Assignment 3

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a)

Differentiating $y = Ce^{3t} + e^{-t}$ we get:

$$\frac{dy}{dt}y = 3Ce^{3t} - e^{-t}$$

Then, we re-arrange our first equation:

$$Ce^{3t} = y - e^{-t}$$

And substitute it into the second equation:

$$y' = 3(y - e^{-t}) - e^{-t}$$
$$y' = 3y - 4e^{-t}$$

Therefore $y = Ce^{3t} + e^{-t}$ is indeed a general analytic solution to the equation $y' = 3y - 4e^{-t}$.

b)

Substituting the values $y = 1$ and $t = 0$ into $y = Ce^{3t} + e^{-t}$

$$1 = Ce^{0} + e^{0}$$
$$1 = C + 1$$

Therefore $C = 0$, substituting this back in the proposed solution we find the analytic solution is indeed $y = e^{-t}$

c)

Using a computer program written in C$^{\#}$ the following values were obtained:

$$
\begin{gathered}
y(0) = 1 \\
y(1) = 0.367741025173408 \\
y(2) = 0.132504642015483 \\
y(3) = -0.00707763902447456 \\
y(4) = -1.1236691550673 \\
y(5) = -22.9270219157003 \\
y(6) = -460.561696136583 \\
y(7) = -9249.21770901647 \\
y(8) = -185746.199126798 \\
y(9) = -3730223.27913773 \\
y(10) = -74911711.536581
\end{gathered}
$$

d)

Around the value of $y(3)$ the estimation suddenly becomes negative. We know from our analytical solution this shouldn't happen as $y(t \to \infty) \to 0$. This is because of rounding errors in floating point numbers. Taking a look at the derivative, it can be seen that if the value of y becomes negative due to rounding at any point, it will cause a small error. This small error will then increase with each iteration causing catastrophic failure.