

# Principles of Programming

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# Java Pseudo Knowledge

Things you need to use now even you do not understand now

```
public class HelloWorld {  
    /**  
     * @param args  
     */  
    public static void main(String[] args) {  
        System.out.println("Hello, World");  
    }  
}
```

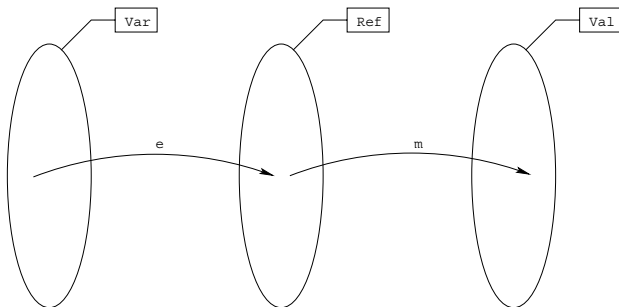
# Semantics of Core - Decomposition of State

$s : \text{State} = \text{FinV} \rightarrow \text{Val}$

$e : \text{Environment} = \text{FinV} \rightarrow \text{Ref}$

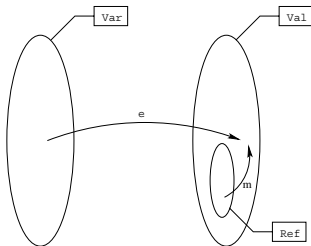
$m : \text{MemoryState} = \text{Ref} \rightarrow \text{Val}$

$s = e; m$



# Decomposition of State (Cont.)

- (Update function on states)  $\oplus = +_e; +_m$  where
  - ▶ (Update function on *environment*):  $e +_e (x = r)$
  - ▶ (Update function on *memory state*):  $m +_m (r = v)$
- Remember *Constants*! The Solutions:
  - ▶ Dependent Type: (overly complicated)  
 $\text{Env} : \{\text{mutable}, \text{constant}\} \times \text{Var} \rightarrow (\text{Ref} \vee \text{Val})$   
 $\text{Env} : (t : \{\{\text{mutable}, \text{constant}\}\}) \times \text{Var} \rightarrow$   
if  $t = \text{mutable} \Rightarrow \text{Ref}$  else  $\text{Val}$
  - ▶  $\text{Ref} \subset \text{Val}$

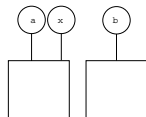


# Visual Representation of a State

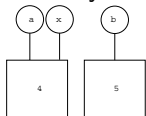
- Reference: by box



- Environment: by adding one or more labels to certain references



- Memory state: by filling each square with a value



- When a variable is associated directly with a value in the environment:



# The Value of Expressions

- $\Theta : Expr \times Env \times Mem \rightarrow Val$

example:  $\Theta(x + 3, [x = r_1, y = r_2], [r_1 = 5, r_2 = 6]) = 8$

- $kind : Var \rightarrow \{mutable, constant\}$

- For Java:

- ▶  $\Theta(x, e, m) = m(e(x))$  if  $kind(x) = mutable$
- ▶  $\Theta(x, e, m) = e(x)$  if  $kind(x) = constant$
- ▶  $\Theta(c, e, m) = c$  if  $c$  is a constant
- ▶  $\Theta(t + u, e, m) = \Theta(t, e, m) + \Theta(u, e, m)$   
(+ in the left side is defined on Expr and + in the right side is the usual one)  
(+ can be replaced by any elements  $\in \{+, -, *, /, \%\}$ )
- ▶  $\Theta((b)?t : u, e, m) =$   
if  $\Theta(b, e, m) = true$  then  $\Theta(t, e, m)$ ,  
if  $\Theta(b, e, m) = false$  then  $\Theta(u, e, m)$ ,

# The Value of Expressions (Cont.)

- For C: the same as Java
- For Caml: the same except  
 $\Theta(x, e, m) = e(x)$  where the variable  $x$  is either mutable or constant.  
Caml also has a construct  $!$  such that  
 $\Theta(!t, e, m) = m(\Theta(t, e, m))$

# Execution of Statements

Now, the declaration of “ $\Sigma$ ” is “ $Stat \times Env \times Mem \rightarrow Mem$ ”

The definition of  $\Sigma(p, e, m)$  depends on  $p$

- a mutable variable declaration of the form “ $\{T\ x = v; s\}$ ”  
 $\Sigma(\{T\ x = v; s\}, e, m) = \Sigma(s, e \oplus (x = r), m \oplus (r = \Theta(v, e, m)))$   
where  $r$  is “fresh”
- a constant variable declaration of the form “ $\{final\ T\ x = v; s\}$ ”  
 $\Sigma(\{final\ T\ x = v; s\}, e, m) = \Sigma(s, e \oplus (x = \Theta(v, e, m)), m)$
- an assignment of the form “ $x = v;$ ”  
 $\Sigma(x = v; , e, m) = m \oplus (e(x) = \Theta(v, e, m))$



# Execution of Statements - Cont.

- a sequence of the form “ $\{s1\ s2\}$ ”  
 $\Sigma(\{s1\ s2\}, e, m) = \Sigma(s2, e, \Sigma(s1, e, m))$
- a test of the form “*if* ( $b$ )  $s1$  *else*  $s2$ ”
  - ① *if*  $\Theta(b, e, m) = \text{true}$  *then*  $\Sigma(s1, e, m)$
  - ② *if*  $\Theta(b, e, m) = \text{false}$  *then*  $\Sigma(s2, e, m)$

# Execution of Statements - Cont.

- a loop of the form “*while (b) s*”

- a. Imaginary Statements

- ① skip; with  $\Sigma(\text{skip};, e, m) = m$
  - ② give-up; with  $\Sigma(\text{give-up};, e, m) = \perp$

- b. Approximation *while (b) s*

$p_0 = \text{if } (b) \text{ giveup; else skip;}$

$p_1 = \text{if } (b) \{ s \ p_0 \} \text{ else skip;}$

$p_2 = \text{if } (b) \{ s \ p_1 \} \text{ else skip;}$

.

$p_{n+1} = \text{if } (b) \{ s \ p_n \} \text{ else skip;}$

If loop terminates then  $\Sigma(p_n, e, m)$  is eventually constant

$$\Sigma(\text{while } (b) \ s, e, m) = \lim_{n \rightarrow \infty} \Sigma(p_n, e, m)$$