




Introduction to Modeling & Simulation

Stochastic Discrete Event Simulation

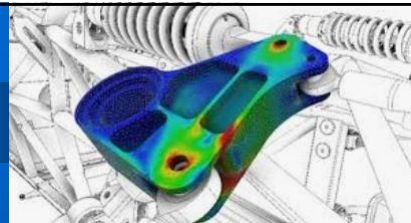
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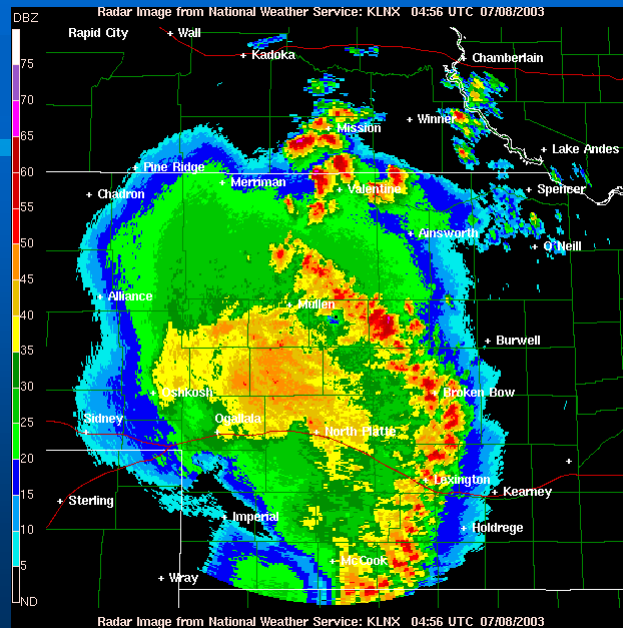
Part I.A

Simulation is ...



- **Simulation**
 - a very broad term – methods and applications to imitate or mimic real systems, usually via computer
- Applies in many fields and industries
- Very popular and powerful method
- In this chapter : general ideas, terminology, examples of applications, why simulation is used

Why not work with the real System?



When it's possible study the real system



- Measure, control, play with the actual system
- Advantage — we are unquestionably looking at the right thing

But it's often impossible to do so in reality with the actual system :

- Often the system doesn't exist – simulation is used to design the future system
- Sometimes it is too expensive, complex or dangerous to design improvements and to check them directly in real life

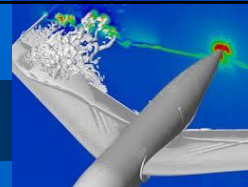
MODEL: A set of assumptions and approximations about how the system works



- Study the model instead of the real system ... usually much easier, faster, cheaper, safer
- Can try a wide-ranging of ideas with the model
 - Make your mistakes on the computer **where they 'don't count'**, rather than for real **where they do count**
- Often, just *building* the model is instructive – regardless of results
- Model *validity*
 - Care in building to meet objectives
 - Level of details : Mimic reality faithfully only if needed
 - Get same conclusions from the model as you would from system

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Broader definition... (1/2) Simulation :



Aeronautic air tremble
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- In Computer Science:
Simulation is the imitation of the operation of a system or a real world process over time.
- The **system** is a collection of interacting **objects** (cf. dictionary definition)
- The system can be existing or not :
 - « A priori » modelling (non existing)
 - « A posteriori » modelling (existing)

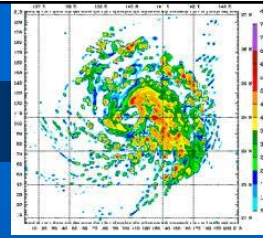
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Broader definitions... (2/2) Models and systems :

- A **model** is a representation of a **system**
- In Matter, Mind and Models (published by MIT Press in 1965) by Marvin L. Minsky we find the following definition :

To an observer B,
an object A* is a model of an object A
to the extent that B can use A*
to answer questions that interest him about A

- It implies that :
 - A model is built with an intended **goal** in mind.
 - A the model should be complex enough to answer the **questions** raised.



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Continuous vs. Discrete Simulation

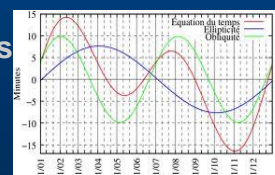
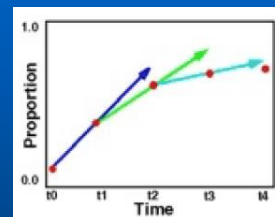
- The **system state** is defined by a collection of **variables** that describe a system at any time :
 - With a discrete **event** simulation, the model **state variables** change only at discrete points in **time**
 - In a continuous simulation the systems state changes continuously according to a mathematical model (equation or set of equations)
- We can find **combined simulation** with both discrete and continuous components
- Remark: At the quantic level, everything is discrete.

$$\frac{\partial \theta}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) = \frac{\partial}{\partial \alpha} \int_{\mathbb{R}^n} f(x, \theta) dx = M \left(\tau(\xi) \frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi) \right)$$

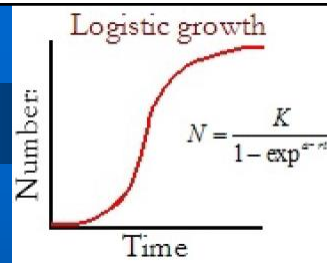
$$\frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) = \frac{(\xi_i - \alpha)}{\sigma^2} f_{\alpha, \sigma^2}(\xi_i) - \frac{1}{2\sigma^2} \left[\frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) \right]$$

$$\frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) = \frac{\partial}{\partial \alpha} \left(\frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) \right) f_{\alpha, \sigma^2}(\xi_i) - \left[\frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) \right] f_{\alpha, \sigma^2}(\xi_i)$$

$$\frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) = \frac{\partial}{\partial \alpha} \left(\frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) \right) f_{\alpha, \sigma^2}(\xi_i) - \left[\frac{\partial}{\partial \alpha} \ln f_{\alpha, \sigma^2}(\xi_i) \right] f_{\alpha, \sigma^2}(\xi_i)$$



Continuous vs. Discrete (Analytical vs. Algorithmic)



- **Analytical Model** : the model rely on a mathematical “formula” often named “closed form” mathematical solution – named analytical solution.
 - Advantage: a fast computing of the solution
 - Drawback: limited to a small set of systems
- **Computer Simulation Model** : we rely on a simulation algorithm to compute a solution for which we do not have an “Analytical solution”
 - Drawback: often slow to compute solutions.
 - Advantage : fits with all types of systems



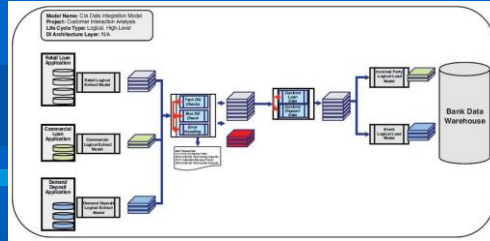
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Types of Models

- **Physical** models
 - Mock-ups...
 - Mechanical Car or flight simulators
- **Logical (mathematical)** models
 - Approximations and assumptions about a system's operation
 - Often represented via computer program
 - Exercise with computer simulation programs to try things, get results, learn about model



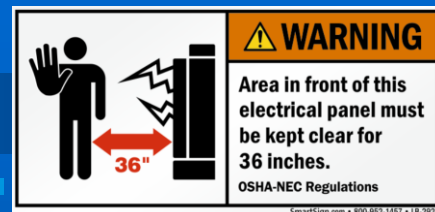
Studying Logical Models



- If model is simple enough, use traditional mathematical analysis ...
- Get exact results, it works with :
 - Queueing theory
 - Differential equations
 - Linear programming
- But complex systems can seldom be **validly** represented by a simple analytic model



Over-simplifying danger



- There is a danger of **over-simplifying assumptions** with analytical models
- **Mathematical representations are efficient** but do not often fit to meet the modeling objectives for complex systems
- Often, a complex system requires a complex model,
- If analytical methods don't apply ...
what can we do?

When analytical models fail

$$M_{\text{ext}} L_{\text{ext}}^2 M_{\text{ext}} L_{\text{ext}}^2 M_{\text{ext}} h_{\text{ext}}^2$$

$$2 \sum_{i=1}^n \frac{K_{\text{ext}} L_{\text{ext}}^2}{\omega_{\text{ext}} \sqrt{G_{\text{ext}}}} \tan \left(\frac{L_{\text{ext}} \omega_{\text{ext}}}{\sqrt{G_{\text{ext}}}} \right) +$$

$$\frac{K_{\text{ext}} L_{\text{ext}}}{\omega_{\text{ext}} \sqrt{G_{\text{ext}}}} \sec \left(\frac{L_{\text{ext}} \omega_{\text{ext}}}{\sqrt{G_{\text{ext}}}} \right) +$$

$$\frac{K_{\text{ext}} L_{\text{ext}}}{\omega_{\text{ext}} \sqrt{G_{\text{ext}}}} \tan \left(\frac{L_{\text{ext}} \omega_{\text{ext}}}{\sqrt{G_{\text{ext}}}} \right) +$$

$$\frac{K_{\text{ext}} L_{\text{ext}}}{\omega_{\text{ext}} \sqrt{G_{\text{ext}}}} \sec \left(\frac{L_{\text{ext}} \omega_{\text{ext}}}{\sqrt{G_{\text{ext}}}} \right)$$

- Broadly interpreted, **computer simulation** refers to methods for studying a wide variety of models of systems
 - Numerically evaluate with a computer (numerical simulations)
 - Computer simulation : algorithm & software to imitate the system operations and characteristics, often over time
- Computer simulation – can be used to study simple models **but should not be used if an analytical solution is available.**

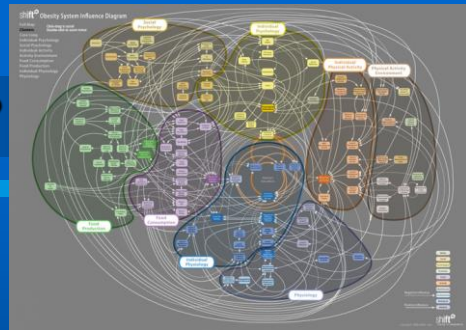
Computer simulation for complex systems



- Can also be used to study simple models but should not be used if an analytical solution is available
- When there is no hope to obtain an analytical solution, computer simulation is the answer.
- The real power of simulation is in studying complex models that cannot be studied otherwise
- Simulation can tolerate complex models

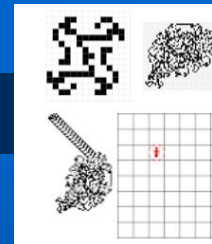
What are complex systems ?

- Complex systems are systems whose behavior are intrinsically difficult to understand
- This is due to the dependencies, competitions, relationships, or other types of interactions between their parts or between a given system and its environment.
- A complex system **can be composed of few** or many components which interact with each other.



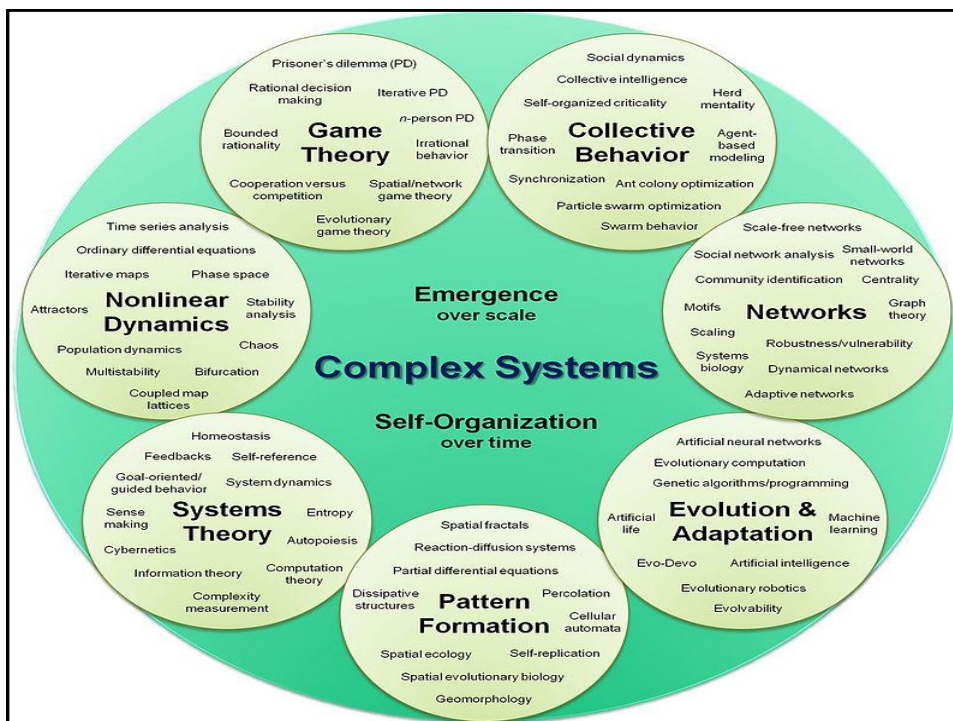
Predictive limit

- Most complex systems have many components, but **even very small systems can be complex** showing the essence of the **predictive limit** and **the absolute need of simulation to understand them**.
- Examples of complex systems are :
 - Earth's global climate, living organisms, the human brain, infrastructure such as power grid, transportation or communication systems, social and economic organizations (like cities), an ecosystem, a living cell, and ultimately the entire universe.

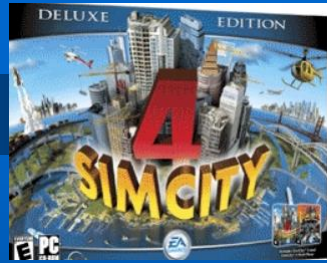


Complex systems properties

- Systems that are "complex" have distinct properties that arise from these relationships, such as:
 - nonlinearity,
 - pattern formation,
 - spontaneous order,
 - emergence,
 - adaptation,
 - feedback loops, among others.



Popularity of Simulation



- Consistently ranked as the most useful, popular tool in the broader area of operations research / management science
- Survey of all large firms, to see which methods are mainly used?
 1. Statistical analysis (93% used it)
 2. Simulation (84%)Followed by LP, PERT/CPM, inventory theory, NLP, ...

Advantages of Simulation

- Flexibility to model things as they are (even if messy and complicated)
 - Avoid *looking only where the light is*
- Allows the modeling of uncertainty and nonstationarity :
 - The only thing that's for sure:
 - nothing is for sure
 - There is danger in ignoring system variability
 - Model validity by confrontation with experts or other validation methods

Advantages of Simulation (cont'd.)

- Advances in computing/cost ratios
 - Estimated that 75% of computing power is used for various kinds of simulations
 - Can use dedicated machines (e.g., real-time control)
- Advances in simulation software
 - Far easier to use (GUIs)
 - No longer as restrictive in modeling constructs
 - Statistical design & analysis capabilities

The Bad News

Diagram illustrating the components of a confidence interval formula:

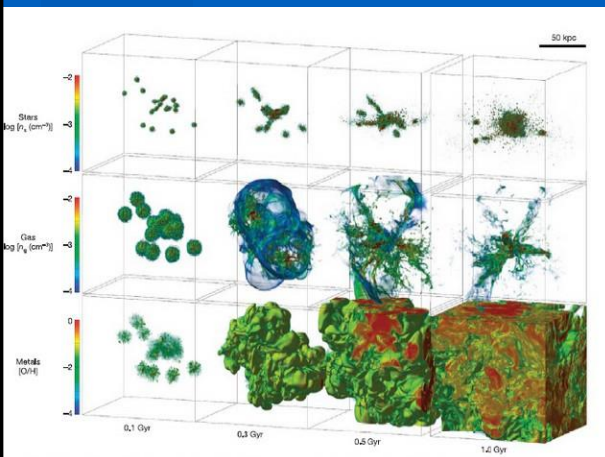
$$\mu = \bar{x} \pm Z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}$$

Labels and arrows:

- Point Estimate: \bar{x}
- Confidence Level: $Z_{\frac{\alpha}{2}}$
- Margin of Error: $\frac{\sigma}{\sqrt{n}}$

- Don't get exact answers, only approximations, estimates
 - Also true of many other modern methods
 - Can bound errors by machine roundoff
- Get random output from stochastic simulations
 - Statistical design, analysis of simulation experiments
 - Exploit: noise control, replicability, sequential sampling, variance-reduction techniques
 - Catch: “standard” statistical methods **seldom** work

Ex: Simulations of galaxies



Sept. 27th 2012,
Two astronomers have made one of the largest ever conducted simulations in astrophysics to model the growth of galaxies. Masao Mori (LA Univ.) and Masayuki Umemura (Tsukuba Univ.) are able to simulate galaxy evolution since 300 million years after the Big Bang until today. Their results show that galaxies may have evolved much faster than previously thought.²³

ISIMA - F2 (inside)



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