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Logarithm

Domain : $(0, \infty)$
Range : $(-\infty, \infty)$

\# Some Real World Examples where we use 'Log'
- → Richter scale to measure earthquakes
- → pH scale to measure basicity & acidity.
- → Decibel scale to measure sound level

\# BASICS OF LOGARITHM

$$\text{Base}^{\text{Power}} = \text{Number} \Leftrightarrow \log_{\text{BASE}} \text{Number} = \text{Power}$$

$$\therefore \quad a^x = N \Leftrightarrow \boxed{\log_a N = x}$$

Hence, logarithm of a number 'N' to some base 'a' is the power 'x' by which the base must be raised in order to get that number.

note : (i) $N > 0$ (ii) $a > 0$, but $a \neq 1$

Examples :
- (i) $\log_2 2 = 1$
- (ii) $\log_2 16 = 4$
- (iii) $\log_2 1 = 0$
- (iv) $\log_2 1/2 = -1$
- (v) $\log_2 1/8 = -3$
- (vi) $\log_2 \sqrt{2} = 1/2$

Question : Find the value of x, if (i) $\log_2 8 = x$ → 3

Solution : $2^x = 8$

$2^x = 2^3$

$\Rightarrow \boxed{x = 3}$

(ii) $\log_{16} x = 1/2$

$\Rightarrow 16^{1/2} = x$

$\Rightarrow \boxed{4 = x}$

(ii) $\log_2 x = 0$

$\Rightarrow 2^0 = x$

$\Rightarrow \boxed{x = 1}$

(iv) $\log_x 27 = 3$

$\Rightarrow x^3 = 27$

$\Rightarrow x^3 = 3^3$

$\Rightarrow \boxed{x = 3}$

(v) $\log_x 1/8 = -3$

$\Rightarrow x^{-3} = 1/8$

$\Rightarrow x^{-3} = 2^{-3}$

$\Rightarrow \boxed{x = 2}$

(vi) $3 + \log_{\sqrt{5}} x = 7$

$\Rightarrow \log_{\sqrt{5}} x = 4$

$\Rightarrow (\sqrt{5})^4 = x$

$\Rightarrow \boxed{x = 25}$

(vii) $\log_3 (\log_2 x) = 1$

$\rightarrow \log_2 x = 3$

$\Rightarrow x = 2^3$

$\Rightarrow \boxed{x = 8}$

(viii) $\log_4 \log_3 \log_2 x = 0$

$\Rightarrow \log_3 \log_2 x = 4^0$

$\Rightarrow \log_2 x = 3^1$

$\Rightarrow x = 2^3$

$\Rightarrow \boxed{x = 8}$

LIMITATIONS OF LOGARITHM

$\log_a N$ is defined only when (i) $N > 0$

(ii) $a > 0$ but $a \neq 1$.

NOTE : * For a given value of N, $\log_a N$ will give us a unique value.

* log of zero does not exist

* Log of -ve real no.s are not defined.

Question : If $\log_2 (x-1)$ is meaningful, then find the value of x.

$x - 1 > 0 \Rightarrow \boxed{x > 1}$

PROPERTIES OF LOGARITHM

(1) $\boxed{\log_N N = 1}$, where $N > 0$, $N \neq 1$
(Logarith of a no. to the same base is 1)

(2) $\boxed{\log_a 1 = 0}$, where $a > 0$, $a \neq 1$
(Logarithm of unity to any base is zero)

(3) $\boxed{\log_{\frac{1}{N}} N = -1}$, where $N > 0$, $N \neq 1$
(log of a no. to its reciprocal as base is -1)

(4) $\boxed{\log_a m.n = \log_a m + \log_a n}$, where $m, n, a > 0$ and $a \neq 1$.

(5) $\boxed{\log_a \frac{m}{n} = \log_a m - \log_a n}$, where $m, n, a > 0$ & $a \neq 1$

(6) $\boxed{\log_a m^p = p\,(\log_a m)}$

(7) $\boxed{\log_{a^q} m = \frac{1}{q}\left(\log_a m\right)}$

(8) Base Changing Theorem $\boxed{\log_a m = \frac{\log_b m}{\log_b a}}$
where $m, a, b > 0$ & $a, b \neq 1$

NOTE : $\boxed{\log_b a = \frac{1}{\log_a b}}$

(9) <u>Chain Rule</u> : $\boxed{\log_b a \cdot \log_c b \cdot \log_d c = \log_d a}$

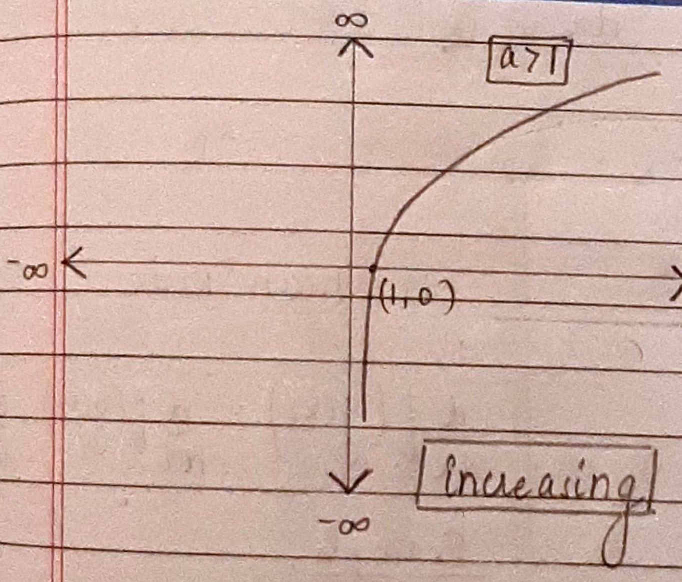
(10) $\boxed{a^{\log_a N} = N}$ and $\boxed{\log_a a^N = N}$

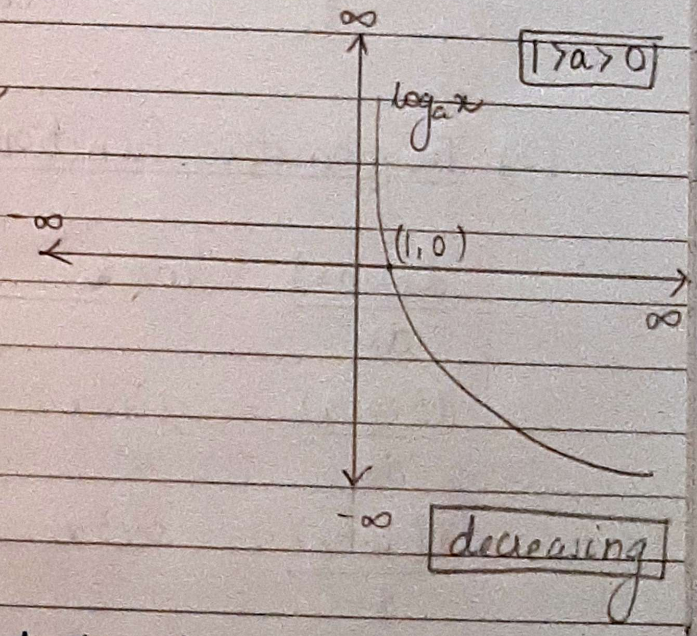
(11) $\boxed{a^{\log_b c} = c^{\log_b a}}$

LOGARITHMIC EQUATIONS

<u>Question</u> : Solve for $x \rightarrow (x+1)^{\log_{10}(x+1)} = 100(x+1)$

GRAPH OF LOG FUNCTIONS





- Whenever the number (x) and the base are on the same side of the unity then logarithm of that no. to that base is positive (+).

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Antilogarithm

The positive real no. 'n' is called the antilogarithm of a number 'm' if $\log n = m$.
Thus, $\log n = m \Leftrightarrow \text{antilog } m = n$

$\boxed{\log_a(n) = m} \Rightarrow \boxed{\text{antilog}_a(m) = n}$

__Question__: Find the antilog of foll. : $\boxed{\text{antilog}_p q = p^q}$

1. $\text{antilog}_8\left(\frac{2}{3}\right) = 4$ 2. $\text{antilog}_{\frac{1}{100}}\left(\frac{-1}{2}\right) = 10$

__Question__: Find the antilog of $\frac{2}{3}$, when base of log is 1331.

→ $\text{antilog}_{1331}\left(\frac{2}{3}\right) = 121$

LOG : Characteristic & Mantissa

__Common Bases of Log__

The base of the log can be any positive number other than 1. But in normal practice, two bases are used often :

BASE 10	BASE e
$\log_{10} x$	$\log_e x$ / $\ln x$
'Common log or Briggs log'	'Natural log or Napierian log'
	[$e = 2.718$ approx.]

Remember :

$\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, $\log_{10} 10 = 1$

Example : Find $\log_{10} 5$

$\Rightarrow \log_{10}\left(\frac{10}{2}\right) = \log_{10} 10 - \log_{10} 2 = 1 - 0.3010$

$\Rightarrow \log_{10} 5 = 0.6990$

CONVERSION OF BASE 'e' to BASE 10 & Vice versa

Remember : $\ln 2 = 0.693$ $\ln 10 = 2.303$

$$\log_e a = \frac{\log_{10} a}{\log_{10} e} = \log_{10} a \cdot \log_e 10 = 2.303 \log_{10} a$$

$$\therefore \boxed{\log_e a = 2.303 \log_{10} a}$$

Characteristic & Mantissa

For any given positive number N,

$\log_a N$ = Integral part (Characteristic) + Fractional Part (+ve) (Mantissa)

{ $0 \le$ Mantissa < 1 }

Observations :

1. characteristic of $\log_{10}$ of single digit is always 0, double digit is 1, three digit is 2 and so on.

NOTE : If the characteristic of $\log_{10} N$ be n, then the no. of digits before decimal in N is $= n+1$.

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	Characteristic	Mantissa
$\log_{10} 0.1 = -1$	-1	0
$\log_{10} 0.01 = -2$	-2	0
$\log_{10} 0.001 = -3$	-3	0

Note : If the characteristic of $\log_{10} N$ be $(-n)$, then no. of zeroes after decimal & before first significant digit is $N = |-n+1|$