$

$$\alpha a = ka$$

$$\alpha = k$$

$$\alpha^2 = k^2$$

$$\frac{2mE}{\hbar^2} = \left(\frac{2\pi}{\lambda} \right)^2 = \frac{4\pi^2}{\lambda^2}$$

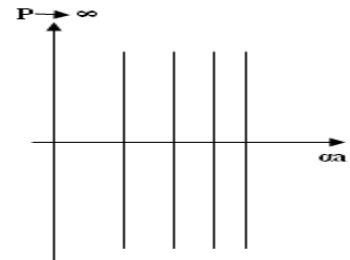
$$E = \frac{4\pi^2 \hbar^2}{2m\lambda^2}$$

$$E = \frac{4\pi^2 \hbar^2}{2m\lambda^2 4\pi^2}$$

$$E = \frac{1}{2} mv^2$$

It gives us kinetic energy of an electron. It means zone theory supports classical free electron theory at this situation electron completely free electron not bounded with allowed and forbidden gaps (and no energy level exists).

Thus by varying P from 0 to ∞ , we find that the completely free electron(s) becomes completely bound to Brillouin Zone.

Brillouin Zone OR E-K diagram:

The Brillouin zone are the boundaries that are marked by the values of wave vector k , in which electrons can have allowed energy values. These represent the allowed values of k of the electrons in 1D, 2D, & 3D.

We have, the energy of the electron in a constant potential box is,

$$E = \frac{n^2 h^2}{8ma^2} \dots (1) \text{ where } a = \text{length of the box.}$$

But,

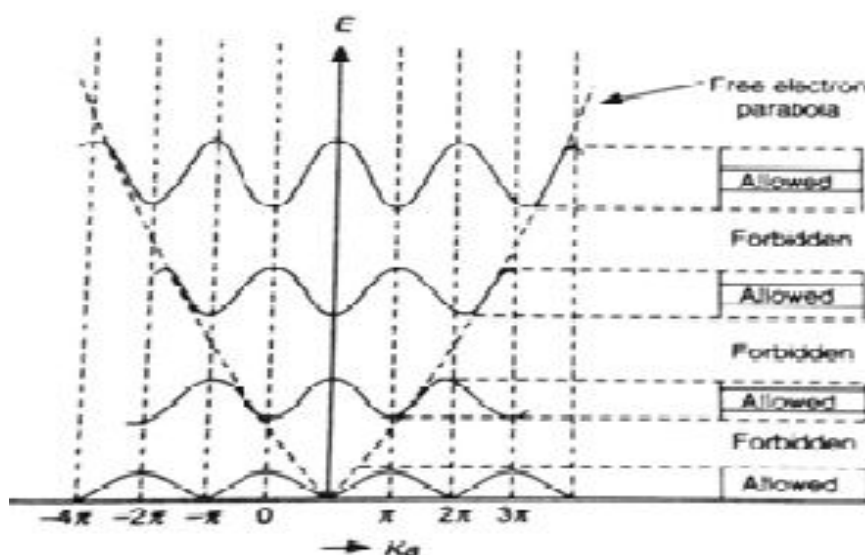
$$k = \frac{n\pi}{a} \Rightarrow k^2 = \frac{n^2 \pi^2}{a^2}$$

$$\frac{n^2}{a^2} = \frac{k^2}{\pi^2} \dots \dots \dots (2)$$

Substitute eqn (2) in (1), we get $E = \frac{k^2 h^2}{8m\pi^2}$; $E \propto k^2$. It represents parabolic equation.

A graph is drawn between the total energy (E) and the wave vector k , for various values of k .

i.e. $k = \frac{n\pi}{a}$; $n = \pm 1, \pm 2, \pm 3, \dots$



It is the energy spectrum of an electron moving in presence of a periodic potential field and is divided into allowed energy regions (allowed zones) or forbidden energy gaps (forbidden zones).

Allowed energy values lie in the region $k = -\pi/a$ to $+\pi/a$. This zone is called the first Brillouin zone. After a break in the energy values, called forbidden energy band, we have another allowed zone spread from $k = -\pi/a$ to $-2\pi/a$ and $+\pi/a$ to $+2\pi/a$. This zone is called the second Brillouin zone. Similarly, higher Brillouin zones are formed

Concept of effective mass of electron:

When an electron in a periodic potential of lattice is accelerated by an known electric field or magnetic field, then the mass of the electron is called effective mass and is represented by m^* .

To explain, let us consider an electron of charge 'e' and mass 'm' moving inside a crystal lattice of electric field E .

Then by taking known expression $F=ma$, can be considered here as $F = m * a \dots\dots(1)$

The acceleration $a = \frac{eE}{m}$ is not constant in the periodic lattice but varies due to the change in electronic mass.

If free electron under wave packet, the group velocity V_g corresponding to the particle's velocity can be written as

$$v_g = \frac{d\omega}{dk} = 2\pi \frac{dv}{dk} = \frac{2\pi}{h} \frac{dE}{dk} = \frac{1}{h} \frac{dE}{dk} \dots\dots (2) \text{ where } E = hv$$

The rate of change of velocity is known as

$$\text{Acceleration, } a = \frac{dv_g}{dt} = \frac{1}{h} \frac{d^2E}{dk dt}$$

$$a = \frac{1}{h} \frac{dE}{dt} \frac{dk}{dk} = \frac{1}{h} \frac{d^2E}{dk^2} \frac{dk}{dt} \dots\dots (3)$$

$$\text{From quantum mechanics relation, } p = \hbar k \dots (4)$$

$$\text{and } F = \frac{dp}{dt} \dots\dots (5)$$

By differentiating eq (4) w.r.to t, and by substituting eq(5)

$$\hbar \frac{dk}{dt} = \frac{dp}{dt} = F \Rightarrow \frac{dk}{dt} = \frac{F}{\hbar} \dots\dots (6)$$

by substituting eq.(6) in eq (3),

$$\therefore a = \frac{1}{h} \frac{d^2E}{dk^2} \frac{F}{\hbar}$$

By rearranging the above term and by comparing with eq.(1)

$$F = \left[\frac{\hbar^2}{\left(\frac{d^2E}{dk^2} \right)} \right] a$$

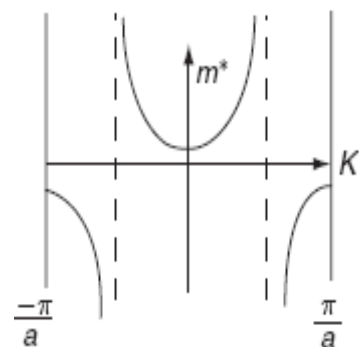
$$\therefore m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2} \right)}$$

Is known as expression for m^* and it is depends on E and K.

Variation of m^* with k:

The graph shows variation of m^* with k. Near $k=0$, effective mass approaches m . As the k value increases m^* increases, reaching its maximum value known as infinite effective mass.

Above the point of inflexion, m^* is negative and as k tends to $\frac{\pi}{a}$, it decreases to a small negative value called negative effective mass in the lower region. The positively charged particle which can be located in the lower region called negative effective mass



Origin of energy band formation in Solids:

The band theory of solids explains the formation of energy bands and determines whether a solid is a conductor, semiconductor or insulator.

The existence of continuous bands of allowed energies can be understood starting with the atomic scale. The electrons of a single isolated atom occupy atomic orbitals, which form a discrete set of energy levels.

When two identical atoms are brought closer, the outermost orbits of these atoms overlap and interact. When the wave functions of the electrons of different atoms begin to overlap considerably, the energy levels corresponding to those wave functions split.

If more atoms are brought together more levels are formed and for a solid of N atoms, each of the energy levels of an atom splits into N energy levels. These energy levels are so close that they form an almost continuous band. The width of the band depends upon the degree of overlap of electrons of adjacent atoms and is largest for the outermost atomic electrons.

As a result of the finite width of the energy bands, gaps are essentially leftover between the bands called forbidden energy gap.

The electrons first occupy the lower energy levels (and are of no importance) then the electrons in the higher energy levels are of important to explain electrical properties of solids and these are called valence band and conduction band.

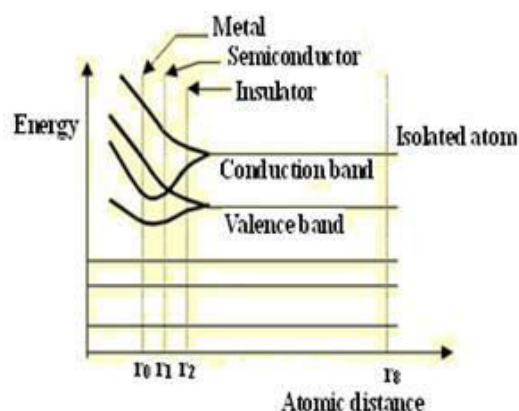
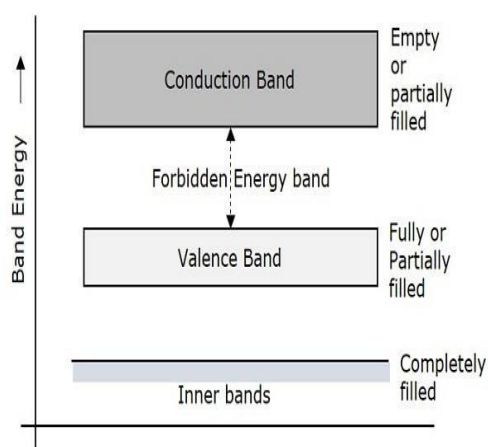


Fig: Formation of energy bands when atoms are closer

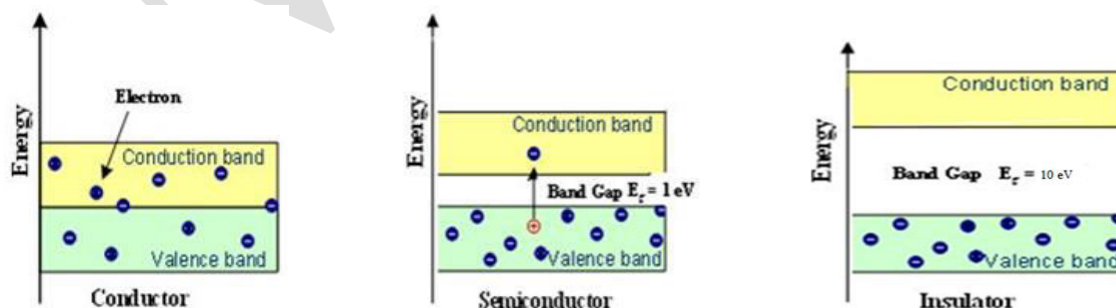
Valence band: A band occupied by valence electrons and is responsible for electrical, thermal and optical properties of solids and it is filled at 0K.

Conduction band: A band corresponding to outer most orbit is called conduction band and is the highest energy band and it is completely empty at 0K.

The **forbidden energy gap** between valence band conduction band is known as the energy band gap. By this solids are classified in to conductors, semiconductors and insulators.

Classification of solids into conductors , semiconductors & insulators:

Based on the energy band diagram materials or solids are classified as follows:



Conductors: In this kind of materials, there is no forbidden gap between the valence band and conduction band. It is observed that the valence band overlaps with the conduction band in metals as shown in figure. There are sufficient numbers of free electrons, available for electrical conduction and due to the overlapping of the two bands there is an easy transition

of electrons from one band to another band takes place, and there no chance for the presence of holes. Resistivity of conductors is very small and it is very few milli ohm meters. ($\Omega \text{ m}$).

Examples: All metals (Na, Mg, Al, Cu, Ni Cu, Ag, Li, Ar etc)

Semiconductors:

In semiconductors, there is a band gap exists between the valence band and conduction band and it is very less and it is the order of -1 to 2 eV are known as semiconductors. It will conduct electricity partially at normal conditions. The electrical resistivity values are 0.5 to 10^3 ohm meter. Due to thermal vibrations within the solid, some electrons gain enough energy to overcome the band gap (or barrier) and behave as conduction electrons. Conductivity exists here due to electronics and holes.

Examples: Silicon, Germanium, Ga As.

Insulators:

In insulators, the width of forbidden energy gap between the valence band and conduction band is very large. Due to large energy gap, electrons cannot jump from V.B to C.B. Energy gap is of the order of ~ 10 eV and higher than semiconductors. Resistivity values of insulators are 10^7 to 10^{12} ohm-m. Electrons are tightly bound to the nucleus, no valence electrons are available.

Examples: Wood, rubber, glass.

Unit III

SEMICONDUCTOR PHYSICS

SEMICONDUCTORS: The substances whose conductivity lies in between conductors and insulators are called as *semiconductors*.

Properties:

1. The conductivity of a semiconductors depends on it's temperature and it is increased with increase in temperature.
2. The energy gap of a semiconductor decreases with rise in temperature.
3. At 0 K a semiconductor becomes an insulator.
4. The absence of an electron in the VB of a semiconductor is called hole. Hole occur only in the VB. Or An empty electron state is called a hole.
5. Like electrons, the hole in the VB also conduct electricity in case of a semiconductor.
6. The electric current in a semiconductor is the sum of the currents due to electron and hole.
7. Hole is a positive charge carrier of current.

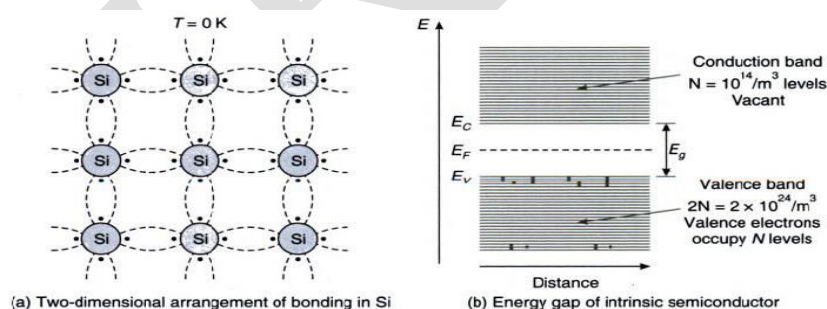
TYPES:

Depending on the relative concentration of holes and electrons the semiconductors are classified into two types.

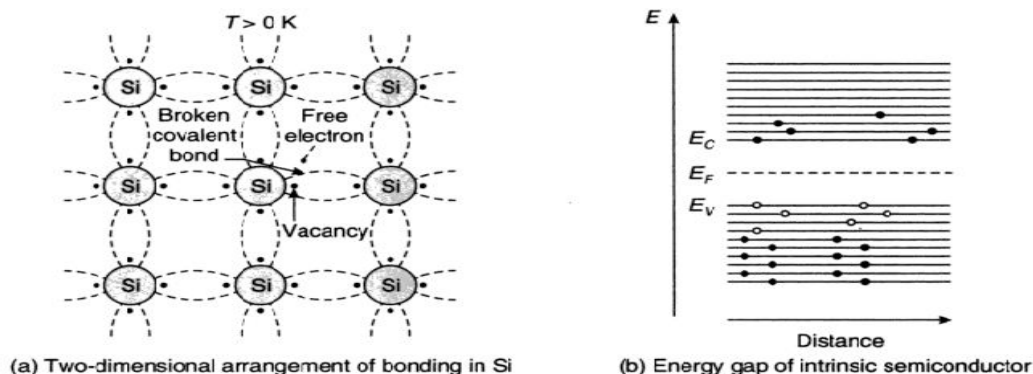
1. Intrinsic Semiconductor
2. Extrinsic Semiconductor.

Intrinsic Semiconductor: The semiconductor which is pure and having the number of electrons in CB equal to number of holes in VB is called as ***intrinsic semiconductor***. The examples of intrinsic semiconductor are pure silicon and pure germanium crystals.

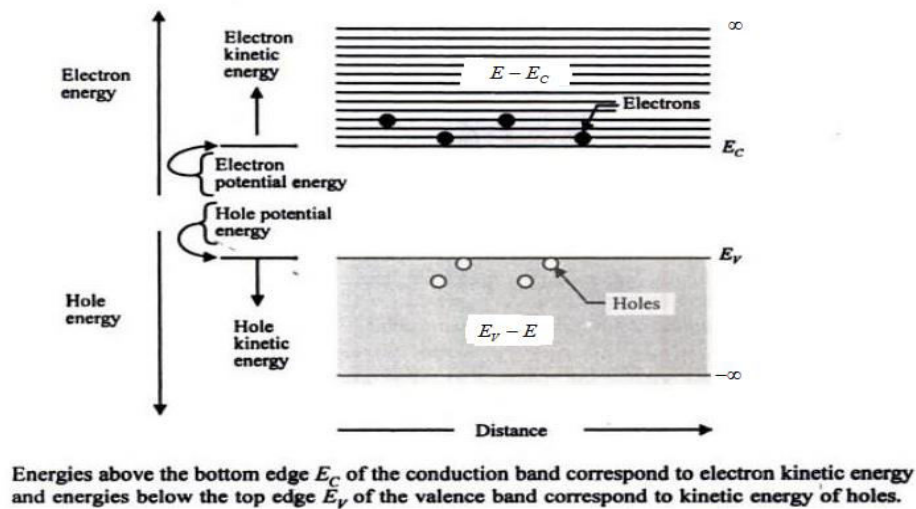
The intrinsic semiconductors at 0 K - The valence band is completely filled at 0 K and all the states in the conduction bands are vacant.



The intrinsic semiconductors at $T > 0\text{ K}$ - The valence band is not completely filled when $T > 0\text{ K}$ and some of the states in the conduction bands are occupied.



CARRIER CONCENTRATION IN AN INTRINSIC SEMICONDUCTOR:



The carrier concentration in an intrinsic semiconductor is determined by finding the concentration of electrons in the conduction band and the concentration of holes in the valence band and by applying the law of mass action.

At temperature above 0 K two types of free charge carriers exist in a semiconductor, electrons in the conduction band and holes in the valence band. In general, it is taken that the conduction band is extended from E_C to $+\infty$ and the valence band is extended from $-\infty$ to E_V .

Concentration of electrons in the conduction band:

In a semiconductor, the conduction band is completely empty and the valence band is completely filled at 0 K . If the temperature becomes greater than 0 K , some of the electrons in valence band get excited into the conduction band.

Let us calculate the electrons available in the conduction band, when $T > 0\text{ K}$. The electron concentration in the conduction band can be obtained using the equation

$$n = \int_{\text{Bottom of CB}}^{\text{Top of CB}} (\text{density of states})(\text{fermi function})$$

$$n = \int_{E_C}^{\infty} g(E)F(E) dE \quad (1)$$

Where $g(E) = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2}$ and $F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$

The electrons are moving in energy band but not in free space, so 'm' should be replaced by the effective mass (m_e^*). The term 'E' is replaced by $(E - E_C)$ because the bottom edge in the conduction band ' E_C ' is the potential energy of an electron at rest in the conduction band then $(E - E_C)$ represents the kinetic energy of the electron at high energy level.

$$n = \int_{E_C}^{\infty} \left[\frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_C)^{1/2} \right] \left[\frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)} \right] dE \quad (2)$$

Since, $E > E_F$, the term $\exp\left(\frac{E-E_F}{KT}\right) > 1$, Therefore, neglecting 1. Eq. (2) can be written as

$$n = \int_{E_C}^{\infty} \left[\frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_C)^{1/2} \right] \exp\left[-\left(\frac{E - E_F}{KT}\right)\right] dE \quad (3)$$

The above integral is solved by substituting $E - E_C = x$, then $dE = dx$, the lower limit of this integral becomes zero and the upper limit is ' ∞ '. Eq. (3) becomes

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_0^{\infty} [(x)^{1/2}] \exp\left[-\left(\frac{x + E_C - E_F}{KT}\right)\right] dx \quad (4)$$

Equation (4) can be written as

$$\begin{aligned} n &= \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_0^{\infty} [(x)^{1/2}] \exp\left[-\left(\frac{E_C - E_F}{KT}\right)\right] \exp\left(\frac{-x}{KT}\right) dx \\ n &= \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp\left(\frac{E_F - E_C}{KT}\right) \int_0^{\infty} (x)^{1/2} \exp\left(\frac{-x}{KT}\right) dx \end{aligned} \quad (5)$$

The integral $\int_0^{\infty} (x)^{1/2} \exp\left(\frac{-x}{KT}\right) dx$ can be evaluated by substituting $\frac{x}{KT} = y$
Then $KT dy = dx$. There is no change in the upper and lower limit values.

$$\begin{aligned} \int_0^{\infty} (x)^{1/2} \exp\left(\frac{-x}{KT}\right) dx &= \int_0^{\infty} (yKT)^{1/2} \exp(-y) (KT) dy \\ &= (KT)^{3/2} \int_0^{\infty} (y)^{1/2} \exp(-y) dy \end{aligned} \quad (6)$$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp\left(\frac{E_F - E_C}{KT}\right) (KT)^{3/2} \int_0^{\infty} (y)^{1/2} \exp(-y) dy \quad (7)$$

The integral $\int_0^{\infty} (y)^{1/2} \exp(-y) dy$ can be solved using the Gamma function.

The Gamma function is

$$\begin{aligned} \int_0^{\infty} y^{(n-1)} \exp(-y) dy &= \Gamma(n) \\ \int_0^{\infty} y^{(3/2-1)} \exp(-y) dy &= \Gamma\left(\frac{3}{2}\right) \\ &= \Gamma\left(\frac{1}{2} + 1\right) \\ &= \frac{1}{2} \Gamma\left(\frac{1}{2}\right) \end{aligned} \quad \boxed{\Gamma(n+1) = n \Gamma(n)} \quad \boxed{\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}}$$

Substituting $\int_0^{\infty} y^{(3/2-1)} \exp(-y) dy$ in equation (7)

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp\left(\frac{E_F - E_C}{KT}\right) (KT)^{3/2} \frac{\sqrt{\pi}}{2}$$

$$n = 2 \left[\frac{2\pi m_e^* KT}{h^2} \right]^{3/2} \exp\left(\frac{E_F - E_C}{KT}\right)$$

$$\boxed{n = N_C \exp\left(\frac{E_F - E_C}{KT}\right)} \quad \text{Where } N_C = 2 \left[\frac{2\pi m_e^* KT}{h^2} \right]^{3/2}$$

The above eq. gives the electron concentration in the conduction band.

Concentration of holes in the valence band:

In a semiconductor the conduction band is completely empty and valence band is completely filled at **0 K**. If the temperature becomes greater than **0 K**, some of the electrons in valence band gets excited into the conduction band.

Let us calculate the number of holes available in the valence band, when **T > 0 K**. The concentration of holes in the valence band is calculated by

$$p = \int_{\text{Bottom of VB}}^{\text{Top of VB}} (\text{density of states})(\text{fermi function})$$

$$p = \int_{\text{Bottom of VB}}^{\text{Top of VB}} g(E) F_h(E) dE$$

We know that, the hole and electron occupation probability of the state of energy 'E' is always equal to 1.

It can be written as $F_e(E) + F_h(E) = 1$

The probability of occupancy of state by hole $F_h(E) = 1 - F_e(E)$

$$p = \int_{-\infty}^{E_V} g(E)[1 - F_e(E)] dE \quad (1)$$

Where $g(E) = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2}$ and $F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$

$$1 - F_e(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$$

$$= \frac{\exp\left(\frac{E - E_F}{KT}\right)}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$$

Since, $E < E_F$, the term $\exp\left(\frac{E - E_F}{KT}\right)$ in the denominator is less than 1. So, it is negligible, Therefore

$$1 - F_e(E) = \exp\left(\frac{E - E_F}{KT}\right)$$

Substitute $g(E)$ and $1 - F_e(E)$ values in eq. (1)

$$p = \int_{-\infty}^{E_V} \left[\frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} \right] \left[\exp\left(\frac{E - E_F}{KT}\right) \right] dE \quad (2)$$

The term E in eq. (2) is replaced by $(E_V - E)$ because E_V is the potential energy of a hole at rest. Then $(E_V - E)$ represents the kinetic energy of a hole at higher energy level. Note that the levels below the level E_F in the diagram are higher energy levels for holes. The hole is in energy band not in free space, so its mass is replaced by effective mass m_h^*

Then eq. (2) becomes

$$p = \int_{-\infty}^{E_V} \left[\frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{1/2} \right] \left[\exp\left(\frac{E - E_F}{KT}\right) \right] dE \quad (3)$$

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} \exp\left[-\left(\frac{E_F - E}{KT}\right)\right] dE \quad (4)$$

Let $E_V - E = x$, Then $-dE = dx$

The lower limit of this integral becomes ∞ and the upper limit is zero.

The integral (4) can be written as

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_{-\infty}^0 (x)^{1/2} \exp \left[- \left(\frac{E_F - (E_V - x)}{KT} \right) \right] (-dx) \quad (5)$$

The above eq. (5) can be written as

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_0^{\infty} (x)^{1/2} \exp \left(\frac{E_V - E_F}{KT} \right) \exp \left(\frac{-x}{KT} \right) dx$$

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \exp \left(\frac{E_V - E_F}{KT} \right) \int_0^{\infty} (x)^{1/2} \exp \left(\frac{-x}{KT} \right) dx \quad (6)$$

The integral $\int_0^{\infty} (x)^{1/2} \exp \left(\frac{-x}{KT} \right) dx$ can be evaluated by substituting $\frac{x}{KT} = y$

Then $KT dy = dx$. There is no change in the upper and lower limit values.

$$\begin{aligned} \text{i.e., } \int_0^{\infty} (x)^{1/2} \exp \left(\frac{-x}{KT} \right) dx &= \int_0^{\infty} (yKT)^{1/2} \exp(-y) KT dy \\ &= (KT)^{3/2} \int_0^{\infty} (y)^{1/2} \exp(-y) dy \end{aligned} \quad (7)$$

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \exp \left(\frac{E_V - E_F}{KT} \right) (KT)^{3/2} \int_0^{\infty} (y)^{1/2} \exp(-y) dy \quad (8)$$

The above integral $\int_0^{\infty} (y)^{1/2} \exp(-y) dy$ can be solved using the Gamma functions

The Gamma function is

$$\begin{aligned} \int_0^{\infty} y^{(n-1)} \exp(-y) dy &= \Gamma(n) \\ \int_0^{\infty} y^{(3/2-1)} \exp(-y) dy &= \Gamma\left(\frac{3}{2}\right) \\ &= \Gamma\left(\frac{1}{2} + 1\right) \\ &= \frac{1}{2} \Gamma\left(\frac{1}{2}\right) \end{aligned} \quad \boxed{\Gamma(n+1) = n \Gamma(n)}$$

$$\boxed{\int_0^{\infty} y^{(3/2-1)} \exp(-y) dy = \frac{\sqrt{\pi}}{2}} \quad \boxed{\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}}$$

Eq. (8) can be written as

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \exp \left(\frac{E_V - E_F}{KT} \right) (KT)^{3/2} \frac{\sqrt{\pi}}{2}$$

$$p = 2 \left[\frac{2\pi m_h^* KT}{h^2} \right]^{3/2} \exp \left(\frac{E_V - E_F}{KT} \right)$$

$$\boxed{p = N_V \exp \left(\frac{E_V - E_F}{KT} \right)}$$

$$\text{Where } N_V = 2 \left[\frac{2\pi m_h^* KT}{h^2} \right]^{3/2}$$

The above equations gives the hole concentration in the valence band.

FERMI LEVEL IN INTRINSIC SEMICONDUCTOR

In an intrinsic semiconductor the concentration of in the valence band is equal to the concentration of electron in the conduction band.

$$\text{i.e., } n = p \quad (1)$$

Substituting n & p values in eq. (1)

$$N_C \exp \left(\frac{E_F - E_C}{KT} \right) = N_V \exp \left(\frac{E_V - E_F}{KT} \right)$$

$$\frac{N_C}{N_V} = \exp \left(\frac{E_V - E_F - E_F + E_C}{KT} \right)$$

$$\frac{N_C}{N_V} = \exp \left(\frac{E_V + E_C - 2E_F}{KT} \right)$$

Taking \log both sides we get,

$$\log\left(\frac{N_C}{N_V}\right) = \frac{E_V + E_C - 2E_F}{KT}$$

$$E_V + E_C - 2E_F = KT \log\left(\frac{N_C}{N_V}\right)$$

$$E_F = \frac{E_V + E_C}{2} - \frac{KT}{2} \log\left(\frac{N_C}{N_V}\right) \quad (2)$$

Now

$$\frac{N_C}{N_V} = \frac{2 \left[\frac{2\pi m_e^* KT}{h^2} \right]^{3/2}}{2 \left[\frac{2\pi m_h^* KT}{h^2} \right]^{3/2}} = \left(\frac{m_e^*}{m_h^*} \right)^{3/2} \quad (3)$$

From eq. (2) and eq. (3) we get

$$E_F = \frac{E_V + E_C}{2} - \frac{3KT}{4} \log\left(\frac{m_e^*}{m_h^*}\right) \quad (4)$$

The above eq. (4) gives the value of the Fermi level in intrinsic semiconductor.

As KT is small and the effective mass m_e^* and m_h^* do not usually differ much, the second term in the eq. (4) may be ignored.

If the effective mass of a free electron is assumed to be equal to the effective mass of a hole.

$$m_e^* = m_h^*$$

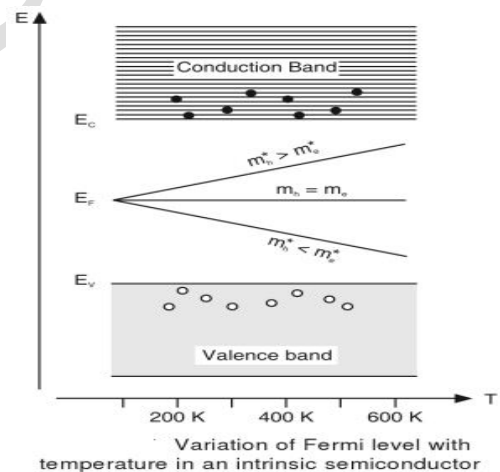
We get
$$E_F = \frac{E_V + E_C}{2}$$

Thus, the Fermi level in a semiconductor lies at the center of the energy gap.

Variation Of Fermi Level With Temperature:

The Fermi level in an intrinsic semiconductor may be considered as independent of temperature as staying in the middle of the band gap.

- ✓ If $m_e^* > m_h^*$, E_F displaced downward to the top edge of the valence band.
- ✓ If $m_e^* < m_h^*$, with increase in temperature, the Fermi level gets displaced upward to the bottom edge of the conduction band.



EXTRINSIC SEMICONDUCTOR

When small quantities of selected impurities are added to an intrinsic semiconductor it becomes an extrinsic semiconductor. Depending upon the type of impurity extrinsic semiconductors are of two types, namely

1. P – type semiconductor.
2. N – type semiconductor.

Carrier Concentration:

The number of charge carriers present per unit volume of a semiconductor material is called **carrier concentration**.

Carrier Concentration In Extrinsic Semiconductor:

Consider an extrinsic semiconductor in which N_A acceptor atoms and N_D donor atoms are doped per unit volume of the semiconductor. Then at some room temperature T K the material contains n , p , N_A^- and N_D^+ .

The material will be electrically neutral if

$$n + N_A^- = p + N_D^+$$

Where $n \rightarrow$ concentration of free electrons.

$p \rightarrow$ concentration of holes.

$N_A^- \rightarrow$ concentration of acceptor ions per unit volume.

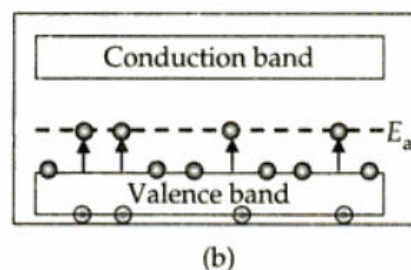
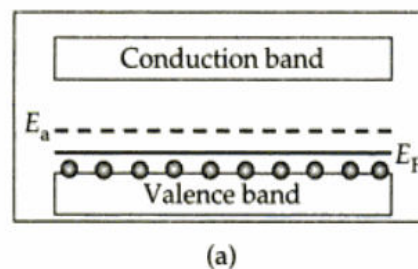
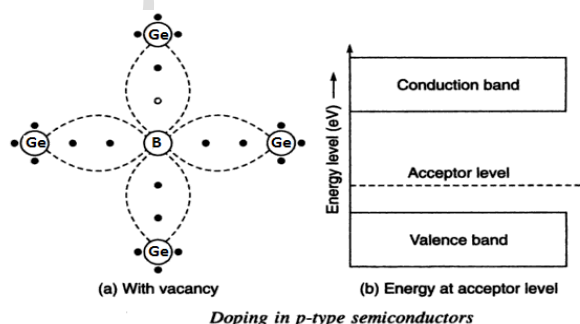
$N_D^+ \rightarrow$ concentration of donor ions per unit volume.

That is total concentration of negative charge in the semiconductor is equal to total concentration of positive charge in the semiconductor. The above equation is called **charge neutrality equation**.

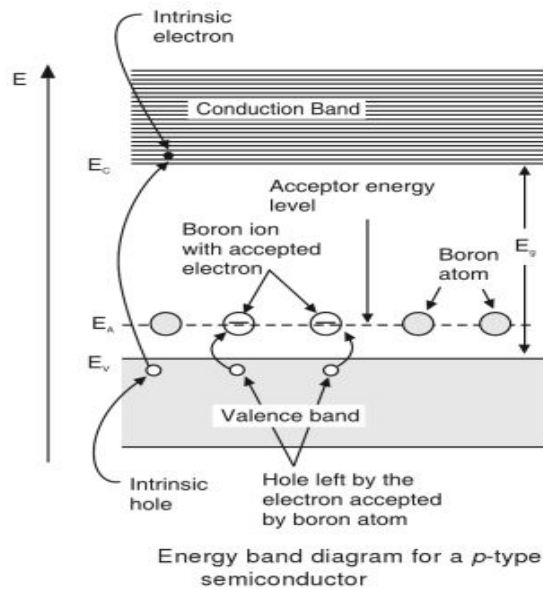
P – TYPE SEMICONDUCTOR:

Generally pure semiconductor has 4 valence electrons and can form four covalent bonds. When a trivalent impure atoms (which have three valence electrons) like Gallium (*Ga*), *Al*, *In* or *Boron (B)* is added, these three valence electrons form three covalent bonds with, say *Ge* atom. But the 4th electron doesn't have a pair, so a '**hole**' exists. This means that it's ready to 'accept' an electron to fill the hole. Thus a small amount of trivalent impurity creates '**majority of holes**', which are positive, and are called as '**p-type semiconductor**' or '**Acceptors**'.

In a P - type semiconductor holes are majority current carriers and **electrons** are minority current carriers. In a P- type semiconductor, **the energy level of this acceptor impurity lies just above the valence band**. This energy level is called **acceptor level** and it is represented as E_A .



(a) $T = 0$ K and (b) $T > 0$ K

CARRIER CONCENTRATION IN P- TYPE SEMICONDUCTOR:

In a *p – type* semiconductor, the doping concentration of acceptor atom is very large in comparison to the donor atoms, So N_D^+ is negligible. Further, in *p – type* semiconductors ‘*n*’ is due to thermal generation only, so it will also be very small in comparison to N_A^- . Therefore, the charge neutrality equation for the *p – type* semiconductor as

$$P = N_A^- \quad (1)$$

Where

$p \rightarrow$ concentration of holes $N_A^- \rightarrow$
concentration of acceptor ions available in the acceptor level
or Ionised acceptor impurity.

The concentration of acceptor ion N_A^- is equal to the product of concentration of acceptor atoms N_A and the probability of finding an electron in acceptor level $f(E_A)$.

$$N_A^- = N_A f(E_A) \quad (2)$$

Where, N_A is acceptor concentration.

$f(E_A)$ is the probability that a quantum state is occupied by an electron in the acceptor level

From the Fermi dirac distribution function

$$f(E_A) = \frac{1}{1 + \exp\left(\frac{E_A - E_F}{KT}\right)} \quad (3)$$

Substituting eq. (3) in eq. (2) we get,

$$N_A^- = \frac{N_A}{1 + \exp\left(\frac{E_A - E_F}{KT}\right)} \quad (4)$$

Since, the acceptor level lies above the Fermi level, $E_A - E_F$ is positive.

Therefore, the term 1 is neglected, compared to $\exp\left(\frac{E_A - E_F}{KT}\right)$. Therefore,

Eq. (4) can be written as

$$N_A^- = N_A \exp\left[-\left(\frac{E_A - E_F}{KT}\right)\right] \quad (5),$$

Sub eq. (5) in eq. (1)

$$P = N_A \exp \left[- \left(\frac{E_A - E_F}{KT} \right) \right] \quad (6)$$

The concentration of holes in the valence band is

$$p = N_V \exp \left(\frac{E_V - E_F}{KT} \right) \quad (7)$$

Sub eq. (7) in eq. (6)

$$N_V \exp \left(\frac{E_V - E_F}{KT} \right) = N_A \exp \left[- \left(\frac{E_A - E_F}{KT} \right) \right]$$

$$\text{i.e., } \exp \left[\frac{E_V - E_F + E_A - E_F}{KT} \right] = \frac{N_A}{N_V}$$

In the above eq, taking log on both sides, we get,

$$E_F = \frac{E_V + E_A}{2} - \frac{KT}{2} \log \left(\frac{N_A}{N_V} \right) \quad (8)$$

Eq. (8) represents the Fermi level in a *p* – *type* semiconductor. At $T = 0$ K, Eq. (8) becomes

$$E_F = \frac{E_V + E_A}{2} \quad (9)$$

At $T = 0$ K, the Fermi energy level lies at the middle of the top of the VB and the acceptor level.

The concentration of hole in the valence band in an intrinsic semiconductor is

$$p = N_V \exp \left(\frac{E_V - E_F}{KT} \right) \quad (10)$$

Sub eq. (8) in eq. (10)

$$\begin{aligned} p &= N_V \exp \left[\frac{E_V}{KT} - \frac{E_V + E_A}{2KT} - \frac{KT}{2KT} \log \left(\frac{N_A}{N_V} \right) \right] \\ &= N_V \exp \left[\frac{E_V - E_A}{2KT} \right] \left(\frac{N_A}{N_V} \right)^{1/2} \\ &= (N_A N_V)^{1/2} \exp \left[\frac{E_V - E_A}{2KT} \right] \end{aligned} \quad (11)$$

Substituting the value of N_V in eq. (11) we get.

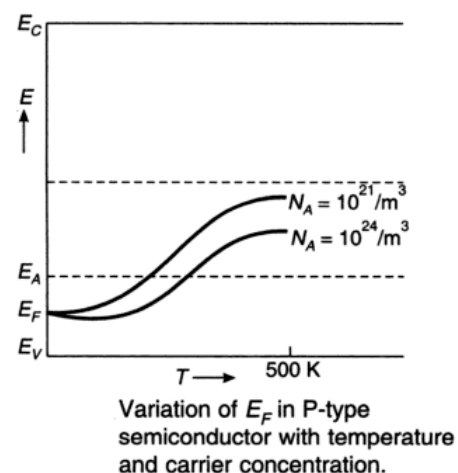
$$p = (2N_A)^{1/2} \left[\frac{2\pi m_h^* KT}{h^2} \right]^{3/4} \exp \left[\frac{E_V - E_A}{2KT} \right]$$

The above eq. represents the concentration of carriers in a *p* – *type* semiconductor.

VARIATION OF FERMI LEVEL WITH TEMPERATURE AND CONCENTRATION

p – type SEMICONDUCTOR

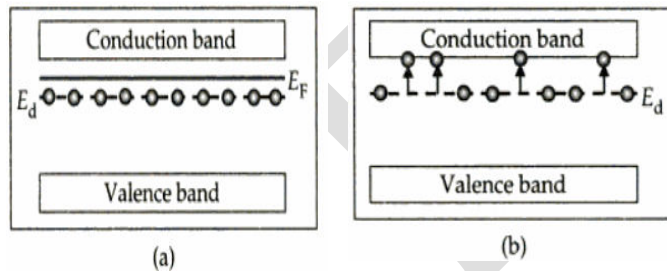
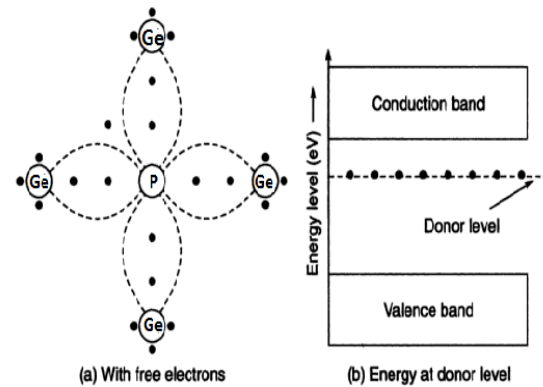
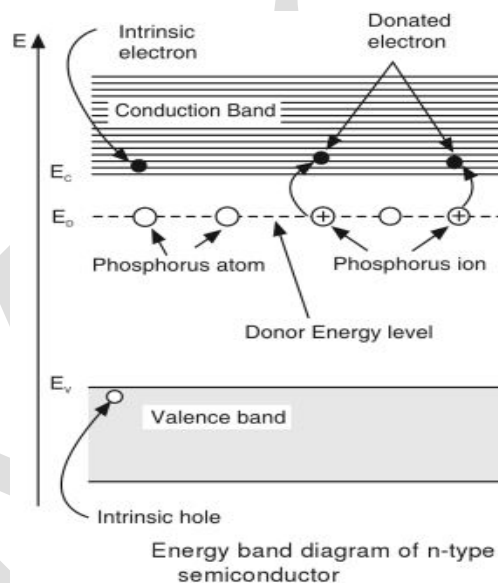
- ✓ At $T = 0$ K, the Fermi energy level lies at the middle of the VB and the acceptor level.
- ✓ At very high temperatures, it lies at middle of the energy gap. It shows that at very high temperature, the *p* – *type* material behave as an intrinsic material.
- ✓ As the acceptor concentration is increased the increase in Fermi energy is minimized and hence the Fermi level is moves down.



N – TYPE SEMICONDUCTOR:

Generally pure semiconductors has **four** valence electrons and can form **four** covalent bonds. When a penta valent impurity, like **P** or **Antimony (Sb)**, (which have five valence electrons) is added to any Ge, the **four** electrons of Ge bond with 4 electrons of 'P' and one extra electron is left out alone. That, it is ready to '**donate**' this electron. Such type of semiconductor is called '**N-type semiconductor**' or '**DONOR**'.

In a N - type semiconductor holes are minority current carriers and electrons are majority current carriers. In a N- type semiconductor, the energy level of this donor impurity lies just *below* the conduction band of the semiconductor. This energy level is called **donor level** and it is represented as E_D .

(a) $T = 0 \text{ K}$ and (b) $T > 0 \text{ K}$ **CARRIER CONCENTRATION IN N - TYPE SEMICONDUCTOR:**

In **n – type** semiconductor, there is no acceptor atoms (and hence the acceptor ions), so acceptor ions ($N_A^- = 0$) are negligible. Further in **n – type** semiconductors, holes are due to thermal generation only, therefore '**p**' will be very small in comparison to N_D^+ , so we can write the charge neutrality equation for the **n – type** semiconductor as

$$\boxed{n = N_D^+} \quad (1)$$

Where $n \rightarrow$ concentration of free electrons

$$n \rightarrow N_C \exp\left(\frac{E_F - E_C}{KT}\right)$$

$N_D^+ \rightarrow$ concentration of donor ions.

Concentration of donor ions N_D^+ is equal to the product of concentration of donor atoms N_D and the probability of the absence of electrons in the donor level E_D .

$$N_D^+ = N_D[1 - f(E_D)] \quad (2)$$

$$1 - f(E_D) = 1 - \frac{1}{1 + \exp\left(\frac{E_D - E_F}{KT}\right)} = \frac{\exp\left(\frac{E_D - E_F}{KT}\right)}{1 + \exp\left(\frac{E_D - E_F}{KT}\right)}$$

Since, $E_D < E_F$, $\exp\left(\frac{E_D - E_F}{KT}\right) < 1$ and hence the term $\exp\left(\frac{E_D - E_F}{KT}\right)$ in the denominator is negligible.

$$1 - f(E_D) = \exp\left(\frac{E_D - E_F}{KT}\right) \quad (3)$$

Sub eq. (3) in eq. (2)

$$N_D^+ = N_D \exp\left(\frac{E_D - E_F}{KT}\right)$$

Substitute n and N_D^+ values in eq. (1)

$$N_C \exp\left(\frac{E_F - E_C}{KT}\right) = N_D \exp\left(\frac{E_D - E_F}{KT}\right)$$

$$\begin{aligned} \text{i.e., } \frac{N_C}{N_D} &= \exp\left(\frac{E_C - E_F}{KT}\right) \exp\left(\frac{E_D - E_F}{KT}\right) \\ &= \exp\left(\frac{-2E_F + E_C + E_D}{KT}\right) \end{aligned} \quad (4)$$

Taking **log** on both sides of eq. (4) we get

$$\begin{aligned} \frac{-2E_F + E_C + E_D}{KT} &= \log\left(\frac{N_C}{N_D}\right) \\ E_F &= \frac{E_C + E_D}{2} - \frac{KT}{2} \log\left(\frac{N_C}{N_D}\right) \end{aligned} \quad (5)$$

Eq. (5) gives the equation for the Fermi level in an n - type semiconductor.

The concentration of electron in the conduction band is

$$n = N_C \exp\left(\frac{E_F - E_C}{KT}\right) \quad (6)$$

Sub eq. (5) in eq. (6)

$$\begin{aligned} n &= N_C \exp\left(\frac{E_C + E_D}{2KT} - \frac{E_C}{KT} - \frac{1}{2} \log \frac{N_C}{N_D}\right) \\ &= N_C \exp\left(\frac{E_C + E_D}{2KT}\right) \left(\frac{N_D}{N_C}\right)^{1/2} \\ n &= (N_C N_D)^{1/2} \exp\left(\frac{E_C + E_D}{2KT}\right) \end{aligned} \quad (7)$$

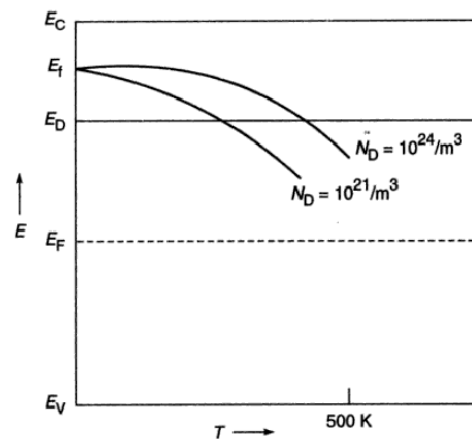
Substituting the value of $N_C = 2 \left[\frac{2\pi m_e^* KT}{h^2} \right]^{3/2}$ in the eq. (7)

$$n = (2N_D)^{1/2} \left[\frac{2\pi m_e^* KT}{h^2} \right]^{3/4} \exp\left[\frac{E_D - E_C}{2KT}\right] \quad (8)$$

Eq. (8) gives the carrier concentration of electron in an n - type semiconductor material.

VARIATION OF FERMI LEVEL WITH TEMPERATURE AND CONCENTRATION**n – type SEMICONDUCTOR**

- ✓ At **0 K** the Fermi level lies in between the conduction band and the donor level.
- ✓ As the temperature increases from **0K**, the Fermi level falls.
- ✓ At higher temperature, it falls below the donor level and hence, it approaches the centre of the forbidden gap. It shows that the material behaving as an intrinsic semiconductor.
- ✓ As the donor concentration is increased, the Fermi level will move up.



Variation of fermi level in an n-type semiconductor with temperature

DIRECT AND INDIRECT BAND GAP SEMICONDUCTORS:

According to the band theory of solids, the energy spectrum of electrons consists of a large number of energy bands and are separated by forbidden regions. The band gap is the difference in the energy between the lowest point of conduction band and highest point of valence band.

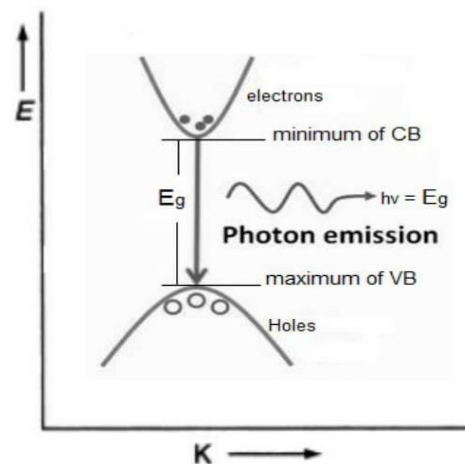
Based on the structure of energy bands and type of energy emission, semiconductors are classified into two types.

- *Direct band gap semi conductor*
- *Indirect band gap semiconductor*

DIRECT BAND GAP SEMICONDUCTOR:

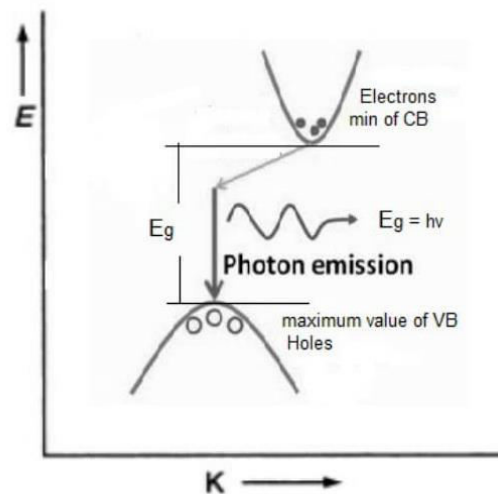
- ✓ The maximum of **VB** and the minimum of **CB** exists at the same value of wave number (**K**). Such semiconductors are called **direct band gap semiconductor**.
- ✓ In this type during the recombination of holes and electron, a **photon of light** is released. This process is known as **radiative recombination**. Also called as **spontaneous emission**.
- ✓ It is more effective, because the direction of motion of electron remains unchanged.
- ✓ In this type of semiconductors life time(*i.e., recombination time*) of charge carrier is very less.
- ✓ Electron – hole pair can be easily generated because it requires less momentum.
- ✓ Recombination probability is much high.
- ✓ These are used to fabricate LEDs and laser diodes.
- ✓ These are mostly from the compound semiconductors.

Ex: InP, GaAs



INDIRECT BAND GAP SEMICONDUCTOR:

- ✓ The maximum value of **VB** and the minimum value of **CB** exists at the different values of wave number (K). Such semiconductors are called **Indirect Band Gap Semiconductor**.
- ✓ In this type, **heat energy** is produced during the recombination of holes and electrons. This process is known as **non radiative** recombination.
- ✓ It is less efficient, because the direction of motion of electron changed.
- ✓ In this type of semiconductors life time (*i.e., recombination time*) of charge carrier is high.
- ✓ The incident process is slower, because it requires an electron, hole and phonon for interaction.
- ✓ Recombination occurs through some defect states.
- ✓ Due to the longer life time of charge carriers, these are used to amplify the signals as in the case of diodes and transistors.
- ✓ These are mostly from the elemental semiconductors.
Ex: Si(0.7 eV), Ge(1.12 eV).

**CARRIER TRANSPORT**

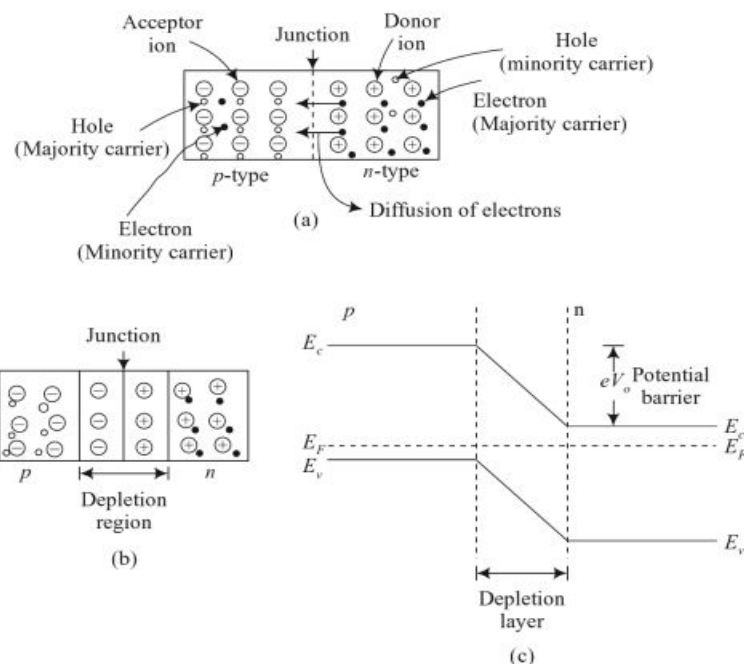
DIFFUSION CURRENT: The current that flows across the junction of a diode due to the difference in the concentration of holes or conduction electrons on it's either side is called diffusion current.

DRIFT CURRENT: The current that flows through the diode due to the electric field is called drift current.

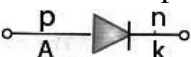
JUNCTION DIODE:

Diode is a device with two electrodes (*di means two and – ode means electrode*).

Formation of p – n Junction diode:



p-n junction diode without biasing (a) When p-type and n-type are brought in contact (b) Depletion region in p-n junction (c) Energy band diagram

- ✓ When a p – type semiconductor is suitably joined with n – type semiconductor a p – n junction diode is formed and the symbol of the p – n junction diode is 
- ✓ Because of the contact, some of the electrons from n – side enter into p – side and holes from p – side goes to n – side.
- ✓ At junction the electrons and hole combine and neutralize each other.
- ✓ Later in the region close to the junction only negative ions are left behind on p – side and only positive ions on n – side.
- ✓ As a result a narrow charged region on either side of the junction is formed where there are no mobile charge carriers.
- ✓ Since this region is depleted of mobile charge carriers it is called **depletion region or charge free region or space charge region or transition region**.
- ✓ The thickness of this region is usually in the order of $0.5 \mu\text{m}$ or 10^{-6} to 10^{-7} m .
- ✓ The depletion layer contains positive and negative immobile ions on either side of the p – n junction. These ions set up a potential difference across the p – n junction which opposes the further diffusion of electrons and holes through the junction. This potential difference is called ‘Potential barrier’.

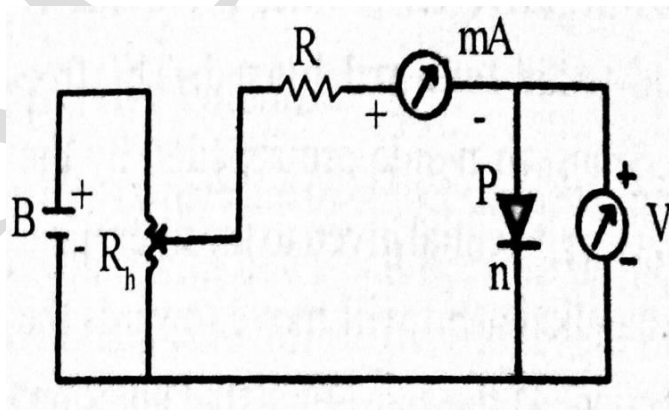
The potential barrier is about **0.7 V** for ‘Si’ and **0.38 V** for ‘Ge’ at room temperature.

USES:

1. It is used as an electronic switch.
2. In a rectifier circuit, it is used to convert *a. c* to *d. c*
3. Certain p – n diodes glow on supply of electricity.

I – V CHARACTERISTICS:

Semiconductor diode is connected to a battery through a rheostat R_h . A ‘mA’ is connected in series to the diode to measure current (I) and a voltmeter is connected across the diode to measure the voltage (V). The diode is forward biased and by varying the voltage across the diode, the current is measured. The polarity of diode is reversed so that it gets reverse biased and **mA** is replaced with micro ammeter μA . The voltage across the diode is varied and the corresponding current is measured.

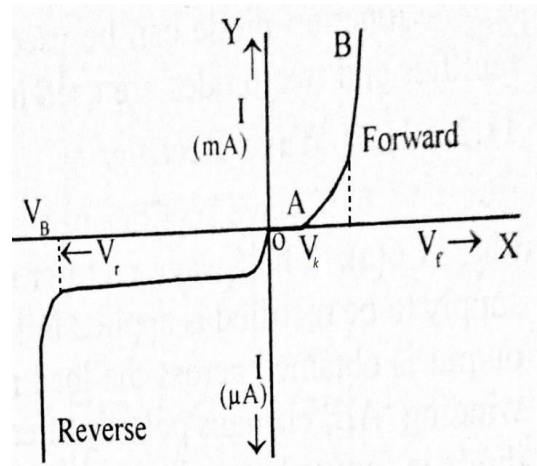
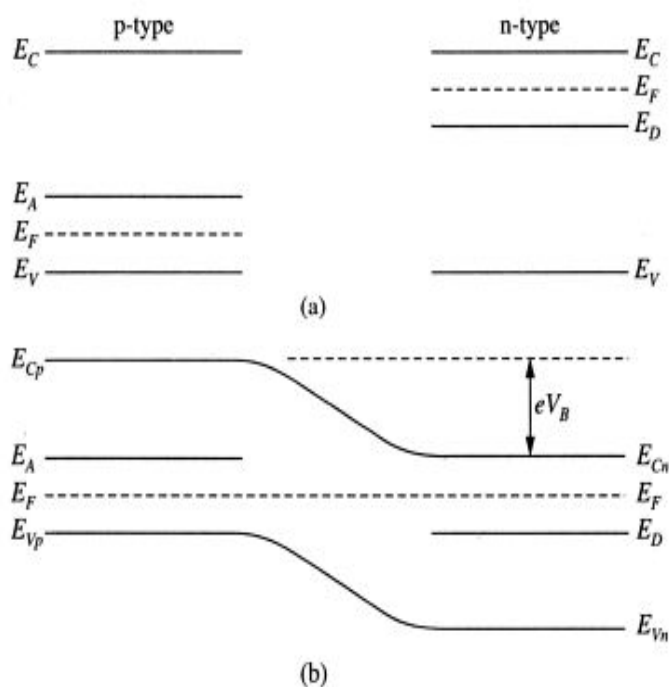


FORWARD BIAS CHARACTERISTICS:

When the battery voltage is zero diode does not conduct and current is zero. As the forward battery voltage increases, the barrier potential starts decreasing and a small current begins to flow. The forward current increases slowly at first but as soon as the battery voltage increases, the forward current increases rapidly. The battery voltage at which the forward current starts increasing rapidly is known as ‘**knee voltage V_K or threshold or cut in or offset voltage**’. The value of cut in voltage for **Ge** is around **0.2 V** to **0.3 V** and for **Si** it is around **0.6 V** to **0.7 V**. If the voltage is increased beyond threshold voltage, the forward current increase exponentially and large current flows.

REVERSE BIAS CHARACTERISTICS:

In the reverse bias, small current flows in the circuit due to minority charge carriers. The applied reverse voltage appears as forward bias to these minority charge carriers. Therefore a small current flows in the reverse direction. This current is called reverse saturation current. If the reverse voltage is increased further after some voltage, there will be a sudden rise of reverse current. This region is named as **break down region**.

**ENERGY LEVEL DIAGRAMS OF p – n JUNCTION DIODE:**

(a) Energy levels in p-and n-type semiconductors before contact (b) Energy levels in the junction after p-and n-type semiconductors are put in contact.

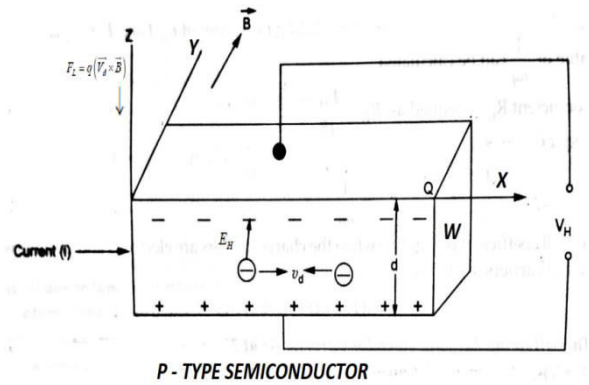
HALL EXPERIMENT:

When magnetic field is applied perpendicular to a current carrying conductor (metal or semiconductor) then a voltage is developed in the material perpendicular to both magnetic field and current in the conductor. This effect is known as **Hall effect** and the voltage is developed is known as **Hall voltage (V_H)**.

Explanation:

Consider a rectangular block of extrinsic semiconductor. Let a current ' i ' is passed in the conductor along **X – direction** and magnetic field (\vec{B}) is established along **Y – direction**, then the charge carrier experience a Lorentz force (F_L) along the **Z – direction**. The direction of this force is noted by Fleming left hand rule. As a result of this force (downward), the charge carriers are forced down into the bottom surface.

If the specimen is a **P – type semiconductor**, Since the holes are the charge carriers, they are forced down into the bottom surface and hence the upper surface is occupied by the electrons. Therefore, a potential difference is developed between the upper and bottom surface. This Hall potential V_H can be measured by connecting two surfaces to a voltmeter.



This separation of charge carriers creates an electric field (E_H) in the upward direction (*positive z – direction*).

These charge carriers are moving in a magnetic field in the semiconductor, they experience Lorentz force

$$F_L = q(\vec{V}_d \times \vec{B}) \quad (V_d = \text{drift velocity of the carriers})$$

$$= qV_d B \sin \theta,$$

$$= qV_d B \sin 90^\circ$$

90° because force acting on charge perpendicularly)

$$= qV_d B \quad (q = e)$$

$$\boxed{F_L = eV_d B}$$

The electric force due to electric field created by the surface charges.

$$F_H = qE$$

$$\boxed{F_H = eE_H}$$

Where, E_H is the hall electrical field.

After some time both the forces become equal in magnitude and act in opposite direction, the situation is said to be under equilibrium.

At equilibrium, $F_H = F_L$

$$eE_H = eV_d B$$

Cancelling common terms, we get

$$E_H = V_d B \quad (1)$$

The current density is

$$J = neV_d \quad (2)$$

From eq. (2) and (3)

$$E_H = \frac{BJ}{ne} \quad (3)$$

The current density is also given by

$$J = \frac{I}{A} \quad (4)$$

From eq. (3) and (4)

$$E_H = \frac{BI}{neA} \quad (5)$$

If E_H be the Hall voltage in equilibrium, the Hall electrical field $E_H = \frac{V_H}{d}$

Substituting $E_H = \frac{V_H}{d}$ in equation (5)

$$\frac{V_H}{d} = \frac{BI}{neA} \quad (6)$$

Eq. (6) can be written by taking $R_H = \frac{1}{ne}$ (where R_H is the Hall coefficient) as

$$\frac{V_H}{d} = \frac{BI}{A} R_H \quad (7)$$

By substituting $A = w \times d$ (Area of cross section = width \times thickness) in eq. (8)

$$\frac{V_H}{d} = \frac{BI}{wd} R_H$$

Cancelling common terms, we get

$$V_H = \frac{BI}{w} R_H \quad (8)$$

From eq. (8) the Hall coefficient is

$$R_H = \frac{V_H w}{BI} \quad (9)$$

Eq. (9) gives the value of the R_H by measuring I, B, w , and V_H the Hall coefficient is determined. From $R_H = \frac{1}{ne}$ one can find the value of the concentration of the carriers.

By knowing the concentration of the carriers the mobility of the charge carriers is determined using the relation.

$$\sigma = ne\mu$$

$$\mu = R_H \sigma$$

Where σ is the electrical conductivity of the material.

The above R_H equations are derived by assuming that the velocity of the electron is constant. But due to thermal agitation the velocity of the electron is not a constant. It is randomly distributed. So, a correction factor $\left(\frac{3\pi}{8}\right)$ is introduced in the Hall coefficient. Therefore R_H can be written as

$$R_H = -\frac{3\pi}{8} \left(\frac{1}{ne} \right) \quad (\text{for } N - \text{type})$$

$$R_H = \frac{3\pi}{8} \left(\frac{1}{pe} \right) \quad (\text{for } P - \text{type})$$

$$R_H = -\frac{1.18}{ne} = \frac{1.18}{pe}$$

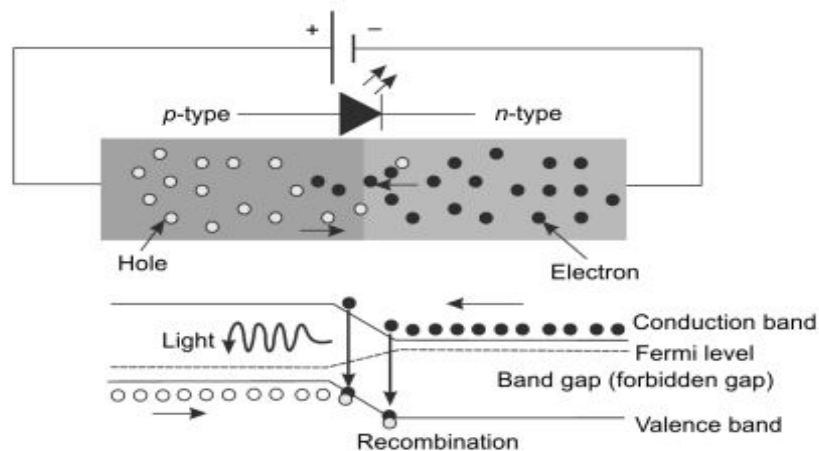
For $N - \text{type}$ material, Since, the charge is negative, R_H also negative and for a $P - \text{type}$ material, R_H is positive.

Uses:

- ✓ It is used to find out whether the given semiconductor is $N - \text{type}$ or $P - \text{type}$.
- ✓ It is used to measure carrier concentration, mobility and conductivity of a semiconducting material.
- ✓ Hall voltage is produced of two input quantities namely the current and the magnetic field. Using this principle, the Hall effect device is used as a multiplier.
- ✓ It is used as a magnetic field sensor. Using the Hall effect devices, the magnetic field ranging from $1\mu T$ to $1T$ is sensed.

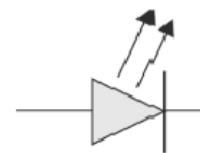
SEMICONDUCTOR MATERIALS FOR OPTOELECTRONIC DEVICES

Light Emitting Diode (LED):



Representation of recombination of electron-hole pairs in LEDs

LED is a $p - n$ junction device which emits light when forward biased, by a phenomenon called electro luminescence in the UV, Visible or IR regions of the electromagnetic spectrum. The quanta of light energy released are approximately proportional to the band gap of the semiconductors. The symbol of LED is



Principle & Working:

When a voltage is applied across a $p - n$ junction (*forward biased*), the electron crosses $p - n$ junction from the $n - type$ semiconducting material to $p - type$ semiconductor material. These free electrons stay in conduction band (*higher energy state*) for a few seconds and while holes present in the valence band. The electron make downward transition from conduction band, recombine with the majority holes in the valence band (*spontaneous emission*). During the recombination, the difference in the energy is given up in the form of heat and light radiation. i.e. photon. In normal diode this energy release in the form of heat.

The energy of light radiation depends on the strength of the recombination. The emitted light is very small in intensity and is of the order of microampere range. Similar, action takes place in $n - region$ also. Under reverse bias no photons (*radiation*) emitted.

The wavelength of emitted photon is

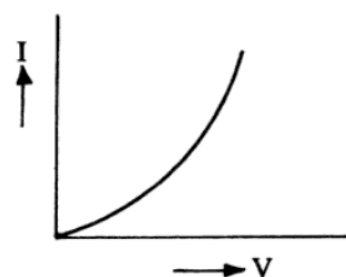
$$\lambda = \frac{hc}{E_g}$$

Where, $h = \text{plancks constant } (6.625 \times 10^{-34} \text{ JS})$

$$\lambda = \frac{hc}{E_g} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8 \times 10^{10}}{1.602 \times 10^{-19} E_g} = \frac{12400}{E_g} \text{ \AA}$$

Where, E_g is the energy gap in electron volt.

Therefore, colour of the emitted light depends on the type of material used.



Applications:

- Indicator light.
- Large scale video displays.
- Status indicator on equipment.
- Message display at airport and railway station.

SOLAR CELLS:

Def: A special $p-n$ junction diode which converts sun light into electrical energy is known as **solar cell or photo voltaic device**. The symbol of solar cell is

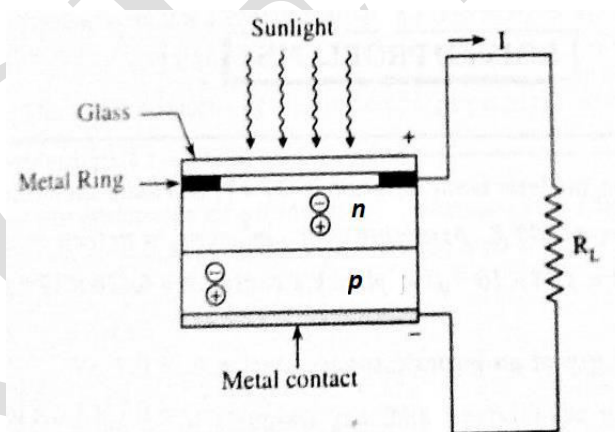
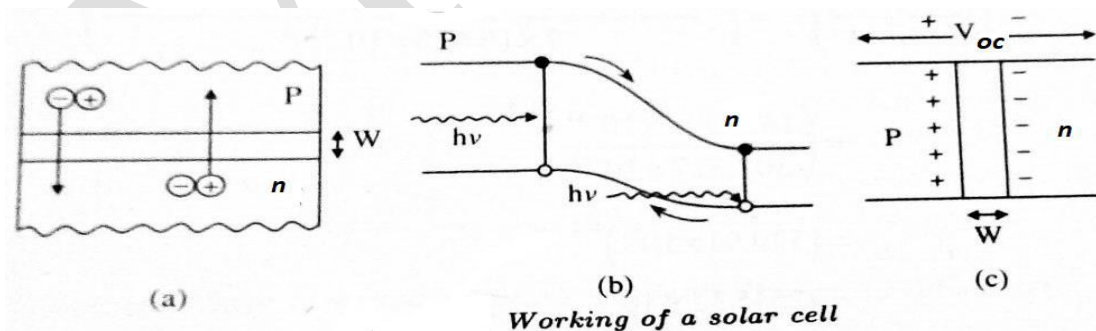
**Materials:**

Main considerations while selecting a material for solar cell fabrication:

- Band gap (1 to 2 eV) :
Semiconductors commonly used for making solar cells are
 $Si(1.1\text{ eV})$, $GaAs(1.43\text{ eV})$,
 $CdTe(1.45\text{ eV})$, $Cu(InGa)Se_2(1.01 - 1.5\text{ eV})$.
- High optical absorption.
- Electrical Conductivity.

Construction:

1. A simple solar cell consist of a $p-n$ junction diode having large junction surface to caught large radiation.
2. In this $p-n$ junction diode n -region ($0.3 \times 10^{-6}m$) is very thin and p -region ($100 \times 10^{-6}m$) is thick.
3. In the solar cell the thin region is called the emitter and the other base, light incident on the emitter.
4. Ni plated contacts are connected through a load resistance.

**Working:**

Three steps are involved in working of a solar cell, when light falls on it.

1. Generation of charge carriers (electron – hole pair):

When light energy falls on a $p-n$ junction diode, photon collide with valence electrons and impart them sufficient energy enabling them to leave their parent atoms. Thus, electron – hole pairs are generated in both p and n sides of the junction.

2. Separation of charge carriers:

The electron from p – region diffuse through the junction to a n – region and holes from n – region diffuse through the junction to the p – region due to electric field of depletion layer. Thus, hole and electrons are separated out. The accumulation of electron and holes on the two sides of the junction gives rise to open circuit voltage(V_{oc}).

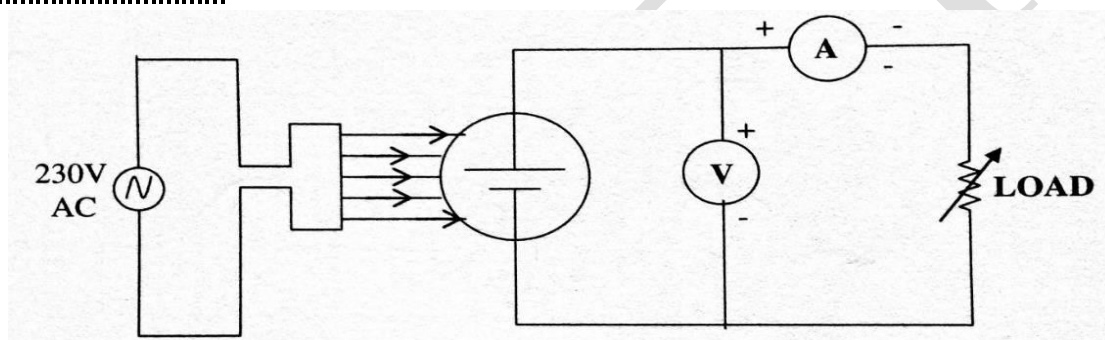
For better hole, electron separation under the effect of junction field, it is required that photo emission takes place in the junction area only.

3. Collection of charge carriers:

The flow of electrons and holes constitutes the minority current. The **d.c** is collected

by the metal electrodes and flows through the external load (R_L). The **d.c** is directly proportional to the illumination and also depends on the surface are being exposed to light.

I – V Characteristics:



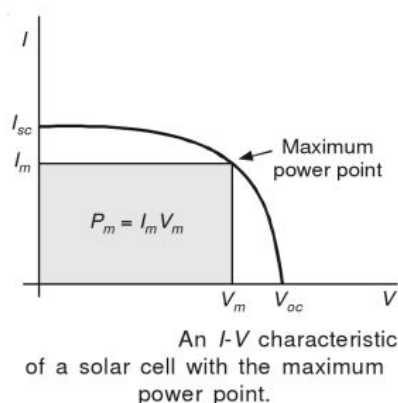
The I – V characteristics of solar cell can be determined by connecting load resistance across the voltmeter in series with ammeter. By keeping the light intensity constant we will vary the load resistance in a sequential manner to observe corresponding voltmeter and ammeter reading. A graph is plotted by taking voltage and current along x and y axis with the given scale. An exponential curve decay in power is obtained. The curve passes through three significant points.

A. **Short circuit current(I_{sc})**: It occurs on a point of the curve where the voltage is zero. At this point the power output of the solar cell is zero.

B. **Open circuit voltage (V_{oc})**: It occurs on a point of the curve where the current is zero. At this point the power output of the solar cell is zero.

The product of above two quantities($I_{sc} \times V_{oc}$) gives the **ideal power** of the cell.

C. **Maximum Power (P_{max})**: The cell delivers the maximum power, where the product $I_m V_m$ is maximum. The position of the maximum power is the area of the largest rectangle that can be formed in the V – I curve.



UNIT IV

DIELECTRIC PROPERTIES

Introduction:

All dielectric materials are insulators but all insulators are not perfect dielectric materials. There are no free charges available for conduction in a dielectric. All the electrons are tightly bound to their nucleus of the atoms. When the dielectric is placed in an electric field, then separation of positive and negative charges takes place in a dielectric, causing polarization of dielectric. Dielectrics are characterized by polarization & dielectric constant. Dielectrics materials store large amounts of electro-static fields.

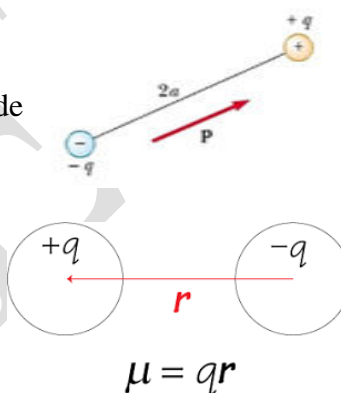
There are two principal methods by which a dielectric can be polarized: stretching and rotation. Stretching an atom or molecule results in an induced dipole moment added to every atom or molecule. Rotation occurs only in polar molecules — those with a permanent dipole moment like the water molecule shown in the diagram below.

Basic Definitions:

(1) Electric Dipole: Two opposite charges of equal magnitude separated by a finite distance constitutes an electric dipole.

(ii) Dipole moment(μ): Electric dipole moment is defined as the product of one of the charges and the charge separating distance.

Units: coulomb meter or Debye, $1 \text{ Debye} = 3.3 \times 10^{-30} \text{ C-m}$



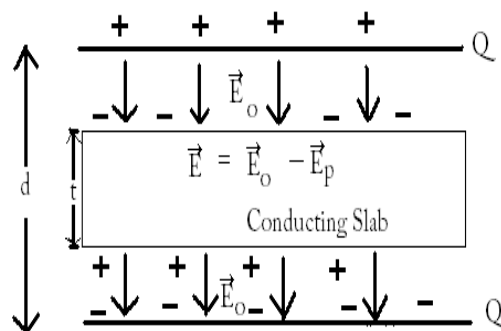
(iii) Permittivity (ϵ): Permittivity represents the easily polarizable nature of the dielectric or medium.

ϵ_0 = permittivity of free space = $8.854 \times 10^{-12} \text{ F/m}$

(iv) Dielectric constant(ϵ_r): Dielectric constant is the ratio between the permittivity of the medium and the permittivity of free space.

Let us take a parallel plate capacitor. Suppose the separation distance between the plates is d . Use air or vacuum as a medium for this experiment.

Suppose $+Q$ is the charge on one plate and $-Q$ is charge on the second plate. Bring a rectangular slab made up of conducting material between the plates of the capacitor. The thickness of



the slab must be less than the distance between the plates of the capacitor. When the electric field will be applied then polarization of molecules will be started. The polarization will take place in the direction same as that of electric field. Consider a vector that must be polarized, name it as P . The polarization vector must be in the direction of electric field E_o . Then this vector will start its functioning and will produce an electric field E_p in the opposite direction to that of E_o . The net electric field in the circuit is shown by the figure.

$$\epsilon_r = \epsilon / \epsilon_0$$

$$E = E_o - E_p$$

Dielectric constant, property of an electrical insulating material (a dielectric) equal to the ratio of the capacitance of a capacitor filled with the given material (c) to the capacitance of an identical capacitor (c_0) (in a vacuum without the dielectric material).

$$\epsilon_r = c/c_0$$

(v) Dielectric polarization (p): Dielectric polarization or Electric polarization is the induced dipole moment (μ) per unit volume (v) of the dielectric in the presence of an electric field.

Electric polarization, slight relative shift of positive and negative electric charge in opposite directions within an insulator, or dielectric, induced by an external electrical field.

$$P = \mu/v$$

Units: $C\cdot m^{-2}$

If “ μ ” is the average dipole moment per molecule and “ N ” is the number of molecules per unit volume then

$$P = N \mu$$

(vi) Dielectric polarisability (α):

Di-electric polarisability is the net dipole moment induced per unit applied electric field.

$$P = N \mu = N \alpha E$$

Units: $F\cdot m^2$

(vii) Electric susceptibility (χ):

It measures the amount of polarization in a given electric field produced in a dielectric. Polarization proportional to the product of ‘ ϵ_0 ’ and applied electric field.

$$\chi = P / \epsilon_0 E$$

(viii) Electric flux density or Electric displacement (D):

It is defined as the total number of electric lines of force passing through the dielectric material is known as electric flux density (D).

Mathematically it is represented by the following equation

$$D = \epsilon E + p$$

$$\epsilon E = \epsilon_0 E + p$$

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + p$$

$$p = \epsilon_0 E (\epsilon_r - 1)$$

Types of polarization in dielectrics:

There are four types of polarization processes in dielectric.

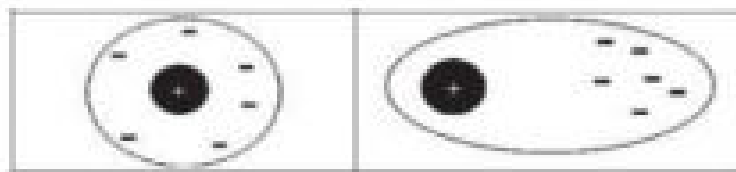
1. Electronic –polarization(p_e)
2. Ionic – polarization(p_i)
3. Orientation-polarization or di-polar polarization(p_o)
4. Space-charge polarization(p_s)

1. Electronic –polarization(p_e) or Atomic Polarization:

It is defined as polarization of rare-gas atoms in presence of an static electric field or polarization due to displacement of an electron-cloud from positive charge of nucleus in an atom in the presence of an electric field in a dielectric material is known as electronic-

polarization(p_e). This involves the separation of the centre of the electron cloud around an atom with respect to the centre of its nucleus under the application of electric field

Expression for Electronic polarization of a dielectric material:



Dielectric Polarization in Nonpolar Molecules. The electric field causes the shifting of charges

The displacement of the positively charged nucleus and the (negative) electron of an atom in opposite directions, on application of an electric field, result in electronic polarization.

On application of external electric field E , the electron cloud around the nucleus readily shifts towards the positive end of the field. Because of this, dipole moment is created, within each atom due to the separation of positive and negative charges. This separation or shift is proportional to field strength (E).

∴ Dipole moment is the product of charge and the separation between the charges.

∴ Induced dipole moment $\mu \propto E$

$$\mu = \alpha_e E$$

α_e is constant of proportionality or electronic polarizability and it is independent of temperature.

Calculation of Electronic polarizability:

By taking classical model of an atom, the nucleus of charge Ze is surrounded by an electron cloud of charge $-Ze$ distributed in a sphere of radius R .

Then the charge density (Z – atomic number) is given by

$$\rho = \frac{-Ze}{\frac{4}{3}\pi R^3} = -\frac{3}{4}\left(\frac{Ze}{\pi R^3}\right) \rightarrow (1)$$

When an external electric field E is applied, the nucleus and electrons experience Lorentz forces of magnitude ZeE in opposite direction. Because of this, the nucleus and electron cloud are pulled apart. If separation is there in between nucleus and electron cloud, a coulomb force develops which is attractive.

When Lorentz and Coulomb force are equal and opposite, equilibrium is reached.

Let ' x ' be the displacement under this condition. Here, nucleus is much heavier than the electron cloud; it is assumed that only the electron cloud is displaced when the external field is applied.

$$\text{Lorentz force} = -ZeE$$

$$\text{Coulomb force} = Ze \times \frac{\text{charge enclosed in the sphere of radius } x}{4\pi\epsilon_0 x^2}$$

$$\text{The charge enclosed} = \frac{4}{3}\pi x^3 \rho$$

$$= \frac{4}{3}\pi x^3 \left(\frac{-Ze}{\frac{4}{3}\pi R^3}\right) = \frac{4}{3}\pi x^3 \left(\frac{-3}{4}\right) \frac{Ze}{\pi R^3} = \frac{-Zex^3}{R^3}$$

$$\text{Coulomb force} = Ze \times \left(\frac{-Zex^3}{R^3} \right) \times \frac{1}{4\pi\epsilon_0 x^2} = \frac{-Z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

$$\text{In the equilibrium position, } -ZeE = \frac{-Z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

$$\text{or } x = \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

Thus, displacement of electron cloud is proportional to applied field

$$x \propto E$$

∴ The electric charges $+Ze$ and $-Ze$ are separated by a distance x under the influence of E constituting induced dipoles (moment).

Induced electric dipole moment,

$$\begin{aligned}\mu_e &= Zex \\ &= Ze \cdot \frac{4\pi\epsilon_0 R^3 E}{Ze} \\ &= 4\pi\epsilon_0 R^3 E\end{aligned}$$

$$\text{i.e. } \mu_e = \alpha_e E$$

where $\alpha_e = 4\pi\epsilon_0 R^3$ is called electronic polarizability

It is the dipole moment per unit volume & is independent of temperature.

$$\begin{aligned}P_e &= N\mu_e \\ &= N\alpha_e E \quad (\text{where, } N \text{ is the no. of atom/m}^3) \\ P_e &= \epsilon_0 E (\epsilon_r - 1) = N\alpha_e\end{aligned}$$

$$\epsilon_r - 1 = \frac{N\alpha_e}{\epsilon_0}$$

Hence

$$\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N}$$

2. Ionic – polarization (p_i):

The polarization that occurs due to relative displacement of ions of the molecules in the presence of an external electrical field. This happens in solids with ionic bonding which automatically have dipoles but which get cancelled due to symmetry of the crystals. Here, external field leads to small displacement of ions from their equilibrium positions and hence inducing a net dipole moment. **Ex:** NaCl molecule

Expression for Ionic polarization of a molecule:

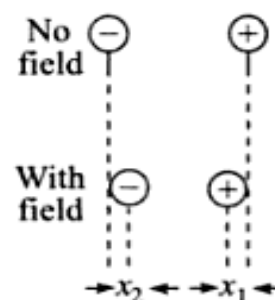
The ionic polarization is due to the displacement of cation and anions in opposite direction and occurs in an ionic solid.

Consider an electric field is applied in the positive x direction then the positive ions move to the right by x_1 and the negative ions move to left by x_2 . If we assume, each unit cell has one cation and one anion.

The resultant dipole moment per unit cell due to ionic displacement is

$$\mu = e(x_1 + x_2)$$

If β_1 and β_2 are restoring force constants of cation and anion, Newton is the force due to applied field. Then



$$F = \beta_1 x_1 = \beta_2 x_2$$

$$\text{Hence, } x_1 = \frac{F}{\beta_1}$$

Restoring force constants depends upon the mass of the ion and angular frequency of the molecule in which the ions are present.

$$x_1 = \frac{eE}{m\omega_0^2} \text{ \& } x_2 = \frac{eE}{M\omega_0^2}$$

where m - mass of the positive ion & M-mass of the negative ion

$$x_1 + x_2 = \frac{eE}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)$$

$$\mu = e(x_1 + x_2) = \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)$$

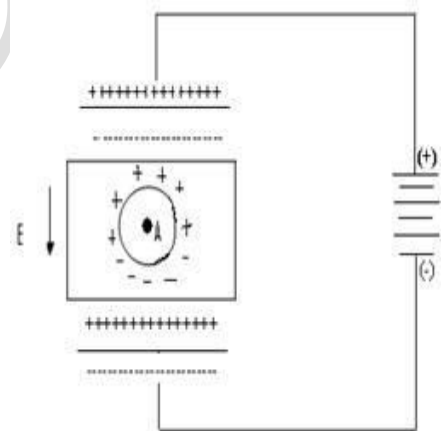
$$\boxed{\alpha_i = \frac{\mu}{E} = \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)}$$

The polarizability α_i is inversely proportional to the square of natural frequency of the ionic molecule and to its reduced mass which is equal to

$$\left(\frac{1}{m} + \frac{1}{M} \right)^{-1}$$

Internal fields in solids [Lorentz Method]:

Now a dielectric material is placed between the two parallel plate capacitor and there be an imaginary spherical cavity around the atom 'A' inside the dielectric. By taking that the radius of the cavity is large compared to the radius of the atom. Now, the internal field at the atom site 'A' can be considered to be made up of four components namely E_1 , E_2 , E_3 and E_4



Field E_1 : -

E_1 is the field intensity at A due to the charge density on the plates. From the field theory.

$$E_1 = \frac{D}{\epsilon_0}$$

$$D = P + \epsilon_0 E$$

$$\therefore E_1 = \frac{P + \epsilon_0 E}{\epsilon_0}$$

$$\boxed{\therefore E_1 = E + \frac{P}{\epsilon_0}} \rightarrow (1)$$

Field E_2 : -

E_2 is the field intensity at A due to charge density induced on the two sides of the dielectric.

Therefore,

$$E_2 = \frac{-P}{\epsilon_0} \rightarrow (2)$$

Field E_3 :-

E_3 is the field intensity at A due to other atoms contained in the cavity. If we assume a cubic structure then $E_3=0$ because of symmetry.

Field E_4 :-

E_4 is the field density due to polarization charges on the surface of the cavity and it is calculated by Lorentz. The enlarged view of the cavity is shown.

If dA is the surface area of the sphere of radius r lying between θ and $\theta+d\theta$ (where θ is the direction with reference to the direction of the applied force) Then $dA = 2\pi(PQ)(QR)$

$$\sin\theta = \frac{PQ}{r} \Rightarrow PQ = r\sin\theta$$

$$\text{and } d\theta = \frac{QR}{r} \Rightarrow QR = r d\theta$$

Hence,

The charge dq on the surface dA is equal to the normal component of the polarization multiplied by the surface area.

$$\therefore dq = p \cos\theta dA = P(2\pi r^2 \sin\theta \cos\theta d\theta)$$

Field due to this charge at the centre 'A' is denoted by dE_4 and is obtained by imagining a unit charge at point 'A'

$$dE_4 = \frac{dq \times l \times \cos\theta}{4\pi\epsilon_0 r^2}$$

$$= \frac{P(2\pi r^2 \sin\theta \cos\theta d\theta) \cos\theta}{4\pi\epsilon_0 r^2}$$

$$dE_4 = \frac{P}{2\epsilon_0} \cos^2\theta \sin\theta d\theta \rightarrow (3)$$

Thus, the total field E_4 due to the charges on the surface of the entire cavity is obtained by integrating

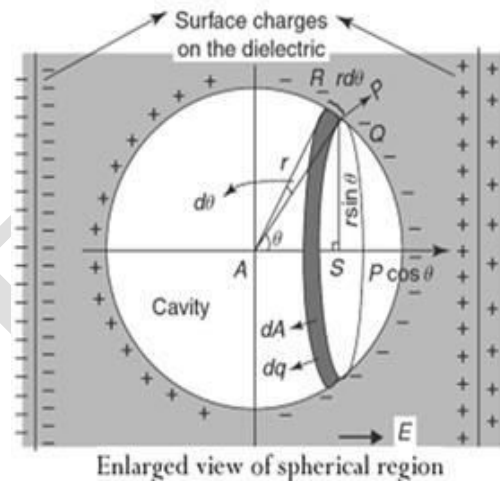
$$\begin{aligned} \therefore \int dE_4 &= \frac{P}{2\epsilon_0} \int_0^\pi \cos^2\theta \sin\theta d\theta \\ &= \frac{P}{2\epsilon_0} \int_0^\pi \cos^2\theta d(-\cos\theta) \\ &= -\frac{P}{2\epsilon_0} \left(\frac{\cos^3\theta}{3} \right)_0^\pi = -\frac{P}{6\epsilon_0} (-1 - 1) = \frac{P}{3\epsilon_0} \end{aligned}$$

$$\text{i.e. } \boxed{E_i = E + \frac{P}{3\epsilon_0} \rightarrow (4)}$$

where E_i is the internal field (or) Lorentz field.

Clausius – Mosotti relation: -

The relation between the dielectric constant and the polarizability of a polarized dielectric is known as Clausius – Mosotti equation. Consider the dielectric having cubic structure, here there are no ions and permanent dipoles in these materials, the ionic polarizability α_i and orientational polarizability α_0 are zero.



i.e. $\alpha_i = \alpha_0 = 0$

$$\begin{aligned} \text{Polarization } P &= N\alpha_e E_i \\ &= N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right) \\ P \left(1 - \frac{N\alpha_e}{3\epsilon_0} \right) &= N\alpha_e E \quad (\text{or}) \quad P = \frac{N\alpha_e E}{\left(1 - \frac{N\alpha_e}{3\epsilon_0} \right)} \quad \rightarrow (1) \end{aligned}$$

We have, $D = P + \epsilon_0 E$

$$P = D - \epsilon_0 E$$

$$\frac{P}{E} = \frac{D}{E} - \epsilon_0$$

$$= \epsilon - \epsilon_0$$

$$= \epsilon_0 \epsilon_r - \epsilon_0$$

$$\therefore \boxed{P = E\epsilon_0(\epsilon_r - 1)} \quad \rightarrow (2)$$

Using (1) and (2):

$$\begin{aligned} P &= E\epsilon_0(\epsilon_r - 1) = \frac{N\alpha_e E}{\left(1 - \frac{N\alpha_e}{3\epsilon_0} \right)} \\ 1 - \frac{N\alpha_e}{3\epsilon_0} &= \frac{N\alpha_e}{\epsilon_0(\epsilon_r - 1)} \\ 1 &= \frac{N\alpha_e}{3\epsilon_0} \left(1 + \frac{3}{\epsilon_r - 1} \right) \\ \Rightarrow \frac{N\alpha_e}{3\epsilon_0} &= \frac{1}{\left(1 + \frac{3}{\epsilon_r - 1} \right)} \\ &= \frac{\epsilon_r - 1}{\epsilon_r + 2} \end{aligned}$$

N is the no. of atoms by knowing ϵ_r , we can determine α_e

MAGNETISM

Introduction:

Magnetism has its own application in the field of physics. Magnetic properties have been the subject of special interest because of the information they yield about the constitution of matter. Magnetics have wide range of application in electrical machinery and in magnetic tapes in computers. Magnetic materials are classified into three, namely diamagnetic, paramagnetic and ferromagnetic materials.

Magnetic field: The 3D region around a magnet in which it's magnetic force is felt is called magnetic field of that magnet. It is a scalar quantity.

Magnetic dipole: It's a system consisting of two equal and opposite magnetic poles separated by a small distance of $2lm$

Or

Two poles of equal strength separated by a very small distance is called a magnetic dipole.

Pole strength (m): The ability of a pole of magnet to attract or repel another magnetic pole is called it's pole strength.

Magnetic dipole moment (μ): It can define in three ways depending on situation.

1. In case of bar magnet it is defined as the product of pole strength and distance between them.

$$\mu = 2lm$$

2. In case of current carrying conductor, it is the product of current (i) and cross section area (A) of the conductor.

$$\mu = iA$$

3. In case of atom, it is the product of current (i) developed by orbital motion of electron and the area covered by the orbital.

$$\mu = iA$$

It is a vector quantity. Its direction is from south pole to the north pole.

Its SI unit is ampere/m².

Magnetic flux(ϕ): The total number of magnetic lines of force passing normal to a surface in a magnetic field is called magnetic flux.

Or

The number of magnetic lines of force passing through a point is called magnetic flux.

Its SI unit is weber or tesla – m²

Magnetic field induction or magnetic flux density (B): The magnetic flux passing through a unit normal area of substance is defined as magnetic flux density and is denoted by B. It is commonly known as magnetic induction. Let ϕ be the flux passing through a substance of area A, then flux density is

$$B = \frac{\phi}{A} = \frac{F}{m} \text{ or } \phi = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta.$$

It is a vector quantity.

Its SI unit is weber/m² or tesla and CGS unit is Gauss(= maxwell/cm²).

1Tesla= 10⁴Gauss.

Magnetising field(force) or Magnetic intensity(H): The force acting on a unit north pole placed at the point. It is independent of the medium.

$$B = \mu H$$

It is vector quantity. Its SI unit is ampere/m.

Permeability: It's ability to allow the magnetic lines of force to pass through it **or** to allow itself to be influenced by magnetic field (μ).

Relative magnetic permeability(μ_r): The ratio of its absolute magnetic permeability(μ) of the material to the magnetic permeability of free space (μ_0).

$$\mu_r = \frac{\mu}{\mu_0}$$

It is purely a number, it has no units.

The permeability of free space or vacuum is $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Magnetization: The ability of a material to get magnetized when placed in an external magnetic field is called magnetization.

It is measured by a quantity called intensity of magnetization(I).

The net magnetic dipole moment acquired by a substance per unit volume is defined as intensity of magnetization (I).

$$I = \frac{\text{net magnetic dipole moment of material}}{\text{Volume of the material}} \quad \text{A/m}$$

It is vector quantity. Its SI unit is A/m .

Relation between B,H and I:

The flux density B in a material due a to magnetizing force H, as the sum of the flux density B_0 in vacuum produced by magnetizing force and flux density B_m due to magnetization of material.

Therefore

$$B = B_0 + B_m$$

Also we know that $B_0 = \mu_0 H$ and $B_m = \mu_0 I$

Therefore,

$$B = \mu_0 H + \mu_0 I$$

$$B = \mu_0 (H + I)$$

It is the relation in SI unit

Magnetic Susceptibility(χ): The ratio of magnetization to the magnetic intensity.

$$\chi = \frac{I}{H}$$

It has no units when a material has high susceptibility then it can be easily magnetized.

Relation between Permeability and Susceptibility:

We know that $B \propto (H+I)$

$$B = \mu_0 (H+I)$$

In vacuum $B = \mu_0 H + \mu_0 I$

$$\frac{B}{H} = \mu_0 + \mu_0 \frac{I}{H}$$

Also,

$$\frac{B}{H} = \mu \text{ (permeability of the material) and}$$

$$\chi = \frac{1}{H} \text{ (Susceptibility)}$$

$$\therefore \mu = \mu_0 + \mu_0 \chi \text{ or } \mu = \mu_0 (1 + \chi)$$

$$\frac{\mu}{\mu_0} = (1 + \chi)$$

$$\mu_r = (1 + \chi)$$

$$\mu_r - 1 = \chi$$

$$\boxed{\mu_r = \chi + 1}$$

Origin of magnetic moment:

Materials are made up of atoms. These atoms consist of positively charged nucleus, surrounded by cloud of electrons. In all atoms electrons are revolving around the nucleus in different orbits and also spin about their own axis. These revolving electrons constitute an electrical current in the orbits. These revolving electrons produces its own orbital magnetic dipole moment, measured in Bohr magneton (μ_B) and there is also a spin magnetic moment associated with it (spin of orbital electrons and spin of nucleus). In most materials there is no resultant magnetic moments, due to the electrons being grouped in pairs causing the magnetic moment to be cancelled by its neighbour. Under external applied magnetic field these dipoles experience torque in the direction of applied field and the atom acquires certain magnetism.

So, the magnetic moment of an atom is due to

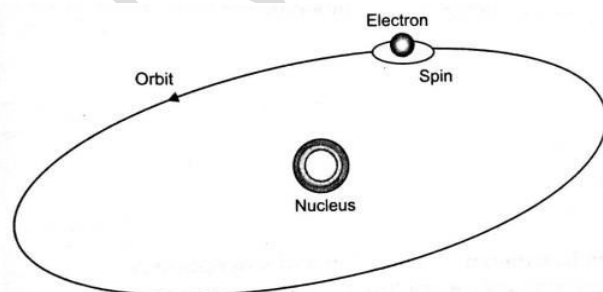
1. Orbital magnetic moment of the electrons.
2. Spin magnetic moment of the electrons.
3. Spin magnetic moment of the nucleus.

The orbital magnetic moment of the electrons(Bohr Magnetron):

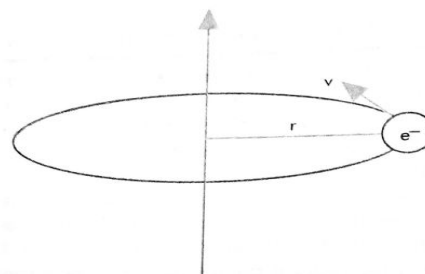
The magnetic moment contributed by a single electron is known as Bohr Magnetron. Mathematically it can express as

$$\boxed{\mu_B = \frac{e\hbar}{2m}}$$

Proof:



The orbit of a spinning electron about the nucleus of an atom



Motion of electron in a orbital

Let us consider an electron revolving in a circular orbit of radius r with a speed v . Consider a point p on the circle. The electron crosses this point once in every revolution. In one revolution the electron travels $2\pi r$ distance.

The current i due to the electron motion in the circular loop (orbit) is

$$i = \frac{\text{charge of electron}(q)}{\text{time}(T)} = \frac{-e}{T}$$

Where, T is time for one revolution of electron about the nucleus.

$$T = \frac{\text{distance travelled by an electron}}{\text{speed of an electron}} = \frac{2\pi r}{v}$$

$$i = \frac{-e}{\left[\frac{2\pi r}{v}\right]} = \frac{-ev}{2\pi r}$$

The magnetic moment μ_l associated with the orbit due to orbital motion of electron is

$$\mu_l = iA$$

Where, ' i ' is current produced by the orbital motion of the electron and A is area covered by the orbital (πr^2).

$$\mu_l = \frac{-ev}{2\pi r} \times \pi r^2$$

Dividing and multiplying above eq. by the mass of the electron

$$\mu_l = \frac{-evr}{2} \times \frac{m}{m} = \left(\frac{-e}{2m}\right) [mvr] = \left(\frac{-e}{2m}\right) [L]$$

w.k.t the angular momentum of an electron revolving in circular orbit is equal to integral multiple of $\frac{h}{2\pi}$

$$L = mvr = \frac{lh}{2\pi}$$

$$\mu_l = \left(\frac{-eh}{4\pi m}\right) l$$

$$\mu_l = \mu_B l \quad \text{where } l = 0, 1, 2, \dots$$

Where, l = orbital quantum number.

$$\mu_B = \frac{-eh}{4\pi m}$$

Bhor magneton

$$= \frac{1.6 \times 10^{-19} \times 6.626 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} = 9.27 \times 10^{-34} \text{ A - m}$$


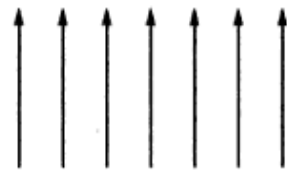
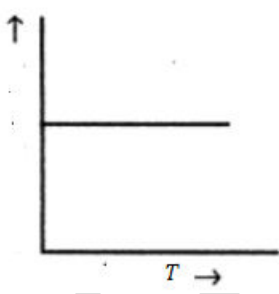
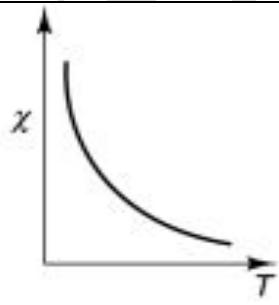
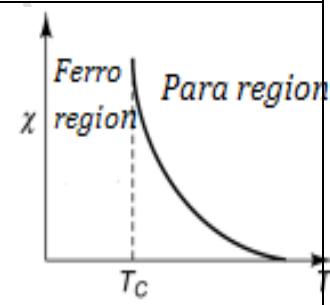
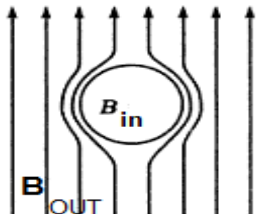
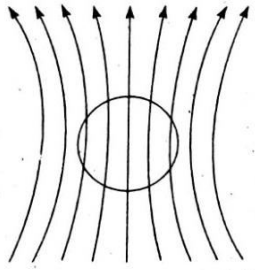
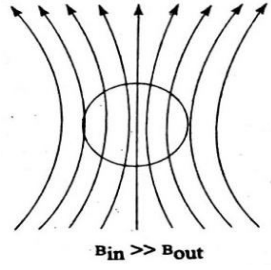
Here – sign indicates that magnetic moment is anti parallel to the angular momentum L .

CLASSIFICATION OF MAGNETIC MATERIAL:

Depending on the direction and magnitude of magnetization and also the effect of temperature on magnetic properties, all magnetic materials are classified into dia, para and ferro materials. Depending upon the spin orientation of the electrons, ferro magnetic materials are classified into two types. They are

- Antiferro magnetic material.
- Ferrimagnetic materials.

PROPERTY	DIA MAGNETIC SUBSTANCES	PARA MAGNETIC SUBSTANCES	FERRO MAGNETIC SUBSTANCES
Defination	The substance in which the resultant magnetic moment of individual atoms is zero.	The substance in which the resultant magnetic moment of individual atoms is not zero.	The substance in which the resultant magnetic moments of individual atoms align themselves in parallel by giving rise to spontaneous magnetization

Cause	Orbital motion of electrons.	Spin motion of electrons.	Formation of domains.
Susceptibility (χ)	Low and negative	Low and positive.	High and positive.
χ depend on T	Does not depend on T . except Bi at low T .	$\chi = \frac{c}{T}$ (curie law) c is curie constant.	$\chi = \frac{c}{T-T_c} : T > T_c$ (curie wisers law) T_c is curie temperature.
Relative permeability (μ_r)	< 1	> 1	$\gg 1$
spin alignment	No spin alignment is present.	All spins or magnetic moments are randomly oriented. 	All spins or magnetic moments are orderly oriented. 
$\chi - T$ curve			
Transition of material at T_c	Do not change.	On cooling these are converted into ferromagnetic material.	Converted into para magnetic material above T_c .
In the presence of magnetic field.	The magnetic lines of force are pulled out from the material. Hence, magnetic flux density is greater, outside the material than inside.  Diamagnetic material	The magnetic lines of force are attracted towards the centre of the material and hence the B_{in} material is greater than the B_{out} .  $B_{in} > B_{out}$	The magnetic lines of force are highly attracted towards the centre of the material and hence the B_{in} material is very greater than the B_{out} .  $B_{in} \gg B_{out}$
Example:	Bi, Zn, H_2O , Gold.....	Al, Pt, Mn, $CuCl_2$	Fe, Ni, Co, MnO, Fe_2O_3

Antiferro magnetic materials:

These are the ferromagnetic materials in which the magnetic interaction between any two dipoles aligns themselves anti parallel to each other. All the dipoles are equal in magnitude. Therefore the resultant magnetization is zero.

Properties:

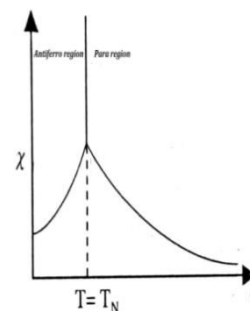
1. Susceptibility is small and positive for these materials.
2. Spin alignment is systematic, but in anti parallel manner.
3. Initially, the susceptibility increases slightly as the temperature increases, and beyond a particular temperature known as Neel temperature the susceptibility decreases with temperature.
4. The temperature at which anti ferromagnetic material converts into paramagnetic material is known as Neel's temperature.
5. Susceptibility is inversely proportional to the temperature. The variation of susceptibility with temperature is expressed as

$$\chi = \frac{C}{T + T_N} \quad \text{When } T > T_N$$

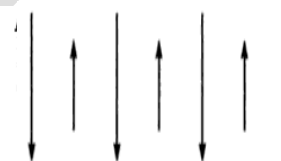
$$\chi \propto T \quad \text{When } T < T_N$$

Where 'c' is the curie constant and the T_N is Neel temperature.

Ex: FeO, MnO, Cr₂O₃

**Ferrimagnetic materials:**

These are the ferromagnetic materials in which the magnetic interaction between any two dipoles aligns themselves anti parallel to each other. But the magnitudes of any two adjacent dipoles are not equal. Therefore, if we apply a small value of magnetic field, it will produce a large value of magnetization.

**Properties:**

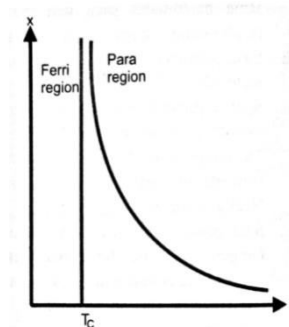
1. Susceptibility is positive and very large for these materials.
2. Spin alignment is systematic, but in anti parallel of different magnitudes.
3. Ferrimagnetic materials possess net magnetic moment.
4. Above curie temperature becomes paramagnetic while below it behaves as ferrimagnetic material.
5. Ferrimagnetic domains become magnetic bubbles to act as memory elements.
6. Susceptibility is inversely proportional to the temperature. The variation of susceptibility with temperature is

$$\chi = \frac{C}{T \pm T_N} \quad \text{for } T > T_N$$

Where, C is curie constant and T_N is Neel temperature.

7. Ferrites are the best examples of ferromagnetic materials.
8. General formula of ferrites are $\text{Me}^{2+}\text{OFe}_2\text{O}_3$ or $\text{Me}^{2+}\text{Fe}_2\text{O}_4$

Examples: $\text{Zn}^{2+}\text{Fe}_2\text{O}_4$, CuFe_2O_4



DOMAIN THEORY OF FERROMAGNETISM:

The concept of domains was proposed by Weiss in order to explain the properties of ferromagnetic materials and their hysteresis effects.

Magnetic Domains: Every ferromagnetic material is made of a very large number of miniature (very small) regions which are known as domains. The boundaries separating the domains are called domain walls, also known as Bloch walls.

In each domain the magnetic dipoles align parallel to each other and produce spontaneous magnetization. The direction of spontaneous magnetization varies from domain to domain.

Process of Domain magnetization:

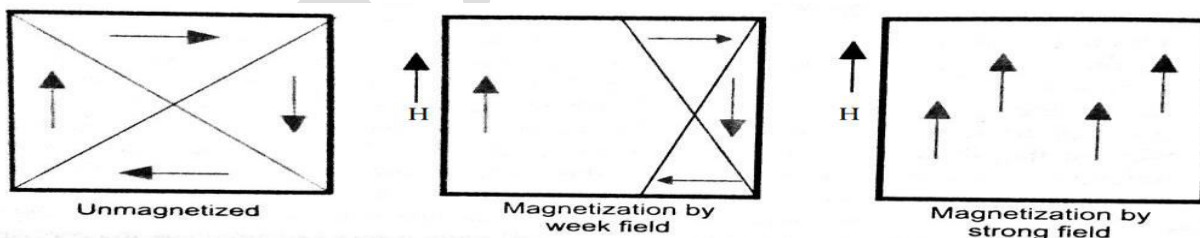
When the external field is applied there are two possible ways of alignment of random domains.

1. By the motion of domain walls.
2. By rotation of domains.

Motion of Domain walls: When a small amount of magnetic field is applied on a ferromagnetic material then the domains of the material whose magnetic moments are parallel or nearly parallel to the direction of applied field can grow in size whereas the domains are not parallel to field can diminish in size. This change produces large magnetization for the bulk material.

Rotation of Domains:

When the magnetic field is increased further to a large value (i.e., near saturation), further domains growth becomes impossible and hence fully



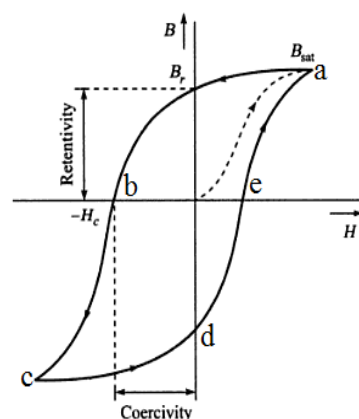
Magnetization of a ferro magnetic material

grown domains can rotate into the field direction and specimen is said to be saturated.

HYSTERESIS CURVE:

It means retardation. [or] Lagging of an effect behind the cause of the effect [or] The phenomenon of B lagging H . [or] It is also defined as a phenomenon of flux density (B) with the change in magnetic field strength (H) in a ferromagnetic material.

Explanation: When the magnetic field H is applied on an unmagnetized ferromagnetic material the magnetic induction increases first rapidly and then slowly from o to a . The increase is non linear, after that the rate of induction slows down and attaining a saturation value B_{sat} ,



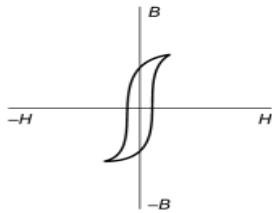
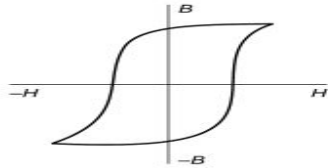
with further increase in H , there is no increase in B . If applied magnetic field H on the material is decreased at this stage, naturally B decreases and B will not travel in that initial path, creates a new path.

When H is reduced to zero, B does not vanish, the value of B that remains in the material is called as retentivity B_r or reduce magnetisation. To remove retentivity in the material sufficient negative magnetic field $-H_c$ or H_c is applied in opposite direction then the retentivity B_r becomes zero. This is (field) known as coercive field. On increasing H further B reaches saturation in the opposite direction denoted by ' c '. Then reducing H to zero B reaches ' d ' and then increasing H in the positive direction B reaches again to B_{sat} and a curve is obtained. This completes a closed loop called hysteresis loop. It includes some area. This area indicates the amount of energy wasted in one cycle of operation.

SOFT AND HARD MAGNETIC MATERIALS:

Ferromagnetic materials are classified into two types based on the characteristic parameters such as hysteresis and magnetisation. They are

1. Hard magnetic materials
2. Soft magnetic materials

S.NO	SOFT MAGNETIC MATERIAL	HARD MAGNETIC MATERIAL
1	Materials which can be easily magnetized and demagnetized are called Soft magnetic materials.	Materials which can't be easily magnetized and demagnetized are called hard magnetic materials.
2	The nature of hysteresis loop is very steep 	The nature of hysteresis loop is very large. 
3	They are prepared by annealing process.	They are prepared by quenching process.
4	Due to small hysteresis loop area, they have small hysteresis loss.	Due to large hysteresis loop area, they have large hysteresis loss.
5	They have large value of permeability and susceptibility.	They have low value of permeability and susceptibility.
6	The coercivity and retentivity are small.	The coercivity and retentivity are large.
7	They are free from irregularities.	They have large amount of impurities and lattice defects.
8	They are used <ul style="list-style-type: none"> To produce temporary magnets. In the preparation of magnetic core materials used in transformers, electric motors, magnetic amplifiers, magnetic switching circuits, etc., 	They are used <ul style="list-style-type: none"> To produce permanent magnets. In loud speakers, toys, in measuring meters, microphones, magnetic detectors, magnetic separators, etc.,
9	Ex: Iron and silicon alloys, Nickel – Iron alloy, Iron – Cobalt alloy.	Ex: High carbon steel, Cobalt steel, Barium ferrite.

UNIT V

LASER & FIBER-OPTICS

INTRODUCTION:

LASER stands for Light Amplification by Stimulated Emission of Radiation. Laser technology started with Albert Einstein in 1917, he has given theoretical basis for the development of Laser. The technology further evolved in 1960 when the very first laser called Ruby Laser was built at Hughes Research Laboratories by T.H. Mainmann.

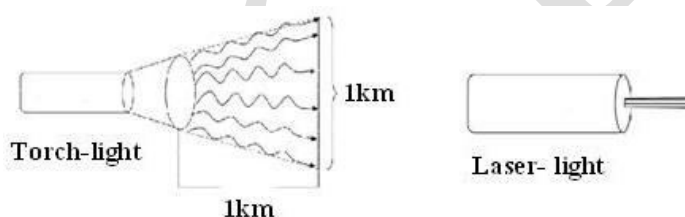
CHARACTERISTIC OF LASER RADIATION:

The laser beam has the properties given below which distinguish it from an ordinary beam of light. Those are

1. Highly directional
2. Highly monochromatic
3. Highly intense
4. Highly coherence

1. Highly directional:

A conventional light source emits light in all directions. On the other hand, Laser emits light only in one direction. The width of Laser beam is extremely narrow and hence a laser beam can travel to long distances without spreading.



The directionality of laser beam is expressed in terms of divergence

$$\Delta\theta = \frac{r_2 - r_1}{d_2 - d_1}$$

Where r_1 and r_2 are the radii of laser beam spots at distances of d_1 and d_2 respectively from laser source.

2. Highly monochromatic:

A monochromatic source is a single frequency or single wavelength source of light. The laser light is more monochromatic than that of a conventional light source. This may be due to the stimulated characteristic of laser light. The band width of conventional monochromatic light source is 1000\AA . But the band width of ordinary light source is 10\AA . For high sensitive laser source is 10^{-8}\AA .

3. Highly intense:

Laser light is highly intense than the conventional light. A one milli-Watt He-Ne laser is highly intense than the sun intensity. This is because of coherence and directionality of laser. Suppose when two photons each of amplitude 'A' are in phase with other, then young's principle of superposition, the resultant amplitude of two photons is $2A$ and the intensity is $4a^2$. Since in laser many numbers of photons are in phase with each other, the amplitude of the resulting wave becomes ' nA ' and hence the intensity of laser is proportional to n^2A^2 . So 1mw He-Ne laser is highly intense than the sun.

4. Highly coherence

A predictable correlation of the amplitude and phase at any one point with other point is called coherence. In case of conventional light, the property of coherence exhibits between a source and its virtual source where as in case of laser the property coherence exists between any two sources of same phase.

There are two types of coherence

- i) Temporal coherence
- ii) Spatial coherence.

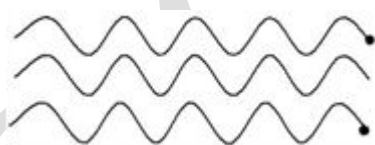
Temporal coherence (or longitudinal coherence):

The predictable correlation of amplitude and phase at one point on the wave train w.r.t another point on the same wave train, then the wave is said to be temporal coherence.



Spatial coherence (or transverse coherence):

The predictable correlation of amplitude and phase at one point on the wave train w.r.t another point on a second wave, then the waves are said to be spatial coherence (or transverse coherence). Two waves are said to be coherent when the waves must have same phase & amplitude.



INTERACTION OF LIGHT WITH MATTER AND THE THREE QUANTUM PROCESSES:

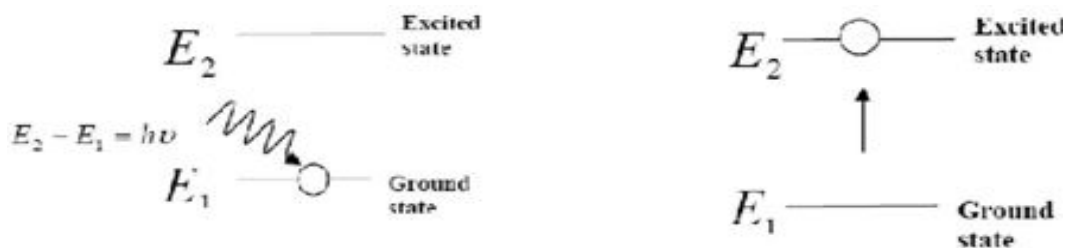
When the radiation interacts with matter, results in the following three important phenomena. They are

- (i) Induced or Stimulated Absorption
- (ii) Spontaneous Emission
- (iii) Stimulated Emission

STIMULATED ABSORPTION (OR) INDUCED ABSORPTION (OR) ABSORPTION:

An atom in the lower energy level or ground state energy level (E_1) absorbs the incident photon and goes to excited state (E_2) as shown in figure below. This process is called induced or stimulated absorption.

Let E_1 and E_2 be the energies of ground and excited states of an atom. Suppose, if a photon of energy $E_2 - E_1 = h\nu$ interacts with an atom present in the ground state, the atom gets excitation from ground state E_1 to excited state E_2 . This process is called stimulated absorption.



Stimulated absorption rate depends upon the number of atoms available in the lowest energy state as well as the energy density of photons.

SPONTANEOUS EMISSION:

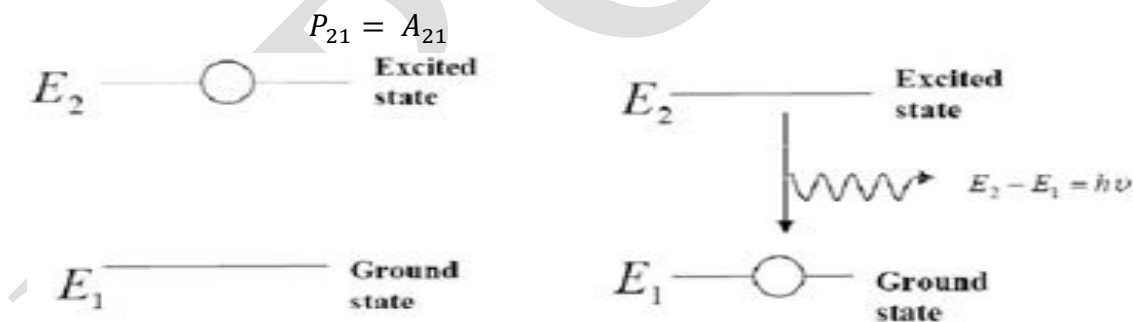
The atom in the excited state returns to ground state emitting a photon of energy $(E) = E_2 - E_1 = h\nu$, without applying an external energy spontaneously is known as spontaneous emission.

Let E_1 and E_2 be the energies of ground and excited states of an atom. Suppose, if photon of energy $E_2 - E_1 = h\nu$ interacts with an atom present in the ground state, the atom gets excitation from ground state E_1 to excited state E_2 .

The excited atom does not stay for a long time in the excited state. The excited atom gets de-excitation after its life time by emitting a photon of energy $E_2 - E_1 = h\nu$. This process is called spontaneous emission.

The spontaneous emission rate depends up on the number of atoms present in the excited state.

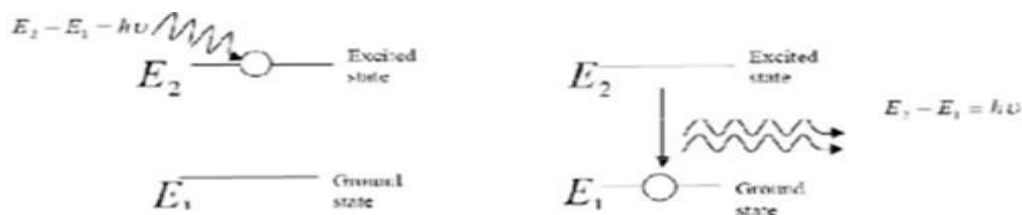
The probability of spontaneous emission (P_{21}) is independent of $u(\nu)$.



STIMULATED-EMISSION:

The atom in the excited state can also returns to the ground state by applying external energy or inducement of photon thereby emitting two photons which are having same energy as that of incident photon. This process is called as stimulated emission.

Stimulated emission was postulated by Einstein. Let E_1 and E_2 be the energies of ground and excited states of an atom. Let a Photon of energy $E_2 - E_1 = h\nu$ interacts with the excited atom with in their life time The atom gets de-excitation to ground state by emitting of another photon. These photons have same phase and it follows coherence. This phenomenon is called stimulated emission



Stimulated emission rate depends upon the number of atoms available in the excited state as well as the energy density of photons.

Comparison between Spontaneous and Stimulated emission:

Spontaneous emission	Stimulated emission
1. The spontaneous emission was Postulated by Bohr.	1. The stimulated emission was Postulated by Einstein.
2. Additional photons are not required in spontaneous emission.	2. Additional photons are required in Stimulated emission.
3. One Photon is emitted in spontaneous emission.	3. Two photons are emitted in stimulated emission.
4. The emitted radiation is incoherent.	5. The emitted radiation is coherent.
5. The emitted radiation is less intense.	6. The emitted radiation is high intense.

Light Amplification:

Light amplification requires stimulated emission exclusively. In practice, absorption and spontaneous emission always occur together with stimulated emission. The laser operation is achieved when stimulated emission exceeds the other two processes due to its higher transitions rates of atomic energy levels.

EINSTEIN COEFFICIENTS & THEIR RELATIONS

Einstein coefficients:

Einstein mathematically expressed the statistical nature of the three possible radiative transition routes (spontaneous emission, stimulated emission, and absorption) with the so-called Einstein coefficients and quantified the relations between the three processes.

Let N_1 be the number of atoms per unit volume with energy E_1 and N_2 be the number of atoms per unit volume with energy E_2 . Let 'n' be the number of photons per unit volume at frequency ' ν ' such that $E_2 - E_1 = h\nu$. Then, the energy density of photons $E = h\nu$.

- **Stimulated absorption:** When the photons interact with the atoms it leads to absorption transition which is called as stimulated absorption. Stimulated absorption rate depends upon the number of atoms available in the lowest energy state as well as the energy density photons.

$$\begin{aligned}\text{Stimulated absorption rate} &\propto N_1 \\ &\propto u(\nu) \\ &= B_{12} N_1 u(\nu)\end{aligned}$$

Where B_{12} is the Einstein coefficient of stimulated absorption.

- **Spontaneous emission:** The atom in the excited state returns to ground state emitting a photon of energy $(E) = E_2 - E_1 = h\nu$, without applying an external energy spontaneously is known as spontaneous emission. The spontaneous emission rate depends up on the number of atoms present in the excited state.

$$\begin{aligned}\text{Spontaneous emission rate} &\propto N_2 \\ &= A_{21} N_2\end{aligned}$$

Where A_{21} is the Einstein coefficient of spontaneous emission.

- **Stimulated emission:** The atom in the excited state can also returns to the ground state by applying external energy or inducement of photon thereby emitting two photons which are having same energy as that of incident photon. This process is called as stimulated emission. Stimulated emission rate depends upon the number of atoms available in the excited state as well as the energy density of incident photons.

$$\begin{aligned}\text{Stimulated emission rate} &\propto N_2 \\ &\propto u(\nu) \\ &= B_{21} N_2 u(\nu)\end{aligned}$$

Where B_{21} is the Einstein coefficient of stimulated emission.

If the system is in equilibrium, the rates of absorption transitions (Stimulated absorption) are equal to emission transitions (The rate of Spontaneous + Stimulated emission).

The rate of Absorption = The rate of Spontaneous + Stimulated emission

$$\begin{aligned}B_{12}N_1u(\nu) &= A_{21}N_2 + B_{21}N_2u(\nu) \\ B_{12}N_1u(\nu) - B_{21}N_2u(\nu) &= A_{21}N_2 \\ u(\nu)(B_{12}N_1 - B_{21}N_2) &= A_{21}N_2 \\ u(\nu) &= \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2} \\ u(\nu) &= \left(\frac{A_{21}}{B_{21}} \times \frac{1}{\left[\frac{N_1}{N_2} \times \frac{B_{12}}{B_{21}} - 1 \right]} \right)\end{aligned}$$

According to Boltzmann distribution law

$$\begin{aligned}N &= N_0 e^{-\frac{E}{k_B T}} \\ N_1 &= N_0 e^{-\frac{E_1}{k_B T}} \\ N_2 &= N_0 e^{-\frac{E_2}{k_B T}}\end{aligned}$$

$$\frac{N_1}{N_2} = \frac{N_0 e^{-\frac{E_1}{k_B T}}}{N_0 e^{-\frac{E_2}{k_B T}}} = e^{\frac{E_2 - E_1}{k_B T}} = e^{\frac{h\nu}{k_B T}}$$

$$u(\nu) = \frac{A_{21}}{B_{21}} \times \frac{1}{\left[e^{\frac{h\nu}{k_B T}} \left(\frac{B_{12}}{B_{21}} \right) - 1 \right]} \quad \text{---(1)}$$

According to Planck's radiation formula the energy density of photon is given by,

$$u(\nu) = \frac{8\pi h \nu^3}{c^3} \times \left(\frac{1}{\left[e^{\frac{h\nu}{k_B T}} - 1 \right]} \right) \quad \text{---(2)}$$

Comparing eq.(1) & eq.(2), we get

$$(i) \quad \frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \quad \text{or} \quad \frac{A_{21}}{B_{21}} \propto \nu^3$$

$$(ii) \quad \frac{B_{12}}{B_{21}} = 1 \quad \text{or,} \quad B_{12} = B_{21}$$

POPULATION INVERSION:

The number of atoms present in the excited (or higher) state is greater than the number of atoms present in the ground energy state (or lower state) is called population inversion.

Let us consider two level energy systems of energies E_1 and E_2 as shown in figure. Let N_1 and N_2 be the population (means number of atoms per unit volume) of E_1 and E_2 respectively.

According to Boltzmann's distribution the population of an energy level E , at temperature T is given by

$$N_i = N_0 e^{\left(-\frac{E_i}{k_B T}\right)} \quad \text{where } i = 1, 2, 3, \dots, N_i$$

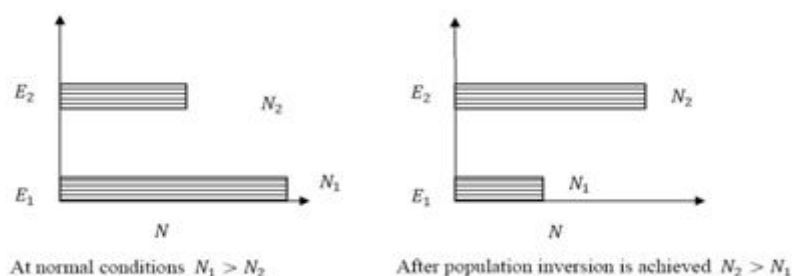
where ' N_0 ' is the number of atoms in ground or lower energy states & k is the Boltzmann constant.

From the above equation the population of energy levels E_1 & E_2 are given by

$$N_1 = N_0 e^{\left(-\frac{E_1}{k_B T}\right)}$$

$$N_2 = N_0 e^{\left(-\frac{E_2}{k_B T}\right)}$$

At ordinary conditions $N_1 > N_2$ i.e., the population in the ground or lower state is always greater than the population in the excited or higher states. The stage of making, population of higher energy level greater than the population of lower energy level is called population inversion i.e., $N_2 > N_1$.



META STABLE STATE:

In general the number of excited particles in a system is smaller than the non excited particles. The time during which a particle can exist in the ground state is unlimited. On the other hand, the particle can remain in the excited state for a limited time known as life time. The life time of the excited hydrogen atom is of the order of 10^{-8} sec. However there exist such excited states in which the life time is greater than 10^{-8} sec. These states are called as Meta stable states.

PUMPING MECHANISMS (OR TECHNIQUES):

Pumping:

The process of rising more no of atoms to the excited state by artificial means is called pumping.

A system in which population inversion is achieved is called as an active system. The method of raising the particles from lower energy state to higher energy state is called pumping. (or the process of achieving of population inversion is called pumping). This can be done by number of ways. The most commonly used pumping methods are

- Optical pumping
- Electrical discharge pumping
- Chemical pumping
- Injection current pumping

Optical pumping:

Optical pumping is used in solid laser. Xenon flash tubes are used for optical pumping. Since these materials have very broad band absorption, sufficient amount of energy is absorbed from the emission band of flash lamp and population inversion is created. Examples of optically pumped lasers are ruby, Nd: YAG Laser ($Y_3 Al_5 G_{12}$) (Neodymium: Yttrium Aluminum Garnet), Nd: Glass Laser

Electrical discharge pumping:

Electrical discharge pumping is used in gas lasers. Since gas lasers have very narrow absorption band pumping them any flash lamp is not possible. Examples of Electrical discharge pumped lasers are He-Ne laser, CO_2 laser, argon-ion laser, etc

Chemical pumping:

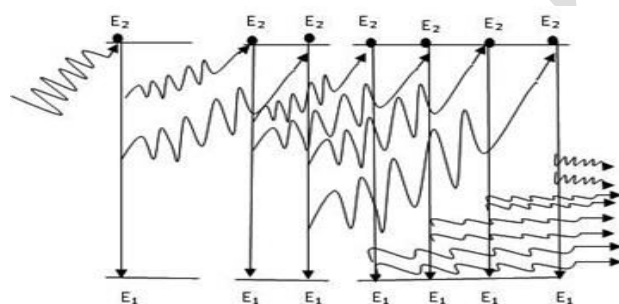
Chemical reaction may also result in excitation and hence creation of population inversion in few systems. Examples of such systems are HF and DF lasers.

Injection current pumping:

In semiconductors, injection of current through the junction results in creates of population inversion among the minority charge carriers. Examples of such systems are InP and GaAs.

PRINCIPLE OF LASER/LASING ACTION:

Let us consider many no of atoms in the excited state. Now the stimulating photon interacts with any one of the atoms in the excited state, the stimulated emission will occur. It emits two photons, having same energy & same frequency move in the same direction. These two photons will interact with another two atoms in excited state & emit 8-photons. In a similar way chain reaction is produced this phenomenon is called “Principle of lasing – action”. We get a monochromatic, coherent, directional & intense beam is obtained. This is called laser beam. This is the principle of working of a laser.

**Components of a LASER:**

Any laser system consists of 3-important components. They are

- (i) Source of energy or pumping source
- (ii) Active-medium (Laser Material)
- (iii) Optical cavity or resonator

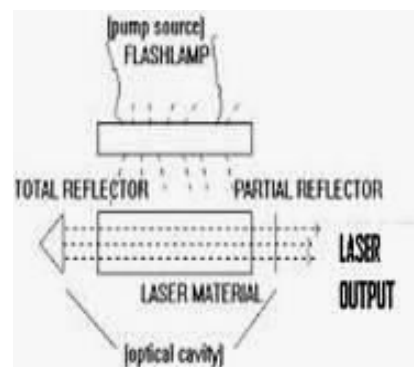
(i) Energy Source : It supply energies & pumps the atoms or molecules in the active medium to excited states. As a result we get population inversion in the active medium which emits laser.

Ex: Xenon flash lamp, electric field.

(ii) Active medium: The medium in which the population inversion takes place is called as active medium.

Active-centre: The material in which the atoms are raised to excited state to achieve population inversion is called as active center.

(iii) Optical-cavity or resonator: The active medium is enclosed between a fully reflected mirror & a partially reflective mirror. This arrangement is called as cavity or resonator. As a result, we get highly intense monochromatic, coherence laser light through the non-reflecting portion of the mirror.



DIFFERENT TYPES OF LASERS

On the basis of active medium used in the laser systems, lasers are classified into several types

- | | |
|-------------------------|--|
| I. Solid lasers | : Ruby laser, Nd:YAG laser, Nd:Glass |
| II. Liquid lasers | : Europium Chelate laser, SeOCl_2 |
| III. Gas lasers | : CO_2 , He-Ne, Argon-Ion Laser |
| IV. Dye lasers | : Rhodamine 6G |
| V. Semiconductor lasers | : InP, GaAs. |

RUBY LASER

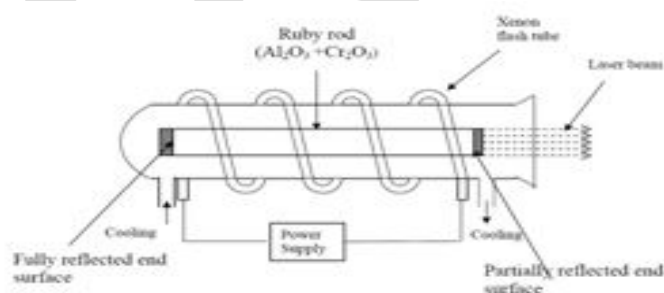
Ruby laser is a three level solid state laser and was developed by Mainmann in 1960. Ruby ($\text{Al}_2\text{O}_3 + \text{Cr}_2\text{O}_3$) is a crystal of Aluminium oxide, in which 0.05% of Al^{+3} ions are replaced by the Cr^{+3} ions. The colour of the ruby rod is pink. The active medium in the ruby rod is Cr^{+3} ions.

Principle or Characteristics of a ruby laser:

Due to optical pumping, the chromium atoms are raised to excited states then the atoms come to metastable state by non-radiative transition. Due to stimulated emission the transition of atoms takes place from metastable state to ground state and gives a laser beam.

Construction:

- In ruby laser 4cm length and 5mm diameter rod is generally used.
- Both the ends of the rods are highly polished and made strictly parallel.
- The ends are silvered in such a way, one becomes partially reflected and the other end fully reflected.
- The ruby rod is surrounded by xenon flash tube, which provides the pumping light to excite the chromium ions in to upper energy levels.

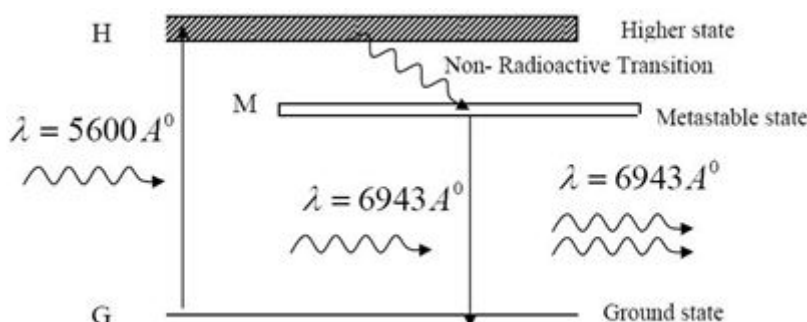


- Xenon flash tube emits thousands joules of energy in few milli seconds, but only a part of that energy is utilized by the chromium ions while the rest energy heats up the apparatus.
- A cooling arrangement is provided to keep the experimental set up at normal temperatures.

Working:

- The energy level diagram of chromium ions is shown in figure.
- The chromium ions get excitation into higher energy levels by absorbing of 5600\AA of wave length radiation.
- The excited chromium ions stay in the level H for short interval of time (10^{-8} Sec).

- After their life time most of the chromium ions are de-excited from H to G and a few chromium ions are de-excited from H to M.



- The transition between H and M is non-radioactive transition i.e. the chromium ions gives their energy to the lattice in the form of heat.
- In the Meta stable state the life time of chromium ions is 10^{-3} sec.
- Due to the continuous working of flash lamp, the chromium ions are excited to higher state H and returned to M level.
- After few milli seconds the level M is more populated than the level G and hence the desired population inversion is achieved.
- The state of population inversion is not a stable one.
- The process of spontaneous transition is very high.
- When the excited chromium ion passes spontaneously from H to M it emits one photon of wave length 6943\AA .
- The photon reflects back and forth by the silver ends and until it stimulates an excited chromium ion in M state and it to emit fresh photon in phase with the earlier photon.
- The process is repeated again and again until the laser beam intensity is reached to a sufficient value.
- When the photon beam becomes sufficient intense, it emerges through the partially silvered end of the rod.
- The wave length 6943\AA is in the red region of the visible spectrum on returning to ground state (G).

Uses of Ruby laser:

- Used in distance measurement using 'pulse echo technique'
- Used for measurement of plasma properties such as electron density and temperature.
- Used to remove the melanin of the skin.
- Used for recording pulsed holograms.
- Used as target designators and range finders in military.

Draw backs of Ruby laser:

- It requires high pumping power.
- The efficiency of ruby laser is very small. It is a pulse laser.

He-Ne LASER

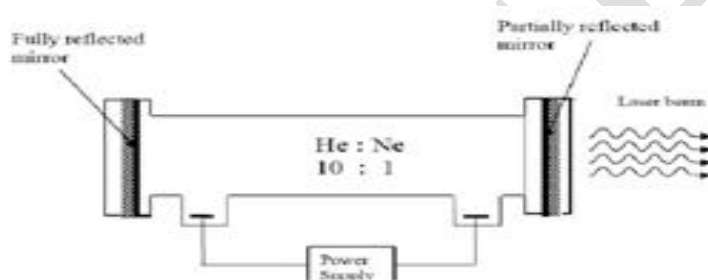
It was discovered by A. Javan & his co-workers in 1960. It is a continuous wave gas laser. It consists of mixture of He & Ne in 10:1 ratio as a active medium.

Principle/Characteristics of He-Ne laser:

This laser is based on the principle of stimulated emission, produced in the He & Ne. The population inversion is achieved due to the interaction between He & Ne gases. Using gas lasers, we can achieve highly coherent, directional and high monochromatic beam.

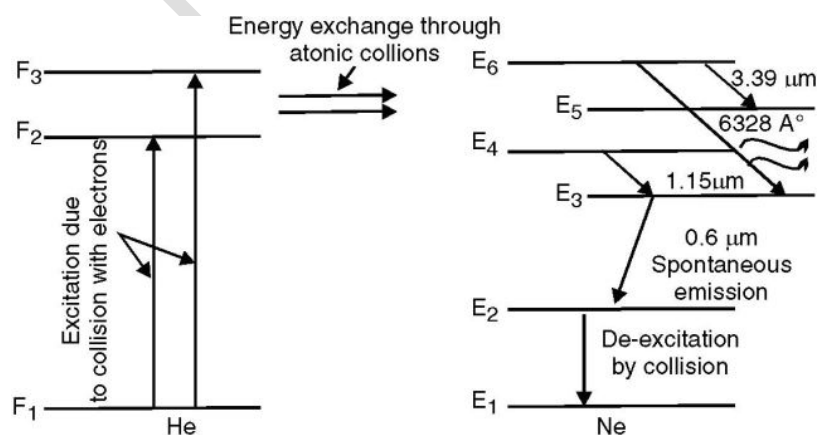
Construction:

- In He-Ne gas laser, the He and Ne gases are taken in the ratio 10:1 in the discharge tube.
- Two reflecting mirrors are fixed on either ends of the discharge tube, in that, one is partially reflecting and the other is fully reflecting.
- In He-Ne laser 80cm length and 1cm diameter discharge tube is generally used.
- The output power of these lasers depends on the length of the discharge tube and pressure of the gas mixture.
- Energy source of laser is provided by an electrical discharge of around 1000V through an anode and cathode at each end of the glass tube.



Working:

- When the electric discharge is passing through the gas mixture, the electrons accelerated towards the positive electrode.
- During their passage, they collide with He atoms and excite them into higher levels.
- F_2 and F_3 form F_1 . In higher levels F_2 and F_3 , the life time of He atoms is more.
- So there is a maximum possibility of energy transfer between He and Ne atoms through atomic collisions.
- When He atoms present in the levels F_2 and F_3 collide with Ne atoms present ground state E_1 , the Ne atoms gets excitation into higher levels E_4 and E_6 .



- Due to the continuous excitation of Ne atoms, we can achieve the population inversion between the higher levels E_4 (E_6) and lower levels E_3 (E_5).
- The various transitions $E_6 \rightarrow E_5$, $E_4 \rightarrow E_3$ and $E_6 \rightarrow E_3$ leads to the emission of wavelengths 3.39A^0 , 1.15A^0 and 6328A^0 .
- The first two corresponding to the infrared region while the last wavelength is corresponding to the visible region.
- The Ne atoms present in the E_3 level are de-excited into E_2 level, by spontaneously emission of photon.
- When a narrow discharge tube is used, the Ne atoms present in the level E_2 collide with the walls of the tube and get de-excited to ground level E_1 .

Uses of He-Ne laser:

- Used in laboratories for all interferometric experiments.
- Used widely in metrology in surveying, alignment etc.
- Used to read barcodes and He-Ne laser scanners also used for optical character recognition.
- Used in holography.

APPLICATIONS OF LASERS

Due to high intensity, high mono-chromaticity and high directionality of lasers, they are widely used in various fields like

1. communication
2. computers
3. chemistry
4. photography
5. industry
6. medicine
7. military
8. scientific research

1. Communication:

- In case of optical communication, semiconductor laser diodes are used as optical sources.
- More channels can be sent simultaneously. Signal cannot be tapped as the bandwidth is large, more data can be sent.
- A laser is highly directional and has less divergence, hence it has greater potential use in space crafts and submarines.

2. Computers :

- In LAN (local area network), data can be transferred from memory storage of one computer to other computer using laser for short time.
- Lasers are used in CD-ROMS during recording and reading the data.

3. Chemistry :

- Lasers are used in molecular structure identification.
- Lasers are also used to accelerate some chemical reactions.
- Using lasers, new chemical compounds can be created by breaking bonds between atoms or molecules.

4. Photography :

- Lasers can be used to get 3-D lens less photography.
- Lasers are also used in the construction of holograms.

5. Industry :

- Lasers can be used to blast holes in diamonds and hard steel.
- Lasers are also used as a source of intense heat.
- Carbon dioxide laser is used for cutting drilling of metals and nonmetals, such as ceramics plastics, glass etc.
- High power lasers are used to weld or melt any material.
- Lasers are also used to cut teeth in saws and test the quality of fabric.

6. Medicine :

- Pulsed neodymium laser is employed in the treatment of liver cancer.
- Argon and carbon dioxide lasers are used in the treatment of liver and lungs.
- Lasers used in the treatment of Glaucoma.

7. Military :

- Lasers can be used as a war weapon.
- High energy lasers are used to destroy the enemy air-crafts and missiles.
- Lasers can be used in the detection and ranging like RADAR.

8. Scientific research:

- Lasers are used in the field of 3D-photography.
- Lasers used in Recording and reconstruction of hologram.
- Lasers are employed to create plasma.
- Lasers used to produce certain chemical reactions.
- Lasers are used in Raman spectroscopy to identify the structure of the molecule.
- Lasers are used in the Michelson- Morley experiment.
- A laser beam is used to confirm Doppler shifts in frequency for moving objects.

FIBER-OPTICS

INTRODUCTION TO OPTICAL FIBER:

Fiber optics is a branch of physics which deals with the transmission & reception of light waves using optical fibers which acts as a guiding media. The transmission of light waves by fiber optics was first demonstrated by John Tyndall in 1870.

Optical Fiber:

Optical fiber is a thin & transparent guiding medium or material which guides the information carrying light waves. It is a cylindrical wave-guide system which propagates the data & speech signals in the optical frequency range.

A light beam acting as a carrier wave is capable of carrying more information than radio waves & microwaves because of its high frequency as shown below.

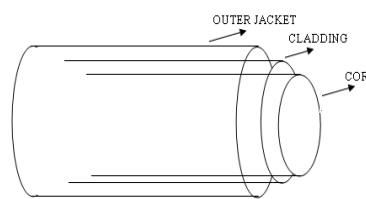
Radio waves - 10^4 Hz , Micro waves - 10^{10} Hz, Light waves - 10^{15} Hz

Construction:

An optical fiber is a very thin, flexible transparent made with plastic or glass.

It has cylindrical shape consisting of three layers or sections

- 1) The Core
- 2) The Cladding
- 3) The Outer jacket or Buffer jacket



1) The Core: It is the central layer surrounded by another layer called cladding. Light is transmitted within the core which has refractive index (n_1). It is a denser medium. Core is made of silica (SiO_2).

2) The Cladding: It is the second layer, surrounded by a third layer called the outer jacket. It has refractive index n_2 which is less than the refractive index of core i.e ($n_1 > n_2$). It acts as a rarer medium. It keeps the light within the core because $n_1 > n_2$. To lower the refractive index of cladding the silica is doped with phosphorous or bismuth material.

3) The Outer or Buffer Jacket: It is the third layer it protects the fiber from moisture & abrasion. To provide necessary toughness & tensile strength, a layer of strength member is arranged surrounding buffer jacket. It is made of polyurethane material.

Working Principle of Optical Fiber:

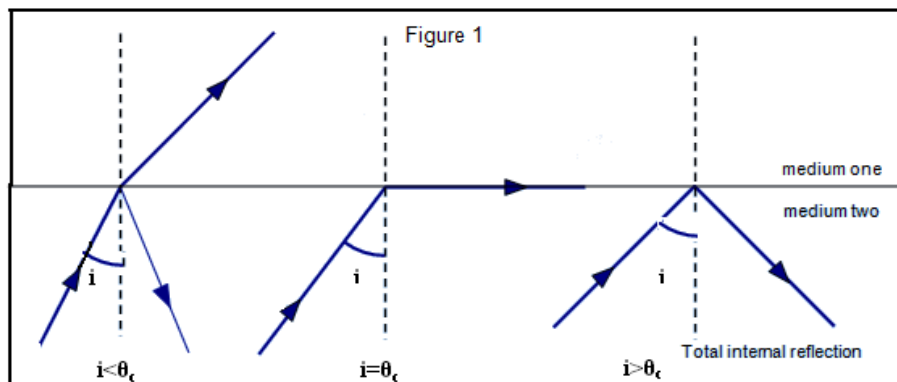
Total Internal Reflection: The principle of optical fiber is total internal reflection.

Condition for Total Internal Reflection:

- 1) The light ray should move from denser to rarer medium.
- 2) The refractive index of core must be greater than cladding i.e. $n_1 > n_2$
- 3) The angle of incidence (i) must be greater than the critical angle (θ_c) i.e. $i > \theta_c$.
- 4) The critical angle $\theta_c = \sin^{-1} \frac{n_2}{n_1}$.

Explanation:

Let us consider a denser medium & rarer medium of refractive indices n_1 & n_2 respectively and $n_1 > n_2$. Let a light ray move from denser to rare medium with 'i' as the angle of incidence & 'r' as angle of refraction. The refracted ray bends away from the normal as it travels from denser to rarer medium with increase of angle of incidence 'i'.



In this we get three cases

Case-1: When $i < \theta_c$, Then the light ray refracts into rarer medium as shown in figure.

Case-2: When $i = \theta_c$, then the light ray traverses along the two media as shown in figure.

For the two media, applying Snell's law

$$n_1 \sin i = n_2 \sin r$$

$$(\because i = \theta_c \text{ \& } r = 90^\circ)$$

$$n_1 \sin \theta_c = n_2 \sin 90$$

$$n_1 \sin \theta_c = n_2$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

Where $n_1 > n_2$

For air, $n_2 = 1$,

$$\theta_c = \sin^{-1} \frac{1}{n_1}$$

Case-3: When $i > \theta_c$, then the light ray reflected back into the medium as shown in figure

ACCEPTANCE ANGLE & ACCEPTANCE CONE:

Def: Acceptance angle is the maximum angle of incidence at the core of an optical fiber so that the light can be guided through the fiber by total internal reflection. This angle is called as acceptance angle. It is denoted by ' α_i '.

- Consider a cross-sectional view of an optical fiber having core & cladding of refractive indices n_1 and n_2 .
- Let the fiber be in air medium (n_0). The incident light while entering into the core at 'A' makes an incident angle of ' α_i ' with the fiber-axis.

- In core it travels along AB & is incident at part B on cladding interface.
- Let α_r be the angle of refraction at part 'A' & ' θ ' be the angle of incidence at 'B'.
- When ' θ ' is greater than the critical angle ' θ_c ', then total internal reflection takes place into the core & light takes the path BD.
- Due to multiple total internal reflections the propagation of light ray takes place through the fiber.
- Applying Snell's law at AC core-air interface :-

$$\frac{\sin \alpha_i}{\sin \alpha_r} = \frac{n_1}{n_o}$$

$$n_o \sin \alpha_i = n_1 \sin \alpha_r \rightarrow (1)$$

- Let a normal 'BC' be drawn from the point 'B' to the fiber axis. Then from ΔABC , we get

$$\alpha_r = 90^\circ - \theta \rightarrow (2)$$

Substitute eq - (2) in eq - (1)

$$n_o \sin \alpha_i = n_1 \sin(90^\circ - \theta)$$

$$n_o \sin \alpha_i = n_1 \cos \theta \rightarrow (3)$$

- To get total internal reflection at point B

(Core-Cladding Interface) i.e. $\theta > \theta_c$

- Let the maximum angle of incidence at point A be $\alpha_i(\max)$ for which $\theta > \theta_c$.

From eqn (3), we get

$$n_o \sin \alpha_i = n_1 \cos \theta \rightarrow (4)$$

$$\alpha_i(\max) = \alpha_i, \text{ when } \theta = \theta_c$$

$$\sin \alpha_i(\max) = \frac{n_1}{n_o} \cos \theta_c \rightarrow (5)$$

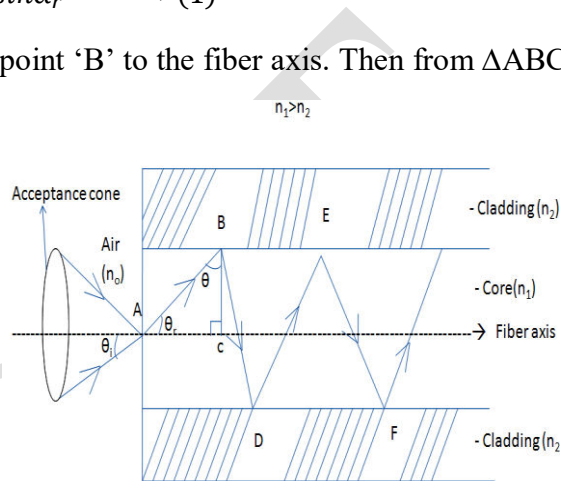
$$\text{We know that } \sin \theta_c = \frac{n_2}{n_1}$$

$$\cos \theta_c = \sqrt{(1 - \sin^2 \theta_c)} = \sqrt{\left(1 - \frac{n_2^2}{n_1^2}\right)}$$

$$\Rightarrow \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_1}$$

$$\therefore \cos \theta_c = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_1} \rightarrow (6)$$

Substitute the eq(6) in eq(5), we get



$$\sin \alpha_i(\max) = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin \alpha_i(\max) = \sqrt{\frac{n_1^2 - n_2^2}{n_0^2}}$$

(OR)

$$\alpha_i(\max) = \sin^{-1} \sqrt{\frac{n_1^2 - n_2^2}{n_0^2}} \rightarrow (7)$$

For air medium, $n_0=1$

$$\alpha_i(\max) = \sin^{-1} \sqrt{n_1^2 - n_2^2} \rightarrow (8)$$

Fractional Index Change (Δ):

It is the ratio of refractive index difference in core & cladding to the refractive index of core.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$n_1 - n_2 = \Delta n_1 \rightarrow (1)$$

NUMERICAL APERTURE (N.A.):

Def: It is defined as light accepting efficiency of the fiber and is equal to sine of the acceptance angle of the fiber i.e. $N.A. = \sin \alpha_i(\max)$

$$N.A. = \sin \alpha_i(\max) = \sqrt{\frac{n_1^2 - n_2^2}{n_0^2}}$$

$$\text{We know that } \Delta = \frac{n_1 - n_2}{n_1}$$

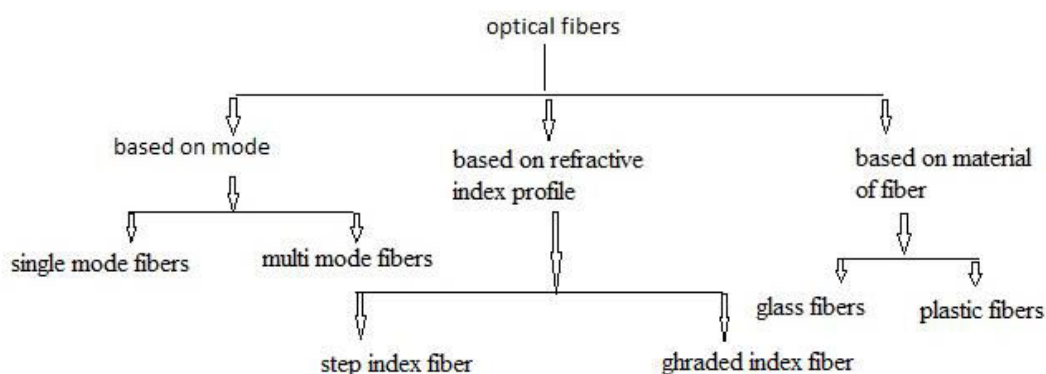
$$n_1 - n_2 = \Delta n_1 \rightarrow (2)$$

$$\text{We know, } N.A. = \sin \alpha_i(\max) = \sqrt{n_2^2 - n_1^2} \quad \text{for air } n_0=1$$

$$N.A. = \sqrt{(n_1 + n_2)(n_1 - n_2)}$$

$$\text{If } n_1 = n_2, \text{ then } N.A. = \sqrt{2n_1 \times \Delta n_1}$$

$$N.A. = n_1 \sqrt{2\Delta} \rightarrow (3)$$

TYPES OF OPTICAL FIBERS:

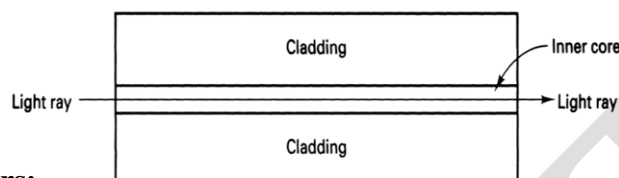
Optical fibers can be classified based on either the mode they support or the refractive index profile of the fiber. They can also be classified based on the material of the fiber.

Based on Mode:

The rays travelling in the fiber by total internal reflection are called modes.

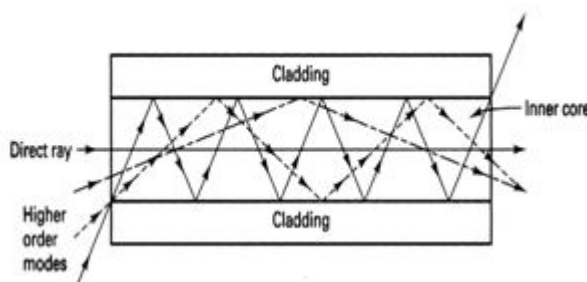
1) Single mode fibers:

- If the thickness of the fiber is so small that it supports only one mode then the fiber is called **single mode fiber** or mono mode fiber.
- The core diameter of this fiber is about 8 to 10 μm and the outer diameter of cladding is 60 to 70 μm .



2) Multi mode fibers:

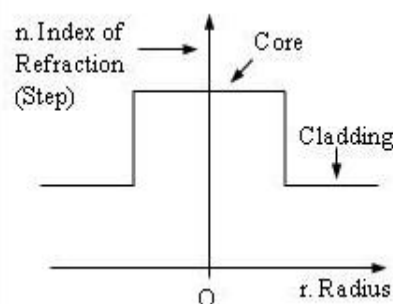
- If the thickness of the fiber is very large that it supports more than one mode then the fiber is called **multi mode fiber**.
- The core diameter of this fiber is about 50 to 200 μm and the outer diameter of cladding is 100 to 250 μm .



Based on refractive index profile:

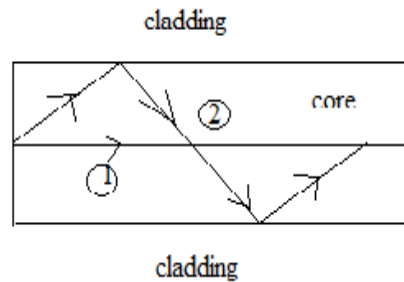
1) Step-Index Optical fiber:

- In a step-index optical fiber, the entire core has uniform refractive index n_1 slightly greater than the refractive index of the cladding n_2 .
- Since the index profile is in the form of a step, these fibers are called step-index fibers.
- The transmission of information will be in the form of signals or pulses.
- These are extensively used because distortion and transmission losses are very less.
- Step-index optical fibers are of two types. They are
 - (i) Single mode step-index fiber
 - (ii) Multi-mode step-index fiber



Transmission / propagation of signal in Step-index fibers:

- Generally the signal is sent through the fiber in digital form i.e. in the form of pulses.
- The same pulsed signal travels in different paths.
- Let us now consider a signal pulse travelling through step index fiber in two different paths (1) and (2).
- The pulse (1) travelling along the axis of the fiber and pulse (2) travelling away from the axis.
- At the receiving end only the pulse (1) which travels along the fiber axis reaches first while the pulse (2) reaches after some time delay.
- Hence the pulsed signal received at the other end is broadened. This is called internal dispersion.
- This reduces transmission rate capacity of the signal.
- This difficulty is overcome by graded index fibers.

**2) Graded index optical fiber:**

- In this fiber, the refractive index of the core varies radially.
- It has maximum refractive index at its centre, which gradually falls with increase of radius and at the core-cladding interface matches with refractive index of cladding.
- Variation of refractive index of the core with radius is given by

$$n(x) = n_1 \left[1 - 2\Delta \left(\frac{x}{a} \right)^p \right]^2$$

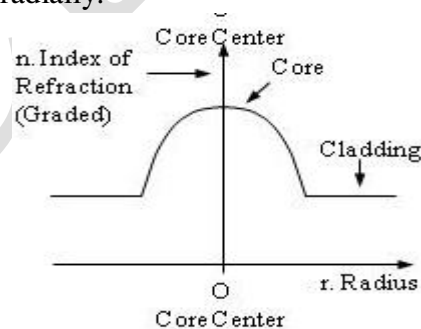
Where n_1 → refractive index at the centre of the core

a → radius of the core

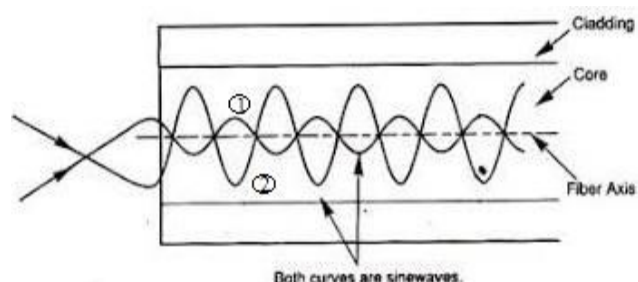
Δ → Fractional index change, $\Delta = \frac{n_1 - n_2}{n_1}$

p → grading profile index number

- This fiber divided into two types.
 - (i) Single-mode graded index fiber
 - (ii) Multi-mode graded index fiber

**Transmission / propagation of signal in Step-index fibers:**

- Let us now consider a signal pulse travelling through graded index fiber in two different paths (1) and (2).
- The pulse (1) travelling along the axis of the fiber though travels along shorter route it travels



through higher refractive index.

- The pulse (2) travelling away from the axis undergo refraction and bend as shown in fig. though it travels longer distance, it travels along lesser refractive index medium.
- Hence both the pulses reach the other end simultaneously.
- Thus the problem of inter modal dispersion can be overcome by using graded index fibers.

Based on types of materials:

- 1) glass-glass optical fiber
- 2) glass-plastic optical fiber
- 3) plastic-plastic optical fiber

ATTENUATION (POWER-LOSS) IN OPTICAL FIBERS

When light propagates through an optical fiber, then the power of the light at the output end is found to be always less than the power launched at the input end. The loss of power is called Attenuation. It is measured in terms of decibels per kilometer.

Attenuation: It is defined as the ratio of the optical power output (P_{out}) from a fiber of length 'L' to the power input (P_{in}).

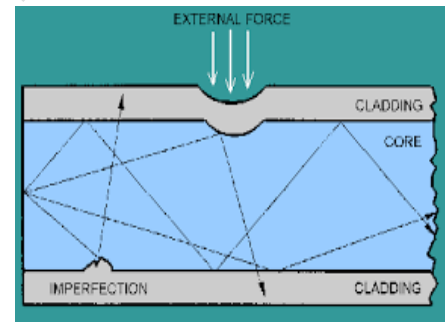
$$Attenuation(\alpha) = -\frac{10}{L} \left(\frac{P_{in}}{P_{out}} \right) dB/km$$

Attenuation occurs because of the following reasons

- (1) Absorption (2) Scattering loss 3) Bending loss

(1) Absorption:

- It occurs in two ways, i) Absorption by impurity or impurity absorption
- ii) Intrinsic absorption or internal absorption
- **Impurity absorption:** The impurities present in the fiber are transition metal ions, such as iron, chromium, cobalt & copper. During signal propagation when photons interact with these impurity atoms, then the photons are absorbed by atoms. Hence loss occurs in light power.
- **Intrinsic absorption or internal absorption:** The fiber itself as a material has a tendency to absorb light energy however small it may be. The absorption that takes place in fiber material assuming that there are no impurities in it, is called intrinsic absorption.



(II) Scattering loss:

- When the signals travels in the fiber, the photons may be scattered due to variations in the refractive index inside the fiber. This scattering is called as Rayleigh scattering. It is also a wavelength dependent loss.

$$Rayleigh \text{ scattering loss} \propto \frac{1}{\lambda^4}$$

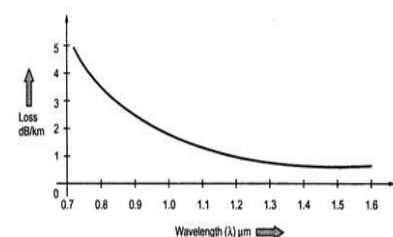
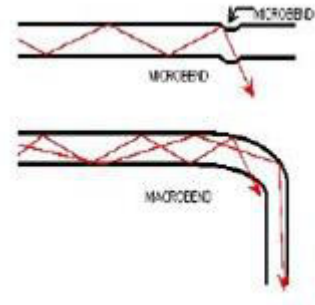


Fig. 1 Scattering loss

(III) Bending losses:

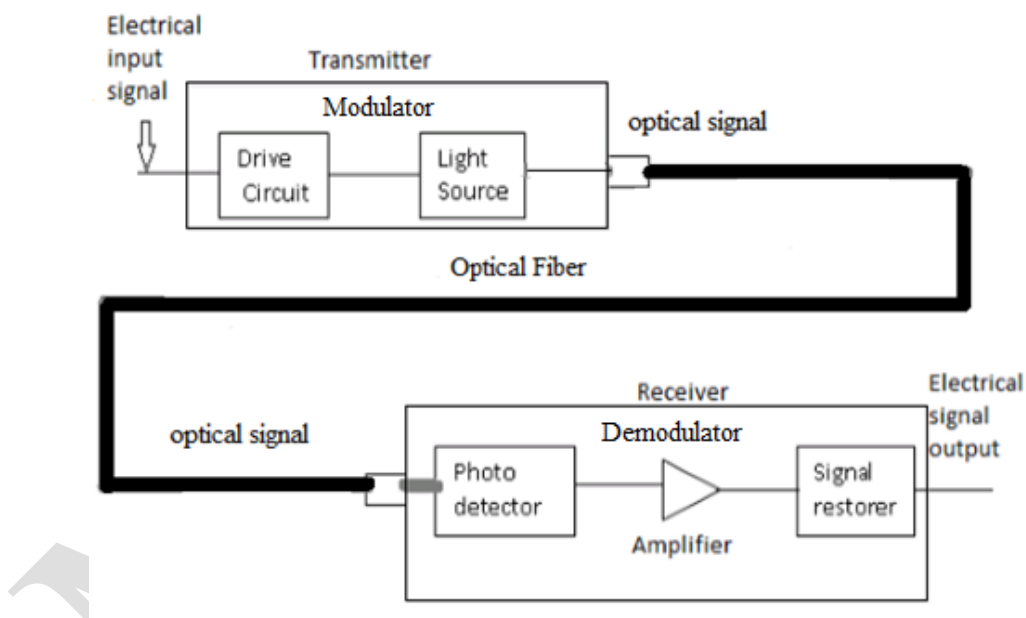
- These losses occur due to (a) Macroscopic bending (b) Microscopic bending
- **Macroscopic bending:** If the radius of core is large compared to fiber diameter causes large curvature at the bends. At these bends, the light will not satisfy the condition for total internal reflection & light escapes out from the fiber. It is called as macroscopic bending.
- **Microscopic bending:** These are caused due to non-uniform pressures created during the cabling of the fiber or during the manufacturing the fiber. It causes irregular reflections. This lead to loss of light by leakage through the fiber.



OPTICAL FIBER IN COMMUNICATION SYSTEM

The most important application of optical fibers occurs in the field of communication.

Fiber optic communication systems comprise of the following units.



Information signal source:

- The information signal to be transmitted may be voice, video or computer data (analog signals).
- In order to communicate through optical fiber, the analog signals are converted into electrical signals. (by Analog to Digital converter)
- The converted electrical signals are passed through the transmitter.

Transmitter:

- The transmitter is a modulator device used to receive electrical input signal, and then modulate it into digital pulses for propagation into an optical fiber.
- The modulator consists of a driver and a light source as shown in fig.
- The driver receives the electrical signals and then converts into the digital pulses.

- These digital pulses are converted into optical signals after passing through a light source, generally either light emitting diodes (LED's) or a semi conductor laser is used as light source.
- The optical signals are then focused into the optical fiber as shown in fig.

Optical Fiber (or) Transmission medium:

- The optical fiber is used as transmission medium between the transmitter and the receiver.
- The optical signals are then fed into an optical fiber cable where they are transmitted over long distances using the principle of total internal reflection.

Receiver:

- The receiver is a demodulator device used to receive the optical signals from the optical fiber and then convert into electrical signals.
- The demodulator consists of a photodetector, an amplifier and a signal restorer.
- The optical signals which are emerging from the optical fiber are received by photo detector.
- The photodetector converts the optical signals into electrical signals.
- The electrical signals are then amplified by the amplifier and the amplified electrical signals are converted into digital form.
- The amplified electrical signals are fed to a signal restorer where the original voice is recovered.

Advantages of Optical Fibers in communication system:

- High data transmission rates and bandwidth.
- Low losses.
- Small cable size and weight.
- Immunity to EM radiations.
- Safety due to lack of sparks.
- Data security.

APPLICATIONS OF OPTICAL FIBERS:

- 1) Due to high band-width, light can transmit at a higher rate up to 10^{14} to 10^{15} Hz. Than radio or micro-frequencies.
- 2) Long distance signal transmission.
- 3) They are used for exchange of information in cable television, space vehicles, sub-marines
- 4) Optical fibers are used in industry in security alarm systems, process control & industrial automation.
- 5) They are used in pressure sensors in biomedical & engine control applications.