最小二乘正规方程组

$$\begin{cases} \sum Y_i = n\hat{\beta}_1 + \sum X_i \beta_2 \\ \sum X_i Y_i = X_i \hat{\beta}_1 + \sum X_i^2 \beta_2 . \end{cases}$$

$$\Rightarrow \quad \beta_2 = \frac{(X-\bar{X})(Y-\bar{Y})}{(X-\bar{X})^2} = \frac{x_i y_i}{x_i^2}$$

$$\beta_1 = \bar{Y} - \beta_2 \bar{X}$$

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + e_i = \hat{Y}_i + e_i$$

$$(Y_i - \bar{Y}) = (\hat{Y}_i - \bar{Y}) + e_i$$

$$\sum (Y_i - \bar{Y})^2 = \sum (\hat{Y}_i - \bar{Y})^2 + \sum (Y_i - \hat{Y}_i)^2$$

$$\underbrace{\sum y_i^2}_{TSS} = \underbrace{\sum \hat{y}_i^2}_{ESS} + \underbrace{\sum e_i^2}_{RSS.}$$

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

$$\hat{\beta}_1 \sim N\left(\beta_1, \sigma^2 \frac{\sum X_i^2}{n \sum x_i^2}\right)$$

$$\hat{\beta}_2 \sim N\left(\beta_2, \frac{\sigma^2}{\sum x_i^2}\right)$$

$$t = \frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)} = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\hat{\sigma}^2 \frac{\sum X_i^2}{n \sum x_i^2}}} \sim t(n-2)$$

$$t = \frac{\hat{\beta}_2 - \beta_2}{\widehat{SE}(\hat{\beta}_2)} = \frac{\hat{\beta}_2 - \beta_2}{\sqrt{\frac{\hat{\sigma}^2}{\sum x_i^2}}} \sim t(n-2)$$

$$t = \frac{\hat{\beta}_2 - \beta_2}{\widehat{SE}(\hat{\beta}_2)} = \frac{\hat{\beta}_2}{\sqrt{\frac{\hat{\sigma}^2}{\sum x_i^2}}} \sim t(n-k)$$

$$t = \frac{\hat{\beta}_2 - \beta_2}{\widehat{SE}(\hat{\beta}_2)} = \frac{\hat{\beta}_2}{\sqrt{\hat{\sigma}^2}} \sim t(n-k)$$

$$\sqrt{\sum x_i^2}$$

$$t = \frac{\hat{\beta}_2}{\sqrt{\frac{\sigma^2}{\sum x_i^2}}} \sim t(n-k)$$

$$SE = \sqrt{\frac{\sigma^2}{\sum x_i^2}} \qquad \sqrt{\sigma^2 \frac{\sum X_i^2}{n \sum x_i^2}}$$

$$\hat{\sigma}^2 = \frac{\sum e_i^2}{n-k}. \qquad \hat{\sigma}^2 = \frac{\sum e_i^2}{n-k}$$

$$\sum (Y_i - \bar{Y})^2 = \sum (\hat{Y}_i - \bar{Y})^2 + \sum (Y_i - \hat{Y}_i)^2$$

$$TSS = ESS + RSS$$

$$n-1 = k-1 + n-k.$$

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = \frac{TSS - RSS}{TSS}$$

$$\bar{R}^2 = 1 - (1-R^2)\frac{n-1}{n-k} = 1 - \frac{RSS/(n-k)}{TSS/(n-1)}$$

$$\bar{R}^2 = 1 - (1-R^2)\frac{n-1}{n-k}$$

F检验

$H_0: \beta_2 = \beta_3 = \dots = \beta_k = 0$

$$F = \frac{ESS/(k-1)}{RSS/(n-k)} \sim F(k-1, n-k)$$

$$F = \frac{n-k}{k-1} \cdot \frac{R^2}{1-R^2}$$