

# BUSINESS ANALYTICS

(SEPTEMBER 2022 TERM)

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Sat  
September 19, 2022

MONDAY

Shivani  
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# WEEK 3

## Determining & Inferring Association

### JOINT PROBABILITY

Joint probability is the probability of two events happening together. The two events are usually designated event A and event B. In probability terms, it is :

$$p(A \text{ and } B) \text{ or } p(A \cap B)$$

Joint probability can also be described as the probability of the INTERSECTION of two (or more) events

$$P(A \cap B) = P(A|B) * P(B)$$

### MARGINAL PROBABILITY

Marginal probability is the probability of an event irrespective of the outcome of another variable.

# CONDITIONAL PROBABILITY

Conditional probability is known as the possibility of an event or outcome happening, based on the existence of a previous event or outcome.

The probability of occurrence of any event A when another event B has occurred in relation to A has already occurred is known as conditional probability. It is depicted by  $P(A|B)$ .

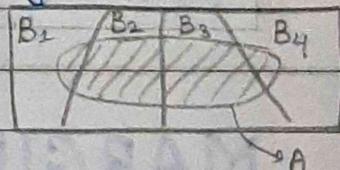
$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{or} \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

probability of A given B

## BAYES' RULE

Bayes rule is used to calculate the posterior probability if we have the initial probability & additional sample information!!

When a sample space is a disjoint union of events & Event A overlaps this disjoint union, then the probability that one of the disjoint partitions events is true, given A is true, is



$$P(B_j|A) = \frac{P(A|B_j) \cdot P(B_j)}{P(A)} \rightarrow P(A \cap B_j)$$

$$\sum_{i=1}^k P(A|B_i) \cdot P(B_i)$$

# CHI-SQUARE DISTRIBUTION

Chi Square Test statistic :

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

- When the  $H_0$  is true, expected and observed frequencies tend to be close for each cell & the test statistic value is relatively small.
- If  $H_0$  is false, at least some cells have a big gap between expected & observed frequencies, leading to a large test statistic value.
- The larger the  $\chi^2$  value, greater is the evidence against the null hypothesis of independence.
- Degrees of freedom for the chi squared distribution is given by the expression :  $df = (r-1) * (c-1)$   
r → no. of rows ; c → no. of columns
- If  $\chi^2 >$  critical value, then reject  $H_0$ .  
(calculated value)                          (tabulated value)
- If p-value < level of significance, then reject  $H_0$ .

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# Week 4

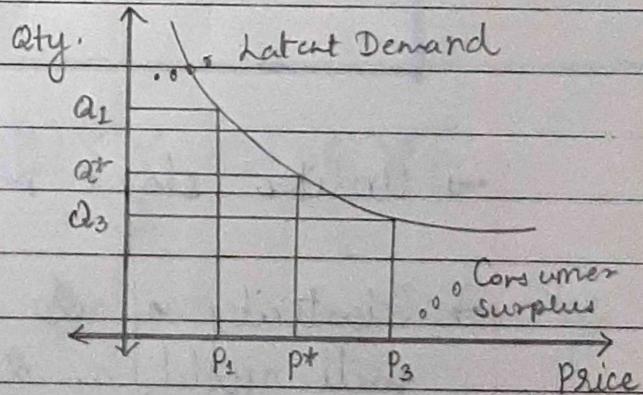
## Demand Response Curve

RELATIONSHIP BETWEEN PRICE & DEMAND

### DEMAND RESPONSE CURVE

- A function that describes how demand for a product  $D(p)$  varies as a function of its price.
- Similar to the Demand curve in Economics, but for a single seller, in a single market
- PROPERTIES :

1. Non-negative
2. Continuous
3. Differentiable
4. Downward Sloping



### PRICE SENSITIVITY

- Slope : Measures how demand changes in response to a price change

$$\partial(p_1, p_2) = \frac{D(p_2) - D(p_1)}{p_2 - p_1}$$

- $p_1 > p_2$  implies that  $D(p_1) < D(p_2)$ , hence slope is always negative.
- Slope can be used as a local estimator of demand change for a small change in price.

• Elasticity : Ratio of the percentage change in demand to the % age change in price.

$$E(p_1, p_2) = \frac{[d(p_2) - d(p_1)] / d(p_1)}{(p_2 - p_1) / p_1}$$

- Unlike slope, elasticity is independent of units.
- Elasticity of 2 means that a 10% red<sup>n</sup> in price will yield a 20% increase in sales.
- Elasticity also depends on TIME. There can be different elasticities of same product for short-term & for long-term.

# LINEAR RESPONSE CURVE

- Simplest Price Response Curve :

$$D(p) = D_0 - m * p$$

where  $D_0$  is the demand at price = 0 (this is called the market size) &  $m$  is the slope.

→ the price at which demand = 0 is called the satiating price,  $p_s = \frac{D_0}{m}$

→ the elasticity of this curve is  $\epsilon = \frac{m * p}{D_0 - m * p}$

We see that  $\epsilon = 0$  when  $p = 0$ .

As  $p \rightarrow p_s$ ,  $\epsilon \rightarrow \infty$ .

# CONSTANT ELASTICITY CURVE

- After algebraic transition, the constant elasticity curve is given by :

$$D = C p^{-\epsilon}$$

where  $C$  is a constant (it is the Demand when price=1)

→ it is not guaranteed that the demand is either finite or satiated ( $D \rightarrow \infty$ , as  $p \rightarrow 0$ ). Also,  $D \neq 0$ , for any  $p$ .

$$\rightarrow \text{Revenue is } R = p * D = C p^{(1-\epsilon)}$$

- We notice that :

- When  $\epsilon < 1$ , (inelastic product demand), the revenue can be increased by simply increasing prices.
  - When  $\epsilon > 1$ , (elastic demand), the revenue can only be increased by setting price close to zero.
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## ESTIMATION PROBLEM

- Assume that we conduct a market experiment where we offer different prices and check the realized demand at that time.
  - So we have the price offered & the corresponding demand values.
  - Price can be considered as an explanatory variable & demand can be considered to be the dependent variable.
  - Can we estimate the slope & the elasticity.
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Wednesday.

# Simple Linear Regression

## LINEAR DEMAND RESPONSE CURVE

- A simple linear Regression (SLR) model can help identify  $\beta_0$  (y-intercept) &  $\beta_1$  (the slope).
- The SLR can tell us if the linear relationship is a good fit for the data available from the market experiment.

## SIMPLE REGRESSION MODEL

### LINEAR ON AVERAGE

- The eq<sup>n</sup> of the SRM describes how the conditional mean of  $Y$  depends on  $X$ .
- The SRM shows that these means lie on a line with the intercept  $\beta_0$  & the slope  $\beta_1$ .

$$My|x = E(Y|X=x) = \beta_0 + \beta_1 x$$

## DEVIATIONS FROM THE MEAN

- The deviations of responses around  $\mu_{Y|x}$  are called errors.
- ERRORS : The vertical gap between the actual value of demand realized at that particular price point & the predicted value of  $y_i$  coming from the SLR is called Error.
- Error, is denoted by  $\epsilon$ , &  $E(\epsilon) = 0$
- In general we minimize the sum of squared errors. That is the objective with which we run the regression model.
- The SRM makes three assumptions about the error term :
  1. Independent
    - Errors are independent of each other.
  2. Equal Variance
    - All errors have the same variance ,  $\text{Var}(\epsilon) = \sigma^2_\epsilon$ .
  3. Normal
    - The errors are normally distributed.

## Coefficient of Determination ( $R^2$ )

• price explains  $R^2 \times 100\%$  of variation/variability in  $y^2$

- Observed values of the response  $Y$  are linearly related to the values of the explanatory variable  $X$  by the equation

$$y = \beta_0 + \beta_1 x + \epsilon$$

↑ y-intercept      ↓ slope  
Market Size

where  $\epsilon \sim N(0, \sigma^2_\epsilon)$   
↑ mean      ↓ variance  
normal distribution

- The observations are independent of one another, have equal variance around the regression line, and are normally distributed around the regression line.