SE 3X03 / CS 4X03 Assignment 3

Released:Nov 20, 2014 Due: Dec 4, 2014 late night

A hard copy of your solution must reach Prof. Franek or one of the TA's by the deadline. In case you cannot hand it in in person to Prof. Franek or any of the TA's, you can slide it under the door of Prof. Franek's office. If Prof. Franek finds it on his arrival in the morning on Dec 5 in his office, he will still accept it, but not later. This assignment comprises 4 questions (a), (b), (c) and (d).

Consider a first order ODE $y' = 3y - 4e^{-t}$.

(a) Verify that $y = Ce^{3t} + e^{-t}$ is a general analytic solution to the equation.

(b) Considering the initial value $y(0) = 1$, show that the analytic solution is e^{-t}.

(c) Use the fourth-order Runge-Kutta method to integrate $y' = 3y - 4e^{-t}$ on the interval $[0, 10]$ in equal steps of size 0.1. Since it is 100 values to compute, you need to use MATLAB or a computer or a calculator to compute them. On the paper give just the values at 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

(d) If you answered (c) correctly you should notice something strange: since we know that e^{-t} is the analytic solution, $|y_n|$ should be decreasing to 0 as t is approaching 10. But the numbers of (c) seem to be increasing in magnitude. Try explain why it is so. *Hint: consider the rounding error of* $y(0)$ *and how it might propagate through the solution.*