STUDY OF THE BREGUET'S EQUATION

Disseny d'Avions

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Contents

Nomenclature

The next list describes the symbols used within the body of the document:

C_D	Drag coefficient
c_e	Specific consumption
C_L	Lift coefficient
C_{d0}	Drag coefficient when lift is 0
D	Drag
e	Oswald factor
g	Earth gravity acceleration
L	Lift
$MTOW$	Maximum take-off weight
N	Number of points for the discretization
R	Range
S_w	Wing reference surface
SPC	Specific Fuel Consumption
T	Thrust
t	Time
TF	Trip fuel
v	Cruise speed
W	Weight

1 Problem definition

In this work an study of the Breguet's equation is presented. First of all, the Breguet's equation is obtained for the airplane of study. Second, a numeric scheme is presented in order to integrate the equation, assuming that the cruise phase is at a constant speed and altitude. Then, a study of the relative error as a function of the number of integration steps is conducted. Finally, a sensitivity analysis is presented in order to extract conclusions in relation to possible improvements that could be made to the design so the efficiency of the airplane is improved.

1.1 Airplane study: data

The plane selected for the range study is the Airbus A-300-600. The technical specifications have been obtained from references [3] and [2], while the SFC (Specific Fuel Consumption) had been retrieved from [4]:

PARAMETER	VALUE
MTOW	171.000 kg
TF	54.793 kg
Cruise Velocity	241,79 m/s
Cruise Mach	0,79
Range	8.440,7 km
Height	11.300 m (FL370)
Thrust	11.300 N/engine
Number of engines	2
Specific fuel consumption	9,4575e-6 kg/(N·s)

For further calculations, some data from the International Standard Atmosphere is needed.

PARAMETER	VALUE
ρ_0	1,225 kg/m3
P_0	101.325 Pa
T_0	288 K
λ	-6,5e-3 K/m
R_{air}	287

1.2 Breguet's equation

In order to obtain the Breguet's equation there has to be taken into account the next conditions:

- Cruise phase at constant speed
- Turbofan airplane

Assuming the conditions before mentioned, it is needed to define the flight equations that will define each cruise phase:

$$x : T = D \tag{1}$$

$$y : W = L \tag{2}$$

Diving (1) by (2) it is obtained:

$$T = \frac{D \cdot W}{L} \tag{3}$$

Knowing that:

$$L = \frac{C_L S_w v^2 \rho}{2} \tag{4}$$

And the same equation applies for the Drag:

$$D = \frac{C_D S_w v^2 \rho}{2} \tag{5}$$

Applying both equations (4) and (5) into (3) it is obtained :

$$T = \frac{C_D \cdot W}{C_L} \tag{6}$$

Breguet's equations it is defined by:

$$R = \int_{t_i}^{t_f} v dt \tag{7}$$

Applying mass conservation through time it can be define the next relationship:

$$\frac{-dW}{g} = T c_e dt \tag{8}$$

Applying the equation before (8) into Breguet's range equation (7) it is found the equation that defined the range as a function of the weight instead of the time:

$$R = \int_{W_i}^{W_f} v \frac{-dW}{g T c_e} \tag{9}$$

The (9) it is now function of T, W and v. Because even though the velocity it is considered to be constant, it can only be considered constant in small sections, but from one discrete section to another one the velocity is a function of the weight.

Applying (4) into (2) it can be found the relationship between the velocity and the weight.

$$v = \sqrt{\frac{2W}{\rho S_w C_L}} \tag{10}$$

Finally applying (10) and (6) it can be found the Breguet's integral equation only as a function of weight:

$$R = \sqrt{\frac{2}{\rho S_w C_L}} \cdot \frac{-C_L}{gTc_eC_D} \int_{W_i}^{W_f} \frac{dW}{\sqrt{W}} \tag{11}$$

1.3 Numeric scheme

In order to integrate the Breguet's equation found on the last section it has been applied the trapezoidal rule.The trapezoidal scheme works by approximating the region under the graph of a function $f(x)$ as a trapezoid and calculating its area. And it is defined as the following equation:

$$\int_a^b f(x)dx \approx (b-a)\frac{f(a)+f(b)}{2} \tag{12}$$

As it can be seen on the last equation (12) it is needed to define a f(x). Applying this scheme into Breguet's equation it can be easily find that:

$$f(x) = \sqrt{\frac{2}{\rho S_w C_L}} \cdot \frac{-C_L}{gTc_eC_D} \frac{1}{\sqrt{W}} \tag{13}$$

Where in this case x is equal to TF.

As it has been explained on the first part of Section **??**, it has been found all the important data of the studied airplane. Because of that it has been fixed the TF and the scheme will be validated by calculating the range and comparing it with the original one. Knowing that it can be defined the next parametrs for the trapeziodal rule:

$$a \longrightarrow TF(i,1) \tag{14}$$

$$b \longrightarrow TF(i+1,1) \tag{15}$$

$$f(a) = \sqrt{\frac{2}{\rho S_w C_L}} \cdot \frac{-C_L}{gTc_eC_D} \frac{1}{\sqrt{W(i,1)}} \tag{16}$$

$$f(a) = \sqrt{\frac{2}{\rho S_w C_L}} \cdot \frac{-C_L}{gTc_eC_D} \frac{1}{\sqrt{W(i+1,1)}} \tag{17}$$

Applying the four last equations into (12) it is defined the numerical equation applied during the development of the problem:

$$k = \sqrt{\frac{2}{\rho S_w C_L}} \cdot \frac{-C_L}{2gTc_eC_D} \tag{18}$$

$$R(i,1) = k \cdot (TF(i+1,1) - TF(i,1)) \cdot (\frac{1}{\sqrt{W(i,1)}} + \frac{1}{\sqrt{W(i+1,1)}}) \tag{19}$$

And the final Range it has been found making the sum of each range step:

$$R = \sum_{i=1}^{N} R(i,1) \tag{20}$$

Where N is the number of points used to do the discretization and i is the current range step.
The (18) defines the Breguet's constant a range step. Taking into account that needs to be constant for a range step, the C_L and C_D will change from one range step into another one. Both of them can be defined as a function of the weight as:

$$C_L(i,1) = \frac{2W(i,1)}{(S_w \rho v^2)} \tag{21}$$

$$C_D(i,1) = \frac{TC_L}{W(i,1)} \tag{22}$$

There has to be taken into account that both TF and W are known because they are data that the manufacturer gives.

And finally the equation that relates the TF and the weight of the plane as a function of the distance W is:

$$W(i,1) = MTOW - TF(i,1); \tag{23}$$

Where the TF vectors increases linearly with i, making that at $i == N$, $W(N,1)$ is equal to the landing weight.

1.4 Numeric scheme diagram

The following diagram describe the logic of the program from the input data until the sensitivity analysis, focusing on the convergence method that uses the trapezoidal rule.

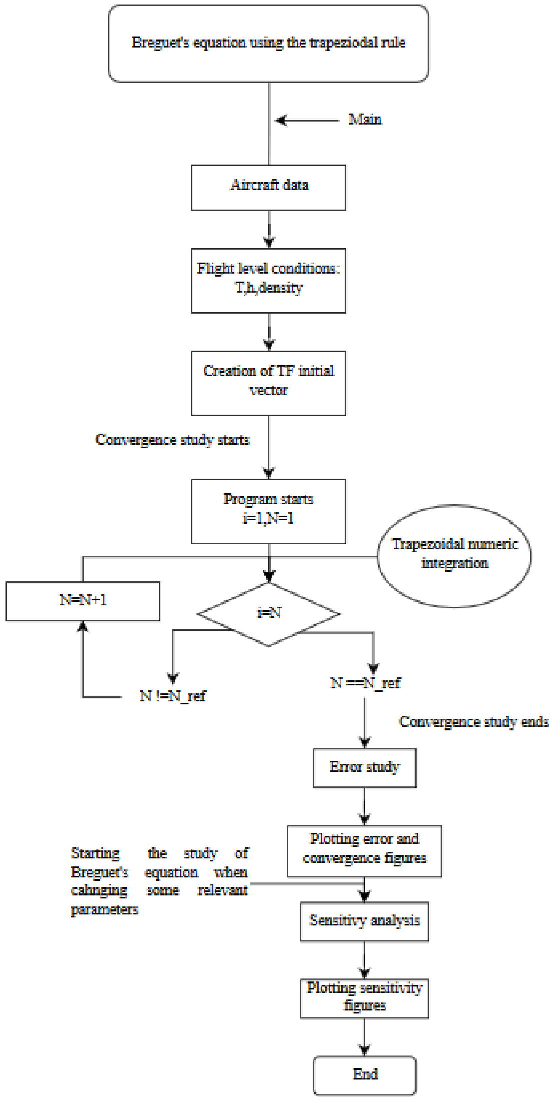

FIGURE 1: Flow chart diagram of the program

2 Results

2.1 Nominal results

After running the code for the specified initial data, a plot was displayed comparing the results depending on the number of steps chosen to discretize the spatial domain. However, in order to estimate the error of a numerical code with respect to the original result, a well controlled initial value should be used for comparison purposes. In this sense, based on data provided by [2], the range for this model was:

$$R(Airliners\ webpage) = 7686\,km$$

But this is a "real" value, that might take into account several phases, such as climb or descend, and velocity might not be constant. Furthermore, this result is for *Maximum Fuel Weight* but no passengers, so it is not used in daily operations.

Having this idea in mind, this result is no longer valid, so for validation, we should compare the numerical results with the range computed with constant velocity across the complete cruise, which is only 1 step, with the prescribed maximum TF given by manufacturer, departing at MTOW. If we observe the typical R-Payload diagram [1] [1], it corresponds to the 2nd characteristic point, where we have reduced payload in order to fill maximum fuel.

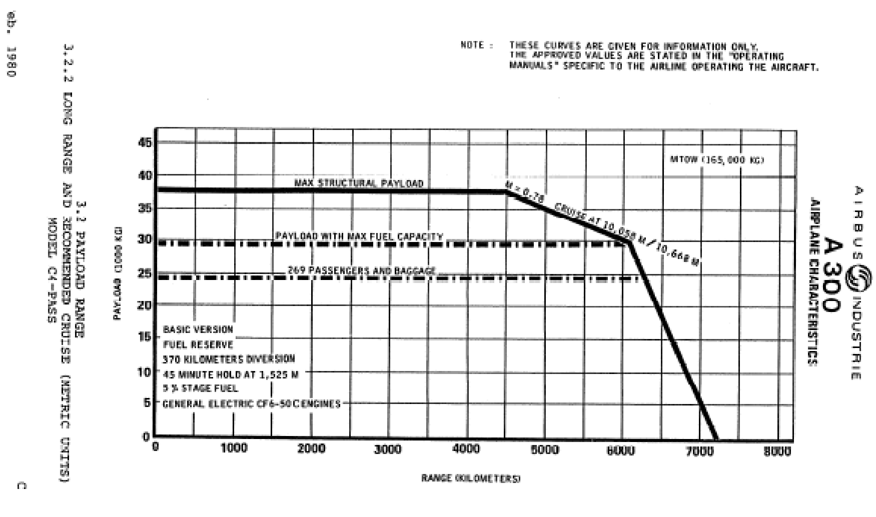

FIGURE 2: Range chart for A300-600, differs from constant velocity results

$$R(v = ct., 1step) = 8441\ km$$

[1] Pag. 74 A300 Airport Planning

This will be taken as the reference range, while numerical scheme will integrate equation (11) in N steps, with constant velocity within each of them, having a much more realistic result, because velocity is not constant over the complete cruise, as long as weight is variable.
In addition, it was shown the difference in results between Trapezoidal integration method and Simpson's rule, as expressed in the following picture:

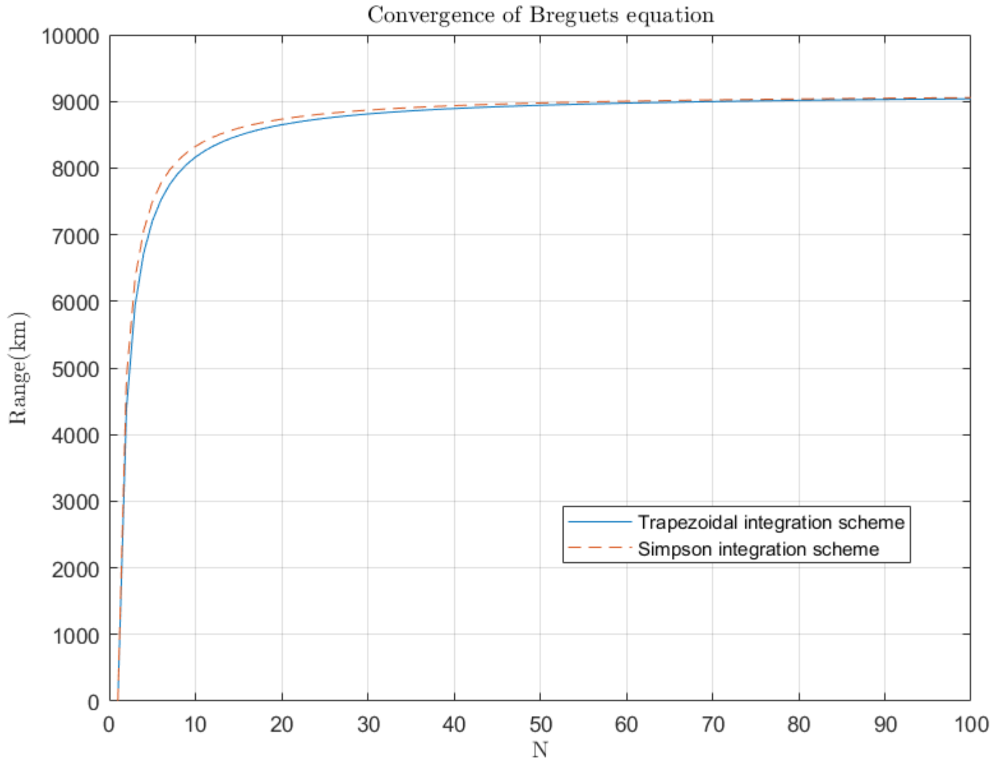

FIGURE 3: Range (km) depending on number of steps chosen

It can be easily concluded that both schemes for integration of differential equation are valid, for which reason trapezoidal scheme will be chosen for future computations, due to its simplicity. Regarding the "grid" convergence for this calculus, it can be concluded that further more than $N = 50$ steps, there is no much difference between results, so we can accept that result as correct.

In addition, the absolute and relative error, compared to the nominal value of range had been computed.

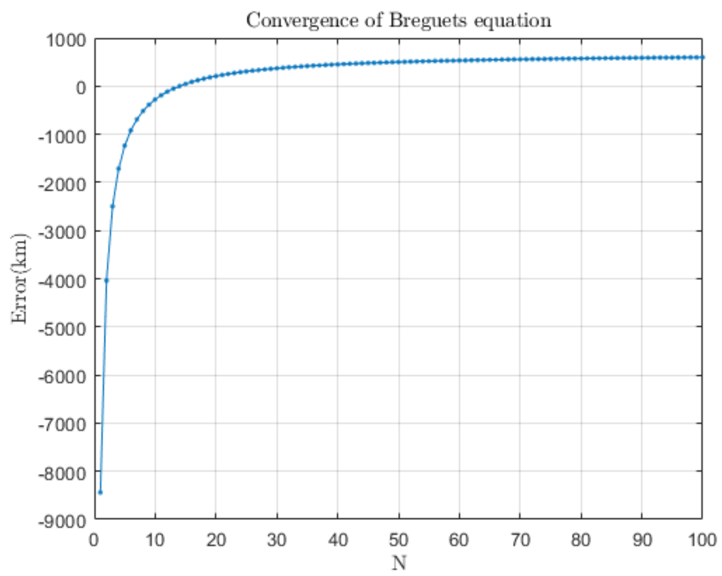

FIGURE 4: Absolute error(km) depending on number of steps chosen

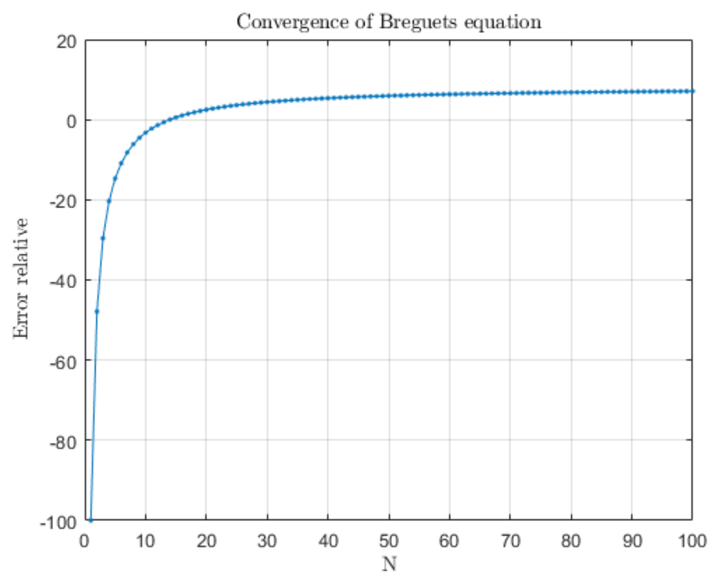

FIGURE 5: Relative error(%) depending on number of steps chosen

At the end, it can be observed that the final result, for 200 steps, ends up with an stationary error about 560 km or 8%, if compared with 1 step result. The reason why this result does not end up until zero is caused by the difference between the reference case where constant velocity over the cruise is assumed, and the real profile, where velocity is only constant within each step, so it adapts to the variation in weight. For this reason, this error should not be taken as a mistake, but the difference in range we're accepting if we assume constant velocity across the complete cruise.

2.2 Sensitivity analysis

2.2.1 Flight modifications: cruise speed

In order to see how does the cruise speed influences the aircraft range, this speed has been incremented by 10% from the original one, and the next graphic has been obtained.

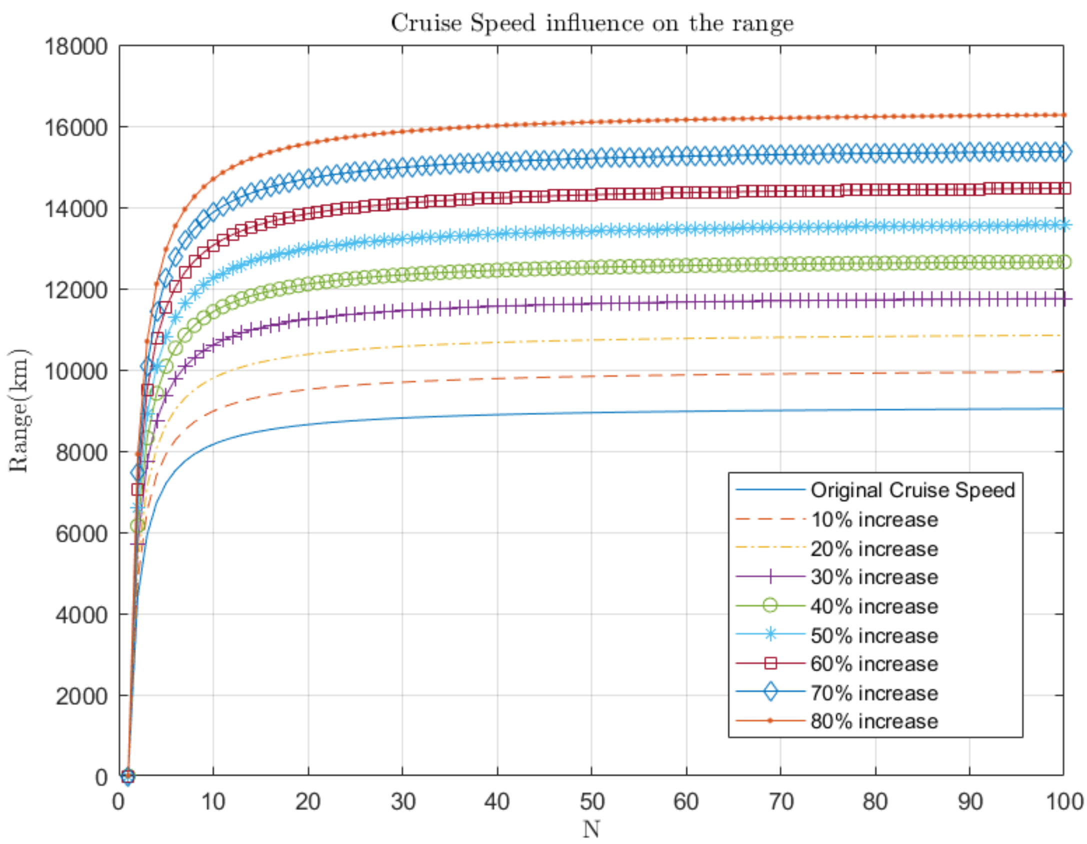

FIGURE 6: Relation between range and cruise speed

As it can be seen in the previous figure, if the cruise speed increases, the range increases as well. The faster the aircraft flies, the further it can go. The original cruise speed is 241.78 m/s, if this speed increases an 80% (459.39m/s), then the range would be nearly the 16000km.

2.2.2 Flight modifications: flight altitude

The flight altitude changes the air density as well as the pressure. These changes modify the lift and drag coefficient. As can be seen in table 1, this are the lift and drag coefficient for the different flight altitudes:

	FL010	FL270	FL290	FL330	FL370	FL410	FL450
CL	0.1566	0.3647	0.3922	0.4522	0.5320	0.6500	0.9553
CD	0.0186	0.0433	0.0465	0.0540	0.0631	0.0771	0.1134

If the aerodynamic efficiency is calculated, the next values are obtained:

	FL010	FL270	FL290	FL330	FL370	FL410	FL450
E	8.4262	8.4262	8.4262	8.4262	8.4262	8.4262	8.4262

As it can be seen in table 1, the efficiency remains constant throughout the different flight altitudes. Then, the range also remains constant. The conclusion is that, even though the flight altitude changes the aerodynamic forces, the efficiency, which is the parameter used to calculate the range, remains constant, and therefore, the range also remains constant.

2.3 Engine modifications: C_e

In order to see how the engine characteristic can modify the airplane range, the Ce has been reduced.
The relation between the Ce and the range can be seen in figure 7:

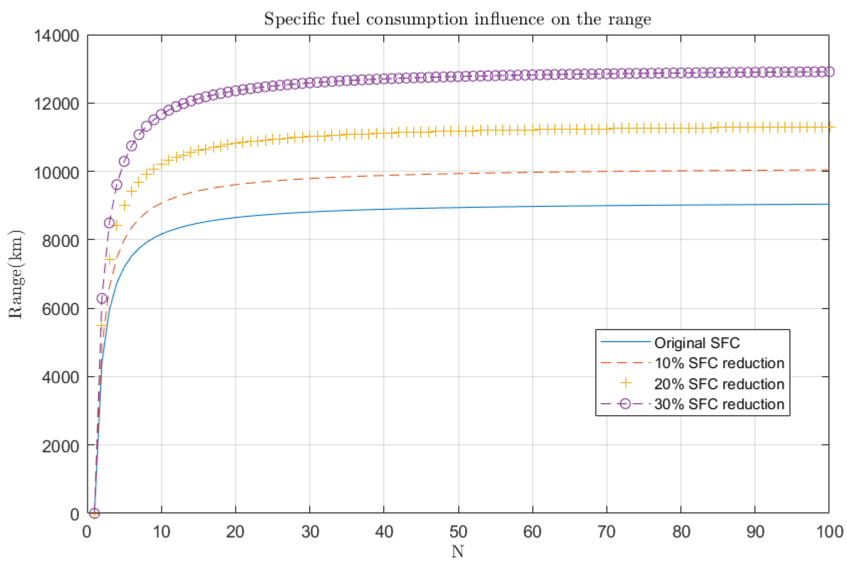

FIGURE 7: Specific fuel consumption influence

For a 30% reduction from the original ce value, the range increases to almost 13000km.

2.3.1 Aerodynamic modifications:Polar distribution of the drag coefficient

Instead of the original distribution of the CD, it has been changed to the following one:

$$CD = Cd0 + k * Cl^2 \tag{24}$$

In [5] [2], the value for CD0 has been found (0.037) and the value for k can be calculated with the next expression:

$$k = 1/(\Pi * AR * e) = 0.054882 \tag{25}$$

Finally, the relation between the range obtained with the original distribution and the polar distribution is shown in the next figure:

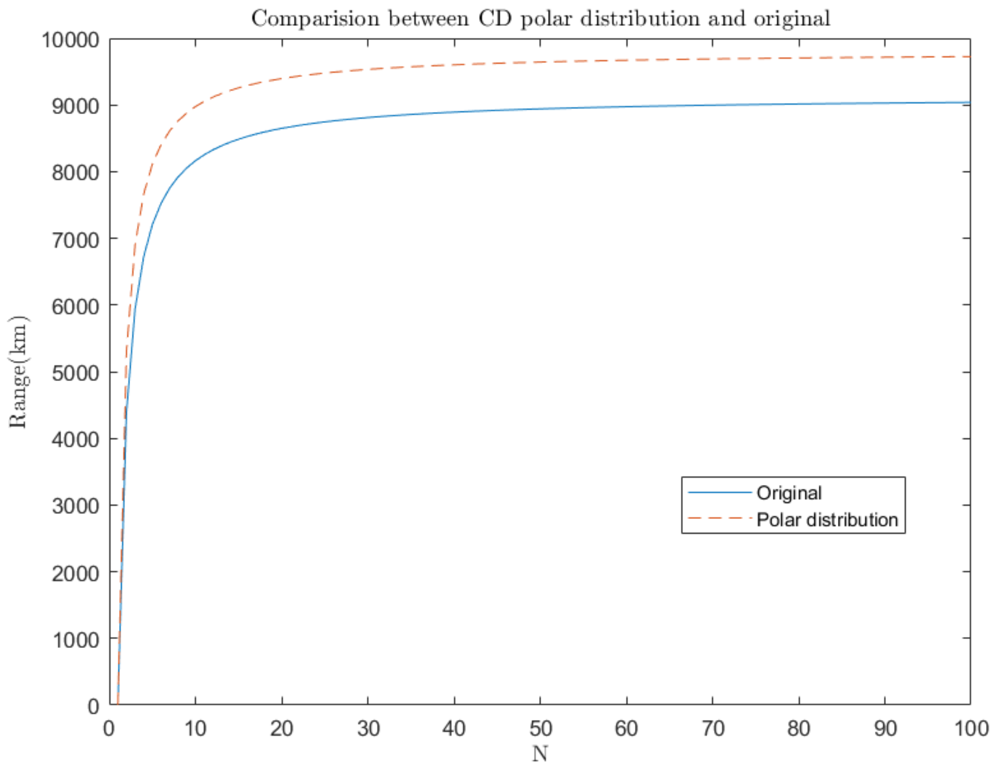

FIGURE 8: How the Cd distribution changes the airplane's range

[2] Pag. 149 orenbeek. *Synthesis of Subsonic Airplane Design*. 1st. Delft University Press.

The range obtained with a polar distribution is higher than the one obtained with the original distribution. This is because with the polar distribution the drag coefficient is lower, and therefore, the aerodynamic efficiency is higher.

2.3.2 Structural modifications: TF

In order to increase the range, the airplane needs more fuel, so the trip fuel weight has to be higher, but without exceeding the maximum takeoff weight. In Figure 5, the results of how the range changes if the trip fuel increases from 10% to 40% are shown:

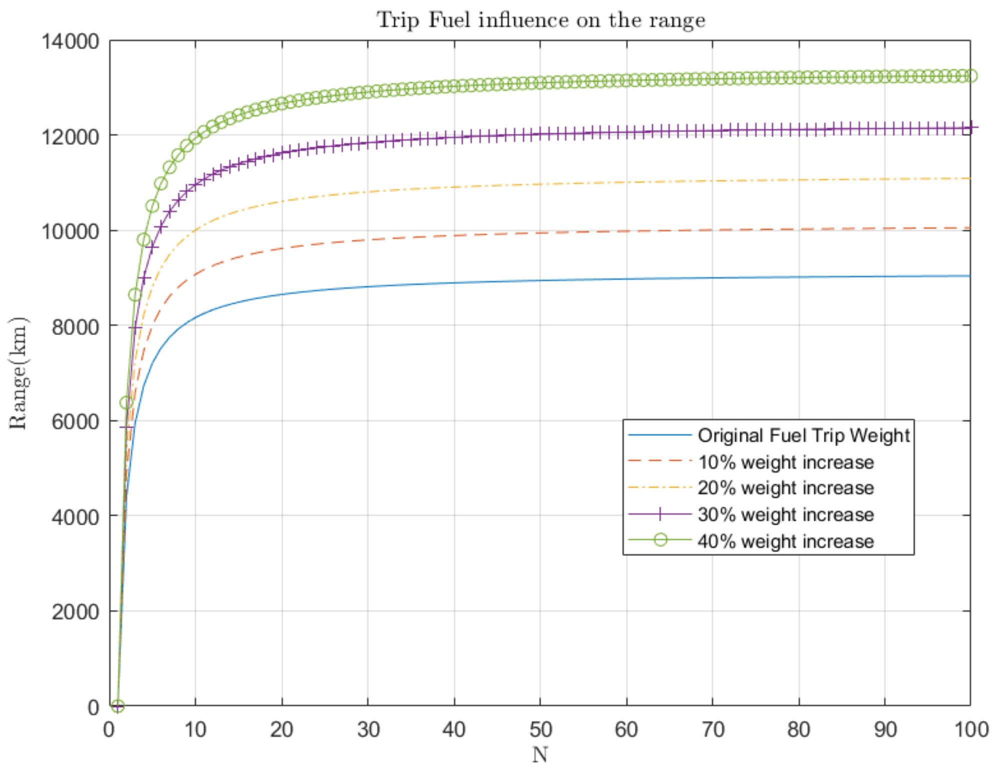

FIGURE 9: Trip Fuel influence on the range

As it was predicted before, the range increases because the amount of fuel available is higher. The weight can only be increased by 40% because of the MTOW limitation.

2.4 Sensibility Conclusions

Analyzing all the parameters changed, the ones that will improve the aircraft performance will be the followings increasing the cruise speed, reducing the specific fuel consumption of the engine or increasing the trip fuel.

One of the alternatives is to increase the cruise speed. As it had been explained before, an increase of the aircraft cruise velocity supposed an increase of the range. However, the maximum speed that an aircraft can achieve is limited due to its Mach transition number, if this velocity is overtaken a lot of aerodynamic problems will appear such as wave resistance, shock wave. So, the cruise speed can be increased as much as aerodynamics parameters are not negatively affected.

Another possibility is using an engine with a lower specific fuel consumption. Using the same amount of fuel, the plane can arrive further. The engines have to be further analyzed in order to know if they can successfully supply the propulsion aspects.

If it is not possible to found a most suitable engine, another solution could be reducing the payload and increasing the amount of fuel. This solution is not appropriate for commercial planes, as its objective is to transport more people as possible, so the best solution will be to minimize the operational empty weight using lightweight materials and components.

3 Conclusions

As a general resume, the development of this numerical code, based on Breguet's equation, has made possible to know the difference in range, for a given aircraft and prescribed fuel TF, with respect to the assumption of constant velocity over the cruise.

The main guideline was to calculate range, based on Breguet's differential equation, but integrated numerically across the nº steps chosen, for MTOW and maximum TF.

Having performed a convergence analysis, it was concluded that **an error about 560 km or 8% is made if a single-phase cruise is used**. In addition, different numerical schemes were used and analyzed, such as Trapezoidal integration scheme or Simpson's rule, resulting in barely the same values.

Finally, a sensitivity analysis was developed, revealing that those parameters that affect range the most are:

- **Cruise speed:** it is limited by shock wave and aeroelastic phenomena. An speed increase of only 10% enlarges range up to 10000km
- **SPC (Specific Fuel Consumption:** dependent on the engine chosen, today, the use of new-trend engines, reducing this value up to 10% would have a traduction of an increase of range up to 10000km, as well.
- **Polar distribution:** for this calculus, it was assumed that the maximum thrust, corrected to the FL, would be the $T_{necessary}$. However, if we make an educated guess of the polar curve of the aircraft, based on [5], we obtain an increase of range about 800km. Of course, for developing a detailed calculation, it would be necessary to know the exact values.
- **Trip Fuel:** this value is limited by size of fuel tanks, if we increase their allowable volume so they can fit 10% more TF, it would end up with a increase of 1000km. However, it can be difficult because it means a new redesign of wing shape as well as structural troubles.

Finally, having all these results into account, it can be affirmed that the numerical convergence plot revealed that the assumption of constant velocity can be only used for small distances, but for commercial aircraft, the complete equation should be solved in steps, with some numerical scheme such as Trapezoidal rule.
Furthermore, sensitivity analysis revealed that performance of the aircraft can be prone to be enlarged, as long as general trend lines in aviation industry, such as increasing cruise speed while reducing fuel consumption.

References

[1] Airbus. "A300 Airport Characteristics for Airport Planning". In: (2011), pp. 1–352. URL: http://www.aircraft.airbus.com/support-services/airport-operations/aircraft-characteristics/.

[2] *Airliners*. URL: http://www.airliners.net/aircraft-data/airbus-a300-600/18 (visited on 11/20/2018).

[3] Eurocontrol. *Aircraft Performance Database*. URL: https://contentzone.eurocontrol.int/aircraftperformance/default.aspx? (visited on 11/19/2018).

[4] *Jet Engine Specification Database*. URL: http://www.jet-engine.net/ (visited on 11/20/2018).

[5] Torenbeek. *Synthesis of Subsonic Airplane Design*. 1st. Delft University Press.

A Source code

```
%-----------------------------------------------------
% Authors: Erik Escalera,Edgar Gago,Anna Gordun
% Ferran Lumbierres, Marta Muoz, Itiziar Ugartemendia
% Date:18/12/2018
%----------------------------------------------------

clc;
clear;
close all;

%Compact way to include scripts
format shortE;
format compact;

%% INPUT DATA
input_data;

%% BREGUET'S EQUATION & NUMERIC SCHEME AND INTEGRATION
for i=2:length(N)
    [Range_num(i,1),Range_num_s(i,1)]= BreguetsEquation(MTOW,rho,...
        V_cruise,c_e,-g,DeltaTF(1,i),Sw,Thrust,N(1,i),N_engines);
end

%% NUMERIC CONVERGENCE AND ERROR
Convergence;

%% SENSITIVITY ANALYSIS
```

```

% Effect of the trip fuel
TripFuelSensitivity;

% Effect of the cruise speed.
CruiseSpeedSensitivity;

%Effect of the specific consume CE
CESensitivity;

%Effect of the flight level
HSensitivity;

%CD POLAR DISTRIBUTION
CDCLSensitivity;

%Comparison of the different analyzed values
Comparison;

%% End
```

```

% INPUT DATA

R_max = 9327; % [km]=5036.177nm, Nominal range (last 4...
% numbers of the ID)

% AIRBUS A-300-600 -> A360
% IATA: AB6 / ABY
% Accommodation: Flightcrew of 2 and 266 passengers in
% typical seating configuration, up to 298 in single ...
% class configuration. Wing span = 44.84m; Wing area ...
% = 260m2; Lenght = 54.1m; Height=16.54m
% Powerplant: 2 x 262kN CF6-80C2 or 2 x 249kN P&W PW4000 turbofans
% Notes: Long range wide-body airliner. In service since 1994,
%developed on the basis of the A-300B4.
% https://contentzone.eurocontrol.int/aircraftperformance/...
% details.aspx?ICAO=A306&
% http://www.airliners.net/aircraft-data/airbus-a300-600/18

% ENGINE CF6-80C2A1 DATA from http://www.jet-engine.net/...
% civtfspec.html
% Thrust(dry) = 59000lbf; SFC(dry) = 0.334 lb/lbf hr;
% Airflow = 1754 lb/s;

MTOW = 171700;            %[kg]
rho_jet = 0.804 ;         % [kg/]
```

```
TF = 68150*rho_jet;        % [kg]
V_cruise = 470*1.852/3.6;     % [m/s]
Mach_cruise = 0.79;
R = 4150*1.852;          % [km]


Sw = 260;                  % [m^2]
h = 11300;               % [m] FL370
N_engines = 2;
Thrust = 262000; % or 59000?;              % [N] Thrust per engine
c_e = 0.334*0.453592/(4.44974115*3600); % [kg/(N*s)] converted...
%from [lb/(lbf*h)]

% ISA ATMOSPHERE

rho_0 = 1.225; % [kg/m3]
P_0 = 101325; % [Pa]
T_0 = 288; % [k]
lambda = -6.5*10^-3; %[k/m]
g = -9.81; % [m/s2]
R_air = 287; % [m2s2/k]

if h ≤ 11000 %Troposphere
    T = T_0 + lambda*h;
    P = P_0*((T_0+lambda*h)/T_0)^(g/(R_air*lambda));
    rho = rho_0*((T_0+lambda*h)/T_0)^(g/(R_air*lambda)-1);
elseif h > 11000 %Tropopause
    T = 216.5;
    P11 = 22552; % P at 11km
    rho11 = 0.3629; % rho at 11km
    P = P11*exp(g*(h-11000)/(R_air*T));
    rho = rho11*exp(g*(h-11000)/(R_air*T));
end


Thrust=Thrust.*(rho./rho_0).^(0.9);

CL_cruise = 2*(-g.*mean([MTOW,(MTOW-TF)]))./...
(Sw*rho*V_cruise^2);

CD_cruise = (N_engines.*Thrust)*(CL_cruise./...
(-g.*mean([MTOW,(MTOW-TF)])));

R=(V_cruise./(-g.*c_e)).*(CL_cruise./CD_cruise).*log(MTOW./...
    (MTOW-TF));
R=R./1000;

% Trip fuel diferentials
N=1:100;
```

```
DeltaTF = TF./N; % for the different scneraios
```

```
function[R,R_s]= BreguetsEquation(MTOW,rho,v,ce,g,DeltaTF...
    ,Sw,Thrust,N,N_engines)

dx = linspace(1,N,N);
DeltaTF_v = dx.*DeltaTF;
W = (MTOW - DeltaTF_v).*g; %W along the diferent weight increments

CL = 2*W./(Sw*rho*v^2);
CD = (N_engines.*Thrust)*(CL./W);

K = (-CL./(g.*ce.*CD)).*sqrt(2./(rho.*Sw*CL)); % Breguet's constant

W=W';
r = (K./sqrt(W));
R=0;
R_s=0;
% Due to the constat TF the Delta R is bigger...
% thatn the one that should be
%so the final range will also be bigger (positive error)

for i=1:N-1
    %Trapezoidal method
    h(i,1) = (W(i+1,1) - W(i,1)) / 2;
    R_sum(i,1) = h(i,1)*(r(i+1,1)+r(i,1));
    R = R_sum(i,1) + R;

    %Simpson method
    h_s(i,1) = (W(i+1,1) - W(i,1)) / 6;
    r_av(i,1) = (r(i+1,1) - r(i,1)) / 2;
    R_sum_s(i,1) = h(i,1)*( r(i,1) + 4*r_av(i,1)+ r(i+1,1));
    R_s = R_sum_s(i,1) + R_s;
end

end
```

```

Range_num = Range_num/1000;
Range_num_s = Range_num_s/1000;
Error=(Range_num)-R;
Error_rel=(Error./R).*100;


figure(1);
plot(N,Range_num);
```

```
grid on
hold on
plot(N,Range_num_s);
title('Convergence of Breguets equation','Interpreter','Latex');
ylabel('Range(km)','Interpreter','Latex');
xlabel('N','Interpreter','Latex')

figure(2)
plot(N,Error,'.-')
grid on
title('Convergence of Breguets equation','Interpreter','Latex');
ylabel('Error(km)','Interpreter','Latex');
xlabel('N','Interpreter','Latex')


figure(3)
plot(N,Error_rel,'.-')
grid on
title('Convergence of Breguets equation','Interpreter','Latex');
ylabel('Error relative','Interpreter','Latex');
xlabel('N','Interpreter','Latex')
```

```
input_data;
j=1;

for V_cruise=241.7889:24.17889:459.3981
    CL_cruise = 2*(-g.*mean([MTOW,(MTOW-TF)]))./...
        (Sw*rho*V_cruise^2);
    CD_cruise = (N_engines.*Thrust)*(CL_cruise./...
        (-g.*mean([MTOW,(MTOW-TF)])));
    R=(V_cruise./(-g.*c_e)).*(CL_cruise./...
        CD_cruise).*log(MTOW./(MTOW-TF));
    R=R./1000;
    N=1:100;
    DeltaTF = TF./N;
    for k=1:length(N)
        [Range_num_se(k,1)]=BreguetsEquation(MTOW,rho,...
            V_cruise,c_e,-g,DeltaTF(1,k),Sw,Thrust,N(1,k),N_engines);
        Range_num_sens(k,j)=Range_num_se(k,1);
    end
    j=j+1;
end
Range_num_sens=Range_num_sens/1000;
figure(2);
plot(N,Range_num_sens);
grid on
title('Cruise Speed influence on the range','Interpreter','Latex');
```

```
ylabel('Range(km)','Interpreter','Latex');
xlabel('N','Interpreter','Latex');
```

```
input_data;
j=1;

fligh_altitudes=[304.8 8229 8839 10058 11227 12496 14935];
for m=1:1:7
    h=fligh_altitudes(m);
    if h ≤ 11000 %Troposphere
        T = T_0 + lambda*h;
        P = P_0*((T_0+lambda*h)/T_0)^(g/(R_air*lambda));
        rho = rho_0*((T_0+lambda*h)/T_0)^(g/(R_air*lambda)-1);
    elseif h > 11000 %Tropopause
        T = 216.5;
        P11 = 22552; % P at 11km
        rho11 = 0.3629; % rho at 11km
        P = P11*exp(g*(h-11000)/(R_air*T));
        rho = rho11*exp(g*(h-11000)/(R_air*T));
    end
    CL_cruise = 2*(-g.*mean([MTOW,(MTOW-TF)]))./(Sw*rho*V_cruise^2);
    CD_cruise = (N_engines.*Thrust)*(CL_cruise./...
        (-g.*mean([MTOW,(MTOW-TF)])));
    R=(V_cruise./(-g.*c_e)).*(CL_cruise./...
        CD_cruise).*log(MTOW./(MTOW-TF));
    R=R./1000;
    N=1:100;
    DeltaTF = TF./N;
    for k=1:length(N)

        [Range_num_se(k,1)]=BreguetsEquation(MTOW,rho,...
            V_cruise,c_e,-g,DeltaTF(1,k),Sw,Thrust,N(1,k),N_engines);
        Range_num_sens(k,j)=Range_num_se(k,1);
    end
    rho_sens(j)=rho;
    R_sens(j)=R;
    CL_cruise_se(j)=CL_cruise;
    CD_cruise_se(j)=CD_cruise;
    E_se(j)=CL_cruise/CD_cruise;%CONSTANT!!!!!!!!!
    j=j+1;
end

%Plotting the results
Range_num_sens=Range_num_sens/1000;
%  figure(4);
%  plot(N,Range_num_sens);
%  grid on
```

```
%  title('Flight altitude influence on the range',...
%'Interpreter','Latex');
%  ylabel('Range(km)','Interpreter','Latex');
%  xlabel('N','Interpreter','Latex')
%flight altitude does not influence ...
%on the range because the airplane's
%efficiency remains constant.
```

```
input_data;
j=1;

for c_e=9.4575e-06:-9.4575e-07:6.62025e-06
    CL_cruise = 2*(-g.*mean([MTOW,(MTOW-TF)]))./...
        (Sw*rho*V_cruise^2);
    CD_cruise = (N_engines.*Thrust)*(CL_cruise./...
        (-g.*mean([MTOW,(MTOW-TF)])));
    R=(V_cruise./(-g.*c_e)).*(CL_cruise./...
        CD_cruise).*log(MTOW./(MTOW-TF));
    R=R./1000;
    N=1:100;
    DeltaTF = TF./N;
    for k=1:length(N)
        [Range_num_se(k,1)]=BreguetsEquation(MTOW,rho,V_cruise...
            ,c_e,-g,DeltaTF(1,k),Sw,Thrust,N(1,k),N_engines);
        Range_num_sens(k,j)=Range_num_se(k,1);
    end
    j=j+1;
end

%Plotting the results

Range_num_sens=Range_num_sens/1000;
figure(3);
plot(N,Range_num_sens);
grid on
title('Specific fuel consumption influence on the range'...
    ,'Interpreter','Latex');
ylabel('Range(km)','Interpreter','Latex');
xlabel('N','Interpreter','Latex');
```

```
input_data;
j=1;
%CD=CD0+K*CL^2
CD0=0.037;
k_par=0.0548826;
CD_cruise=CD0+k_par*(CL_cruise)^2;
```

```
R=(V_cruise./(-g.*c_e)).*(CL_cruise./...
    CD_cruise).*log(MTOW./(MTOW-TF));
R=R./1000;
N=1:100;
DeltaTF = TF./N;
for k=1:length(N)
    [Range_num_se(k,1)]=BreguetsEquation(MTOW,rho,...
        V_cruise,c_e,-g,DeltaTF(1,k),Sw,Thrust,N(1,k),N_engines);
    Range_num_sens(k,j)=Range_num_se(k,1);
end

%Plotting the results
Range_num_sens=Range_num_sens/1000;
figure(5);
plot(N,Range_num_sens);
grid on
title('blabla','Interpreter','Latex');
ylabel('Range(km)','Interpreter','Latex');
xlabel('N','Interpreter','Latex')
%change of the k parameter
j=1;
```

```
k_or=[0
4400.578387
5943.101015
6728.107388
7203.461294
7522.165797
7750.691168
7922.564833
8056.528043
8163.87708
8251.825971
8325.197668
8387.33885
8440.644599
8486.874168
8527.348834
8563.08021
8594.855937
8623.298417
8648.905987
8672.082361
8693.158073
8712.406337
8730.054958
8746.295388
```

```
8761.289699
8775.176013
8788.072764
8800.08209
8811.292539
8821.781259
8831.615771
8840.855433
8849.552629
8857.75377
8865.500119
8872.828484
8879.771809
8886.359661
8892.618658
8898.572824
8904.243898
8909.651601
8914.813865
8919.747026
8924.466007
8928.984458
8933.314896
8937.468816
8941.456796
8945.288585
8948.973182
8952.518908
8955.93347
8959.224013
8962.397173
8965.459121
8968.415603
8971.271977
8974.033241
8976.704068
8979.288831
8981.791622
8984.21628
8986.566408
8988.845391
8991.05641
8993.202462
8995.286368
8997.310789
8999.278236
9001.191081
9003.051567
9004.861813
```

```
9006.623828
9008.339513
9010.010674
9011.639019
9013.226175
9014.773684
9016.283013
9017.755559
9019.19265
9020.59555
9021.965467
9023.303549
9024.610894
9025.888548
9027.137511
9028.358741
9029.553149
9030.72161
9031.86496
9032.984001
9034.079499
9035.152189
9036.202777
9037.231939
9038.240323
9039.228553
];
k_new=[0
5313.318852
6904.854189
7665.528739
8110.669432
8402.740283
8609.068453
8762.561757
8881.203878
8975.650855
9052.617343
9116.544273
9170.485991
9216.612151
9256.506365
9291.351526
9322.049006
9349.297694
9373.647823
9395.538499
9415.324402
9433.295147
```

```
9449.689548
9464.706288
9478.512008
9491.247517
9503.032608
9513.969842
9524.147535
9533.642147
9542.520213
9550.839897
9558.652268
9566.002356
9572.930012
9579.470636
9585.655779
9591.513654
9597.069566
9602.346277
9607.364316
9612.142248
9616.696903
9621.043576
9625.196193
9629.167466
9632.96902
9636.611507
9640.104706
9643.457609
9646.678504
9649.775035
9652.754268
9655.622745
9658.386529
9661.051248
9663.622136
9666.104062
9668.501566
9670.818882
9673.059968
9675.228524
9677.328015
9679.361689
9681.332593
9683.243588
9685.097367
9686.896459
9688.64325
9690.339988
9691.988794
```

```
9693.59167
9695.15051
9696.667103
9698.143144
9699.580239
9700.979907
9702.343593
9703.672667
9704.968429
9706.232117
9707.464906
9708.667918
9709.842216
9710.988819
9712.108692
9713.202762
9714.271909
9715.316975
9716.338765
9717.338048
9718.315559
9719.272003
9720.208053
9721.124354
9722.021525
9722.900159
9723.760825
9724.604067
9725.430411
];

figure(6);
plot(N,k_or);
hold on
plot(N,k_new);
hold off
title('Comparision between CD polar distribution and original',...
    'Interpreter','Latex')
ylabel('Range(km)','Interpreter','Latex');
xlabel('N','Interpreter','Latex');
```