

Interaction Between User and UAV with Unreliable Location Information^{*}

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Abstract. In this work we consider a scenario of data transmission between a user and UAV. During transmission UAV doesn't have any information on the user location but have the information on its mobility model. Transmitted traffic has certain restrictions on SNR and the main objective is to lower the probability of breaching these restrictions. The current study is a proof of concept for the considered model, aimed to propose and verify the described approach through analysis of the derived results.

Keywords: Signal-to-noise ratio· Mobile base station· Analytical model· Probability density function.

1 Introduction

In recent years the idea of using unmanned aerial vehicles (UAV), namely drones, as mobile base stations have become extremely popular. Such development of the industry of UAVs has brought dozens of new scenarios where drones could be used cheaply and efficiently. There are currently many papers studying inter-drone transmissions as D2D[4, 5], or focus on drone battery efficiency [1, 6], some even focus on modeling movements of UAV groups as a single entity[3]. an ongoing research into new use-cases for UAVs is also conducted by Ericsson in [7].

The length of autonomous flight of drones is a key factor withholding further development of the area. Fortunately, there is an ongoing research intended to boost battery capacity of drones, as [6] claims. The authors aim to increase the capacity of drone batteries by utilizing lithium-metal battery production technology, replacing the conventional lithium-ion method.

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The aim of this work is to study a basic drone-user interaction model in terms of SNR, obtain results for mobile user case and compare them against the results of developed simulation model.

2 A basic drone-user interaction model

We consider a simple scenario of continuous data transmission between a mobile base station, located on UAV and mobile user. The transmitted data have requirements on SNR that should be complied. Furthermore, in the considered scenario UAV doesn't have any means to locate user and only have the data of possible user movement model.

Consider a three dimensional euclidean space, where a drone D and a user U are located. The drone is considered to be hovering at a certain altitude h . Let us fix the coordinate grid with drone position at $(0, 0, h)$, so that its position is fixed. Let the user U be initially located directly under the UAV at time $t = 0$. We consider two-dimensional mobility model for user, thus at an arbitrary time t the user is considered to have coordinates $(x(t), y(t), 0)$. Since a user is able to walk not more than a certain distance during a time interval δt , here we consider its coordinates to be random variables $x(t) = \xi(t), y(t) = \eta(t) \sim \text{uniform}(a(t), b(t))$. The considered system is depicted on figure 1. Further we would use denotations without explicit dependency on t while focusing on system behaviour at a single moment of time.

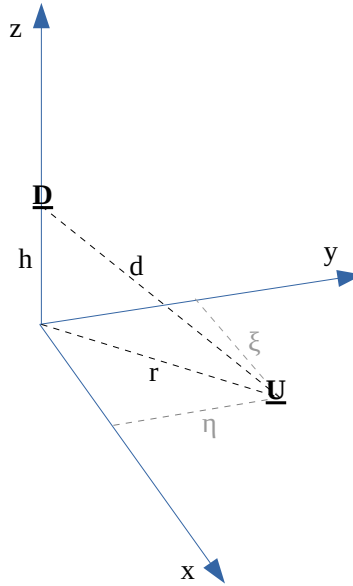


Fig. 1: Mathematical model of drone-user interaction

We consider scenario when drone and user will always have Line of Sight (LoS). This is due to the fact that since base station is located a certain distance from the ground there should not be many obstacles between it and the addressed user. Also, due to the absence of obstacles we can implement Free-Space Path Loss (FSPL) model given in (1) as the default path loss model for the considered model.

$$FSPL(d) = 20 \log_{10} (4\pi\nu d) \quad (1)$$

Here d is the distance between drone and user position at time t , ν is the data transmission frequency and π is a universal constant. To calculate the FSPL function we need to acquire distance $d(t)$. But first, let's have a look at its projection to the ground plane r . The $CDF(r)$ can be stated as:

$$CDF_r(w) = P\{\sqrt{\xi^2 + \eta^2} < w\} = \int_{-\infty}^{\infty} f_{\xi}(x) \int_{-\sqrt{w^2-x^2}}^{\sqrt{w^2-x^2}} f_{\eta}(y) dy dx, \quad (2)$$

where $f_{\xi}(x) = f_{\eta}(y) = \frac{1}{a+b}$ - PDFs for user coordinates.

The $CDF(d)$ can be stated as:

$$CDF_d(w) = P\{\sqrt{h^2 + x^2 + y^2} < w\} = \int_{-\infty}^{\infty} f_{\xi}(x) \int_{-\sqrt{w^2-x^2-h^2}}^{\sqrt{w^2-x^2-h^2}} f_{\eta}(y) dy dx, \quad (3)$$

where h - drone hovering altitude.

One can notice, that F_d can also be expressed through F_r as

$$CDF_d(w) = \begin{cases} CDF_r(\sqrt{w^2 - h^2}), & w > h \\ 0, & w < h \end{cases} \quad (4)$$

Now that we have d in explicit form and can calculate $FSPL(d)$, we can also write an equation, necessary to obtain $SNR(d)$:

$$SNR(d) = P_t - FSPL(d) - N \quad (5)$$

Here the power of drone's transmitting antenna is denoted as P_t in dB, N is noise, also in dB.

Now we can express $CDF_{SNR}(w)$ like this:

$$CDF_{SNR}(w) = 1 - CDF_d\left(\frac{1}{4\pi\nu d} 10^{\frac{P_t - N - w}{20}}\right), \quad (6)$$

It's also possible to consider this model in polar coordinate system. In such a case, an angle ϕ and a radius r will be defining the user's position, and they will also be normally distributed: $PDF_{\phi}(\psi) = 1/2\pi$, $PDF_r(w) = 1/R$, where R is the maximum value of r . For this case, distribution functions for distance and its projection would have following form:

$$CDF_r(X_r) = \int_{w < X_r} \int_0^{2\pi} PDF_r(w) PDF_{\phi}(\psi) dw d\psi = \frac{X_r}{R} \quad (7)$$

$$CDF_d(X_d) = CDF_r\left(\sqrt{X_d^2 - h^2}\right) \quad (8)$$

Now if we consider system dynamic in time, the PDFs of users coordinates can be modified to have an extra parameter t , modeling the time, as was mentioned in the beginning of this section. The idea is that the maximal distance that user can reach increases as time passes. So, for instance, if the considered area is bounded by a square, formed by four points $(-a, -b), (-a, b), (a, -b)$ and (a, b) , and if user's speed is given as parameter v units per unit of time, then at time $t = 0$ units, the user is considered to be right under the drone and nowhere else; at $t = 1$ units, the area where the user can possibly be, is bounded by interval (a', b') on both coordinates, where $a' = \frac{t}{t_{max}}a$, and $b' = \frac{t}{t_{max}}b$. In terms of common sense, if no time passed, user could not change its position, and if only a small fraction of time has passed, the user can not reach the furthest possible point as well. Hence, the corresponding PDFs can be redefined as:

$$PDF_\xi(x, t) = \begin{cases} \frac{t_{max}}{(a+b)t}, & x \in \left[\frac{t}{t_{max}}a, \frac{t}{t_{max}}b\right] \\ 0, & x \notin \left[\frac{t}{t_{max}}a, \frac{t}{t_{max}}b\right] \end{cases} \quad (9)$$

$$PDF_\eta(y, t) = \begin{cases} \frac{t_{max}}{(a+b)t}, & y \in \left[\frac{t}{t_{max}}a, \frac{t}{t_{max}}b\right] \\ 0, & y \notin \left[\frac{t}{t_{max}}a, \frac{t}{t_{max}}b\right] \end{cases} \quad (10)$$

where $t_{max} = \frac{y_{max}}{v}$ in case of 10, and $t_{max} = \frac{x_{max}}{v}$ in case of 9 – is the maximum possible time, in other words, the time, needed for the user to reach the furthest possible point of the considered area, v – user movement speed, x_{max} and y_{max} – maximum coordinates of the user, i.e. the border of the considered area.

In this case the equations for distance and projection CDFs should be modified as follows:

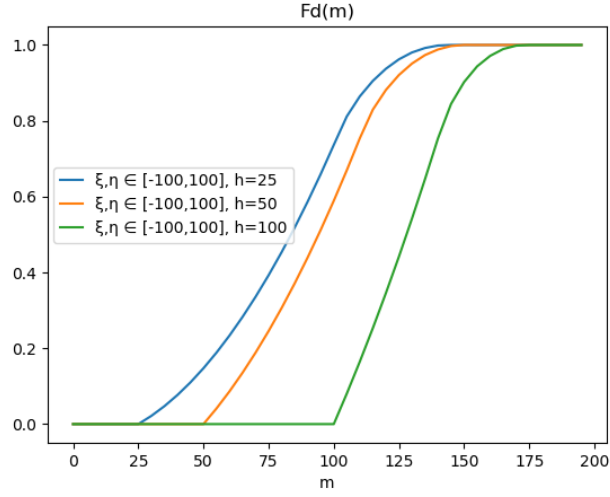
$$CDF_r(w, t) = \int_{-\infty}^{\infty} f_\xi(x, t) \int_{-\sqrt{w^2 - x^2}}^{\sqrt{w^2 - x^2}} f_\eta(y, t) dy dx \quad (11)$$

$$CDF_d(w, t) = \begin{cases} CDF_r(\sqrt{w^2 - h^2}, t), & w > h \\ 0, & w < h \end{cases} \quad (12)$$

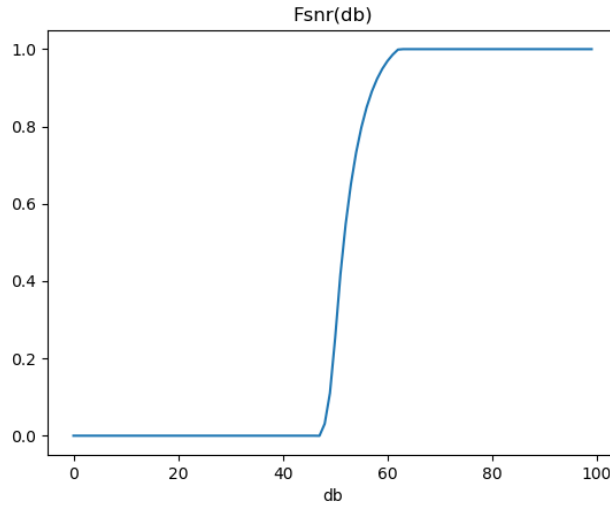
3 Simulation and Performance Analysis

In this section we present numerical results of both mathematical and simulation model. Initial data of the trials can be seen in table 1.

Figures 2a and 2b represent plots for CDF_d and CDF_{SNR} are represented for Cartesian coordinate system and figures 3a and 3b – for polar coordinate system. For both trials drone was positioned at the coordinate zero-point with altitude h . On figure 2a we have considered three cases, where drone is hovering

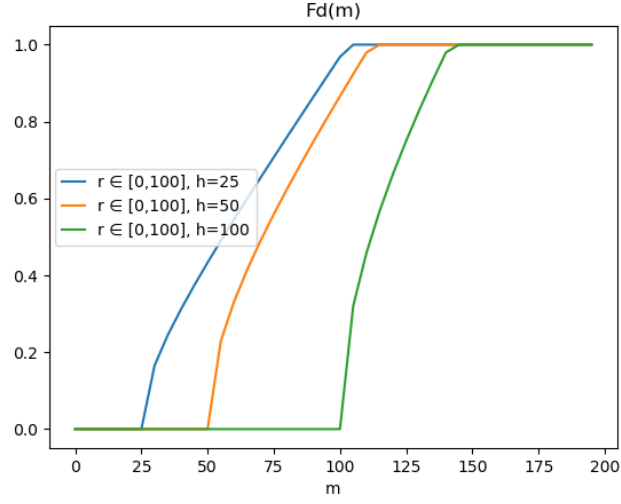


(a) Distance CDF for different drone altitudes

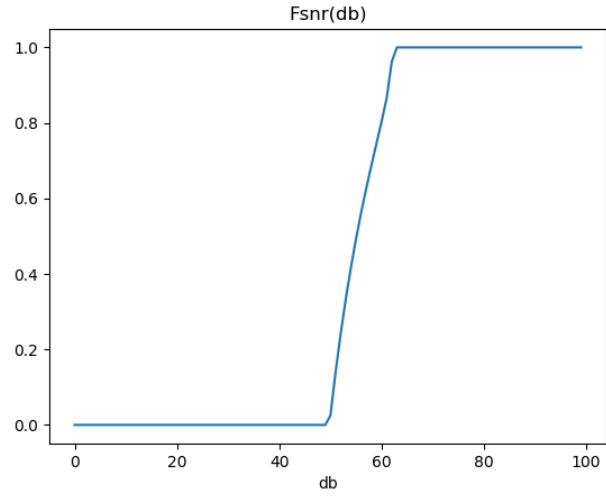


(b) SNR CDF for drone altitude of 25 meters

Fig. 2: Performance metrics for Cartesian system



(a) Distance CDF for different drone altitudes



(b) SNR CDF for drone altitude of 25 meters

Fig. 3: Performance metrics for polar system

Table 1: Initial Data

Parameter	Value	Description
a	-100 m	Minimal coordinate a user can have in Cartesian system
b	100 m	Maximal coordinate a user can have in Cartesian system
ν	750 MHz	Data transmission frequency
P_t	20 dBm	Transmission antenna power
N	-100 dBm	Noise power
h	25, 30, 100 m	Drone altitude
v	2 m/s	User movement speed
R	100 m	Area radius for polar system

on altitude 25,30 and 100 meters. It can be seen that the higher the drone hovers the smaller the considered area becomes. Figure 2b shows F_{snr} for the first of those cases, i.e where drone is hovering at 25 meters. The other two cases were omitted, since their behaviour reflects the one of given plot.

In case of radial coordinate system illustrated on 3a – distance CDF and 3b – SNR CDF, the user is moving inside a circle with radius R . Depending on drone's altitude, distance between the user and the drone is either bounded by interval [25, 130] if the drone is hovering on altitude of 25 m, [50,150] when altitude is 50 m, or [100,170] in case of 100m altitude. One can notice, that the plots on 3a shift to the right with increasing drone altitude. That can be explained by the fact, that the minimal possible distance is defined by its altitude h , since it is the case where user stands in the closest to drone point on the ground, i.e. right underneath of drone. According to figures 2b and 3b, that SNR for both models belongs to interval [85,110].

It should be noted that both systems exhibit very similar, although not identical behaviour. The similarity is due to the fact that the figures illustrate two approaches of obtaining SNR CDF for the same system. The difference between plots shows space curvature between Cartesian and polar systems.

In case of Cartesian model with parameter t , dropping probability, i.e. probability that current SNR does not satisfy the requirements, was also studied as a function of time and required threshold. The results are presented on figure 4. Here the Blocking probability was calculated as a CDF of SNR with respect to t and SNR^* , SNR^* is the threshold, defining acceptable quality of connection. By fixing SNR^* at a certain value, the plot on figure 4 would illustrate, how blocking probability $P\{SNR(t) < SNR^*\}$ changes over time, i.e how probable it is that at certain point of time the quality of connection will become unacceptable. The reason, why for SNR^* located in range of 0 to 80 dBm, the probability is zero, i.e the plot is flat, is because there is no way for the user to go far enough from the drone in the observed period of time, to make the quality of connection unacceptable. For the same reason probability for SNR to be acceptable at a late time tends to zero, since the area where user may be located expands with time.

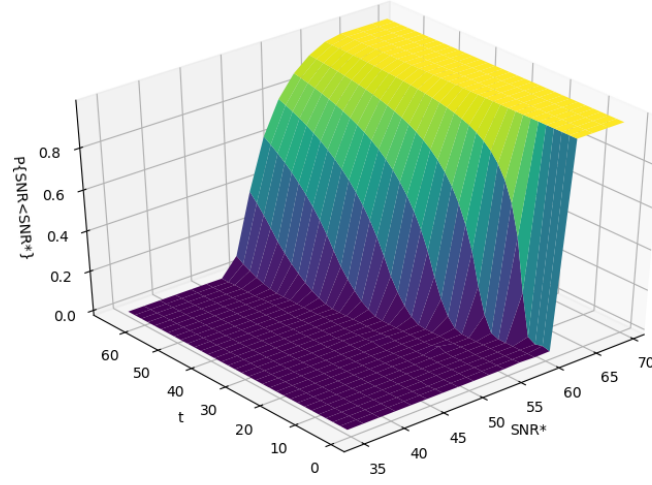


Fig. 4: Dropping probability with respect to time and threshold

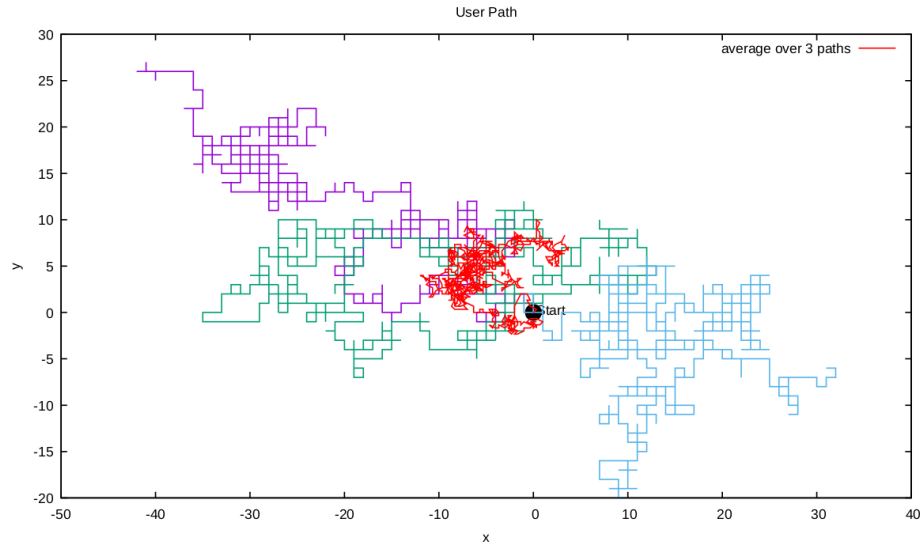


Fig. 5: Mobility model simulation

For the considered system a simulation model was also developed to acquire the same characteristics as the ones obtained in mathematical in order to proof the model reliability. Simulation model consisted of a single drone and user in a closed region. The drone was located directly in the center of the region, while user was situated directly under drone at the start of the simulation. Then simulator implemented random walk model as user mobility model. According to that model at each moment of time t user randomly chooses one of four directions and moves in a straight line. An illustration of simulation for case of three different trials is presented on figure 5. Differently coloured lines illustrate users paths during different trials, red line corresponds to the average position of the user among all the trials. It can be noted, that during each trial user only moves between the nodes of a discrete coordinate grid. This is due to the chosen mobility model. The average value does not belong the aforementioned grid due to the fact that each point of this plot was calculated as average of all the other paths in the trial.

For the described mobility model 500 trials were conducted to acquire the data for average SNR value, the results of these trials are shown on figure 6. The lightly colored part corresponds to the results of each trial, while brightly colored line depicts average SNR value. That way we can both observe average as well as approximate variation on this figure.

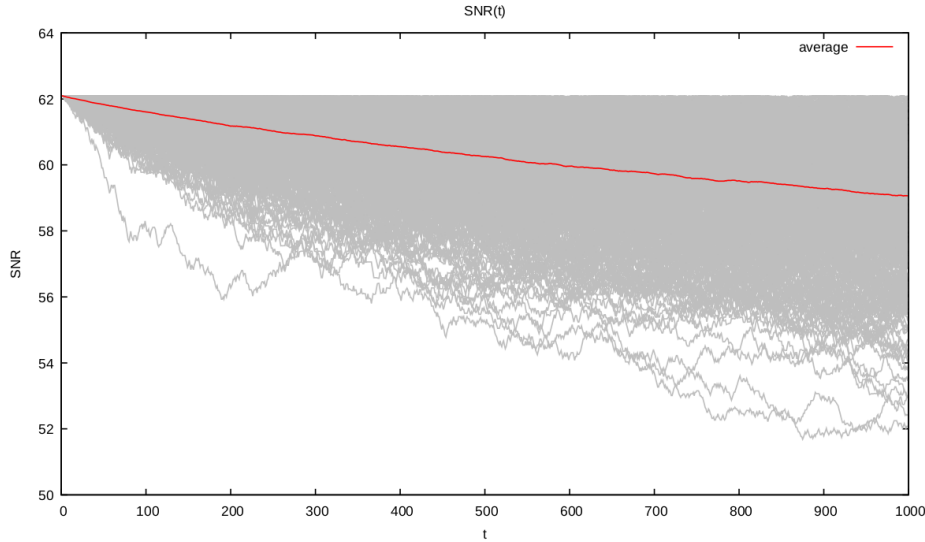


Fig. 6: SNR value for simulation model with 500 trials

4 Conclusion

In this paper we have touched upon a problem of user position in interactions between user and UAV. It was shown that even in case of omnidirectional antennas user mobility greatly impacts system performance. Using the constructed model it is possible to determine minimal and average time in which a user can be lost. Also using this model with knowledge of user mobility model and threshold value it is possible to find a point in time at which probability to drop user would go beyond threshold. That time can serve as an approximation of a maximum time available for the UAV to locate user. In further studies we consider using different motion models as well as having partial information on the user location, e.g. GPS coordinates, given with certain variance. It is also possible to consider case of directional antennas, however it would bring forward deafness problem, which is especially hard to solve in three dimensional dynamic case as was shown in [2]. Also having information of the user location would let the UAV to follow user more effectively.

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