Date
June 7, 2021

WEEK 5

Permutations & Combinations

BASIC PRINCIPLES OF COUNTING

Learning Objectives

- Understanding Basic principles of counting
- Concepts of factorials

(1) Addition Rule of Counting (OR)

- If an action A can occur in n_1 different ways, another action B can occur in n_2 different ways, then the total number of ~~occurs~~ occurience of the actions A or B is $n_1 + n_2$.

(2) Multiplication Rule of Counting (And)

- If an action A can occur is n_1 different ways, another action B can occur in n_2 different ways, then the total number of occurence of the actions A and B together is $n_1 \times n_2$
- Suppose that k actions are to be performed in a

definite order. Further suppose that there are n_1 possibilities for the first action and that corresponding to each of these possibilities are n_2 possibilities for the second action, and so on. Then, there are $n_1 \times n_2 \times \dots \times n_k$ possibilities altogether for the k actions.

EXAMPLE : Application → Creating alpha-numeric code

- Suppose you are asked to create a six digit alpha numeric password with the following requirement:

- The password should have first two letters followed by four numbers.

- Repetition allowed

  ∴ No. of ways = $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6760000$

- Repetition not allowed

  ∴ No. of ways = $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3276000$

FACTORIAL

→ The product of the first n positive integers (counting numbers) is called 'n' factorial and is denoted $n!$.

→ In symbols, $n! = n \times (n-1) \times \dots \times 1$

NOTE: By convention, $0! = 1$

→ In general,

$$n! = n \times (n-1)!$$

→ In general, for $i \leq n$ we have,

$$n! = n \times (n-1) \times \dots \times (n-i+1) \times (n-i)!$$

EXAMPLE : SIMPLIFYING EXPRESSIONS

1. $\frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = 6 \times 5 \times 4 = 120$

2. $\frac{6! \times 5!}{3! \times 4!} = \frac{6 \times 5 \times 4 \times \cancel{3!} \times 5 \times \cancel{4!}}{\cancel{3!} \times \cancel{4!}} = 6 \times 5 \times 4 \times 5 = 600$

3. Express $25 \times 24 \times 23$ in terms of factorial -

$$\frac{25 \times 24 \times 23 \times \dots \times 1}{22 \times 21 \times \dots \times 1} = \frac{25!}{22!}$$

Date
June 8, 2021

Tutorials

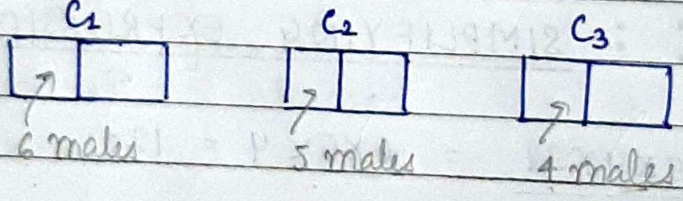
1. In a building 6 men and 4 women are staying. You know there are exactly 3 married couples. In how many ways you can guess who the couples are?

Sol. 1. No. of men = 6
No. of women = 4
No. of married couples = 3

~~No of ways = 6 × 4 = 24 ways~~

m_1 m_2 m_3 m_4 m_5 m_6

${}^4C_3$ → w_1 w_2 w_3 w_4 — only 3 are married!



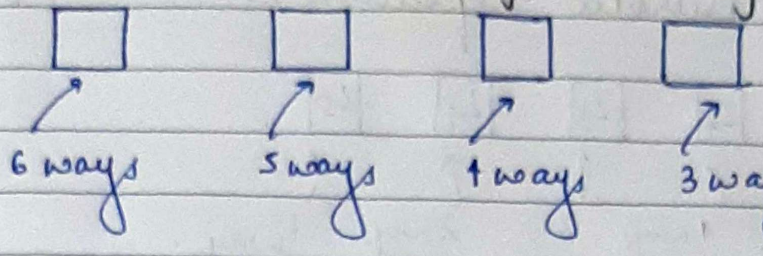
Combination of 3 women out of 4 = ${}^4C_3 = 4$

∴ No. of ways couples can be guessed = $(6 \times 5 \times 4) \times 4$

= 480 ways.

2. Four dice are rolled. A no. is observed on each face of the die. How many ways are there such that each die shows diff no. of on the face?

Sol. 2



∴ No. of ways such that each die show diff no. on the face = $6 \times 5 \times 4 \times 3 = 360$ Ans.

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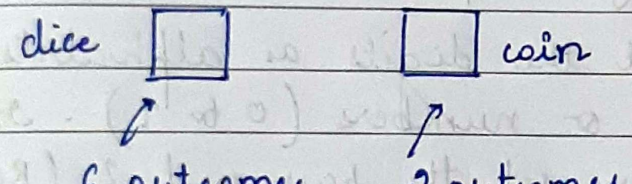
3. A fair die is rolled and a fair coin is tossed. How many different ways of pairs of outcome of a die and a coin are there?

Sol. 3.



∴ No. of ways of pairs of outcome of a die and a coin = $6 \times 2 = 12$ pairs

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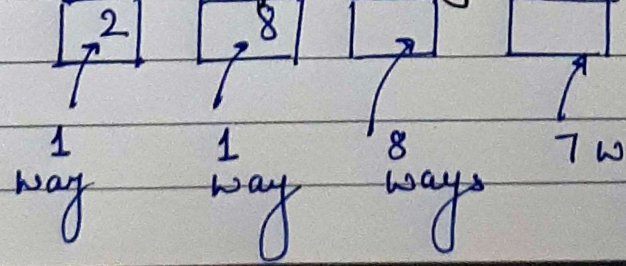
4. How many 4 digit no.s can be formed using the digits 0 to 9 if the first & second numbers are 2 and 8 respectively. (repetition not allowed)?

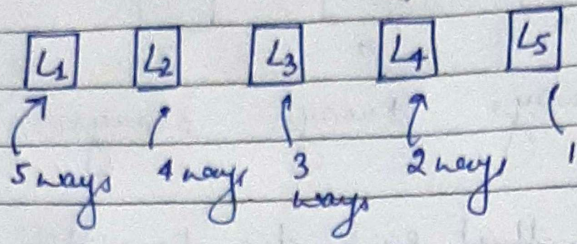
Sol. 4.



∴ No. of 4 digit nos $= 1 \times 1 \times 8 \times 7$
$= 56$ numbers

5. Lakhya has five different letter blocks. How many different words of more than 4 letters (can be meaningless) can be formed by using these five letters without repetition?

Sol. 5.



∴ No. of letters that can be formed = 5! = 120

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6. Nisha wants to set a 4 digit password for her laptop. But there are some conditions she must follow while setting the password.

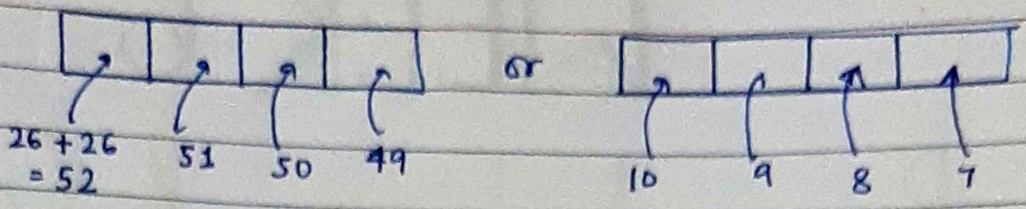
(a) She can choose all the digits as alphabets (both small & capital) or numbers (0 to 9). In how many ways she can set the password? (Repetition is not allowed).

(b) If she wants to use an alphanumeric password then she cannot use the small alphabets and she can use only first digit as an alphabet. In how many ways she can set the password? ~~(Repetition is not allowed)~~

(i) Repetition not allowed

(ii) Repetition allowed.

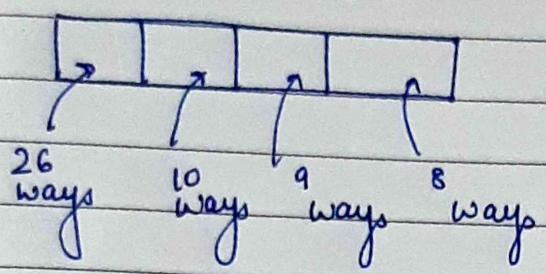
Sol. 6. (a)



if alphabets if numbers

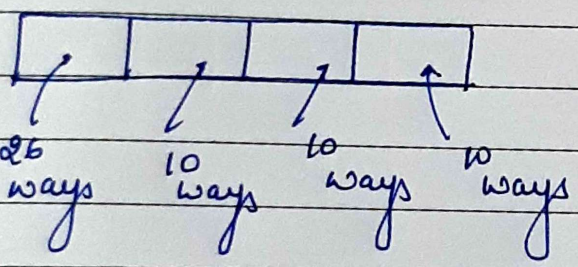
∴ No. of ways = $(52 \times 51 \times 50 \times 49) + (10 \times 9 \times 8 \times 7)$

(b) (i)



∴ No. of ways (w/o repetition) = $26 \times 10 \times 9 \times 8$ ways

(ii)



∴ No. of ways (with repetition) = $26 \times 10 \times 10 \times 10$ ways.