Samizdat

In Defense of the CAPM

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Many textbooks repeat two common complaints about the Capital Asset Pricing Model (CAPM):

1. Evidence that it takes more than one factor to explain the shared, or systematic, risk in securities refutes the CAPM.
2. In demonstrating that the risk premium on an asset depends only on its systematic factor loadings, Arbitrage Pricing Theory (APT) provides investors with a result of great practical value that the CAPM does not provide.

It may not be coincidental that some of the same books that make the first complaint don't actually discuss the CAPM. Some discuss the market model, and some discuss (usually without attribution) the Vasicek-McQuown pricing model. Both build on William Sharpe's Diagonal Model paper, which suggested (following up on a footnote in Harry Markowitz's book) that systematic risk could usefully be accounted for by a single factor.*

Vasicek and McQuown's argument proceeds as follows. Assume a single systematic risk factor exists, such that residual risks can be uncorrelated both with this factor and with each other. By taking appropriately long positions in some securities and appropriately short positions in others, the individual investor can hedge away all systematic risk. Thus he won't bear systematic risk unless it competes successfully with residual risk for inclusion in his portfolio. But if he includes enough positions (long *or* short) in his portfolio, he can drive the portfolio's residual risk to zero. Now, if residual risk is priced, this offers him an infinite reward-to-risk ratio. In order for market risk to compete successfully, it must offer an infinite expected return. Otherwise the investor will choose not to hold *any* systematic risk, and the market won't clear.

The market model takes the assumptions of the Diagonal Model and the result of the Vasicek and McQuown model and concludes that the actual, *ex post* systematic return—surprise plus risk premium—of every asset will be proportional to the asset's market sensitivity. Both Vasicek and McQuown and the market model thus assume a single systematic factor; but the CAPM in its various forms (Sharpe, Treynor, Lintner, Mossin, Black) makes no assumption about factor structure. In particular, it does not make the one-factor, Diagonal Model assumption that Vasicek and McQuown and the market model make.

Actually, a CAPM that abandons the assumptions of homogeneous expectations (as Lintner did) and riskless borrowing and lending (as Black did) doesn't make many other assumptions. All it *does* assume is that asset risk can be adequately expressed in terms of variances and covariances (with other assets); that these measures exist; that investors agree on these risk measures; and that investors can trade costlessly to increase expected portfolio return or reduce portfolio variance, to which they are all averse. Short-selling is permitted.

There is thus nothing about factor structure in the CAPM's *assumptions*. But there is also nothing about factor structure in the CAPM's *conclusions*. Factor structure is about surprise—in particular, the nature of correlations between surprises in different assets. In the Diagonal Model, a single factor accounts for all correlation. In richer, more complex factor structures, correlations in asset surprises can be due to a variety of pervasive, market-wide influences. The conclusion of the CAPM—that an asset's *expected* return is proportional to its covariance with the market portfolio—makes no assertions about the *surprise* in that asset's return. Because expected return and surprise are mutually exclusive elements in *ex post*, actual return, there can be no conflict between the CAPM and factor structure.

This point is the key to the second complaint. In order to demonstrate that APT's principal conclusion—that an asset's risk premium depends only on its systematic factor loadings—also holds for the CAPM, we can assume a perfectly general factor structure without fear of tripping on either the assumptions of the CAPM or its conclusions.

Consider a market in which *absolute* surprise for the ith asset, x_i, obeys:

Eq. 1

$$x_i = \sum \beta_{ij} u_j + e_i,$$

where u_j are the systematic factors in the market and e_i is a residual unique to the ith asset.

* This material will appear as an appendix to the second edition of the *Guide to Portfolio Management* by James L. Farrell Jr., to be published by McGraw-Hill Book Company.

The absolute surprise for the market as a whole, x, is simply the sum of absolute surprises for the individual assets:

$$x = \sum x_i = \sum \beta_{ij} u_j + \sum e_i.$$

The CAPM asserts that any risk premium of the ith asset depends only on its covariance with the market—i.e., on the expectation of the following product:

Eq. 2

$$(\sum \beta_{ij} u_j + e_i)(\sum \beta_{hk} u_k + \sum e_h).$$

Bearing in mind that the u's and e's are surprise, and assuming that all covariances between e's and between u's and e's are zero, we have for this expectation:

Eq. 3

$$E[\sum \beta_{ij} \beta_{hk} u_j u_k + e_i^2],$$

or

Eq. 4

$$\sum \beta_{ij} [\sum \beta_{hk} \sigma_{jk}^2] + s_i^2,$$

where

Eq. 5

$$\sigma_{jk}^2 = E[u_j u_k]$$

and

Eq. 6

$$s_i^2 = E[e_i^2].$$

We see that an asset's covariance with the market portfolio (absolute surprise with absolute surprise) has two terms—one for systematic factors and one for the asset's unique surprise. A key issue is the relative size of these two terms.

We can, of course, make all the covariance terms in the systematic part zero by choosing a set of factors that are orthogonal. Then the covariance expression becomes:

Eq. 7

$$\beta_{i1} \sum \beta_{h1} \sigma_{11}^2 + \beta_{i2} \sum \beta_{n2} \sigma_{22}^2 + \ldots + s_i^2.$$

Among the many possible choices of orthogonal factors, we can choose one in which only one factor correlates with the market portfolio. Assign the factor index to the factors so that the first term in the bracketed expression is the variance term for that factor. Then we can ignore for the moment the other systematic terms and focus on the first and last terms in the covariance expression, Equation (7). Factoring the respective terms into standard deviations gives us:

Eq. 8

$$(\beta_{i1}\sigma_1)(\sum \beta_{h1}\sigma_1) + \ldots + (s_i)(s_i)$$

Consider the first term. One factor reflects the absolute factor weight of the asset, the other the absolute factor weight of the market portfolio. The former is of the same order of magnitude as the standard deviation of the asset's unique risk; the latter is orders of magnitude larger (see Figure A). This is because all the systematic risk in the market portfolio's absolute gain or loss is reflected in the market's factor weight, whereas the unique risk in the second term is that for a single asset. But this means that the systematic product is orders of magnitude larger than the unique product.

Figure A Systematic vs. Unique Risk

Same Order of Magnitude

$$(\beta_{i1}\sigma_1)(\sum \beta_{j1}\sigma_1) + \ldots + (s_i)(s_i)$$

Orders of Magnitude Bigger

To keep our exposition simple, we chose a special set of systematic factors. Had we chosen different systematic factors, the market portfolio could have been correlated with most or all of them, rather than merely one. Then the systematic portion of an asset's covariance with the market would have non-zero terms for more than one factor. Spreading it across several factors, however, won't diminish the magnitude of the aggregate systematic portion. Whether one systematic factor or several are correlated with the market portfolio, then, the systematic portion of an asset's covariance with the market will be orders of magnitude bigger than the unique portion.

If we drop the unique term and relax the assumption that only one systematic factor correlates with the market, then the expression within brackets in Equation (4) will in general have a different value for each of the systematic factors. But the value *doesn't depend on i*—the index that distinguishes between individual assets. What is *outside* the brackets is the factor loadings β_{ij} for the ith individual asset on the factors u_j in the factor structure. But this is the principal conclusion of APT—that an asset's risk premium should depend only on its factor loadings. Any corollaries that flow from this APT result also flow from the CAPM.

One of the differences between APT and the CAPM is the specification by CAPM of how systematic factors should be priced in relation to each other. The expression in brackets in the first term of Equation (4) is, to some constant of proportionality, the price of risk. The value of the expression, hence that price, is generally going to be different for different factors. APT can't specify the value of this expression; therefore it can't specify how factors will be priced in relation to each other.

What the APT *does* do is weaken the assumptions necessary to reach this conclusion. APT appeals to the older, better established tradition of arbitrage arguments exemplified by Modigliani and Miller's 1958 paper. APT joins the idea of specifying security risk in terms of a factor structure (Farrar's 1960 doctoral thesis at Harvard; Hester and Feeney's contemporaneous work at Yale) with Modigliani and Miller's arbitrage argument. Some may argue that, in effect, APT substitutes systematic risk *factors* for Modigliani and Miller's risk *classes*. Others may argue that the real provenance of APT is not Modigliani and Miller, but rather Vasicek and McQuown, with multiple risk factors substituted for their single risk factor.

This writer has no desire to take a position in such controversies. His only purpose is to clear up certain misunderstandings regarding the CAPM.