MLF Cheat Sheet - QUIZ 1 (Week 1-4)

WEEK - 1

1. Loss function of REGRESSION MODEL $= \frac{1}{n}\sum_{i=1}^{n}(f(x_i)-y_i)^2$

→ also known as (i) Sum of squared errors
(ii) Average of squared residuals

2. Loss f^n of CLASSIFICATION MODEL $= \frac{1}{n}\sum_{i=1}^{n}(f(x_i) \neq y_i)$

→ also known as
fraction of misclassified instances

3. Loss f^n of DIMENSIONALITY RED^n MODEL $= \frac{1}{n}\sum_{i=1}^{n}(g[f(x_i)] - x_i)^2$

→ encoder - decoder functions

4. Loss f^n of DENSITY ESTIMATION MODEL $= \frac{1}{n}\sum_{i=1}^{n}[-\log(P(x_i))]$

→ probabilistic model
→ also known as negative log liklihood

WEEK - 2

1. Linear Approximation for Single Variable [$f(x)$ at a paint a]

$$L(x) = f(a) + f'(a)(x-a)$$

also known as eq^n of tangent line

2. Gradient → collection of partial derivatives $\boxed{\nabla f \perp \text{contour}}$

$$\nabla f = [\partial f/\partial x, \partial f/\partial y, \ldots\ldots]^T$$

The gradient of a function points in the direction of <u>STEEPEST ASCENT</u>.

3. Linear Approximation of f^n for Multi-Variables [$f(x,y)$ at point (a,b)]

$$L(x,y) = f(a,b) + \frac{\partial f}{\partial x}(x-a) + \frac{\partial f}{\partial y}(y-b)$$

4. Directional Derivative

$$D_{\vec{u}} f(x,y) = \nabla f \cdot \vec{u} = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] \cdot [u_1, u_2]$$

u is the unit vector of given vector

5. Direction of Steepest Ascent

$$u = \frac{\nabla f}{\| \nabla f \|}$$

6. Direction of Steepest Descent

$$u = -\left(\frac{\nabla f}{\| \nabla f \|} \right)$$

WEEK-3

1. Column space

$C(A) = \{ A\vec{x} \mid \vec{x} \in \mathbb{R}^n \}$

Solving $Ax = b$, $b \in C(A)$ for a solution.

2. Null space

$N(A) = \{ \vec{x} \mid A\vec{x} = 0 \text{ and } \vec{x} \in \mathbb{R}^n \}$

3. Row Space

$R(A) = C(A^T)$

4. Left Null Space

$N(A^T)$

5. Dimensions of Fundamental Subspaces

- Dim. of $C(A)$ = Rank of A = Dim. of $R(A)$ = r
- If A is $m \times n$ matrix, then,

Dim. of $N(A)$ = nullity of A = ~~R~~ No. of Cols − Rank = $n - r$

Dim. of $N(A^T)$ = No. of Rows − Rank = $m - r$

6. Rank-Nullity Theorem

r + nullity ⇒ Rank of A + Nullity of A = No. of Columns of A

7. Orthogonality

$N(A) \perp R(A)$

$N(A^T) \perp C(A)$

8. Length of a vector

$\|x\|^2 = \vec{x} \cdot \vec{x} = x^T x$

9. Orthogonal Vectors

- $\vec{a}$ & $\vec{b}$ are orthogonal if $\vec{a} \cdot \vec{b} = 0$
- If $\{v_1, v_2, \ldots v_n\}$ is a set of mutually orthogonal set of vectors, it means the set is LINEARLY INDEPENDENT.

10. Orthonormal Vectors

$\vec{a} \cdot \vec{b} = 0$ and $\|a\| = \|b\| = 1$

11. Orthogonal Sub-spaces

Two subspaces S_1 and S_2 are orthogonal if,

$x^T y = 0$, $\forall x \in S_1$, $\forall y \in S_2$

12. Projection of $\vec{a}$ along/on $\vec{b}$

$$\text{Proj}_{\vec{b}}(\vec{a}) = \left(\frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}\right)\vec{b}$$

13. Projection matrix (vector on which we project)

→ $$\mathbb{P} = \frac{a a^T}{a^T a}$$

→ Projection of $\vec{b}$ on $\vec{a}$ = $\mathbb{P}\vec{b}$

→ $\mathbb{P}$ is symmetric

→ $\mathbb{P}^2 = \mathbb{P}$

→ Column space of P is line through $\vec{a}$

→ Null space of P is plane orthogonal to $\vec{a}$

14. Projection of a vector $\vec{x}$ on a subspace V

$$\text{Proj}_V(\vec{x}) = A(A^TA)^{-1}A^T\vec{x}$$

where A is the basis of subspace V

15. Least Squares Approximation

$$A^TA\hat{x} = A^Tb$$

solving the above equation we get $\hat{x} = \begin{bmatrix} \text{slope} \\ \text{intercept} \end{bmatrix}$ → for best fit line

- If $Ax = b$ has solⁿ, then $\|Ax - b\|^2 = 0$

WEEK-4

1. Polynomial Regression

$$A^TA\hat{\theta} = A^Tb$$

2. Eigenvalues and Eigenvectors

$A\vec{x} = \lambda\vec{x}$ (to find corresponding eigenvector)

$\det(A - \lambda I) = 0$ (to find the eigenvalues)

Trace of matrix = sum of eigenvalues

Determinant of matrix = product of eigenvalues

3. Diagonalization of Matrix

$S^{-1}AS = \lambda$ → diagonal matrix of eigenvalues of A

S → matrix of eigenvectors of A

4. Orthogonally diagonalizable Matrices

→ If P is orthogonal matrix ⇒ $\boxed{P^TP = I}$ ⇒ $\boxed{P^{-1} = P^T}$
