Submit your answers to Canvas. Word and PDF files are acceptable. Please include your name (family name first) at the beginning of the filename.

Part A

In this part of the assignment, you will construct an alpha factor based on the prior 5-day returns and evaluate the effectiveness of the factor.

- 1. Consider the US market and the years 2005 to 2024 (you will also need some data from 2004 to calculate the alpha factor at the beginning of the year 2005). The universe used will be based on the universe from the start of the year (defined in the univ_h.csv file).
- 2. To construct the factor.
 - a) calculate the volatility $\sigma_i(t)$ using the prior 21 days of daily returns (use the log return $r_i(t') = \ln\left(\frac{p_i(t')}{p_i(t'-1)}\right)$, t' = t 20, ..., t, the return is set to 0 if there is an "NA" in the adjusted prices). If $\sigma_i(t)$ obtained is less than 0.005, set it to 0.005
 - b) calculate the prior 5-day return; you can use the log return $v_i(t)=\ln{(\frac{p_i(t)}{p_i(t-5)})} \ \ (\text{again, the return is set to 0 if the prices are not available})$
 - c) normalize the variable by dividing the volatility $\sigma_i(t)$ obtained in step a), $v_i(t) \leftarrow v_i(t)/\sigma_i(t)$
 - d) subtract out the industry component, $v_i(t) \leftarrow v_i(t) v_i(t)$, where $v_i(t)$ is the simple average of $v_i(t)$ (after step c) over all stocks in the industry that the stock i belongs to.
- 3. Do a cross-sectional regression

$$R_i(t+1) = \beta(t)v_i(t) + \epsilon_i(t), \qquad i = 1, ..., N$$

on everyday t (except for the last day of the available data) and get a time series of $\beta(t)$ and a time series of $R^2(t)$. Here the industry return is removed from $r_i(t+1)$: $R_i(t+1) = r_i(t+1) - r_i(t+1)$

For this simple one-variable regression, you can calculate $\beta(t)$ and $R^2(t)$ directly:

$$\beta(t) = \frac{\sum_{i} R_{i}(t+1)v_{i}(t)}{\sum_{i} v_{i}(t)v_{i}(t)}; \ R^{2}(t) = 1 - \frac{\sum_{i} \epsilon_{i}^{2}(t)}{\sum_{i} R_{i}^{2}(t+1)}$$

4. For the years 2005 to 2024, calculate and list the average of $\beta(t)$ for each year and the corresponding t-stat, $\sqrt{T} \times \text{(the average of } \beta(t)$

 $\beta(t)$ /(the standard deviation of $\beta(t)$), where T is the number of β values obtained for the year. Comment on your results.

5. **Repeat the calculation (Steps 3-4) using a ranked variable**, by adding a sub-step in the construction of the factor:

rank the normalized variable after step 2c) from the largest (rank 1) to the smallest (rank N) and redefine the factor in terms of the rank, k,

 $v \leftarrow (N+1-2k)/(N-1),$

N is the number of stocks in the universe on the day Then proceed to step 2d)

Part B

From the years 2006 to 2024, use the previous year's average beta $\overline{\beta_{\nu}}$, calculated in Part A (for example, for the year 2007, use the average β_{ν} obtained for the year 2006, and evaluate the expected returns for all trading days of the year 2007,

$$R_{Ei}(t, t+1) = \overline{\beta_v} v_i(t)$$

Construct and evaluate the portfolio as follows,

- 1. On each day t, rank the stocks according to the expected returns, and long (with equal weights) the top 20% of the stocks with the largest values of $R_{Ei}(t, t+1)$ and short the bottom 20% of the stocks with the smallest values (most negative values) of $R_{Ei}(t, t+1)$
- 2. Get the portfolio return at each time step t. The return is on the long market value of the portfolio, so it is the sum of the returns on individual positions divided by the number of long positions in the portfolio,

$$r_p(t,t+1) = \frac{1}{N_l} \Big(\sum_{j=1}^{N_l} r_{L(j)}(t,t+1) - \sum_{j=1}^{N_s} r_{S(j)}(t,t+1) \Big),$$

where N_l and N_s are the number of long and short positions (both are equal to $0.2 \times N$, N is the number of stocks in the universe for that year). L(j) is the stock index of the long position j. S(j) is the stock index of the short position j. Note that when calculating the portfolio return, the full return $r_l(t+1) \equiv r_l(t,t+1)$ without subtracting the market return is used.

3. For each year calculate the annual return (assuming the cost of trading is 0, and for simplicity, simply add up all daily portfolio returns to get the annual return) and the annualized return volatility of the portfolio. List your results in a table. Which are the best and the worst years for the strategy?

4. Assume that the percentage trading cost is 5 bps. Calculate the portfolio returns, taking into account the cost. Compare the results to the case when the costs are not taken into account. For simplicity, we assume the LMV (the long market value) of the portfolio is kept the same and we ignore the cost of maintaining the constant LMV.

You will need to briefly describe how you obtained the answers to the above questions