Semantics of Functions

We need to extend Σ and Θ for functions!

1. Global Environment
2. Functions can modify memory (and so expression can too):
   1&2 $\Longrightarrow \Theta : \mathrm{Expr} \times \mathrm{Env}_{\mathrm{local}} \times \mathrm{Mem} \times \mathrm{Env}_{\mathrm{global}} \rightarrow \mathrm{Val} \times \mathrm{Mem}$
3. Execution of "return":
   Example: s_1: a:= 5; return a; s_2: a:= 6;
   $\Sigma(\{\mathrm{s}_1\ \mathrm{s}_2\}, \mathrm{e}, \mathrm{m}, \mathrm{G}) = \Sigma(\mathrm{s}_2, \mathrm{e}, \Sigma(\mathrm{s}_1, \mathrm{e}, \mathrm{m}, \mathrm{G}), \mathrm{G})$
   $\Longrightarrow \quad \Sigma : \mathrm{State} \times \mathrm{Env} \times \mathrm{Env} \rightarrow \{\mathrm{normal}\} \times \mathrm{Mem} \ \uplus \ \{\mathrm{return}\} \times \mathrm{Val} \times \mathrm{Mem}$

Semantics of Functions - The Value of Expressions

recall: $K\ (\text{kind}) : \mathit{Var} \rightarrow \{\text{constant}, \text{mutable}\}$

$$\Theta(x, e, m, G) = \begin{cases} (m(e(x)), m) & \text{if } K(x) = \text{mutable} \\ (e(x), m) & \text{if } K(x) = \text{constant} \end{cases}$$

$\Theta(c, e, m, G) = (c, m)$

$\Theta(t\ \mathbf{op}\ u, e, m, G) = (v\ \mathbf{op}\ w, m'')$ where

op is any logical or arithmetic operation,

$(v, m') = \Theta(t, e, m, G)$ and $(w, m'') = \Theta(u, e, m', G)$

$$\Theta((b)?t : u, e, m, G) = \begin{cases} \Theta(t, e, m', G) & \text{if } \Theta(b, e, m, G) = (\mathit{true}, m') \\ \Theta(u, e, m', G) & \text{if } \Theta(b, e, m, G) = (\mathit{false}, m') \end{cases}$$

$\cdots$ - The Value of Expressions - Cont.

$\Theta(f(t_1, \cdots, t_n), e, m, G) = X$

Let $x_1, \cdots, x_n$ be formal parameters and p the body of f.

Let $(v_1, m_1) = \Theta(t_1, e, m, G)$

$(v_2, m_2) = \Theta(t_2, e, m_1, G)$

$\vdots$

$(v_n, m_n) = \Theta(t_n, e, m_{n-1}, G)$

Let $e' \subseteq G$ be the declaration of (global) variables

Let r_i $(1 \leqslant i \leqslant n)$ be fresh references

Let $a_i = \begin{cases} (x(i) = v_i) & \text{if } K(x_i) = \text{constant} \\ (x(i) = r_i) & \text{if } K(x) = \text{mutable} \end{cases}$

$\cdots$ - The Value of Expressions - Cont.

Let $e'' = e' \oplus a_1 \oplus \cdots \oplus a_n$

Let $J = \{J_1, \cdots, J_k\}$ in the ascendant order, $k \leqslant n$, $J \subseteq \{1, \cdots, n\}$

$$m'' = m_n \oplus (r_{j_1} = v_{j_1}) \oplus \cdots \oplus (r_{j_k} = v_{j_k})$$

- Now, consider $\Sigma(p, e'', m'', G)$
  If it is of the form $(return, v, m''')$ then $X = (v, m''')$
  o.w. Θ is not defined (an error!)
  [Java's type will prevent this!]

- Wed. 9: objects (Chapter 4 of Java Actually book). See code posted separately.
- Fri. 11: classes (Chapter 7 of Java Actually book). See code posted separately.