总体回归模型

$$\boxed{Y_i = \beta_1 + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + \mu_i} \qquad i=1,2\ldots,n$$

k-1为解释变量的数目。习惯上，把常数项看成为虚变量的系数，该虚变量的样本观测值始终取1。于是，模型中解释变量的数目为*k*。

β_j为**回归参数（regression coefficient）**。

①零均值假设

$$\boxed{E(\mu_i | X_i) = 0, i = 1, 2, \ldots, n}$$

②同方差

$$\boxed{Var(\mu_i | X_i) = \sigma^2, i = 1, 2, \ldots, n}$$

$\sigma^2 I_n$.

③ 无自相关

$$\boxed{Cov(\mu_i, \mu_j | X_i, X_j) = 0,\ i, j = 1, 2, \ldots, n, i \neq j}$$

④随机干扰项与解释变量不相关

$Cov(X_{ji}, u_i) = 0.$

⑤无多重共线性

$Rank(X'X) = K.$

⑥正态性假定

$u_i \sim N(0, \sigma^2)$

$e_i = Y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i})$

$\sum e_i^2 = \sum [Y - (\hat{\beta}_1 + \hat{\beta}_2 X_{21} + \hat{\beta}_3 X_{3i})]^2$

令 $\dfrac{\partial \sum e_i^2}{\partial \beta} = 0.$

$\therefore \quad Y = \hat{\beta} X + e$

$$\therefore \quad X'Y = \hat{\beta} XX' + X'e = 0.$$

$$\therefore \quad \hat{\beta} = (XX')^{-1} X'Y.$$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$\sum Y_i = n\hat{\beta}_1 + \hat{\beta}_2 \sum X_i$$

$$\sum X_i Y_i = \sum i \hat{\beta}_1 + \hat{\beta}_2 \sum X_i^2$$

无偏性：

$$\hat{\beta} = (X'X)^{-1}X'Y = (X'X)^{-1}X'(X\beta + U)$$

$$= (X'X)^{-1}X'X\beta + (X'X)^{-1}X'U$$

$$= \beta + (X'X)^{-1}X'U \quad =0.$$

$$E(U)=0.$$

$$E(\hat{\beta}) = \beta = \beta.$$

$$\hat{\beta}_2 = \sum k_i Y_i$$

$$= \sum k_i (\beta_1 + \beta_2 X_i + u_i)$$

$$= \underbrace{\sum k_i}_{0=} \beta_1 + \overbrace{\sum k_i \beta_2 X_i}^{=1} + \sum k_i u_i$$

$$= \beta_2 + \sum k_i u_i$$

$$E(\hat{\beta}) = \beta_2 + \sum k_i \overbrace{E(u_i)}^{=0}$$

$$= \beta_2.$$

$$E(\hat{\beta}_1) = E(\bar{Y} - \hat{\beta}_2 \bar{X})$$

$$= E(\bar{Y}) - \bar{X} E(\hat{\beta}_2)$$

$$= \beta_1 + \cancel{\beta_2 \bar{X} - \bar{X} \beta_2}$$

$$= \beta_1$$