CS/SE 2S03: Tutorial 7 (Semantics)

October 24, 2013

The Questions

Q1. Give the definition of the function Θ for expressions of the form "t & u", "t && u", "t | u", and "t || u".
Remark: &, |, &&, || are called "boolean logical AND", "boolean logical OR", "logical AND", and "logical OR", respectively.

& and |: verify both operands,

&&: stops evaluating if the first operand evaluates to false since the result will be false

||: stops evaluating if the first operand evaluates to true since the result will be true

Q2. Define the incomplete conditional test using a complete test and a statement **skip**.

Q3. Give the definition of the function Σ for "**do** s **while**(b)".

Q4. Define the function Σ for "**for**" construct.

Q5. Give the definition of the Σ function for the declaration of a variable without an initial value.

Q6. Consider the following function:

```
static int f (int x, int y) { return (x+1) * (x+y); }
```

Describe the execution of the statement $u = f(a, b)$ in the environment $e = [a = r_1, b = r_2, u = r_3]$, the memory state $m = [r_1 = 2, r_2 = 3, r_3 = 0]$ and the global environment G composed of the environment e and the function declaration f.

The Answers

Q1.

$\Theta(t \mid u, e, m, G) = (v \vee w, m'')$ where
$(v, m') = \Theta(t, e, m, G)$ and $(w, m'') = \Theta(u, e, m', G)$

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Unlike the boolean operator & that evaluates its two arguments, the operator && evaluates its second argument only if the first argument evaluates to true.The boolean operator || only evaluates its second argument if the first argument evaluates to false.

$$\Theta(t \,\&\&\, u, e, m, G) = \begin{cases} \Theta(u, e, m', G) & \text{if } \Theta(t, e, m, G) = (\mathbf{true}, m') \\ (\mathbf{false}, m') & \text{if } \Theta(t, e, m, G) = (\mathbf{false}, m') \end{cases}$$

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Q2.

$\Sigma(\mathbf{if}\ (\mathrm{b})\ \mathrm{s}, e, m, G) =$

$$\Sigma(\mathbf{if}\ (\mathrm{b})\ \mathrm{s}\ \mathbf{else\ skip};, e, m, G) = \begin{cases} \Sigma(s, e, m', G) & \text{if } \Theta(b, e, m, G) = (\mathbf{true}, m') \\ \Sigma(\mathbf{skip};, e, m', G) & \text{if } \Theta(b, e, m, G) = (\mathbf{false}, m') \end{cases}$$

Q3.

$\Sigma(\mathbf{do}\ s\ \mathbf{while}(b), e, m, G) =$

$$\Sigma(\{\mathrm{s}\ \mathbf{while}(b)\ s\}, e, m, G) = \begin{cases} \Sigma(\mathbf{while}(b)\ s, e, m', G) & \text{if } \Sigma(s, e, m, G) = (\text{normal}, m') \\ (\text{return}, v, m') & \text{if } \Sigma(s, e, m, G) = (\text{return}, v, m') \end{cases}$$

$\Sigma(\mathbf{while}(b)\ s, e, m', G) = \lim_{\mathrm{n}\to\infty} \Sigma(p_n, e, m', G)$ where
$p_0 = \mathbf{if}\ (b)\ \mathbf{giveup};\ \mathbf{else\ skip;}$, $\forall n \geq 0.\ p_{n+1} = \mathbf{if}\ (b)\ \{\ s\ p_n\ \}\ \mathbf{else\ skip};$

Q4.

$\Sigma(\mathbf{for}(s_1;\ b;\ s_2)\ s_3, e, m, G) =$

$$\Sigma(\{s_1\ \mathbf{while}(b)\ \{s_3\ s_2\}\}, e, m, G) = \begin{cases} \Sigma(\mathbf{while}(b)\{s_3\ s_2\}, e, m', G) & \text{if } \Sigma(s_1, e, m, G) = (\text{normal}, m') \\ (\text{return}, v, m') & \text{if } \Sigma(s_1, e, m, G) = (\text{return}, v, m') \end{cases}$$

$\Sigma(\mathbf{while}(b)\{s_3\ s_2\}, e, m', G) = \lim_{\mathrm{n}\to\infty} \Sigma(p_n, e, m', G)$ where
$p_0 = \mathbf{if}\ (b)\ \mathbf{giveup};\ \mathbf{else\ skip;}$, $\forall n \geq 0.\ p_{n+1} = \mathbf{if}\ (b)\ \{\ \{s_3\ s_2\}\ p_n\ \}\ \mathbf{else\ skip};$

Q5.

$\Sigma(T\ x;, e, m, G) = \Sigma(\mathbf{skip};, e \oplus (x = r), m, G)$ where r fresh e, m

Q6. To do so, we start by evaluating a and b, which produces the values 2 and 3, without changing the memory state. And we create an environment $e'' = [a = r_1, b = r_2, u = r_3, x = r_4, y = r_5]$ and the memory state $m'' = [r_1 = 2, r_2 = 3, r_3 = 0, r_4 = 2, r_5 = 3]$

Next, we execute the body of the function, which produces the result $(return, 15, m'')$ and so $\Theta(f(a, b), e, m, G)$ is $(15, m'')$. The result of the execution of the statement $u = f(a, b)$; is then an ordered pair composed of a boolean *normal* and the memory state $m''' = [r_1 = 2, r_2 = 3, r_3 = 15, r_4 = 2, r_5 = 3]$. The value of the variable u in the state e, m''' is 15.