1.概念：

$$Y_t = \beta_1 + \beta_2 X_{2t} + \cdots + \beta_k X_{kt} + \mu_t$$

其他基本假设仍成立，随机扰动项存在序列相关：

$$\boxed{Cov(\mu_i, \mu_j) = E(\mu_i \mu_j) \neq 0}$$

即 $Cov(u_i, u_j) = E(u_i u_j) \neq 0$

$$E(\mu_t \mu_{t-1}) \neq 0$$

$$\mu_t = \rho \mu_{t-1} + \varepsilon_t$$

$-1 \leq \rho \leq 1$

ρ称为**一阶自相关系数**（first-order coefficient of autocorrelation）

$$\rho = \frac{\sum_{t=2}^{n} u_t u_{t-1}}{\sqrt{\sum_{t=2}^{n} u_t^2}\sqrt{\sum_{t=2}^{n} u_{t-1}^2}}$$

二、自相关产生的原因

$\mu_t = \rho \mu_{t-1} + \varepsilon_t$

1) 经济系统的惯性

2) 经济活动的滞后效应

3) 数据处理造成的相关

4) 蛛网现象

5) 模型设定偏误

一阶自回归 $\mu_t = \rho \mu_{t-1} + \varepsilon_t$

二阶 $\mu_t = \rho_1 \mu_{t-1} + \rho_2 \mu_{t-2} + \varepsilon_t$

DW 检验

OLS 对原模型进行估计

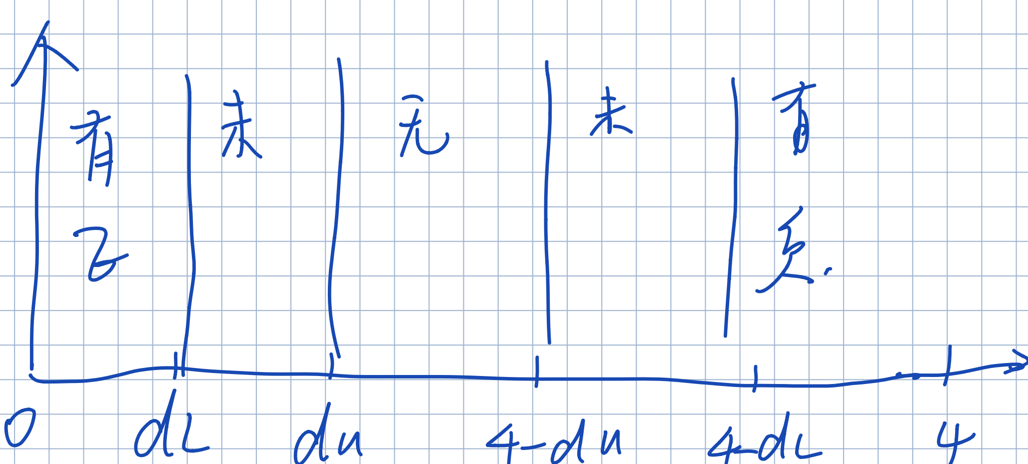
$$D.W. = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=2}^{n} e_t^2}$$

得到 D.W 的结果 $0 \leq 0 \leq 4$.

根据 n 和 k'

n：样本容量

k'：解释变量的数目，n 个 X.



$$D.W. \approx 2\left(1-\frac{\sum_{t=2}^{n}\tilde{e}_t\tilde{e}_{t-1}}{\sum_{t=1}^{n}\tilde{e}_t^2}\right) \approx 2(1-\rho)$$

$D.W \approx 2(1-\rho)$

LM检验. GB检验

- 由布劳殊（Breusch）与戈弗雷（Godfrey）于1978年提出的，也被称为**GB检验**。
- 适合于高阶序列相关以及模型中存在滞后被解释变量的情形。
- **对原模型进行OLS估计，用残差近似值的辅助回归模型的可决系数构造统计量。**

先OLS，得到残差 e_t.

$$e_t = a_1 + a_2 X_{2t} + a_3 X_{3t} + a_4 X_{4t} + \hat{\rho}_1 e_{t-1} + \hat{\rho}_2 e_{t-2} + \varepsilon_t$$

然后计算可决系数 R^2

$LM = TR^2 = nR^2 \sim \chi^2_p$ → 回归阶数
后

$TR^2 > \chi^2_\alpha(2)$，拒绝，就存在自相关

补救

广义差分法

$Y_t = \beta_1 + \beta_2 X_t + \mu_t$

$\mu_t = \rho u_{t-1} + v_t$

$v_t = \mu_t - \rho u_{t-1}$

$$= (Y_t - \beta_1 - \beta_2 X_t) - \rho(Y_{t-1} - \beta_1 - \beta_2 X_{t-1})$$

$$= \underline{\underline{(Y_t - \rho Y_{t-1})}} - \beta_1(1-\rho) - \beta_2(1-\rho X_{t-1})$$

$$\hookrightarrow Y_t - \rho Y_{t-1} = \beta_1(1-\rho) + \beta_2(1-\rho X_{t-1}) + v_t$$

$$Y_t^* = \beta_1^* + \beta_2^* X_t^* + v_t$$

案例分析

Dependent Variable: Y
Method: Least Squares
Date: 12/21/20 Time: 12:45
Sample: 1990 2015
Included observations: 26

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-11.11948	23.46353	-0.473905	0.6399
X	0.783541	0.011695	66.99933	0.0000

R-squared	0.994682	Mean dependent var	1346.764
Adjusted R-squared	0.994460	S.D. dependent var	809.9703
S.E. of regression	60.28520	Akaike info criterion	11.10985
Sum squared resid	87223.33	Schwarz criterion	11.20663
Log likelihood	-142.4281	Hannan-Quinn criter.	11.13772
F-statistic	4488.911	Durbin-Watson stat	0.508796
Prob(F-statistic)	0.000000		

$\hat{\rho} \approx 1-\frac{DW}{2} = 0.7456.$ $DW = 2(1-\hat{\rho})$

Y

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	13.19581	Prob. F(2,22)	0.0002
Obs*R-squared TR^2	14.17977	Prob. Chi-Square(2)	0.0008

$0.0008 < 0.05$

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 12/21/20 Time: 12:56
Sample: 1990 2015
Included observations: 26
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-11.23005	17.69095	-0.634791	0.5321
X	0.008079	0.009339	0.864992	0.3964
RESID(-1)	0.912378	0.213026	4.282949	0.0003
RESID(-2)	-0.162607	0.242997	-0.669172	0.5103

R-squared	0.545376	Mean dependent var	-3.75E-13
Adjusted R-squared	0.483382	S.D. dependent var	59.06719
S.E. of regression	42.45524	Akaike info criterion	10.47542
Sum squared resid	39653.84	Schwarz criterion	10.66897
Log likelihood	-132.1804	Hannan-Quinn criter.	10.53115
F-statistic	8.797205	Durbin-Watson stat	1.988405
Prob(F-statistic)	0.000506		